Lecture 6 Sharpening Filters

1. The concept of sharpening filter
2. First and second order derivatives
3. Laplace filter
4. Unsharp mask
5. High boost filter
6. Gradient mask
7. Sharpening image with MatLab

Sharpening Spatial Filters

- To highlight fine detail in an image or to enhance detail that has been blurred, either in error or as a natural effect of a particular method of image acquisition.

- Blurring vs. Sharpening

  - Blurring/smooth is done in spatial domain by pixel averaging in a neighbors, it is a process of integration

  - Sharpening is an inverse process, to find the difference by the neighborhood, done by spatial differentiation.

Derivative operator

- The strength of the response of a derivative operator is proportional to the degree of discontinuity of the image at the point at which the operator is applied.

- Image differentiation
  - enhances edges and other discontinuities (noise)
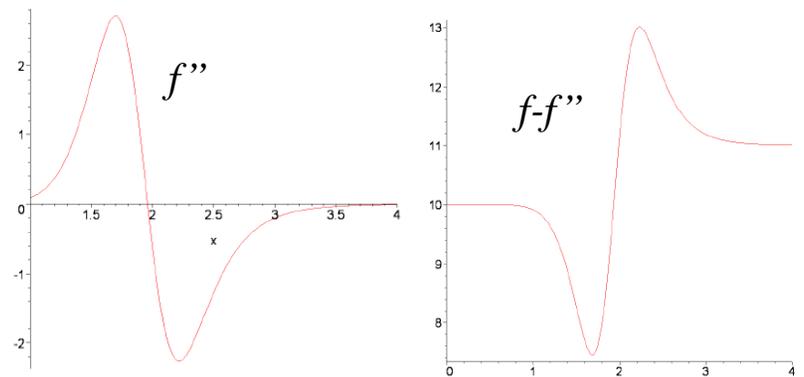 - deemphasizes area with slowly varying gray-level values.

Sharpening edge by First and second order derivatives

- Intensity function $f = 8 + \dfrac{1}{1 + \dfrac{1024}{x^{10}}}$

f

f'

- First derivative $f' = \dfrac{10240}{\left(1 + \dfrac{1024}{x^{10}}\right)^2 x^{11}}$
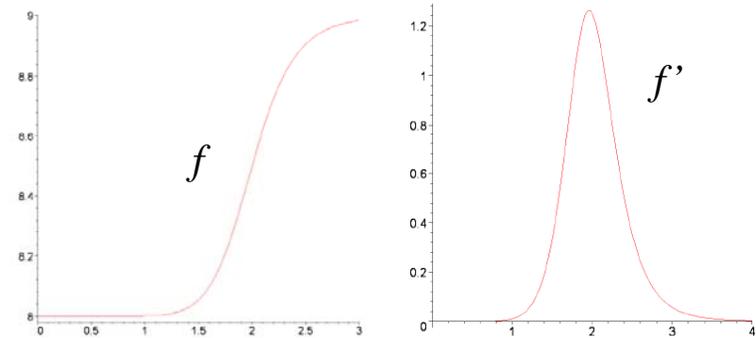
- Second-order derivative

$$f'' = \dfrac{209715200}{\left(1 + \dfrac{1024}{x^{10}}\right)^3 x^{22}} - \dfrac{112640}{\left(1 + \dfrac{1024}{x^{10}}\right)^2 x^{12}}$$

f''

$f\text{-}f''$

- $f\text{-}f'' = 8 + \dfrac{1}{1 + \dfrac{1024}{x^{10}}} - \dfrac{209715200}{\left(1 + \dfrac{1024}{x^{10}}\right)^3 x^{22}} + \dfrac{112640}{\left(1 + \dfrac{1024}{x^{10}}\right)^2 x^{12}}$
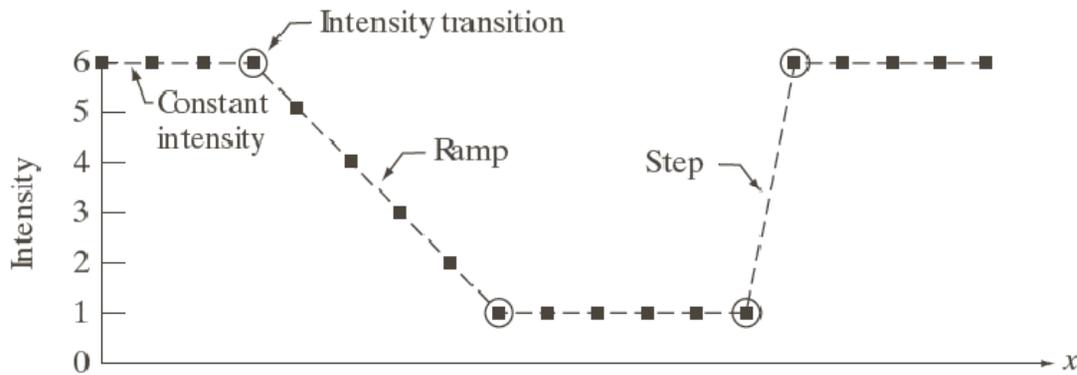
First and second order difference of 1D

- The basic definition of the first-order derivative of a one-dimensional function f(x) is the difference

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

- The second-order derivative of a one-dimensional function f(x) is the difference

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x)$$

Intensity transition

Constant intensity

Ramp

Step

| Scan line | 6 | 6 | 6 | 6 | 5 | 4 | 3 | 2 | 1 | 1 | 1 | 1 | 1 | 1 | 6 | 6 | 6 | 6 | 6 | → x |

1st derivative   0   0 −1 −1 −1 −1 −1   0   0   0   0   0   5   0   0   0   0

2nd derivative 0   0 −1   0   0   0   0   1   0   0   0   0   5 −5   0   0   0

Zero crossing

● First derivative
□ Second derivative
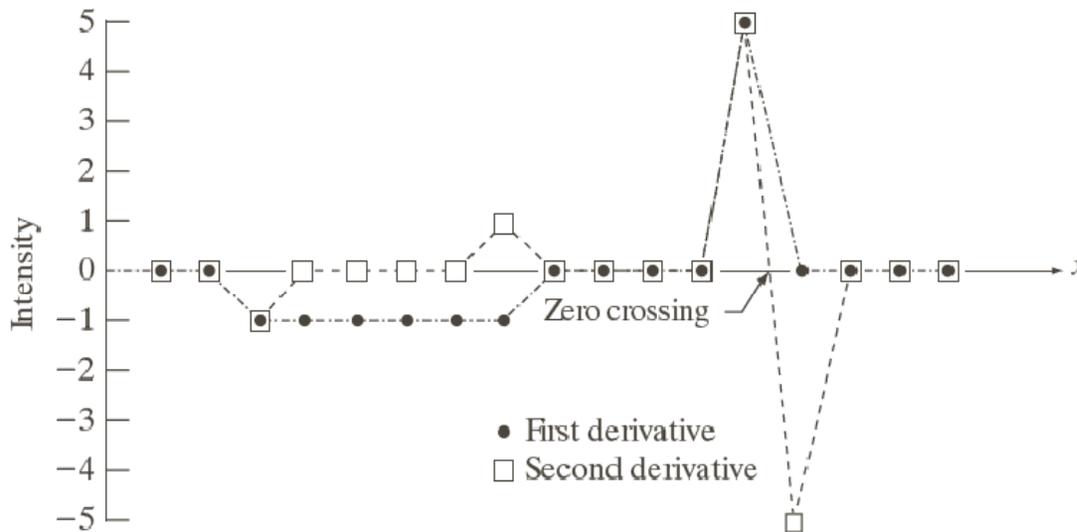
a
b
c

FIGURE 3.36
Illustration of the first and second derivatives of a 1-D digital function representing a section of a horizontal intensity profile from an image. In (a) and (c) data points are joined by dashed lines as a visualization aid.

First and Second-order derivative of 2D

- when we consider an image function of two variables, f(x, y), at which time we will dealing with partial derivatives along the two spatial axes.

Gradient operator

$$\nabla \mathrm{f} = \frac{\partial f(x, y)}{\partial x \partial y} = \frac{\partial f(x, y)}{\partial x} + \frac{\partial f(x, y)}{\partial y}$$

(linear operator)

Laplacian operator (non-linear)

$$\nabla^2 f = \frac{\partial^2 f(x, y)}{\partial x^2} + \frac{\partial^2 f(x, y)}{\partial y^2}$$

Discrete form of Laplacian

from $$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

$$\nabla^2 f = [f(x+1, y) + f(x-1, y)$$
$$+ f(x, y+1) + f(x, y-1) - 4f(x, y)]$$

Result Laplacian mask

0	1	0
1	−4	1
0	1	0

Laplacian mask implemented an extension of diagonal neighbors

1	1	1
1	−8	1
1	1	1

Other implementation of Laplacian masks

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

give the same result, but we have to keep in mind that when combining (add / subtract) a Laplacian-filtered image with another image.

Effect of Laplacian Operator

- as it is a derivative operator,
  - it highlights gray-level discontinuities in an image
  - it deemphasizes regions with slowly varying gray levels
- tends to produce images that have
  - grayish edge lines and other discontinuities, all superimposed on a dark,
  - featureless background.
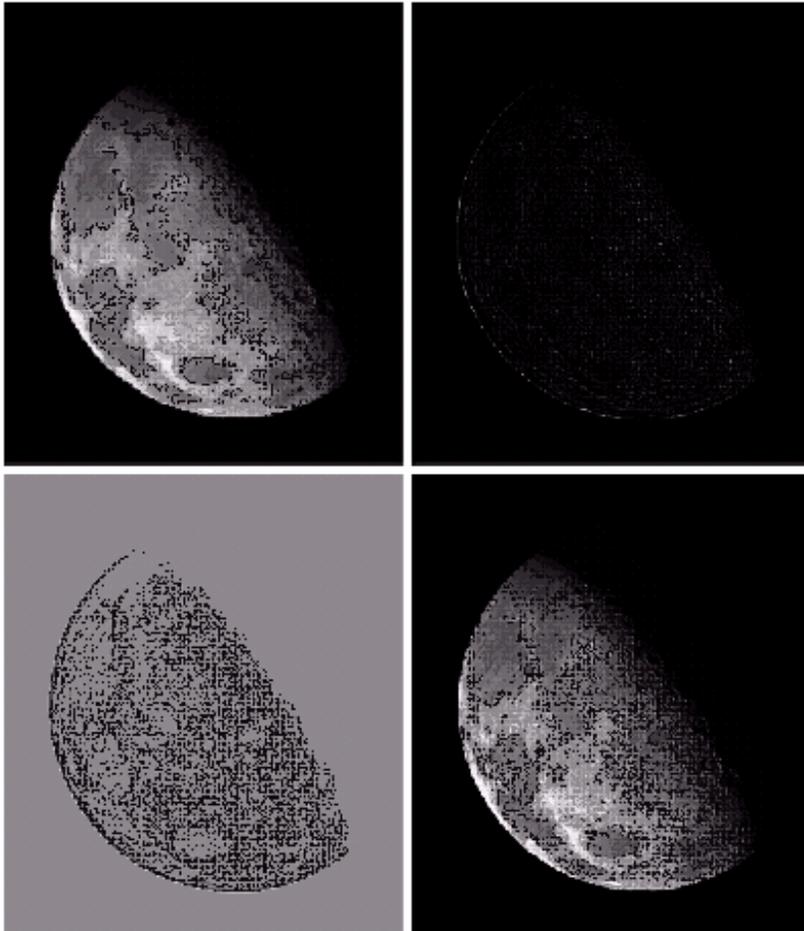
Correct the effect of featureless background

- easily by adding the original and Laplacian image.
- be careful with the Laplacian filter used

$$g(x, y) = \begin{cases} f(x, y) - \nabla^2 f(x, y) \\ f(x, y) + \nabla^2 f(x, y) \end{cases}$$

if the center coefficient of the Laplacian mask is negative

if the center coefficient of the Laplacian mask is positive

Example

- a). image of the North pole of the moon
- b). Laplacian-filtered image with

1	1	1
1	-8	1
1	1	1

- c). Laplacian image scaled for display purposes
- d). image enhanced by addition with original image

Mask of Laplacian + addition

- to simply the computation, we can create a mask which do both operations, Laplacian Filter and Addition the original image.
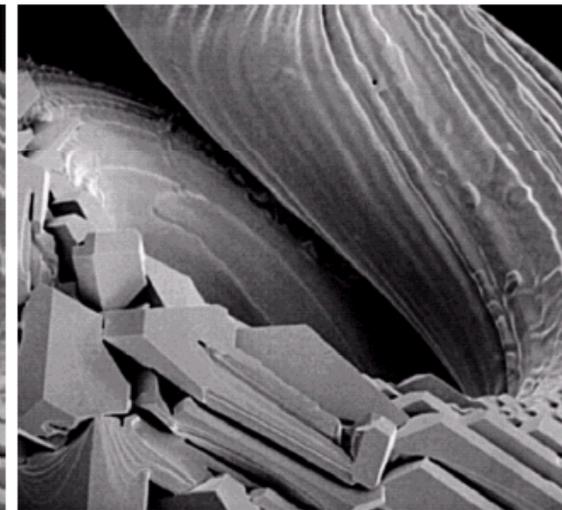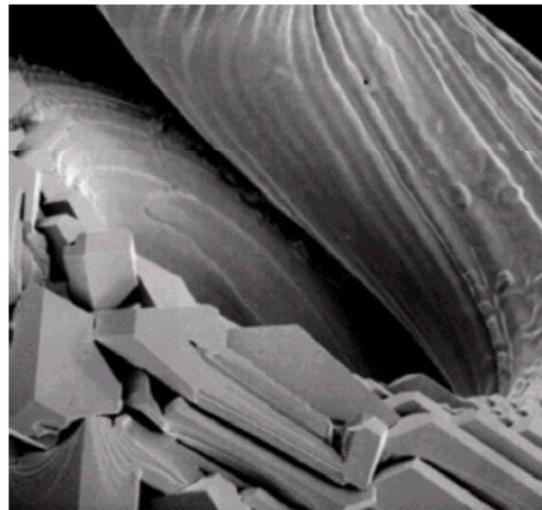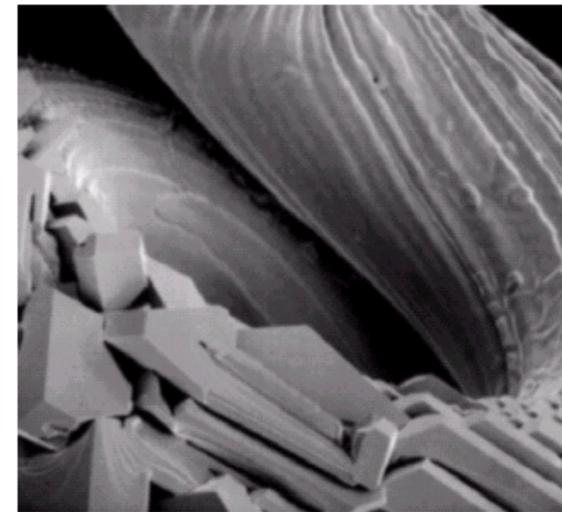
Mask of Laplacian + addition

$$g(x, y) = f(x, y) - [f(x+1, y) + f(x-1, y)$$
$$+ f(x, y+1) + f(x, y-1) + 4f(x, y)]$$
$$= 5f(x, y) - [f(x+1, y) + f(x-1, y)$$
$$+ f(x, y+1) + f(x, y-1)]$$

0	-1	0
-1	5	-1
0	-1	0

Example

0	−1	0
−1	5	−1
0	−1	0

−1	−1	−1
−1	9	−1
−1	−1	−1

a b c
d e

FIGURE 3.41 (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

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Note

$$g(x, y) = \begin{cases} f(x, y) - \nabla^2 f(x, y) \\ f(x, y) + \nabla^2 f(x, y) \end{cases}$$

0	-1	0
-1	5	-1
0	-1	0

=

0	0	0
0	1	0
0	0	0

+

0	-1	0
-1	4	-1
0	-1	0

0	-1	0
-1	9	-1
0	-1	0

=

0	0	0
0	1	0
0	0	0

+

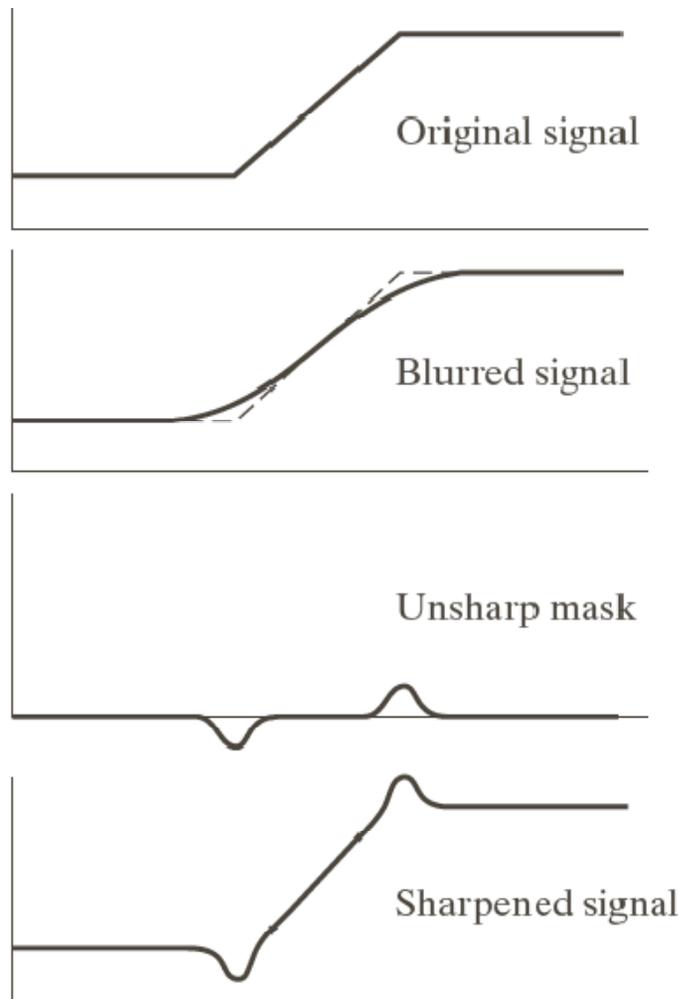| 0	-1	0
-1	8	-1
0	-1	0

Unsharp masking

$$f_s(x, y) = f(x, y) - \bar{f}(x, y)$$
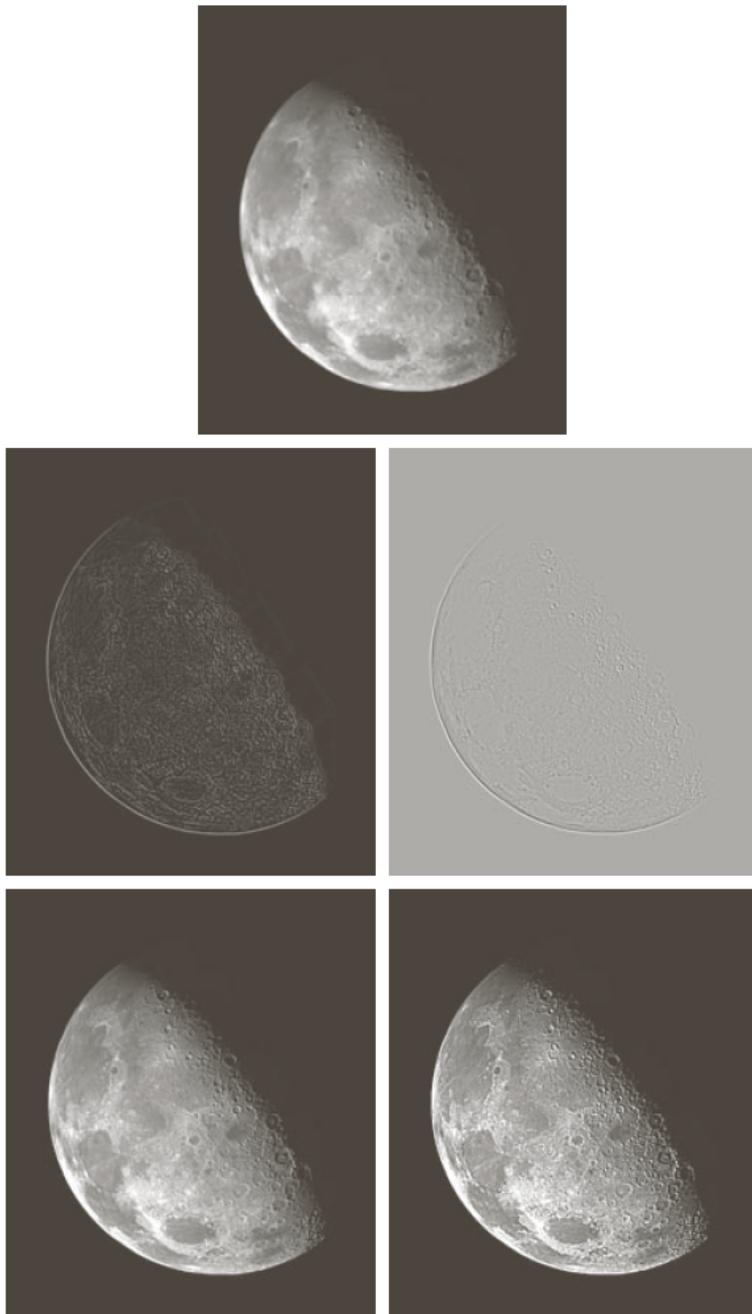
sharpened image = original image − blurred image

- to subtract a blurred version of an image produces sharpening output image.

Unsharp mask

Original signal

Blurred signal

Unsharp mask

Sharpened signal

a
b
c
d

FIGURE 3.39 1-D illustration of the mechanics of unsharp masking. (a) Original signal. (b) Blurred signal with original shown dashed for reference. (c) Unsharp mask. (d) Sharpened signal, obtained by adding (c) to (a).

FIGURE 3.38
(a) Blurred image of the North Pole of the moon.
(b) Laplacian without scaling.
(c) Laplacian with scaling. (d) Image sharpened using the mask in Fig. 3.37(a). (e) Result of using the mask in Fig. 3.37(b). (Original image courtesy of NASA.)

High-boost filtering

$$f_{hb}(x, y) = Af(x, y) - \bar{f}(x, y)$$

$$f_{hb}(x, y) = (A-1)f(x, y) - f(x, y)\bar{f}(x, y)$$
$$= (A-1)f(x, y) - f_s(x, y)$$

- generalized form of Unsharp masking
- $A \geq 1$

High-boost filtering

$$f_{hb}(x, y) = (A - 1) f(x, y) - f_s(x, y)$$

- if we use Laplacian filter to create sharpen image $f_s(x,y)$ with addition of original image

$$f_s(x, y) = \begin{cases} f(x, y) - \nabla^2 f(x, y) \\ f(x, y) + \nabla^2 f(x, y) \end{cases}$$

$$f_{hb}(x, y) = \begin{cases} A f(x, y) - \nabla^2 f(x, y) \\ A f(x, y) + \nabla^2 f(x, y) \end{cases}$$

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High-boost Masks

0	−1	0
−1	$A + 4$	−1
0	−1	0

−1	−1	−1
−1	$A + 8$	−1
−1	−1	−1

- $A \geq 1$
- if $A = 1$, it becomes "standard" Laplacian sharpening
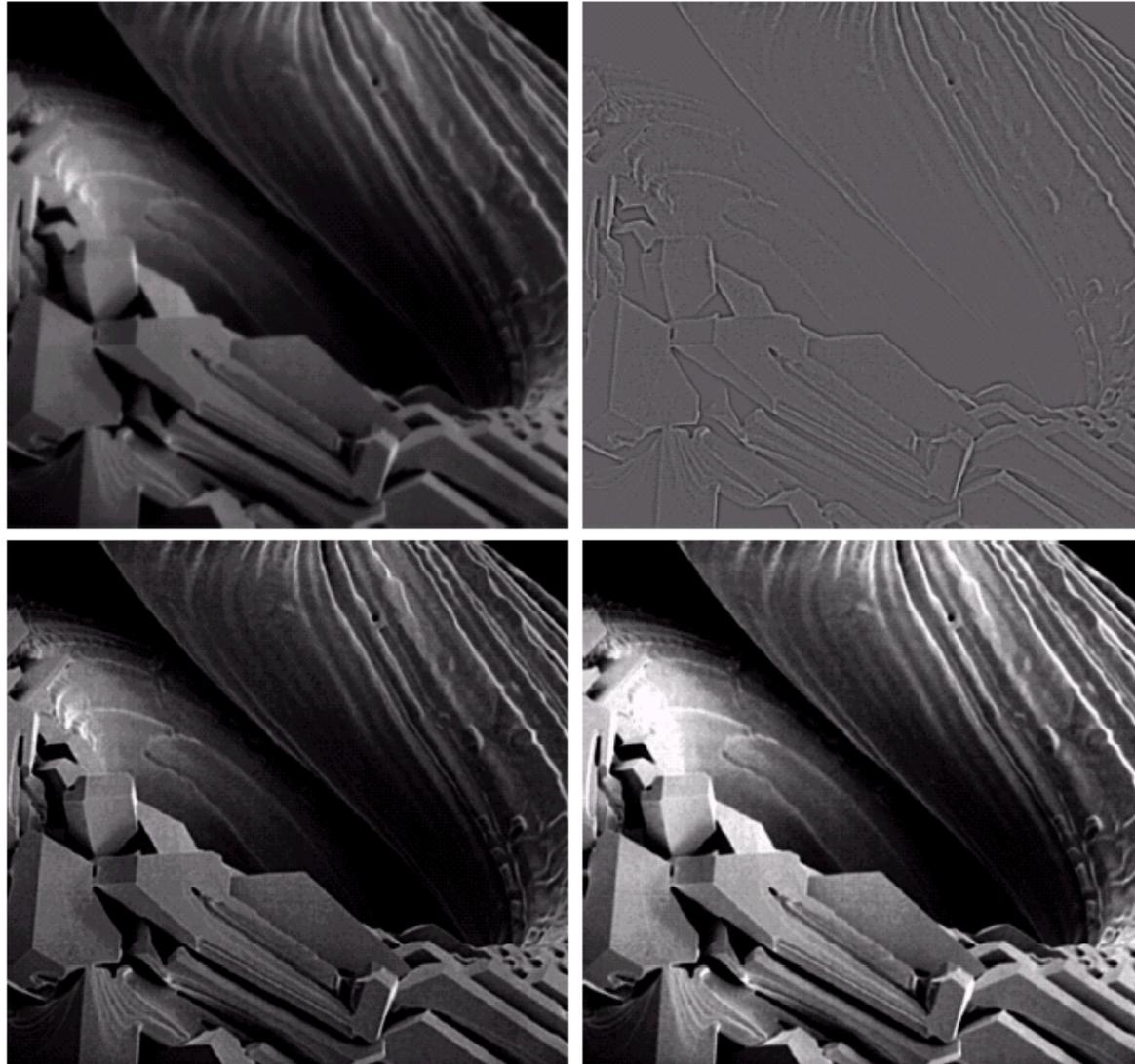
Example

a b
c d

FIGURE 3.43
(a) Same as
Fig. 3.41(c), but
darker.
(a) Laplacian of
(a) computed with
the mask in
Fig. 3.42(b) using
$A = 0$.
(c) Laplacian
enhanced image
using the mask in
Fig. 3.42(b) with
$A = 1$. (d) Same
as (c), but using
$A = 1.7$.

Gradient Operator

- first derivatives are implemented using the magnitude of the gradient.

$$\nabla f = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix}$$

$$\nabla f = mag(\nabla f) = [G_x^2 + G_y^2]^{1/2}$$

$$= \left[\left(\frac{\partial f}{\partial x} \right)^2 + \left(\frac{\partial f}{\partial y} \right)^2 \right]^{1/2}$$

commonly approx.

$$\nabla f \approx |G_x| + |G_y|$$

the magnitude becomes nonlinear

Gradient Mask

- simplest approximation, 2x2

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

$$G_x = (z_8 - z_5) \quad \text{and} \quad G_y = (z_6 - z_5)$$

$$\nabla f = [G_x^2 + G_y^2]^{1/2} = [(z_8 - z_5)^2 + (z_6 - z_5)^2]^{1/2}$$

$$\nabla f \approx |z_8 - z_5| + |z_6 - z_5|$$
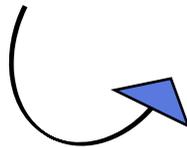
Gradient Mask

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

- Roberts cross-gradient operators, 2x2

$$G_x = (z_9 - z_5) \quad \text{and} \quad G_y = (z_8 - z_6)$$

$$\nabla f = [G_x^2 + G_y^2]^{1/2} = [(z_9 - z_5)^2 + (z_8 - z_6)^2]^{1/2}$$

$$\nabla f \approx |z_9 - z_5| + |z_8 - z_6|$$

-1	0
0	1

0	-1
1	0

Gradient Mask

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

- Sobel operators, 3x3

$$G_x = (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)$$
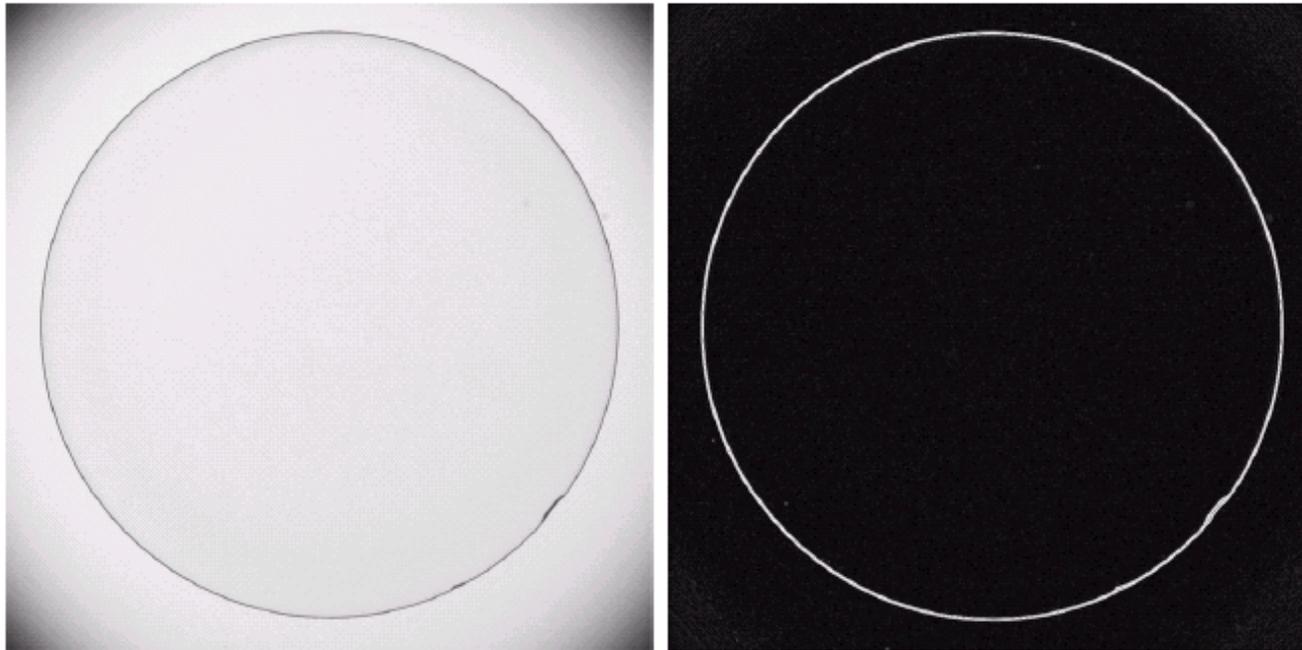
$$G_y = (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)$$

$$\nabla f \approx |G_x| + |G_y|$$

-1	-2	-1
0	0	0
1	2	1

-1	0	1
-2	0	2
-1	0	1

the weight value 2 is to achieve smoothing by giving more important to the center point
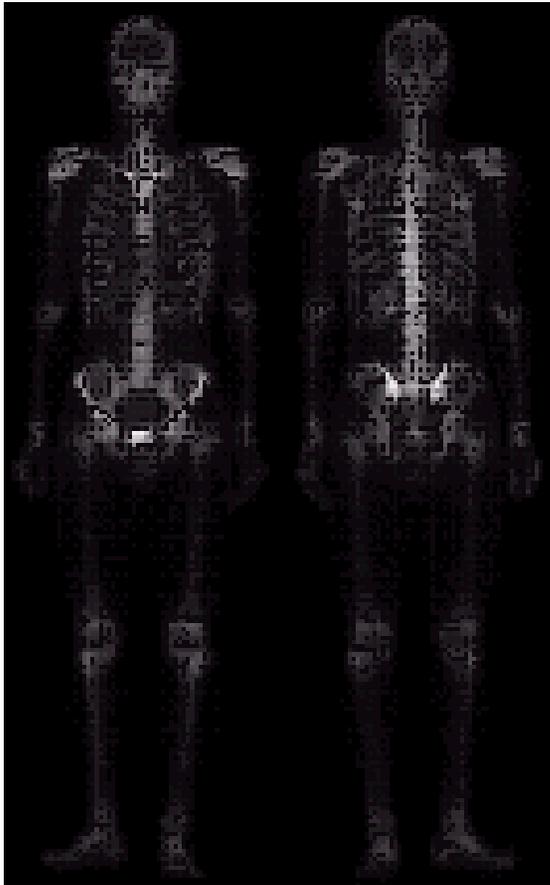
Example

a b

FIGURE 3.45
Optical image of
contact lens (note
defects on the
boundary at 4 and
5 o'clock).
(b) Sobel
gradient.
(Original image
courtesy of
Mr. Pete Sites,
Perceptics
Corporation.)

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Example of Combining Spatial Enhancement Methods

- want to sharpen the original image and bring out more skeletal detail.

- problems: narrow dynamic range of gray level and high noise content makes the image difficult to enhance
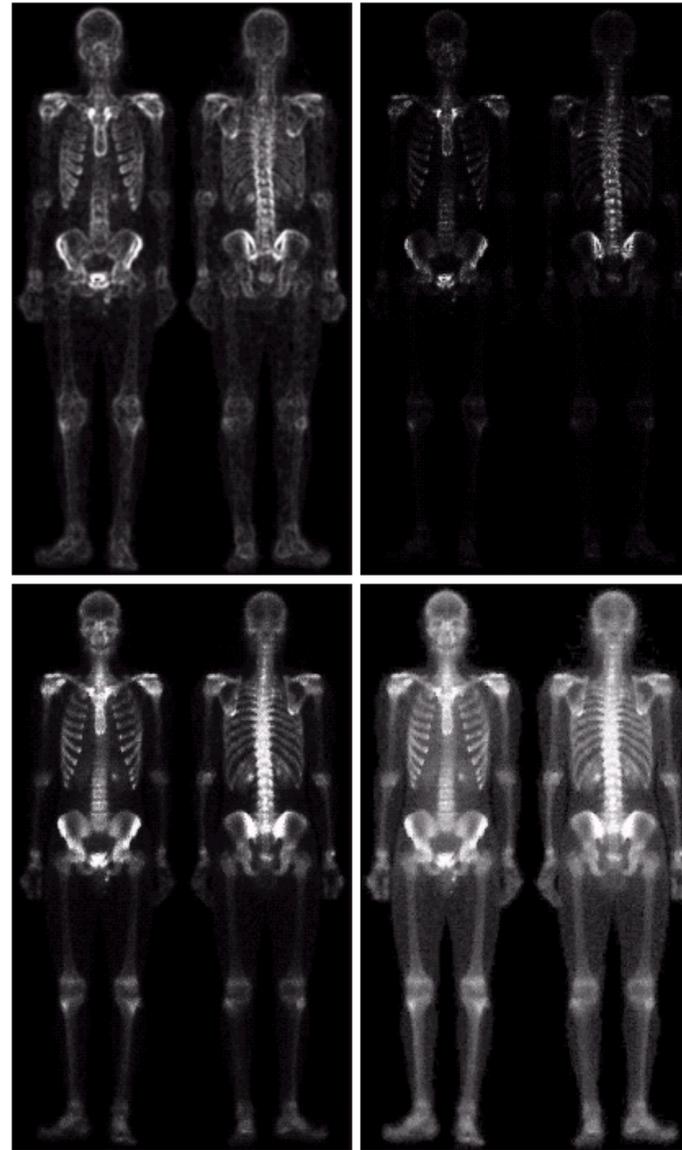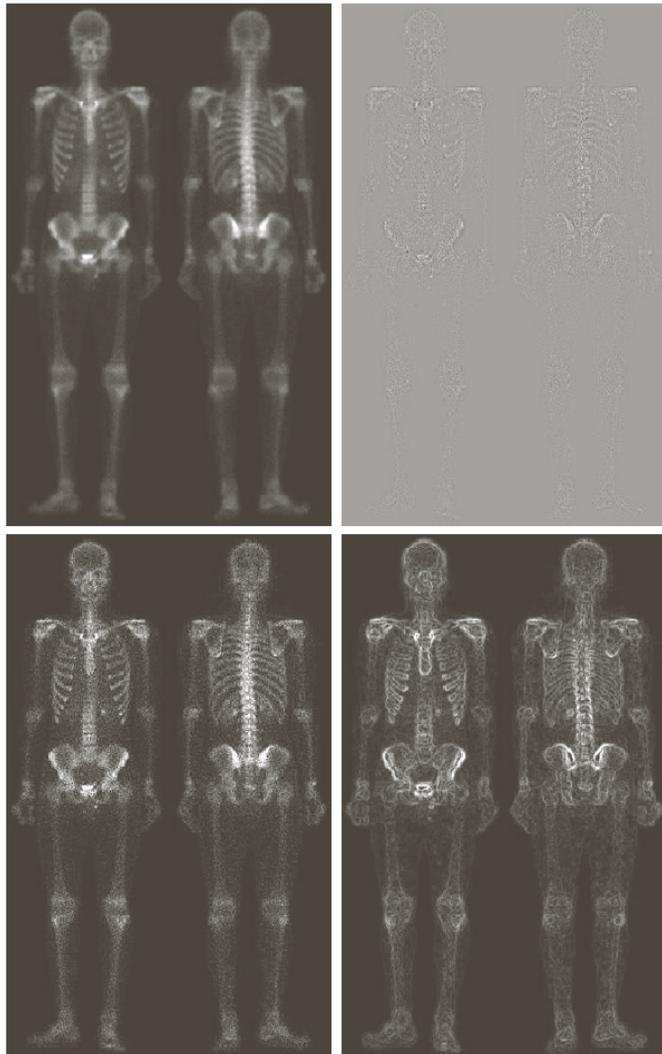
Example of Combining Spatial Enhancement Methods

- solve :

    1. Laplacian to highlight fine detail
    2. gradient to enhance prominent edges
    3. gray-level transformation to increase the dynamic range of gray levels

a b
c d

FIGURE 3.43
(a) Image of whole body bone scan.
(b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b).
(d) Sobel gradient of (a).

e f
g h

FIGURE 3.46
(Continued)
(e) Sobel image smoothed with a 5 × 5 averaging filter. (f) Mask image formed by the product of (c) and (e).
(g) Sharpened image obtained by the sum of (a) and (f). (h) Final result obtained by applying a power-law transformation to (g). Compare (g) and (h) with (a). (Original image courtesy of G.E. Medical Systems.)