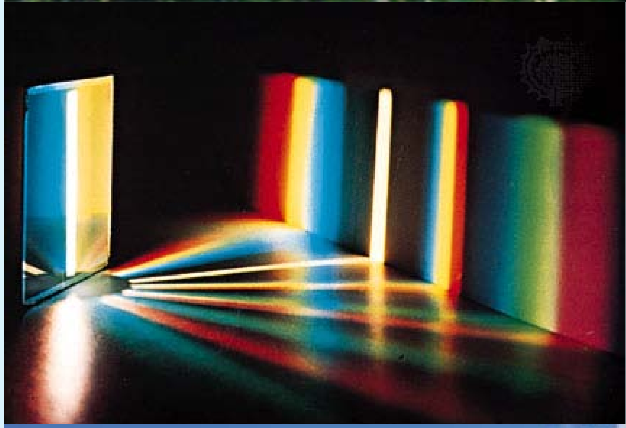
Lecture 9: Introduction to Diffraction of Light



Lecture aims to explain:

1. Diffraction of waves in everyday life and applications

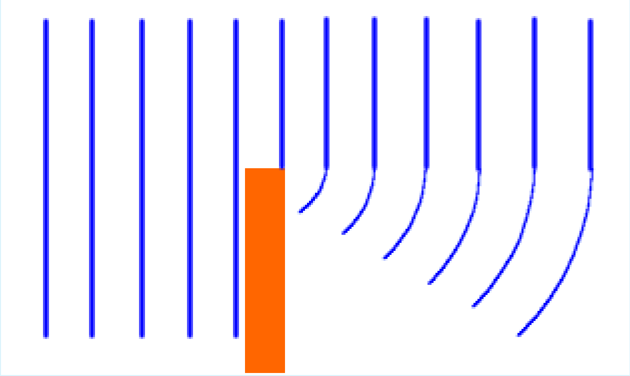
2. Interference of two one dimensional electromagnetic waves

3. Typical diffraction problems: **a slit, a periodic array of slits, circular aperture**

4. Typical approach to solving diffraction problems

Diffraction of waves in everyday life and applications

Diffraction in everyday life



Diffraction in applications

Spectroscopy: physics, chemistry, medicine, biology, geology, oil/gas industry

Communication and detection systems: fibre optics (waveguides), lasers, radars

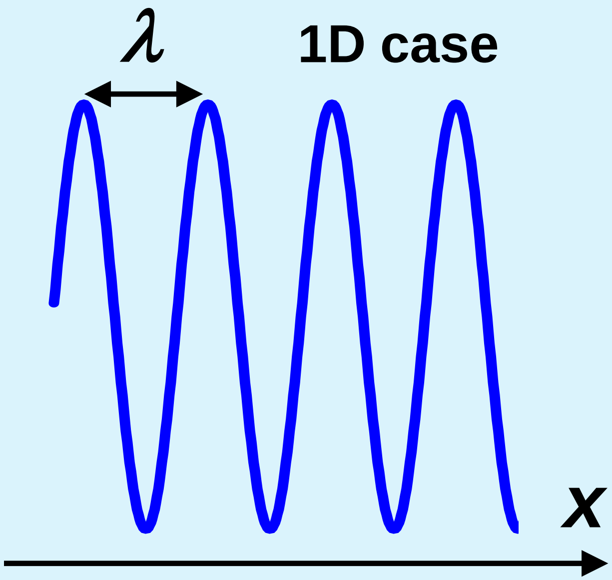
Holography

Structural analysis: X-ray

Must be taken into account in **applications with high spatial resolution:** imaging (astronomy, microscopy including X-ray, electron and neutron scattering), semiconductor device fabrication (optical lithography), CDs, DVDs, BDs

Interference of two one dimensional (1D) electromagnetic waves

Harmonic wave and its detection



$$\sin\left(\frac{2\pi}{\lambda}x-\omega t\right)$$

Oscillating electric field of the wave:

$$E(x,t)=A\sin\left(kx-\omega t\right)$$

In the case of visible light $\omega \sim 10^{15} Hz$

The detectable intensity (irradiance):

$$I=\left\langle E^2\right\rangle_T=\frac{1}{T}\int_0^T E^2dt$$

Superposition of waves

Consider two electromagnetic waves:

$$E_1 = E_{01}\sin(\omega t - kx + \varepsilon_1) \qquad E_2 = E_{02}\sin(\omega t - kx + \varepsilon_2)$$

Intensity on the detector:

$$I = \frac{E_{01}^2}{2} + \frac{E_{02}^2}{2} + E_{01}E_{02}\cos(\delta)$$

Phase shift due to difference in the **optical path** and **initial phase**:

$$\delta = \frac{2\pi}{\lambda}(x_2 - x_1) + (\varepsilon_1 - \varepsilon_2)$$

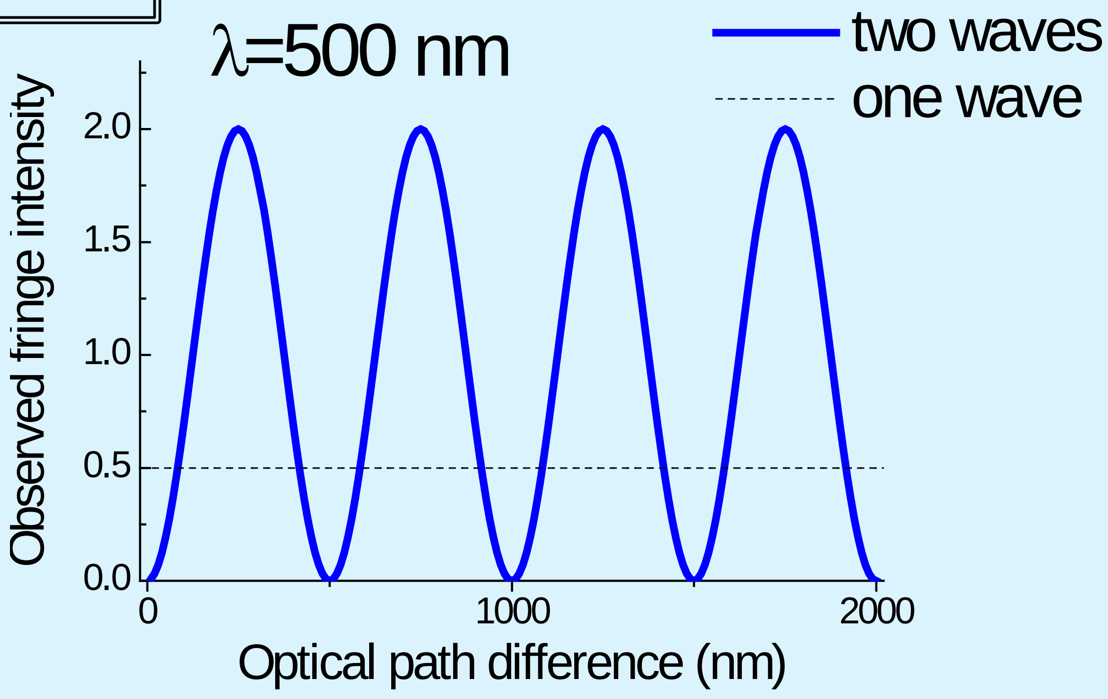
Dependence of intensity on the optical path length difference

$$I = \frac{E_{01}^2}{2} + \frac{E_{02}^2}{2} + E_{01}E_{02}\cos(\delta)$$

For the waves with the same initial phase, the phase difference arises from the difference in the optical path length:

$$\delta = \frac{2\pi}{\lambda}(x_2 - x_1) = \Delta OP$$

Figure shows the dependence of intensity measured by the detector as a function of the optical path difference between the waves of the same amplitude



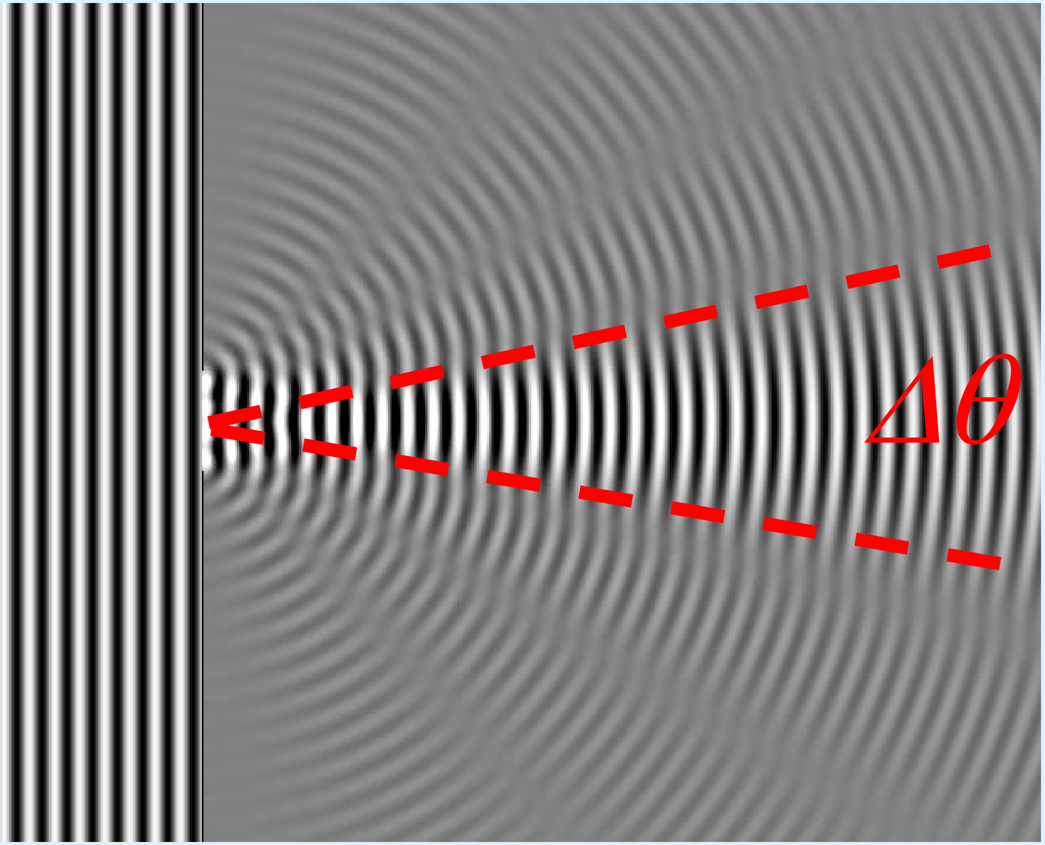
Typical diffraction problems

Diffraction by a slit or periodic array of slits (or grooves)

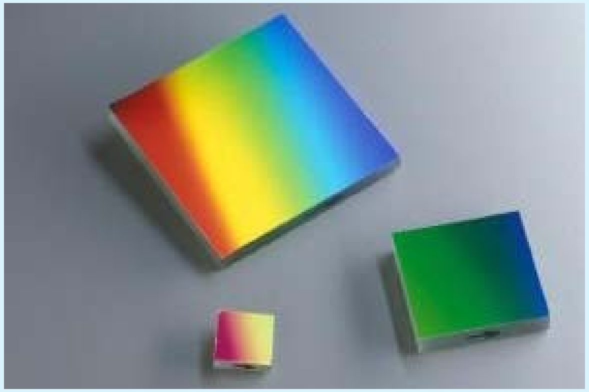
Use in **spectroscopy**: analysis of spectral (“colour”) composition of light

$$\Delta\theta = 2\lambda / b$$

λ- wavelength, b-slit width

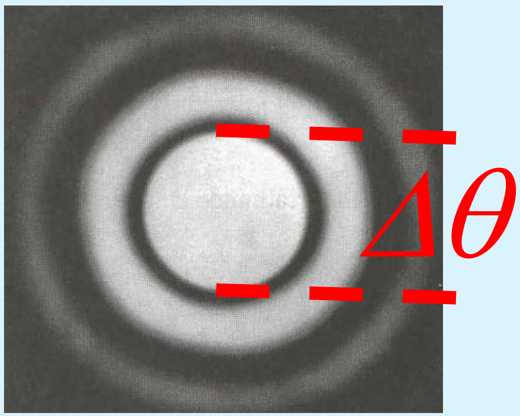


If many slits arranged in a **periodic array**, sharp maxima will appear at different angles depending on the wavelength: **spectral analysis becomes possible**



Diffraction by a circular aperture

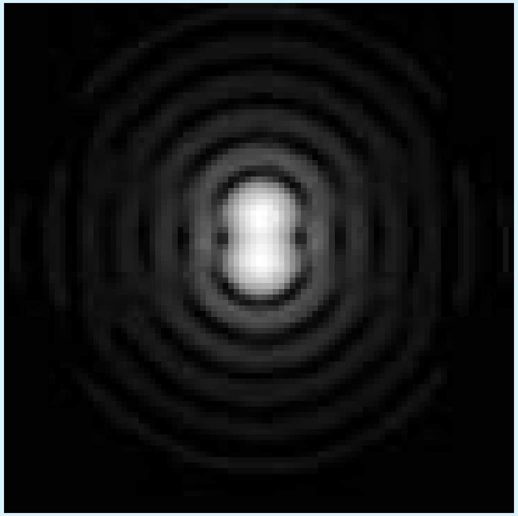
Important in high resolution **imaging** and **positioning**: sets limitations to spatial resolution in astronomy, microscopy, optical lithography, CDs, DVDs, BDs, describes propagation of laser beams



The smallest angular size which can be **resolved** is given by

$$\Delta\theta = 2.44\lambda / D$$

λ- wavelength, *D*-aperture diameter

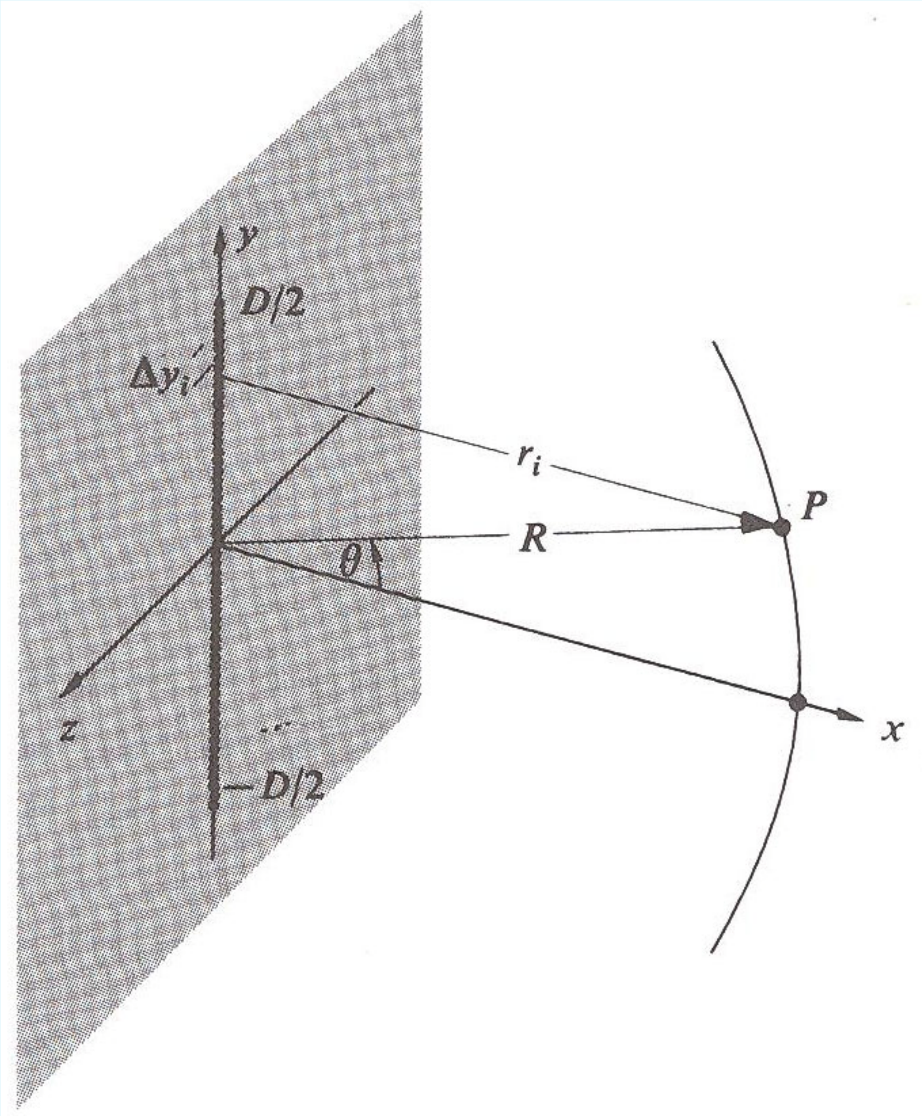


This also defines the **smallest size of a laser spot** which can be achieved by focussing with a lens (see images on the left): roughly **~λ**

Typical approach to solving diffraction problems

Huygens' principle (Lecture 1):
'Each point on a wavefront acts as a source of spherical secondary wavelets, such that the wavefront at some later time is the superposition of these wavelets.'

Extended coherent light source



Each infinitely small segment (each "point") of the source emits a spherical wavelet. From the differential wave equation, the amplitude decays as *1/r*:

$$E_i = \frac{\varepsilon_L}{r_i} \Delta y_i \sin(\omega t - k r_i)$$

ε_L source strength per unit length

Contribution from all points is:

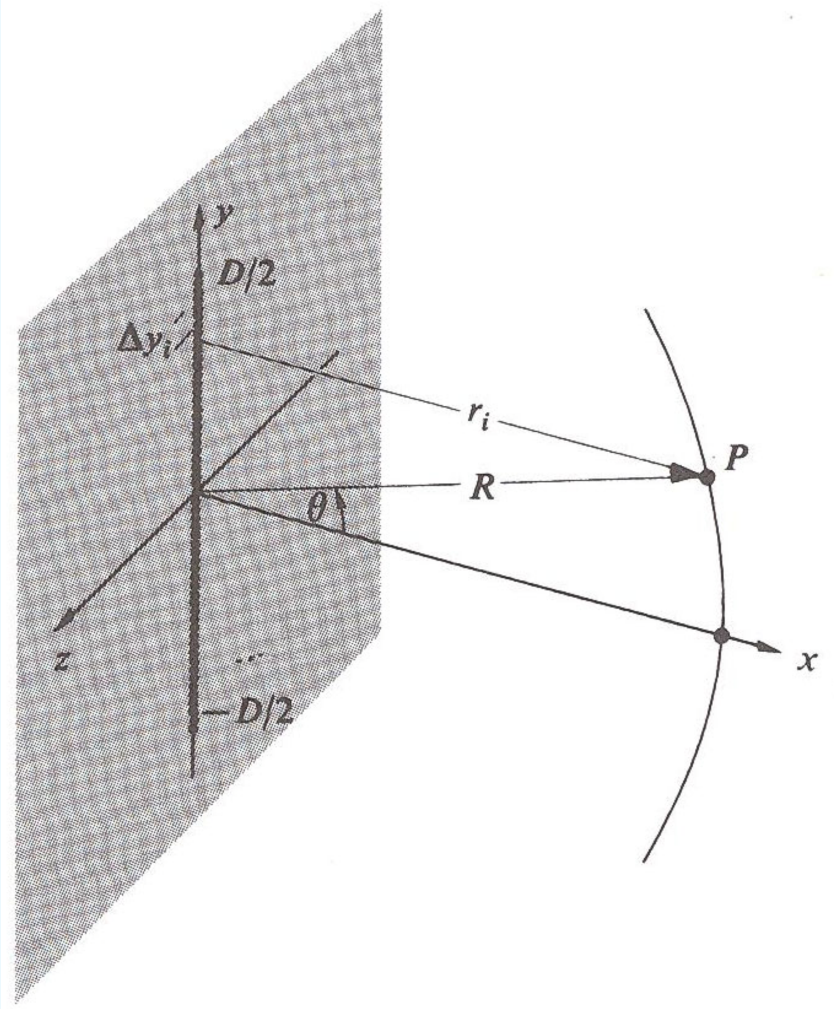
$$E = \varepsilon_L \int_{-D/2}^{D/2} \frac{\sin(\omega t - kr)}{r} dy$$

Fraunhofer and Fresnel diffraction limits

Fraunhofer case: distance to the detector is large compared with the light source *R>>D*. In this case **only dependence of the phase for individual wavelets on the distance to the detector is important**:

$$E = \frac{\varepsilon_L}{R} \int_{-D/2}^{D/2} \sin(\omega t - kr)\, dy$$

Where $r \approx R - y \sin\theta$



Fresnel case: includes “near-field” region, so not only phase but the amplitude is a strong function of the position where the wavelet was emitted originally

$$E = \varepsilon_L \int_{-D/2}^{D/2} \frac{\sin(\omega t - kr)}{r}\, dy$$

SUMMARY

Diffraction occurs due to superposition of light waves. It is used in **spectroscopy**, **communication and detection systems** (fibre optics, lasers, radars), **holography, structural analysis (**X-ray), and defines the limitations in **applications with high spatial resolution:** imaging and positioning systems

The detectable intensity (irradiance) for a quickly oscillating field:

$$I = \left\langle E^2 \right\rangle_T = \frac{1}{T}\int_0^T E^2 dt$$

For the waves with the same initial phase, the phase difference arises from the difference in the optical path length:

$$\delta = \frac{2\pi}{\lambda}(x_2 - x_1) = \Delta OP$$

Typical approach to solving diffraction problems: use Huygens principle and calculate contribution of spherical waves emitted by all “point” emitters. **Fraunhofer diffraction: observation from a distant point**