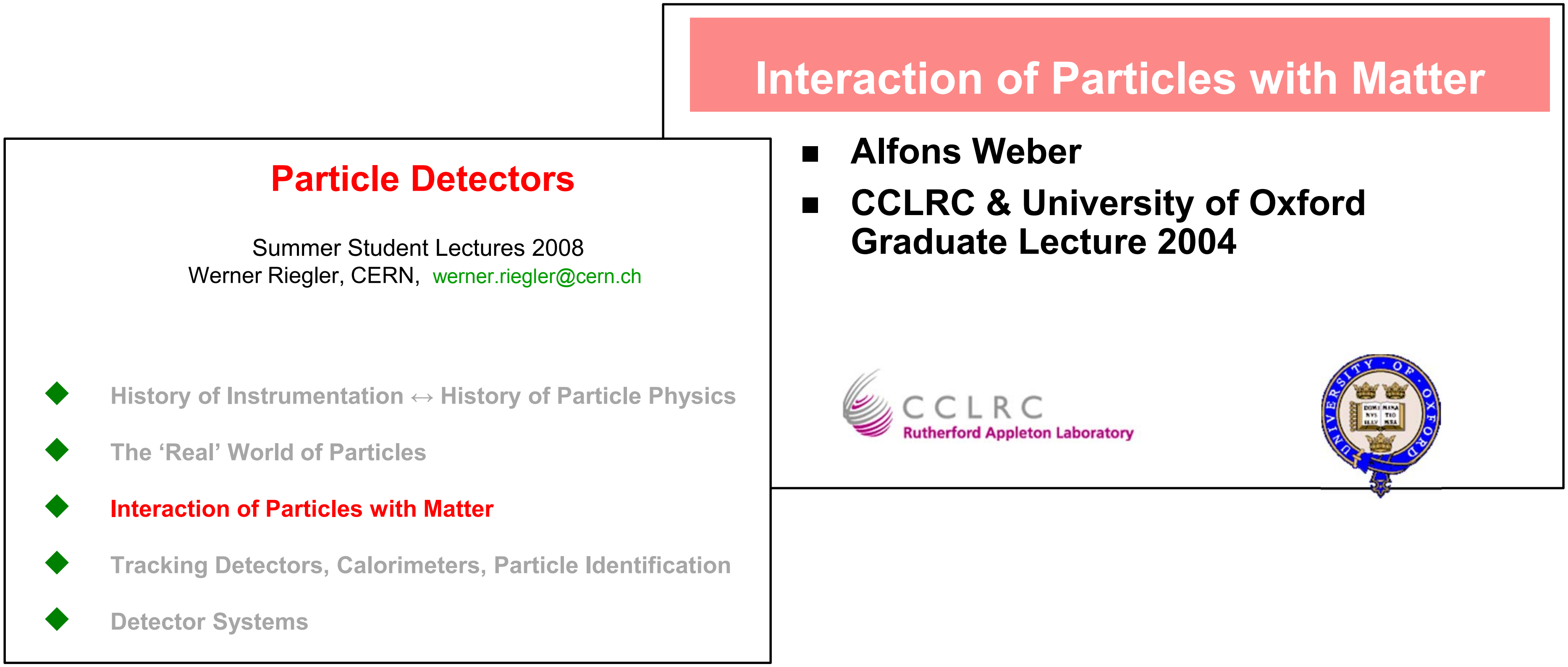
General Information

- Individual Study Projects
  - Schedule of presentations
- Muon Lifetime Update

- Today's Agenda
  - Interaction of Particles with Matter

Interactions of Particles with Matter

- **Many good references available**
  - “Passage of Particles through Matter” section of the Particle Data Book
  - Books by Leo and Gruppen
  - We will follow the approach taken by W. Riegler for the CERN 2008 Summer Student Lecture and A. Weber in his lecture on particle interactions for Oxford graduate students



Particle Detectors

Summer Student Lectures 2008
Werner Riegler, CERN, werner.riegler@cern.ch

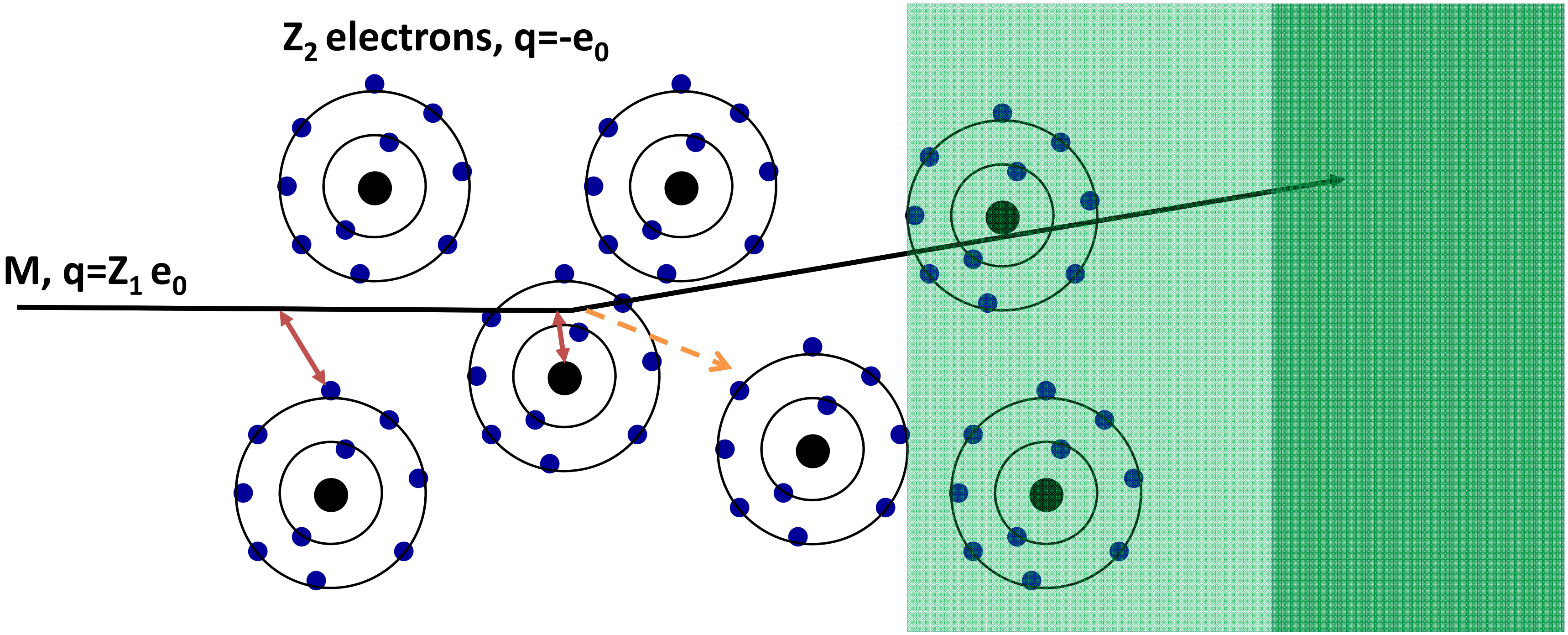
- History of Instrumentation ↔ History of Particle Physics
- The ‘Real’ World of Particles
- Interaction of Particles with Matter
- Tracking Detectors, Calorimeters, Particle Identification
- Detector Systems

Interaction of Particles with Matter

- **Alfons Weber**
- **CCLRC & University of Oxford Graduate Lecture 2004**

CCLRC
Rutherford Appleton Laboratory

Electromagnetic Interaction of Particles with Matter



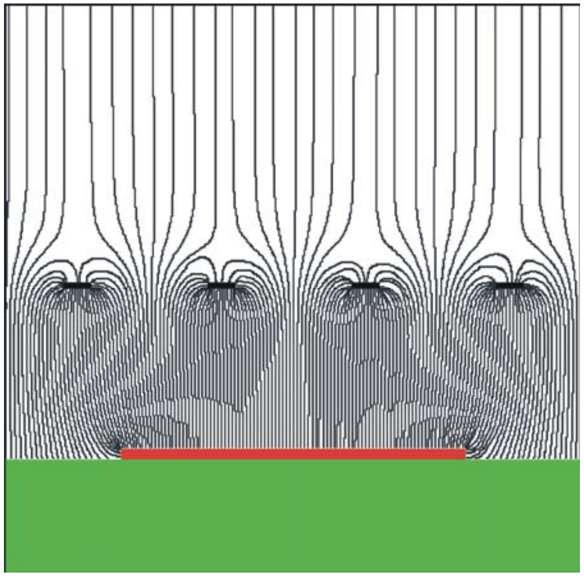
Interaction with the atomic electrons. The incoming particle looses energy and the atoms are excited or ionized.

Interaction with the atomic nucleus. The particle is deflected (scattered) resulting in multiple scattering of the particle in the material. During these scattering events a Bremsstrahlung photons can be emitted.

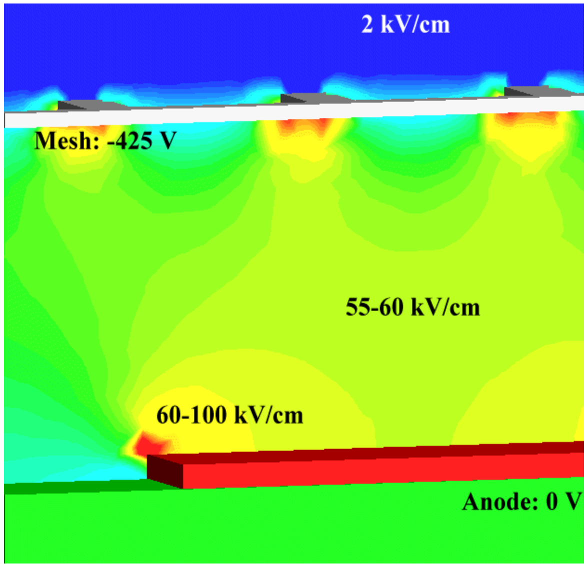
In case the particle's velocity is larger than the velocity of light in the medium, the resulting EM shockwave manifests itself as Cherenkov Radiation. When the particle crosses the boundary between two media, there is a probability of the order of 1% to produce an X ray photon, called Transition radiation.

Particle Detector Simulation Examples

Electric Fields in a Micromega Detector



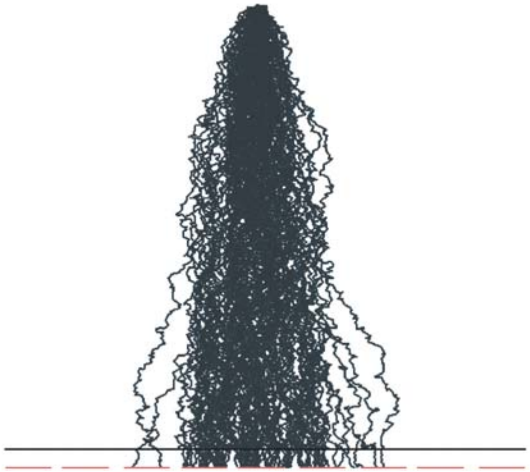
Electric Fields in a Micromega Detector

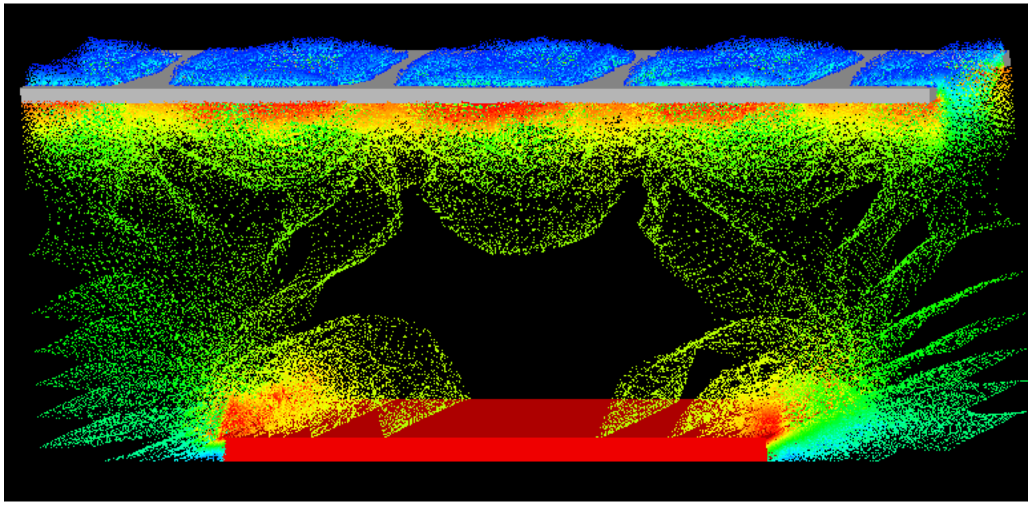


Very accurate simulations of particle detectors are possible due to availability of Finite Element simulation programs and computing power.

Follow every single electron by applying first principle laws of physics.

Electrons avalanche multiplication

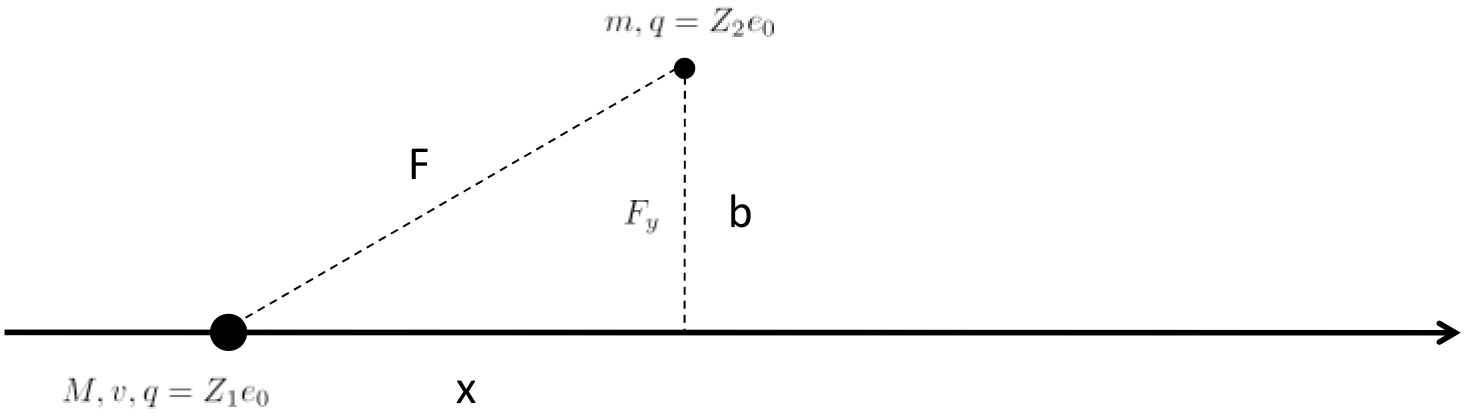




Bethe-Bloch Formula

- Describes how heavy particles ($M>>m_e$) loose energy when travelling through material
- Exact theoretical treatment difficult
  - Atomic excitations
  - Screening
  - Bulk effects
- Simplified semi-classical derivation gives (almost) the correct result
- Phenomenological description

Interaction of Particles with Matter



While the charged particle is passing another charged particle, the Coulomb Force is acting, resulting in momentum transfer

$$F_y = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 r^2}\cos\theta = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0\left(b^2 + (\beta c)^2 t^2\right)}\frac{b}{\sqrt{b^2 + (\beta c)^2 t^2}}$$

$$\Delta p = \int_{-\infty}^{\infty} F_y(t)dt = \frac{2 Z_1 Z_2 e_0^2}{4\pi\varepsilon_0 v b}$$

The relativistic form of the transverse electric field doesn't change the momentum transfer. The transverse field is stronger, but the time of action is shorter

The transferred energy is then

$$\Delta E = \frac{(\Delta p)^2}{2m} = \frac{Z_2^2}{m}\frac{2 Z_1^2 e_0^4}{(4\pi\varepsilon_0)^2 v^2 b^2}$$

$$\Delta E(electrons) = Z_2 \frac{1}{m_e}\frac{2 Z_1^2 e_0^4}{(4\pi\varepsilon_0)^2 v^2 b^2} \qquad \Delta E(nucleus) = \frac{Z_2^2}{2 Z_2 m_p}\frac{2 Z_1^2 e_0^4}{(4\pi\varepsilon_0)^2 v^2 b^2} \qquad \boxed{\frac{\Delta E(electrons)}{\Delta E(nucleus)} = \frac{2 m_p}{m_e} \approx 4000}$$

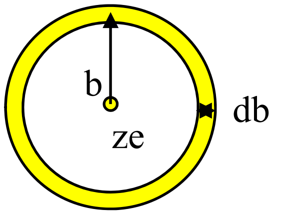
→ The incoming particle transfer energy only (mostly) to the atomic electrons !

Interaction of Particles with Matter

Target material: mass A, Z_2, density ρ [g/cm^3], Avogadro number N_A

A gram → N_A Atoms: **Number of atoms/cm^3 $n_a = N_A \rho/A$ [1/cm^3]**

Number of electrons/cm^3 $n_e = N_A \rho Z_2/A$ [1/cm^3]



$$\frac{dn}{db} = 2\pi b \times (\text{number of electrons / unit area })$$

$$= 2\pi b \times Z \frac{N_A}{A} \rho \Delta x$$

$$\Delta E(electrons) = \frac{2Z_2Z_1^2 m_e c^2}{\beta^2 b^2} \frac{e_0^4}{(4\pi\varepsilon_0 m_e c^2)^2} = \frac{2Z_2Z_1^2 m_e c^2}{\beta^2 b^2} r_e^2$$

Classical electron radius

$$r_e = \frac{e^2}{\left(4\pi\varepsilon_o m_e c^2\right)} = 2.8\times10^{-15}\, m$$

$$dE = \int_{b_{\min}}^{b_{\max}} \frac{dn}{db} \Delta E\, db = \frac{4\pi Z_2 Z_1^2 m_e c^2 r_e^2}{\beta^2} \frac{N_A \rho}{A} dx \int_{b_{\min}}^{b_{\max}} \frac{db}{b}$$

With $\Delta E(b)$ → $db/b = -1/2\, dE/E$ → $E_{max} = \Delta E(b_{min})$ $E_{min} = \Delta E(b_{max})$

$$\frac{dE}{dx} = -2\pi r_e^2 m_e^2 c^2 \frac{Z_1^2}{\beta^2} \frac{N_A Z_2 \rho}{A} \int_{E_{min}}^{E_{max}} \frac{dE}{E} = -2\pi r_e^2 m_e^2 c^2 \frac{Z_1^2}{\beta^2} \frac{N_A Z_2 \rho}{A} \ln \frac{E_{max}}{E_{min}}$$

There must be a limit for E_{min} and E_{max}

All the physics and material dependence is in the calculation of these quantities

Bethe-Bloch (6)

- Simple approximations for E_{max} and E_{min}
  - From relativistic kinematics

$$E_{\max} = \frac{2\gamma^2\beta^2 m_e c^2}{1 + 2\gamma \dfrac{m_e}{M} + \left(\dfrac{m_e}{M}\right)^2} \approx \underset{\nwarrow \text{ If } M \gg 2\gamma m_e}{2\gamma^2\beta^2 m_e c^2}$$

  - Inelastic collision

$$E_{\min} = I_0 \equiv \text{ average ionisation energy}$$

Bethe-Bloch (7)

- This was just a simplified derivation
  - Incomplete
  - Just to get an idea how it is done
- The (approximated) true answer is

$$-\frac{dE}{dx} = 4\pi N_A r_e^2 \frac{m_e c^2}{\beta^2} \frac{Zz^2}{A} \rho \left[\frac{1}{2} \ln\left(\frac{2\gamma^2 \beta^2 m_e c^2 E_{\max}}{I_0^2} \right) - \beta^2 - \frac{\varepsilon}{2} - \frac{\delta(\beta\gamma)}{2} \right]$$

- with
  - ε screening correction of inner electrons
  - δ density correction, because of polarisation in medium

Bethe Bloch Formula, Specific Energy Loss

$$-\frac{1}{\rho}\frac{dE}{dx} = 4\pi N_A r_e^2 \frac{m_e c^2}{\beta^2}\frac{Zz^2}{A}\left[\frac{1}{2}\ln\left(\frac{2\gamma^2\beta^2 m_e c^2 E_{\max}}{I_0^2}\right) - \beta^2 - \frac{\varepsilon}{2} - \frac{\delta(\beta\gamma)}{2}\right]$$

For Z>1, I ≈16Z $^{0.9}$ eV

$1/\beta^2$ at low energies

Log. rise

Screening and density corrections

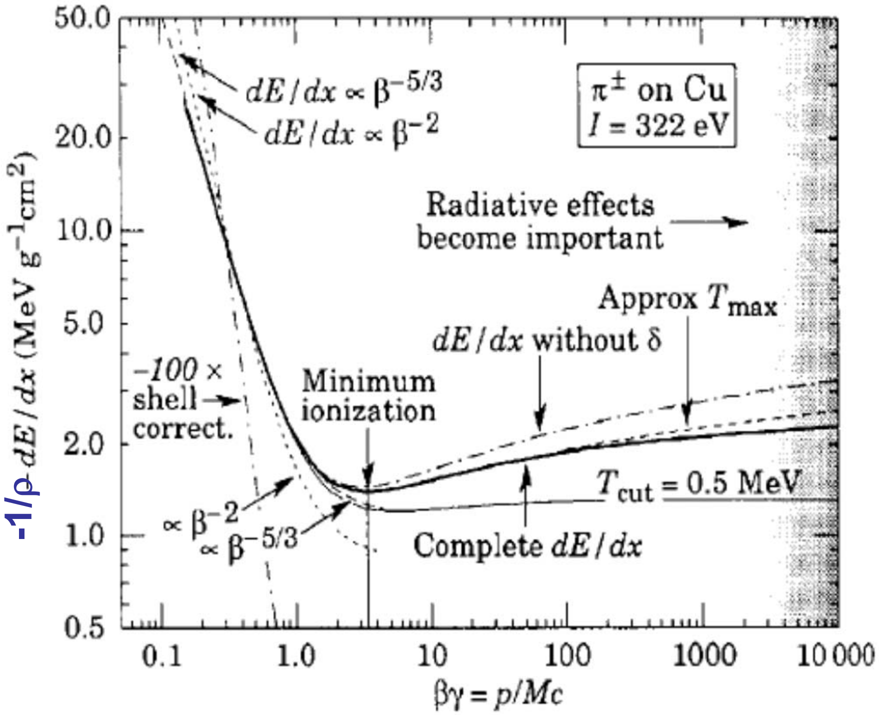
For large βγ the medium is being polarized by the strong transverse fields, which reduces the rise of the energy loss → density effect

At large Energy Transfers (delta electrons) the knocked out electrons can leave the material. In reality, E_{max} must be replaced by E_{cut} and the energy loss reaches a plateau (Fermi plateau).

Characteristics of the energy loss as a function of the particle velocity (βγ)

The specific Energy Loss 1/ρ dE/dx

- first decreases as $1/\beta^2$
- increases with ln γ for β =1
- is ≈ independent of M ($M \gg m_e$)
- is proportional to z^2 of the incoming particle.
- is ≈ independent of the material (Z/A ≈ const)
- shows a plateau at large βγ (>>100)
- Minimum Ionizing: dE/dx ≈ 1-2 x ρ [g/cm³] MeV/cm



Bethe Bloch Formula

Bethe Bloch Formula, a few Numbers:

For $Z \approx 0.5\,A$ (e.g. Fe)
$1/\rho\ dE/dx \approx 1.4$ MeV cm ^{2}/g for $\beta\gamma \approx 3$

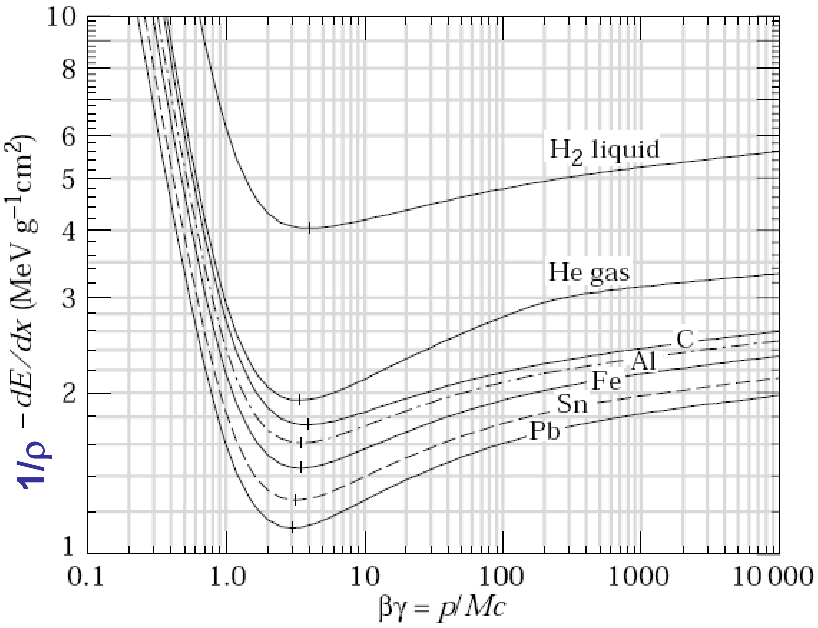
Example :
Iron: Thickness = 100 cm; $\rho = 7.87$ g/cm^3
dE ≈ 1.4 * 100* 7.87 = 1102 MeV

→ A 1 GeV Muon can traverse 1m of Iron

Somewhat material dependent:
for a plastic scintillator
$(1/\rho\ dE/dx)_{min} \approx 2$ MeV cm ^{2}/g

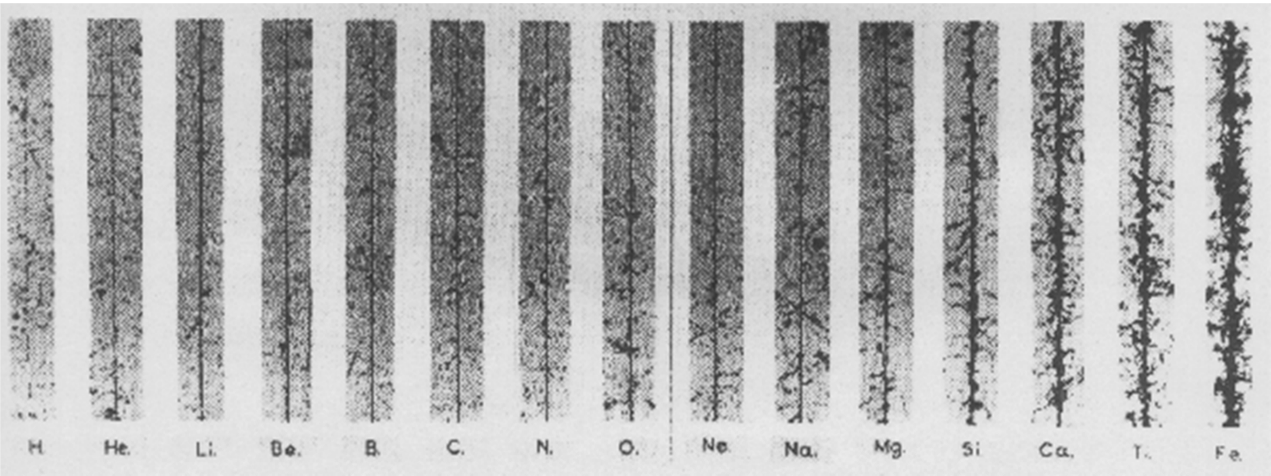
Example :
Scintillator: Thickness = 1 cm; $\rho = 1.0$ g/cm^3
dE ≈ 2 * 1 * 1 = 2 MeV

See materials table in Particle Data Book

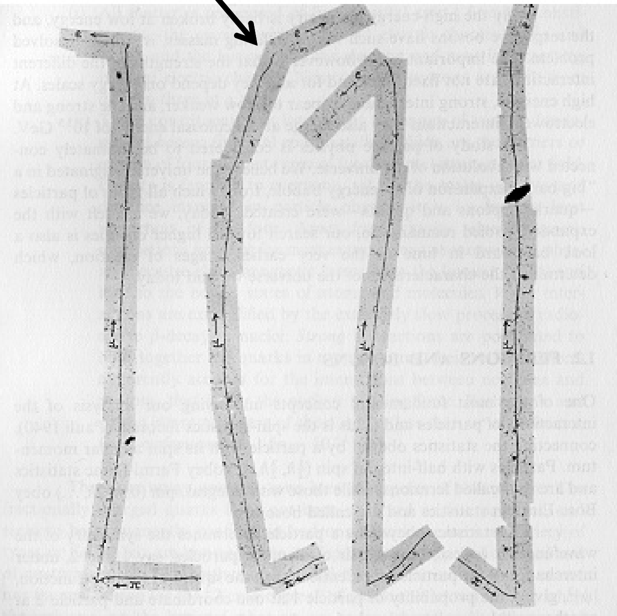


This number must be multiplied with ρ [g/cm^3] of the Material → dE/dx [MeV/cm]

Cosmis rays: dE/dx α Z^2



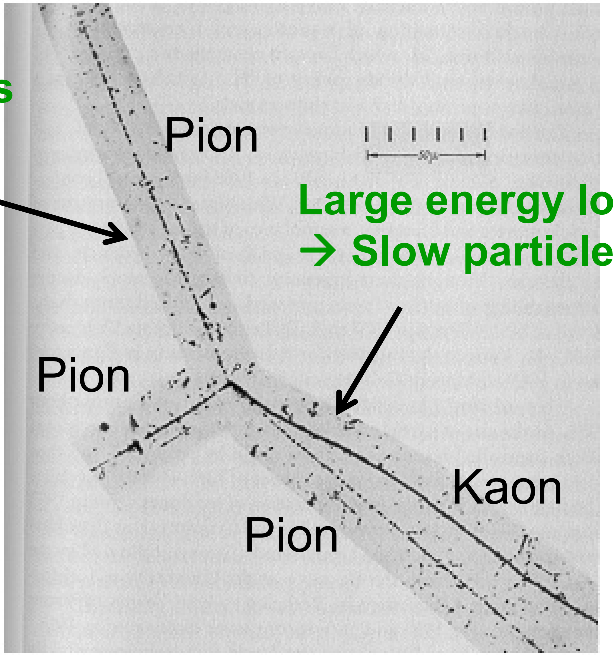
Small energy loss
→ Fast Particle



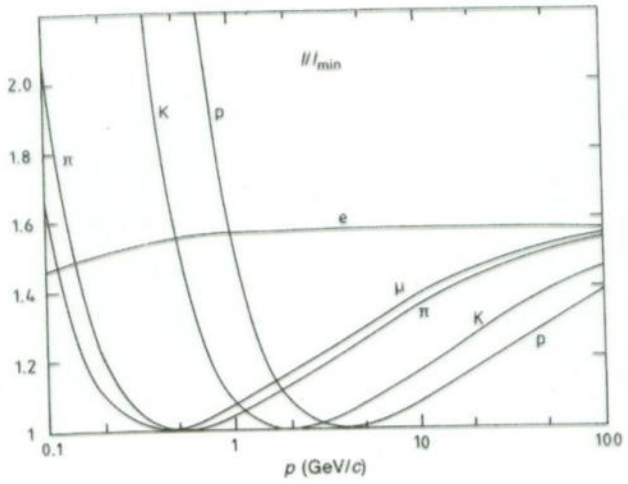
Discovery of muon and pion

Small energy loss
→ Fast particle

Large energy loss
→ Slow particle



Energy Loss as a Function of the Momentum

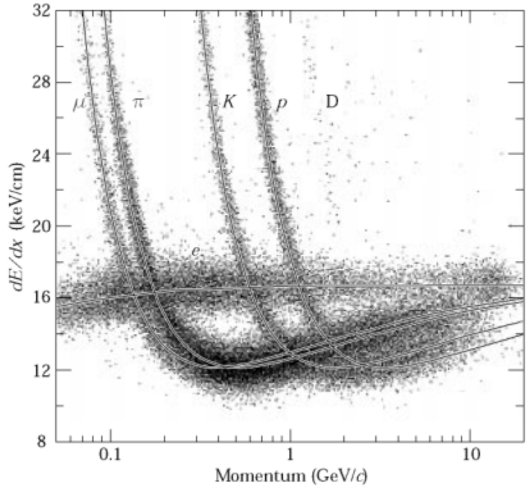


Energy loss depends on the velocity of the particle and is approximately independent of the particle's mass M.

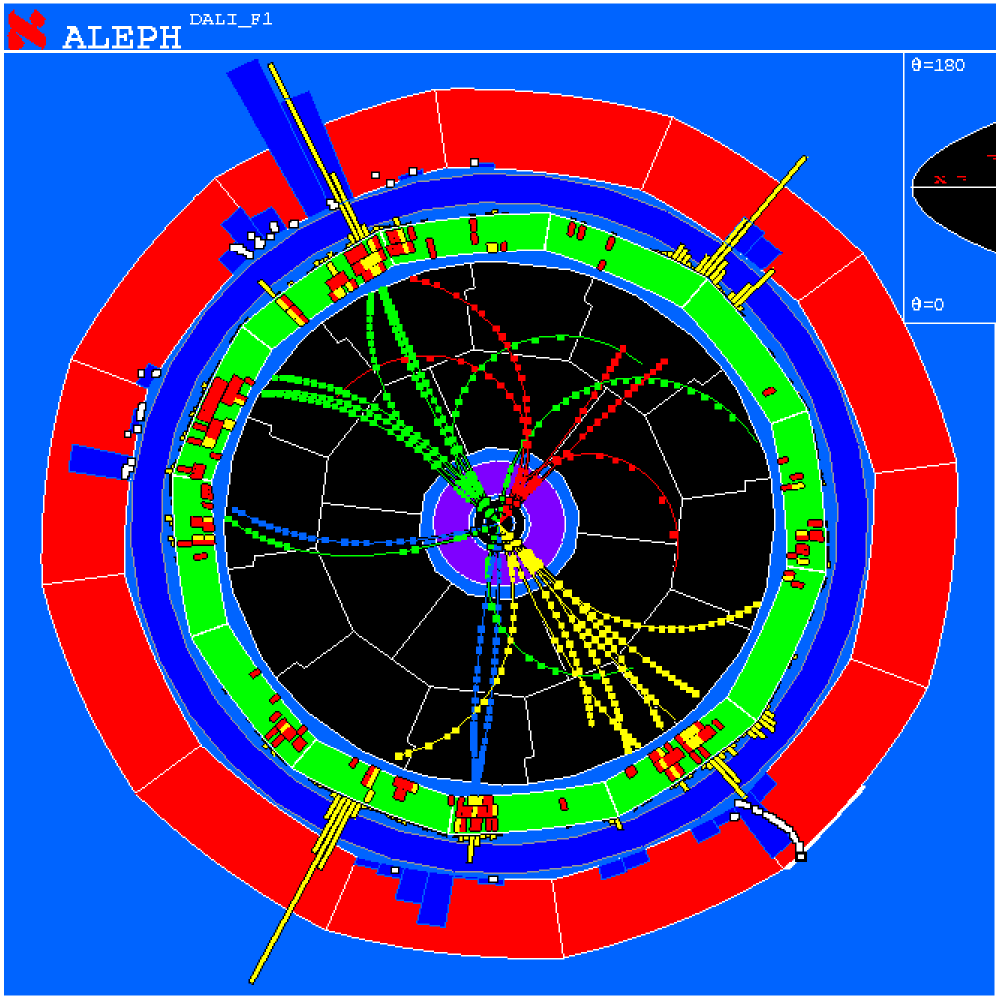
The energy loss as a function of particle Momentum p = Mcβγ however IS mass depending

By measuring the particle's momentum (deflection in a magnetic field) and with a measurement of the energy loss one can determine the mass of the particle.

→ Particle Identification !



Energy Loss as a Function of the Momentum



Measure momentum by curvature of the particle track.

Find dE/dx by measuring the deposited charge along the track.

→Particle ID

Particle Range

A particle of mass M and kinetic Energy E_0 enters matter and looses energy until it comes to rest at distance R.

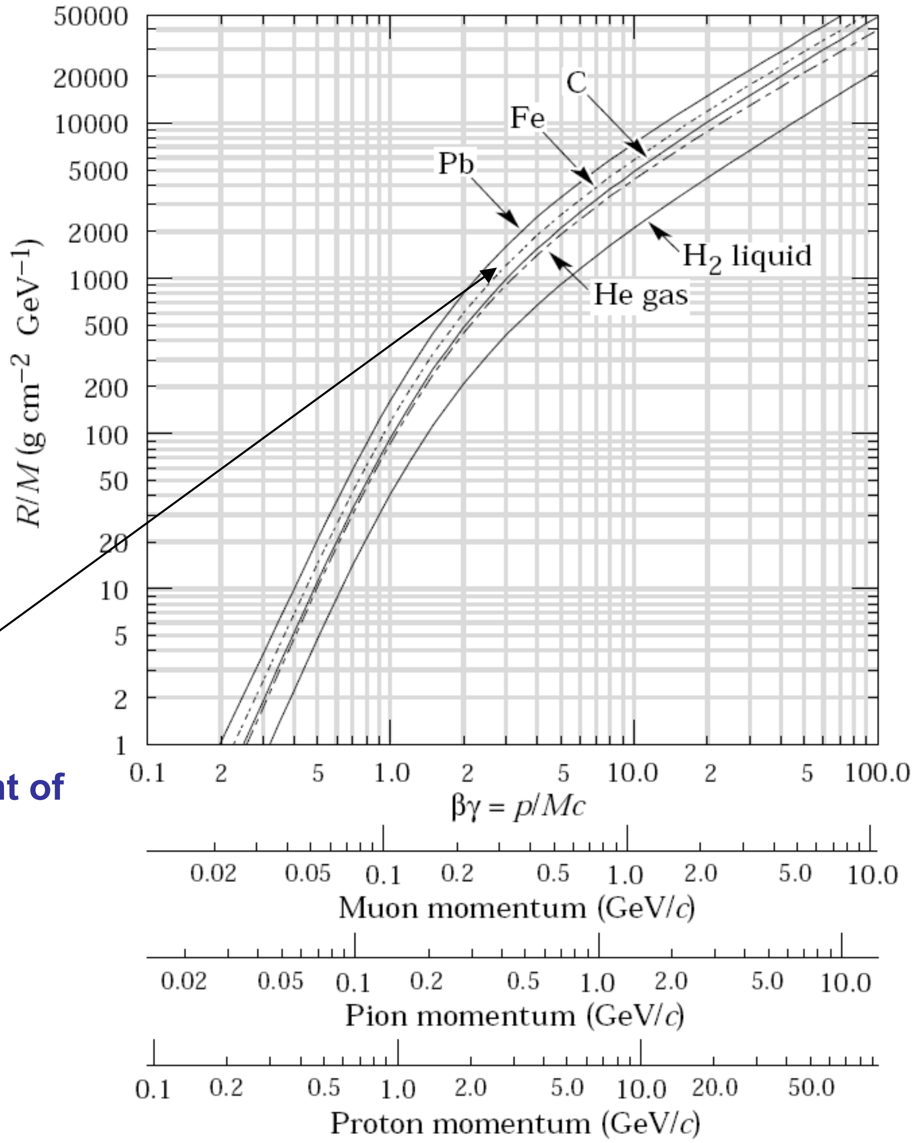
Integrate Bethe-Bloch energy loss:

$$R(E_0) = \int_{E_0}^{0} \frac{-1}{dE/dx} dE$$

$$R(\beta_0\gamma_0) = \frac{Mc^2}{\rho} \frac{1}{Z_1^2} \frac{A}{Z} f(\beta_0\gamma_0)$$

$$\frac{\rho}{Mc^2} R(\beta_0\gamma_0) = \frac{1}{Z_1^2} \frac{A}{Z} f(\beta_0\gamma_0)$$

≈Independent of the material



W. Riegler/CERN

Range of Particles in Matter

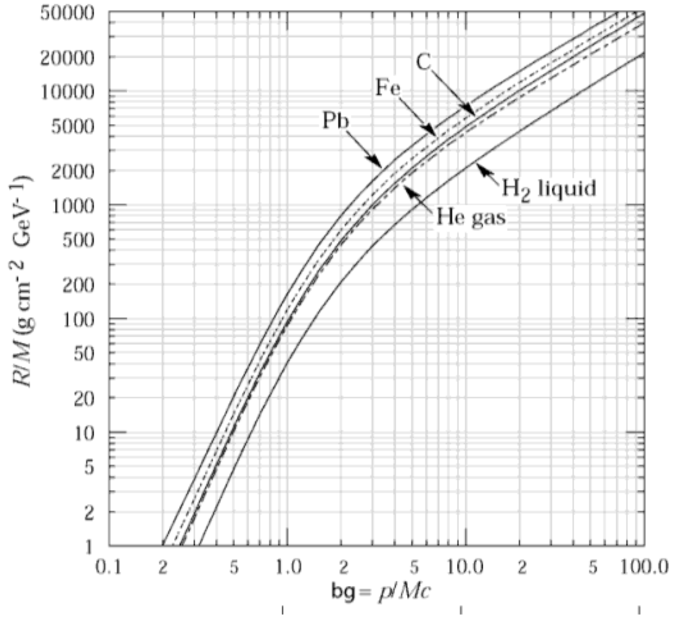
A particle of mass M and kinetic Energy E_0 enters matter and looses energy until it comes to rest at distance R.

$$R(E_0) = \int_{E_0}^{0} \frac{-1}{dE/dx} dE$$

$$R(\beta_0\gamma_0) = \frac{Mc^2}{\rho} \frac{1}{Z_1^2} \frac{A}{Z} f(\beta_0\gamma_0)$$

$$\frac{\rho}{Mc^2} R(\beta_0\gamma_0) = \frac{1}{Z_1^2} \frac{A}{Z} f(\beta_0\gamma_0)$$

≈Independent of the material

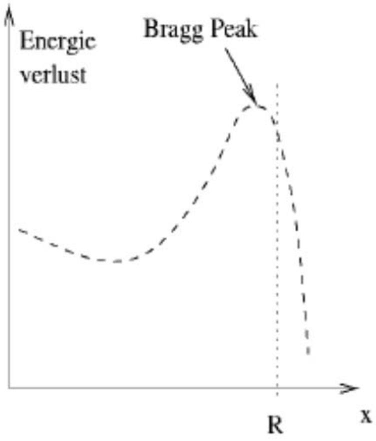


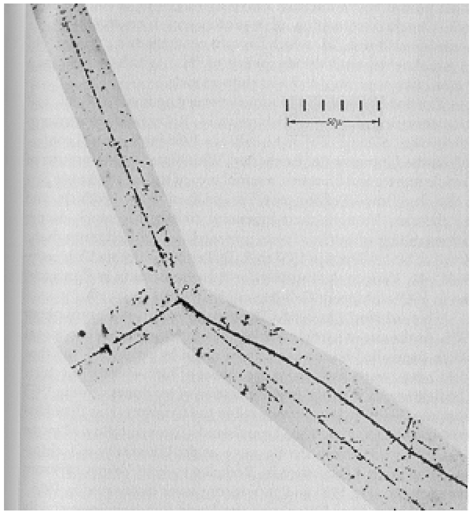
Bragg Peak:

For βγ>3 the energy loss is ≈ constant (Fermi Plateau)

If the energy of the particle falls below βγ=3 the energy loss rises as $1/\beta^2$

Towards the end of the track the energy loss is largest → Proton Cancer Therapy.





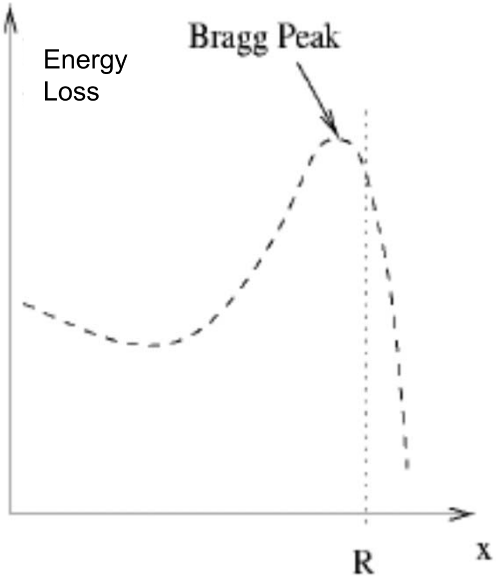
Particle Range, Bragg Peak

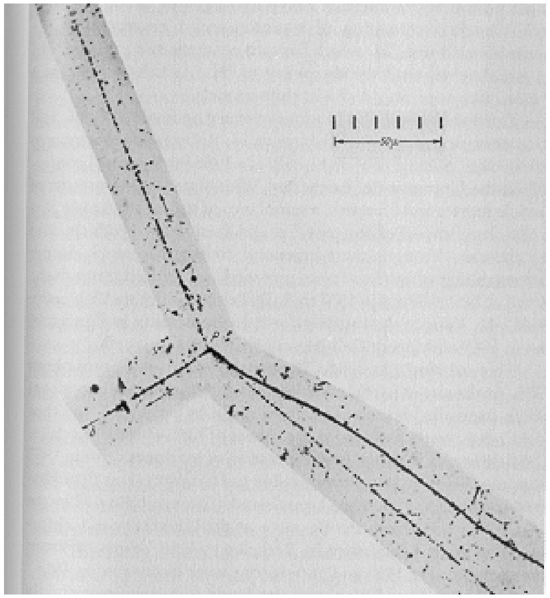
Bragg Peak:

For $\beta\gamma>3$ the energy loss is ≈ constant (Fermi Plateau)

If the energy of the particle falls below $\beta\gamma=3$ the energy loss rises as $1/\beta^2$

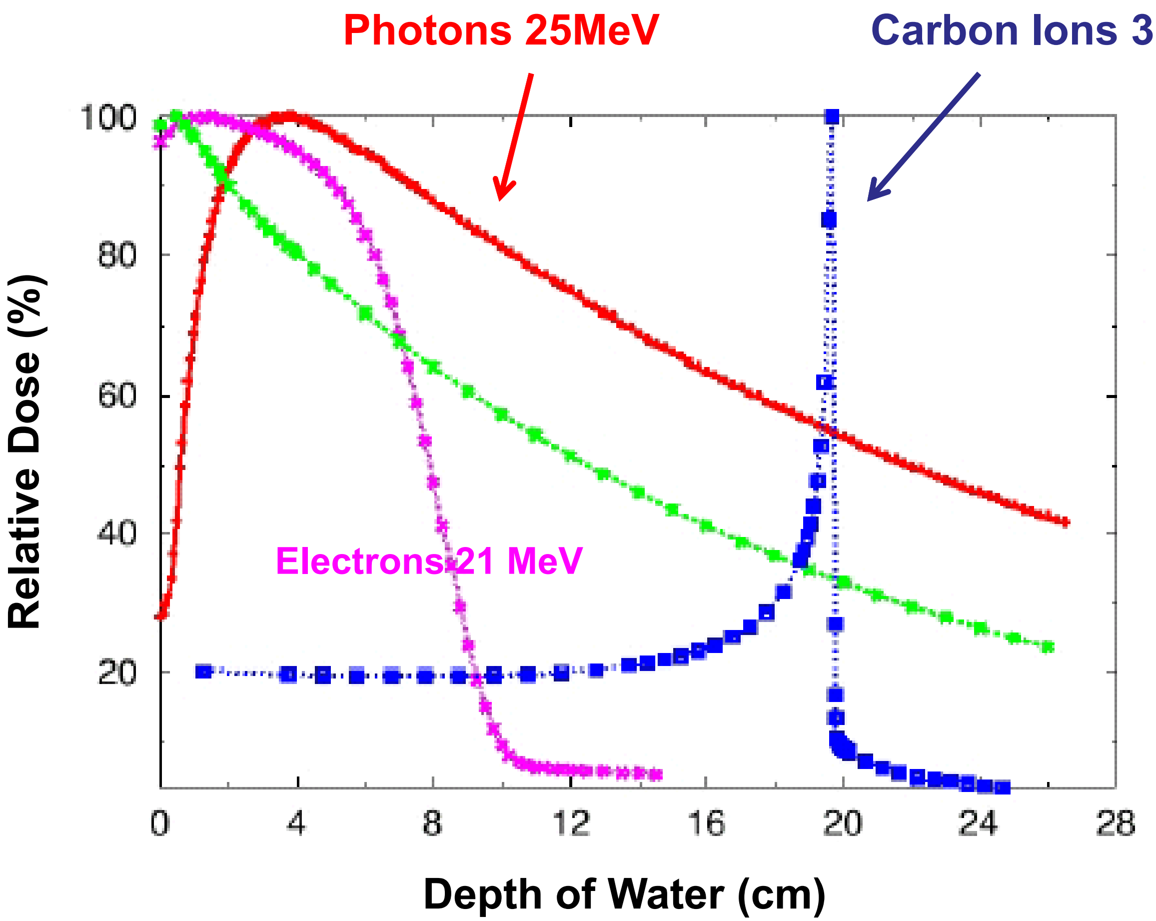
Towards the end of the track the energy loss is largest → Proton Cancer Therapy.





Range of Particles in Matter

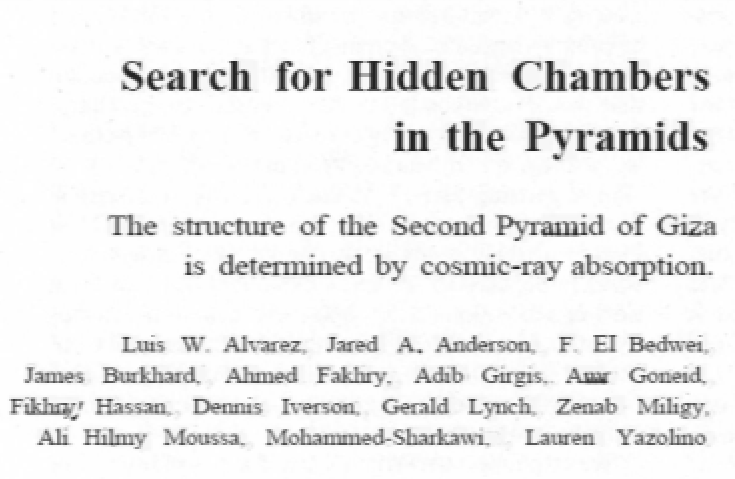
Average Range:
Towards the end of the track the energy loss is largest → Bragg Peak → Cancer Therapy



Photons 25MeV

Carbon Ions 330MeV

Electrons 21 MeV



Search for Hidden Chambers in the Pyramids

The structure of the Second Pyramid of Giza is determined by cosmic-ray absorption.

Luis W. Alvarez, Jared A. Anderson, F. El Bedwei, James Burkhard, Ahmed Fakhry, Adib Girgis, Amr Goneid, Fikhry Hassan, Dennis Iverson, Gerald Lynch, Zenab Miligy, Ali Hilmy Moussa, Mohammed-Sharkawi, Lauren Yazolino

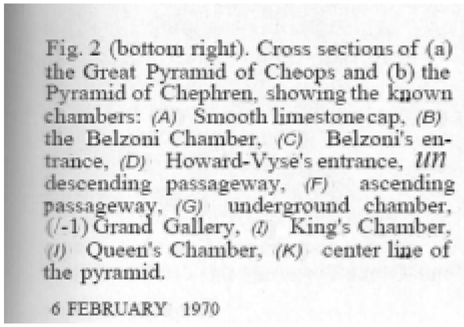
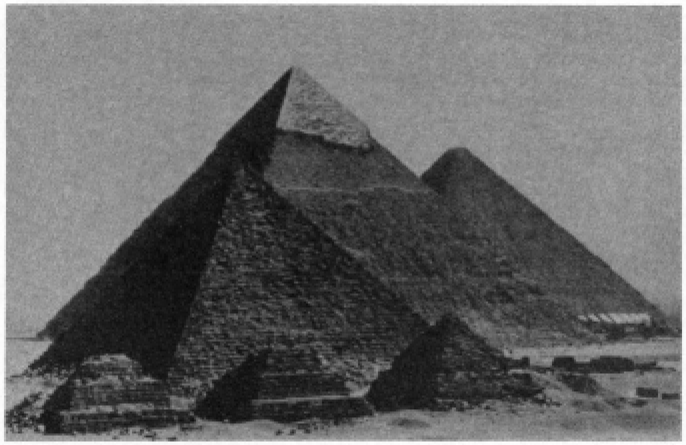


Fig. 2 (bottom right). Cross sections of (a) the Great Pyramid of Cheops and (b) the Pyramid of Chephren, showing the known chambers: *(A)* Smooth limestone cap, *(B)* the Belzoni Chamber, *(C)* Belzoni's entrance, *(D)* Howard-Vyse's entrance, *un* descending passageway, *(F)* ascending passageway, *(G)* underground chamber, (/-1) Grand Gallery, *(I)* King's Chamber, *(J)* Queen's Chamber, *(K)* center line of the pyramid.

6 FEBRUARY 1970



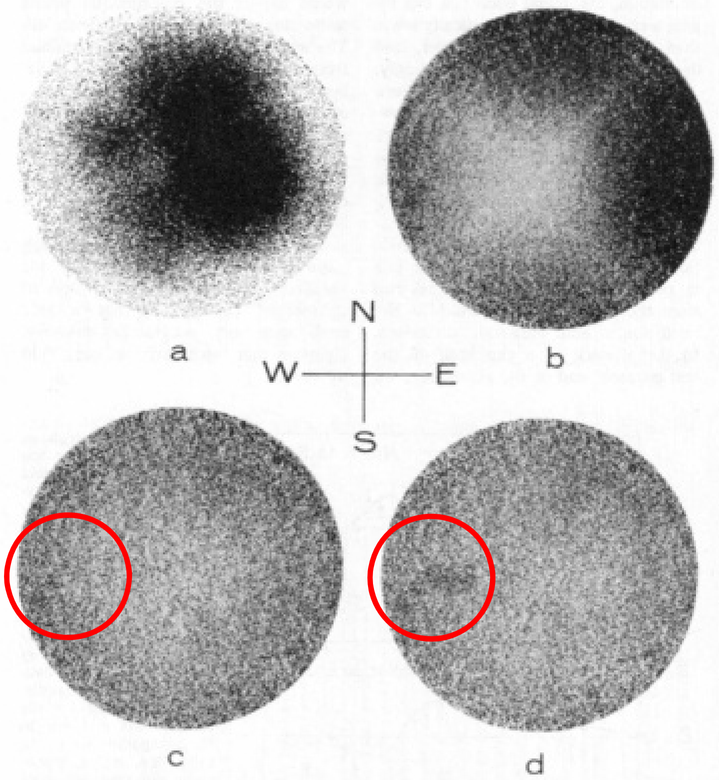
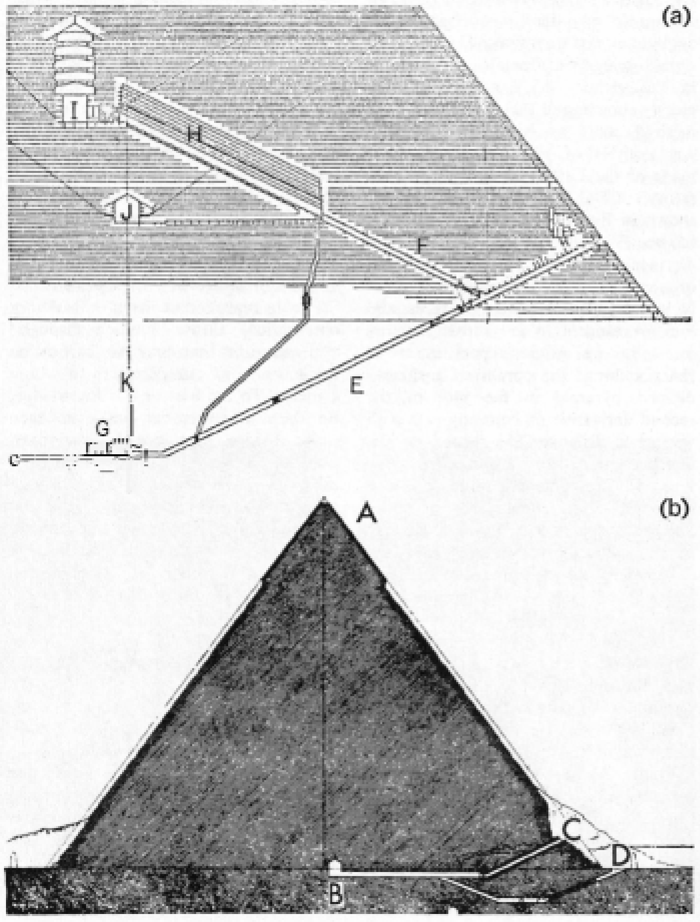


Fig. 13. Scatter plots showing the three stages in the combined analytic and visual analysis of the data and a plot with a simulated chamber. (a) Simulated "x-ray photograph" of uncorrected data. (b) Data corrected for the geometrical acceptance of the apparatus. (c) Data corrected for pyramid structure as well as geometrical acceptance. (d) Same as (c) but with simulated chamber, as in Fig. 12.

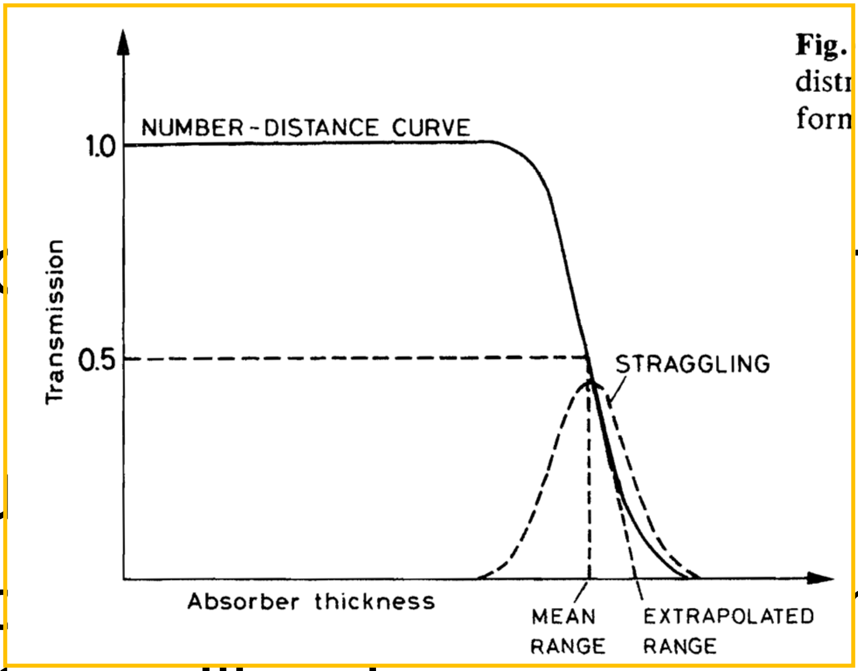
Luis Alvarez used the attenuation of muons to look for chambers in the Second Giza Pyramid → Muon Tomography

He proved that there are no chambers present.

Science, **167** (1970)



Straggling



- So far we have an energy loss
- Actual energy l the mean value
- Difficult to calcu
  - parameterizati me standalone software libraries
  - From of distribution is important as energy loss distribution is often used for calibrating the detector

Nov 2004

More Straggling: Landau dist.

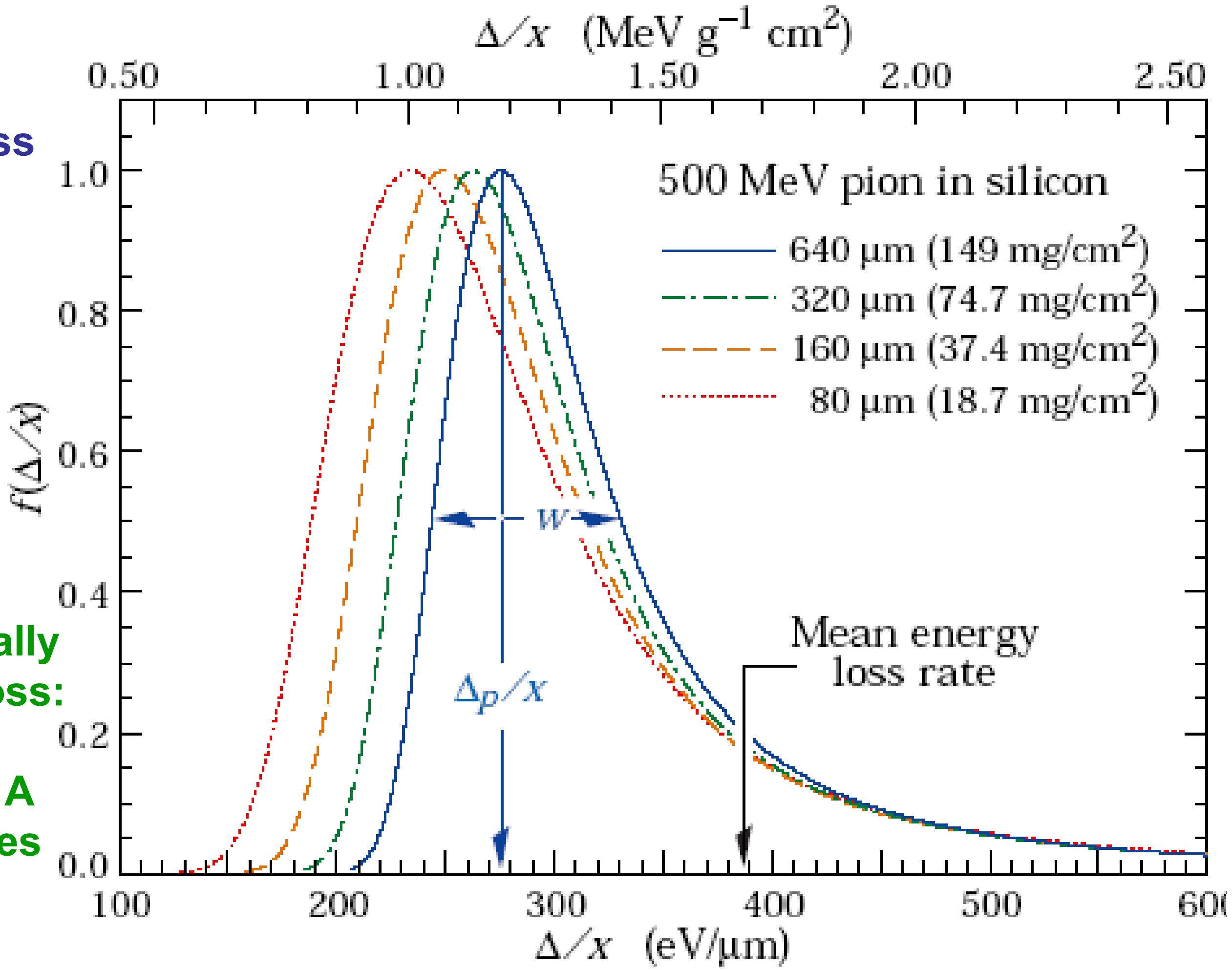
P(Δ): Probability for energy loss Δ in matter of thickness D.

Landau distribution is very asymmetric.

Average and most probable energy loss must be distinguished !

Measured Energy Loss is usually smaller than the real energy loss:

3 GeV Pion: E'_{max} = 450MeV → A 450 MeV Electron usually leaves the detector.



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Straggling (2)

- Simple parameterisation
  - Landau function

$$f(\lambda) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(\lambda + e^{-\lambda}) \right)$$

$$\text{with } \lambda = \frac{\Delta E - \overline{\Delta E}}{C \frac{m_e c^2}{\beta^2} \frac{Zz}{A} \rho \Delta x}$$

Nov 2004

δ-Rays

- Energy loss distribution is not Gaussian around mean.
- In rare cases a lot of energy is transferred to a single electron

δ-Ray

- If one excludes δ-rays, the average energy loss changes
- Equivalent of changing E_{max}

Restricted dE/dx

- Some detector only measure energy loss up to a certain upper limit E$_{cut}$
  - Truncated mean measurement
  - δ-rays leaving the detector

$$\left(\frac{\overline{\Delta E}}{\Delta x}\right)_{E<E_{cut}} = 2C\frac{m_e c^2}{\beta^2}\frac{Zz^2}{A}\rho\left[\frac{1}{2}\ln\left(\frac{2\gamma^2\beta^2 m_e c^2 E_{cut}}{I_0^2}\right) - \beta^2\left(1+\frac{E_{cut}}{E_{\max}}\right) - \frac{\varepsilon}{2} - \frac{\delta(\beta)}{2}\right]$$

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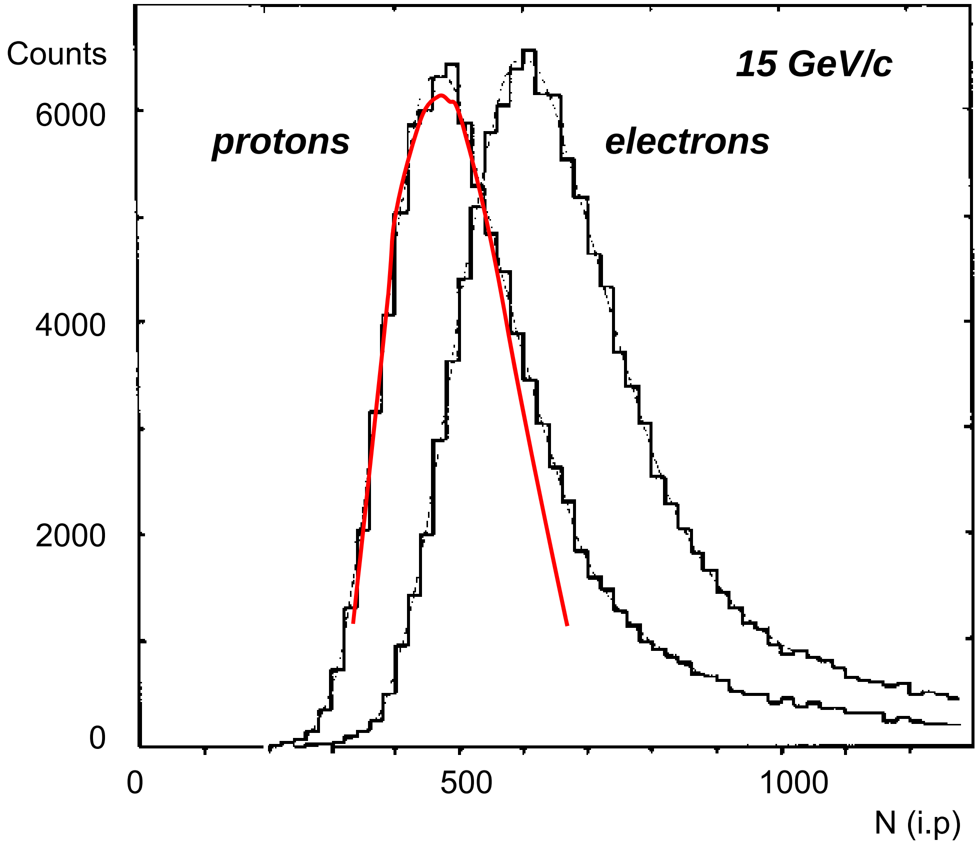
Particle Identification

PARTICLE IDENTIFICATION

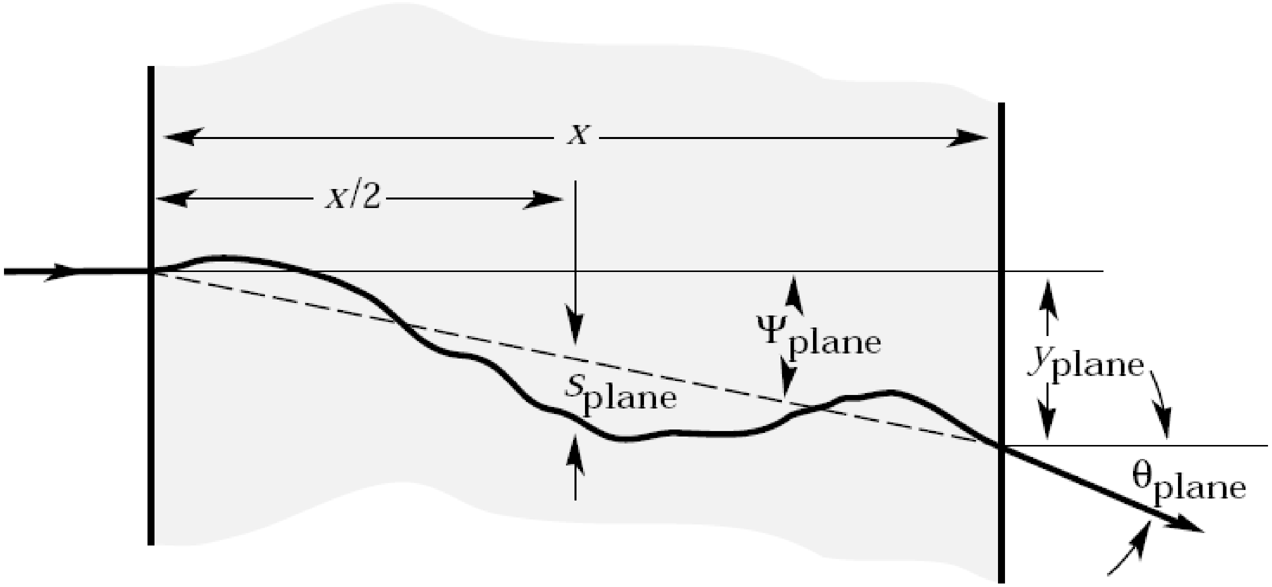
Increase number of samples

Remove outliers (large dE/dx)



Multiple Scattering

Particles don't only loose energy …



… they also change direction

Statistical (quite complex) analysis of multiple collisions gives:
Probability that a particle is defected by an angle θ after travelling a distance x in the material is given by a Gaussian distribution with a mean of 0 and a sigma of:

X_0 ... Radiation length of the material
Z_1 ... Charge of the particle
p ... Momentum of the particle

$$\Theta_0 = \frac{0.0136}{\beta cp[\mathrm{GeV/c}]} Z_1 \sqrt{\frac{x}{X_0}}$$

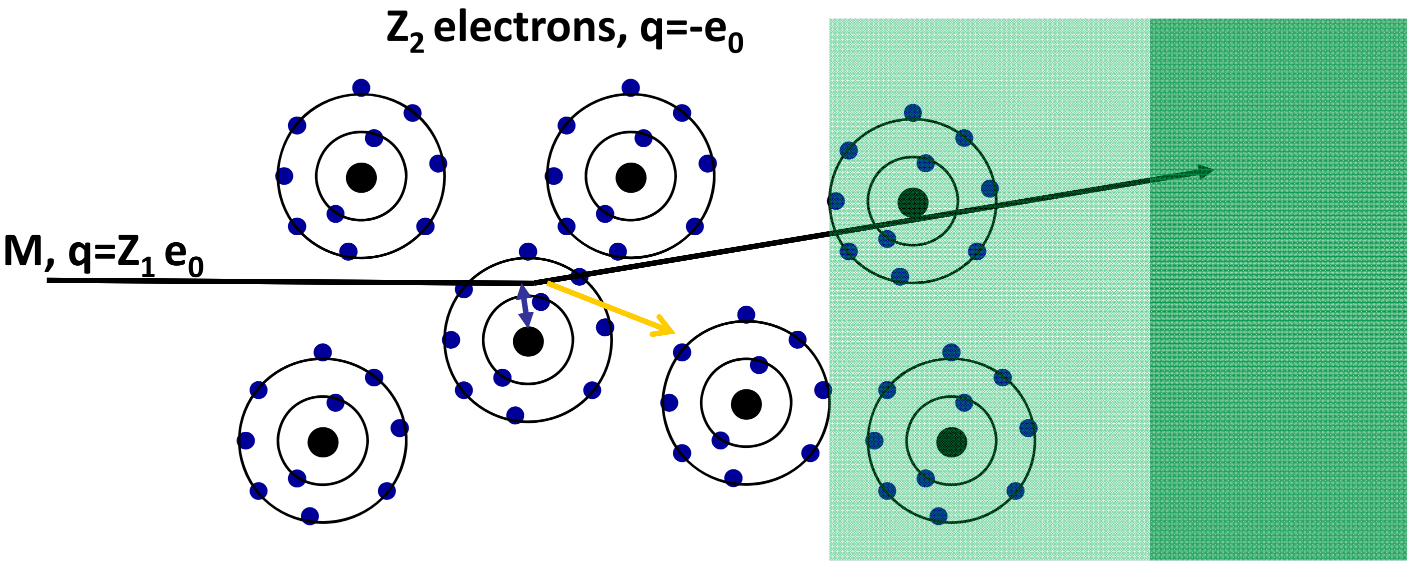
Energy Loss of Electrons

Ionization

Similar to Bethe Bloch but some modifications because (a) electrons are light and (b) we now have collisions between identical particles.
However, the same qualitative behavior

Bremsstrahlung

A charged particle of mass M and charge $q=Z_1e$ is deflected by a nucleus of charge Ze which is partially 'shielded' by the electrons. During this deflection the charge is 'accelerated' and it therefore radiated → Bremsstrahlung.



Bremsstrahlung, Classical

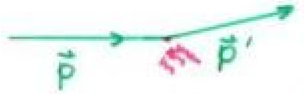
$q = Z_1 \cdot e$

$$\frac{d\sigma}{d\Omega} = \left(\frac{2 Z_1 Z_2 e^2}{4\pi\varepsilon_0 \, p \cdot v}\right)^2 \frac{1}{(2\sin\frac{\theta}{2})^4} \qquad p = Mv\gamma$$

"Rutherford Scattering"

Written in Terms of Momentum Transfer $Q^2 = 2p^2(1-\cos\theta)$

$$\frac{d\sigma}{dQ} = 8\pi \left(\frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 \beta c}\right)^2 \cdot \frac{1}{Q^3}$$



$Q = |\vec{p} - \vec{p}\,'|$

→ From Maxwell's Eq (Jackson)

$$\lim_{\omega \to 0} \frac{dI}{d\omega} \sim \frac{2}{3\pi} \frac{Z_1^2 e^2}{M^2 c^3} \frac{1}{4\pi\varepsilon_0} Q^2$$, Radiated Energy between $\omega, \omega + d\omega$

$$\frac{dE}{dx} = \frac{N_A \rho}{A} \cdot \int_0^{\omega_{max}} d\omega \int_{Q_{min}}^{Q_{max}} dQ \frac{dI}{d\omega} \cdot \frac{d\sigma}{dQ} \quad , \omega_{max} = \frac{E}{\hbar}$$

$$\frac{dE}{dx} = \frac{N_A \rho}{A} \cdot \frac{16}{3} \alpha \cdot Z^2 \cdot \left(\frac{Z_1^2 e^2}{4\pi\varepsilon_0 M c^2}\right)^2 \cdot E \cdot \ln \frac{Q_{max}}{Q_{min}}$$

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \sim \frac{1}{137}$$

A charged particle of mass M and charge q=Z_1e is deflected by a nucleus of Charge Ze.

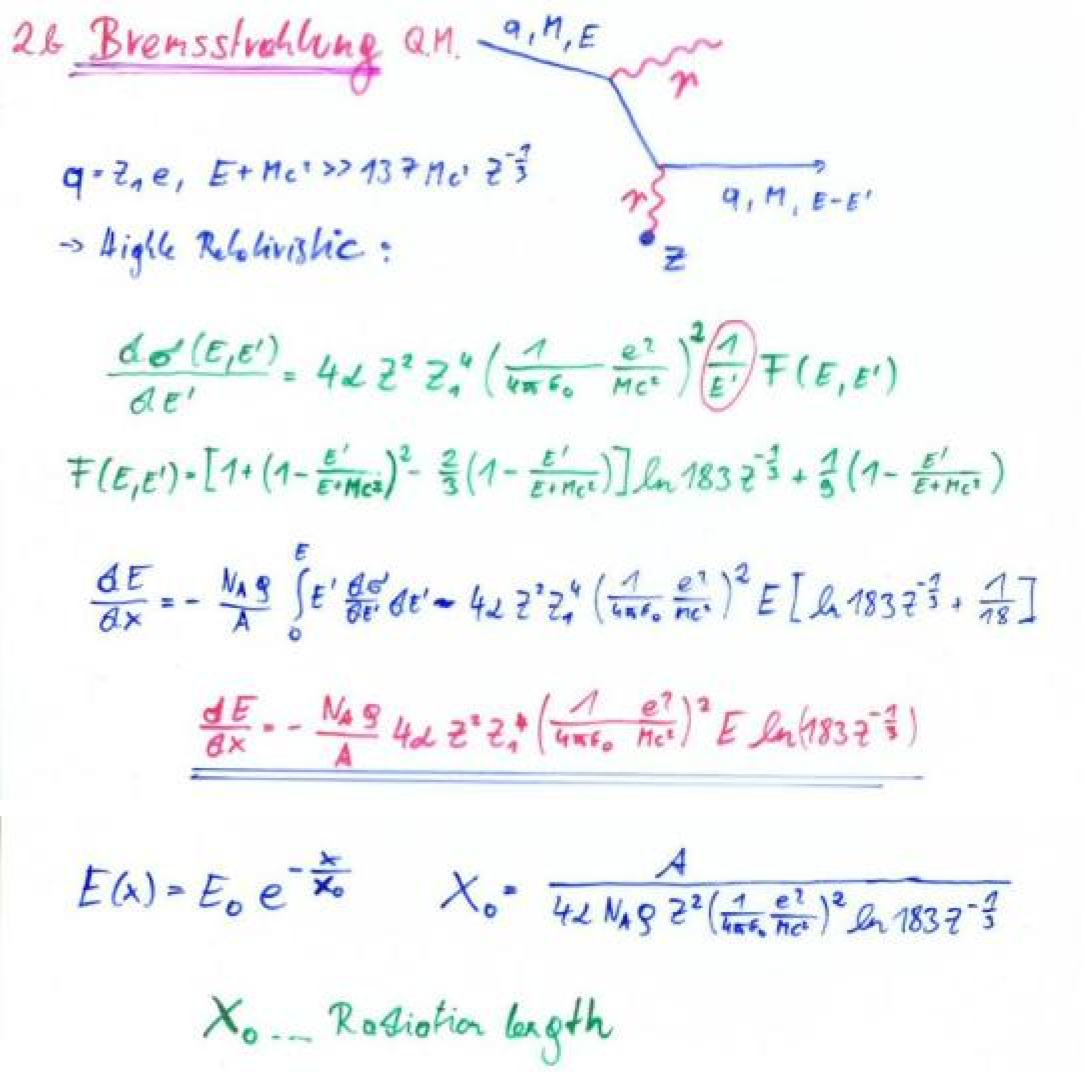
Because of the acceleration the particle radiated EM waves → energy loss.

Coulomb-Scattering (Rutherford Scattering) describes the deflection of the particle.

Maxwell's Equations describe the radiated energy for a given momentum transfer.

→ dE/dx

Bremsstrahlung, QM



2b Bremsstrahlung Q.M.

$q = Z_1 e, \quad E + Mc^2 \gg 137\, Mc^2 \, Z^{-\frac{1}{3}}$

→ Highly Relativistic:

$$\frac{d\sigma(E,E')}{dE'} = 4\alpha Z^2 Z_1^4 \left(\frac{1}{4\pi\varepsilon_0} \frac{e^2}{Mc^2}\right)^2 \boxed{\frac{1}{E'}} F(E,E')$$

$$F(E,E') = \left[1 + \left(1 - \frac{E'}{E+Mc^2}\right)^2 - \frac{2}{3}\left(1 - \frac{E'}{E+Mc^2}\right)\right] \ln 183 Z^{-\frac{1}{3}} + \frac{1}{9}\left(1 - \frac{E'}{E+Mc^2}\right)$$

$$\frac{dE}{dx} = -\frac{N_A \rho}{A} \int_0^E E' \frac{d\sigma}{dE'} dE' \approx 4\alpha Z^2 Z_1^4 \left(\frac{1}{4\pi\varepsilon_0} \frac{e^2}{Mc^2}\right)^2 E \left[\ln 183 Z^{-\frac{1}{3}} + \frac{1}{18}\right]$$

$$\frac{dE}{dx} = -\frac{N_A \rho}{A} 4\alpha Z^2 Z_1^4 \left(\frac{1}{4\pi\varepsilon_0} \frac{e^2}{Mc^2}\right)^2 E \ln\left(183 Z^{-\frac{1}{3}}\right)$$

$$E(x) = E_0 e^{-\frac{x}{X_0}} \qquad X_0 = \frac{A}{4\alpha N_A \rho Z^2 \left(\frac{1}{4\pi\varepsilon_0} \frac{e^2}{Mc^2}\right)^2 \ln 183 Z^{-\frac{1}{3}}}$$

X_0 … Radiation length

Proportional to Z^2/A of the Material.

Proportional to Z_1^4 of the incoming particle.

Proportional to ρ of the material.

Proportional $1/M^2$ of the incoming particle.
$dE/dx_{\text{Muon}} \sim 1/40000 \; dEdx_{\text{Electron}}$

Proportional to the Energy of the Incoming particle →

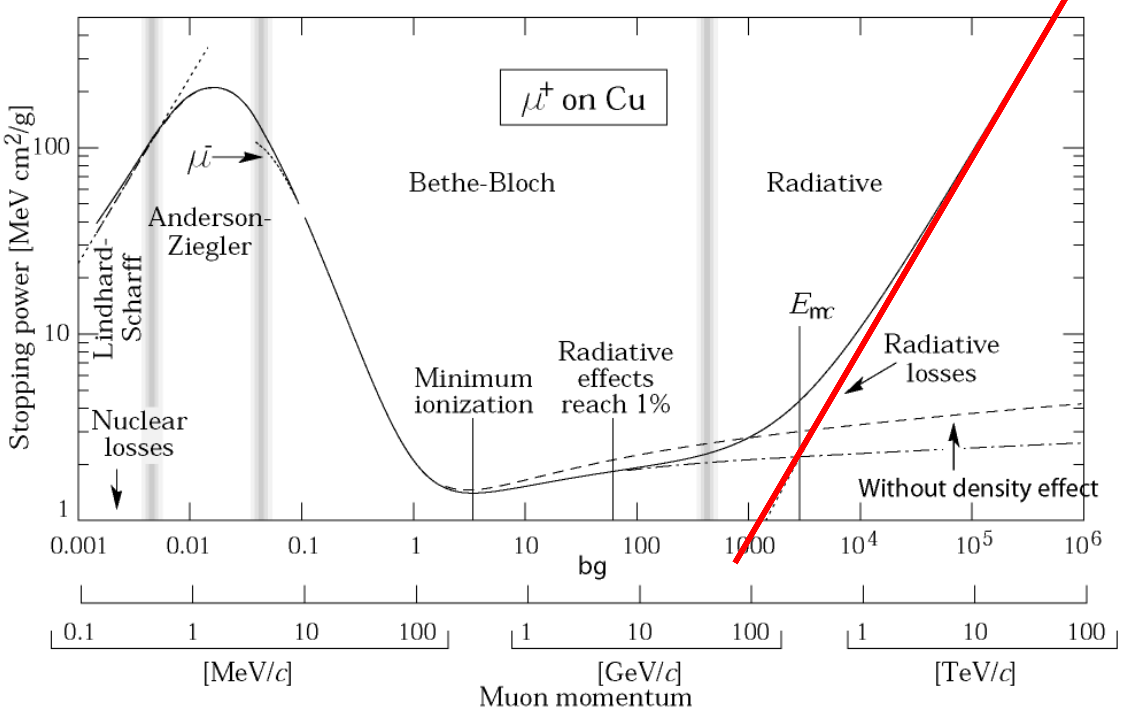
$E(x) = Exp(-x/X_0)$ – 'Radiation Length'

$X_0 \propto M^2 A / (\rho Z_1^4 Z^2)$

X_0: Distance where the Energy E_0 of the incoming particle decreases $E_0 Exp(-1) = 0.37 E_0$.

Critical Energy

such as copper to about 1% accuracy for energies between about 6 MeV and 6 GeV



For the muon, the second lightest particle after the electron, the critical energy is at 400GeV.

The EM Bremsstrahlung is therefore only relevant for electrons at energies of past and present detectors.

Critical Energy: If dE/dx (Ionization) = dE/dx (Bremsstrahlung)

Myon in Copper: p ≈ 400 GeV
Electron in Copper: p ≈ 25 MeV

References used today

- Particle Detectors, CERN Summer Student Lecture 2008, W. Riegler
- Material from the books by Leo and Gruppen
- Particle Data Book