Lecture 10
Loose Ends

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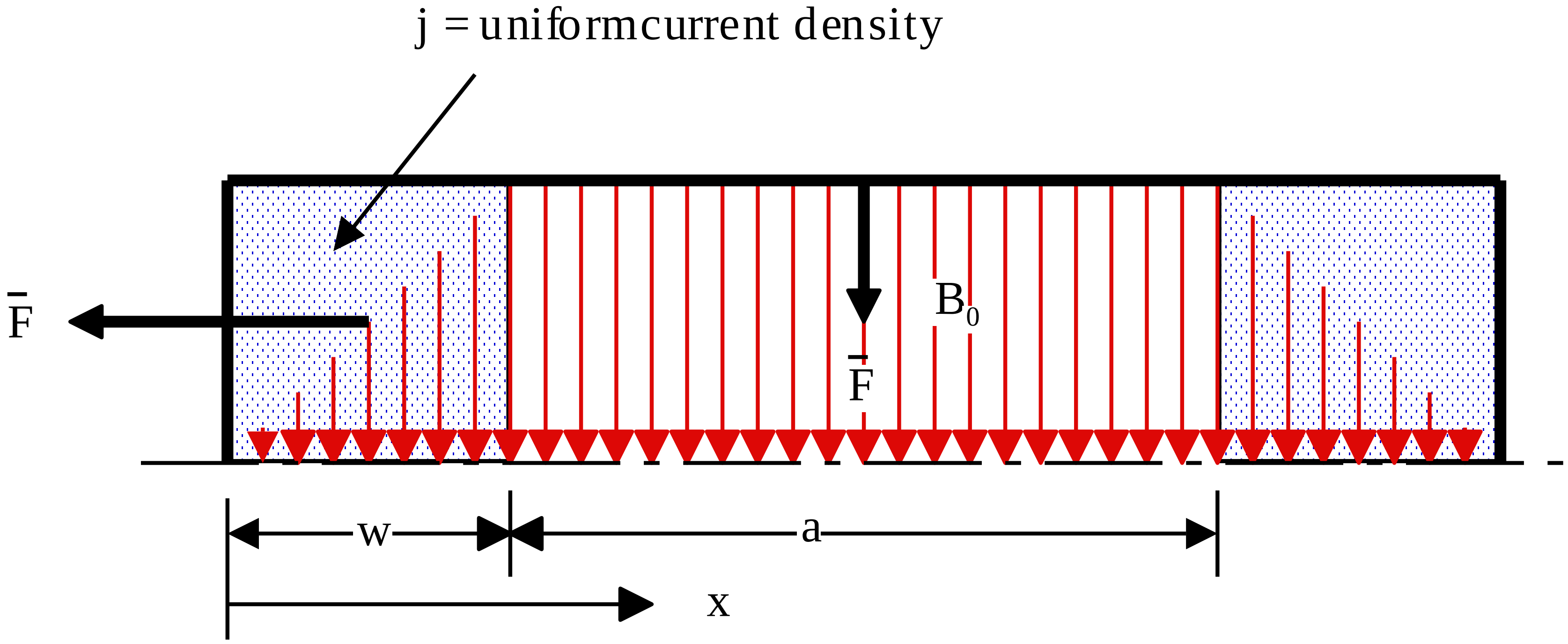
Introduction

- Magnetic Forces
- Magnet Stored Energy and Inductance
- Two and Three Dimensional Fringe Fields
- End Chamfering to Reduce Allowed Multipoles
- Eddy Currents
  - Although Eddy Currents and their effects can be important in the performance of a magnet, it is beyond the mathematical scope of this class. Therefore, there will be no discussion on this subject.

Magnetic Forces

- We use the figure to illustrate a simple example.



Magnetic Force on a Conductor

- We use the expression, $\overline{F} = \int \overline{j} \times \overline{B}\, dv$

Integrated over the volume of the conductor.

$$NI = \frac{B_0 h}{\mu_0} \qquad j = \frac{NI}{wh} = \frac{B_0}{w\mu_0}$$

$$B = B_0 \frac{x}{w} \qquad dv = hL\,dx$$

L is the length *into the paper*.

$$\left|\overline{F}\right| = \int j\, B\, dv = \int \frac{B_0}{w\mu_0} B_0 \frac{x}{w} hL\,dx$$

$$= \frac{B_0^2 hL}{w^2 \mu_0} \int_0^w x\,dx = \frac{B_0^2 hL}{w^2 \mu_0} \frac{w^2}{2} = \frac{B_0^2 hL}{2\mu_0}$$

$$\textit{repulsive pressure} = \frac{Force}{Area} = \frac{\left|\overline{F}\right|}{hL} = \frac{B_0^2}{2\mu_0}$$

Magnetic Force on a Pole

- We use the expression, $\overline{F} = \int \rho \cdot \overline{H}\, dv$

  where the magnetic charge density ρ is given by, $\rho = \dfrac{B_0}{h}$

  The force is in the same direction as the *H* vector and is attractive.

$$H = \frac{B_0}{\mu_0}\frac{y}{h} \qquad dv = aLdy$$

$$\left|\overline{F}\right| = \int \rho H\, dv = \int \frac{B_0}{h}\frac{B_0}{\mu_0}\frac{y}{h} aLdy = \frac{B_0^2 aL}{h^2 \mu_0}\int_0^h ydy = \frac{B_0^2 aL}{h^2 \mu_0}\frac{h^2}{2} = \frac{B_0^2 aL}{2\mu_0}$$

$$attractive\ \ pressure = \frac{Force}{Area} = \frac{\left|\overline{F}\right|}{aL} = \frac{B_0^2}{2\mu_0}$$

Pressure

- The *magnitudes* of the repulsive pressure for the current and the attractive pressure at the pole are *identical*. In general, the pressures *parallel* to the field lines are *attractive* and the forces *normal* to the field lines are *repulsive*. The pressure is proportional to the *flux density squared*.

$$pressure = \frac{B_0^2}{2\mu_0}$$

Units

- The units for the expression for pressure can be derived.

$$\frac{B_0^2}{2\mu_0} = \frac{T^2}{\frac{Tm}{Amp}} = \frac{T - Amp}{m} = \frac{Webers}{m^2}\frac{Amp}{m} = \frac{Volt - \sec.Amp}{m^3}$$

$$= \frac{watt - \sec.}{m^3} = \frac{joules}{m^3} = \frac{Newton - m}{m^3} = \frac{Newton}{m^2}$$

Rule of Thumb

- Let us perform a calculation for a flux density of 5 kG = 0.5T.

$$pressure = \frac{B_0^2}{2\mu_0} = \frac{0.5^2}{2 \times 4\pi \times 10^{-7}} = 99{,}472\,\frac{Newtons}{m^2}$$

$$pressure_{\,@\,0.5\mathrm{T}} = 99{,}472\,\frac{Newton}{m^2}$$

$$= 99{,}472\,\frac{N}{m^2} \times \frac{1\,lb_f}{4.448\ \mathrm{N}} \times \frac{1\,\mathrm{m}^2}{(39.37\ \mathrm{in})^2} = 14.43\,\frac{lb_f}{in^2}$$

$$\approx 1\,\mathrm{atmosphere}$$

Magnet Stored Energy

- The magnet stored energy is given by; $U = \frac{1}{2}\int HBdv$

the volume integral of the product of *H* and *B*.

Consider a *window frame* dipole field (illustrated earlier) with uniform field in the space between the coil. If we ignore the field in the coil and in the iron,

$$U = \frac{B_0^2}{2\mu_0} \times magnetic\ volume$$

$$\frac{B_0^2}{2\mu_0} = \frac{joules}{m^3} = \frac{Newton}{m^2}$$

U is in *joules*.

Magnet Inductance and Ramping Voltage

- Inductance is given by, $L = \frac{2U}{I^2}$

$$V = RI + L\frac{dI}{dt} = RI + \frac{2U}{I^2} \times \frac{I}{\Delta t} = RI + \frac{2U}{I\Delta t}$$

In fast ramped magnets, the resistive term is small.

$$V \approx \frac{2U}{I\Delta t} = \frac{2}{I\Delta t} \times \frac{B_0^2}{2\mu_0} \times volume \qquad I = \frac{B_0 h}{N\mu_0} \qquad volume = ahL$$

Substituting, $$V \approx \frac{1}{\frac{B_0 h}{N\mu_0}\Delta t} \times \frac{B_0^2}{\mu_0} \times ahL = \frac{B_0 NaL}{\Delta t}$$

Units and Design Options

- The units are,

$$V \approx \frac{B_0 NaL}{\Delta t} = \frac{Tm^2}{\sec} = \frac{Webers}{m^2} \times \frac{m^2}{\sec} = \frac{Volt - \sec}{\sec} = Volt$$

$$V \approx \frac{B_0 NaL}{\Delta t}$$

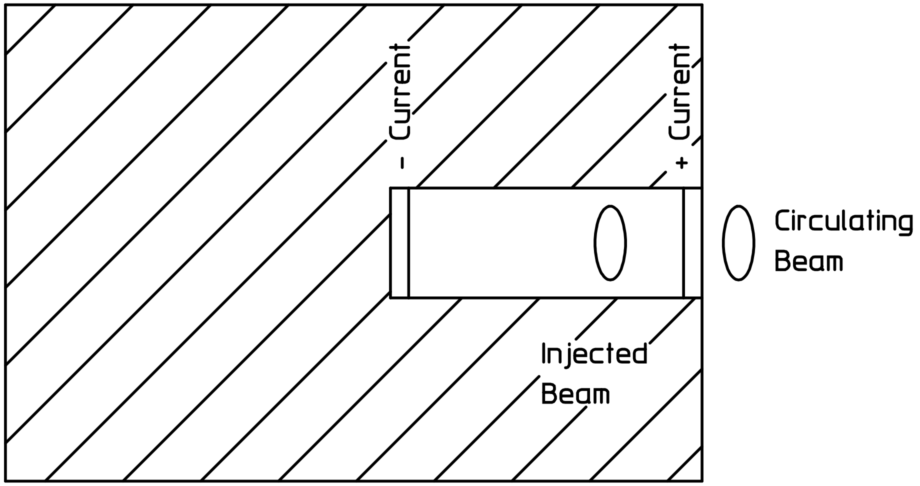
Given the field = B_0, pole width = a, Magnet Length = L and ramp time, Δt, the only design option available for changing the voltage is the number of turns, N.

Other Magnet Geometries

- Normally, the stored energy in other magnets (ie. H dipoles, quadrupoles and sextupoles) is not as easily computed. However, for more complex geometries, two dimensional magnetostatic codes such as *POISSON* will compute the stored energy per unit length of magnet.

Two-Dimensional Fringe Fields

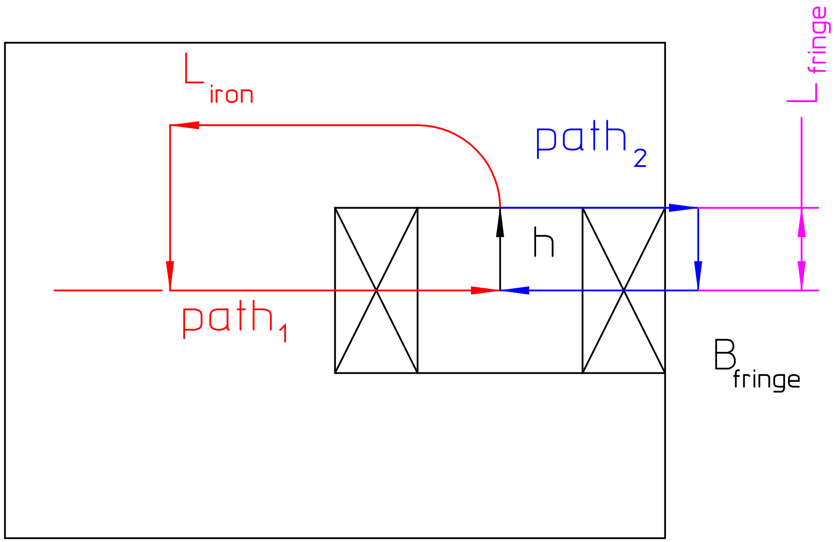
- Fringe fields are often fields where they are not wanted. An example of unwanted fringe fields are those in an injection or extraction septum.



Estimating Septum Fringe Fields

- A simple calculation can be made to estimate the size of the fringe field.
- This same calculation reveals that fringe fields can be reduced by using good design practices.

- The MMF can be calculated around two paths. Both including the half gap, h..
- For path1, the upper horizontal leg contributes to the MMF drop, but the lower leg does not.
- For path2, both horizontal legs do not contribute to the MMF drop.



$$\frac{NI}{2} = \oint_{Path1} H \bullet dl \approx \frac{Bh}{\mu_0} + \frac{B_{iron} L_{iron}}{\mu \mu_0}$$

$$\frac{NI}{2} = \oint_{Path2} H \bullet dl \approx \frac{Bh}{\mu_0} + \frac{B_{fringe} L_{fringe}}{\mu_0}$$

$$\frac{B_{fringe} L_{fringe}}{\mu_0} = \frac{B_{iron} L_{iron}}{\mu \mu_0}$$

- Solving for the fringe field, $$B_{fringe} = \frac{B_{iron}}{\mu} \times \frac{L_{iron}}{L_{fringe}}$$

This expression tells us that B_{fringe} can be reduced by reducing the ratio, $\frac{B_{iron}}{\mu}$

This reduction is accomplished by reducing the saturation by making the yoke and back leg of the septum magnet as thick as possible.

Another expression for calculating the field uses the magnet efficiency. $$\frac{NI}{2} = \frac{Bh}{\eta \mu_0}$$

Homework: Estimate the fringe field using the magnet efficiency.

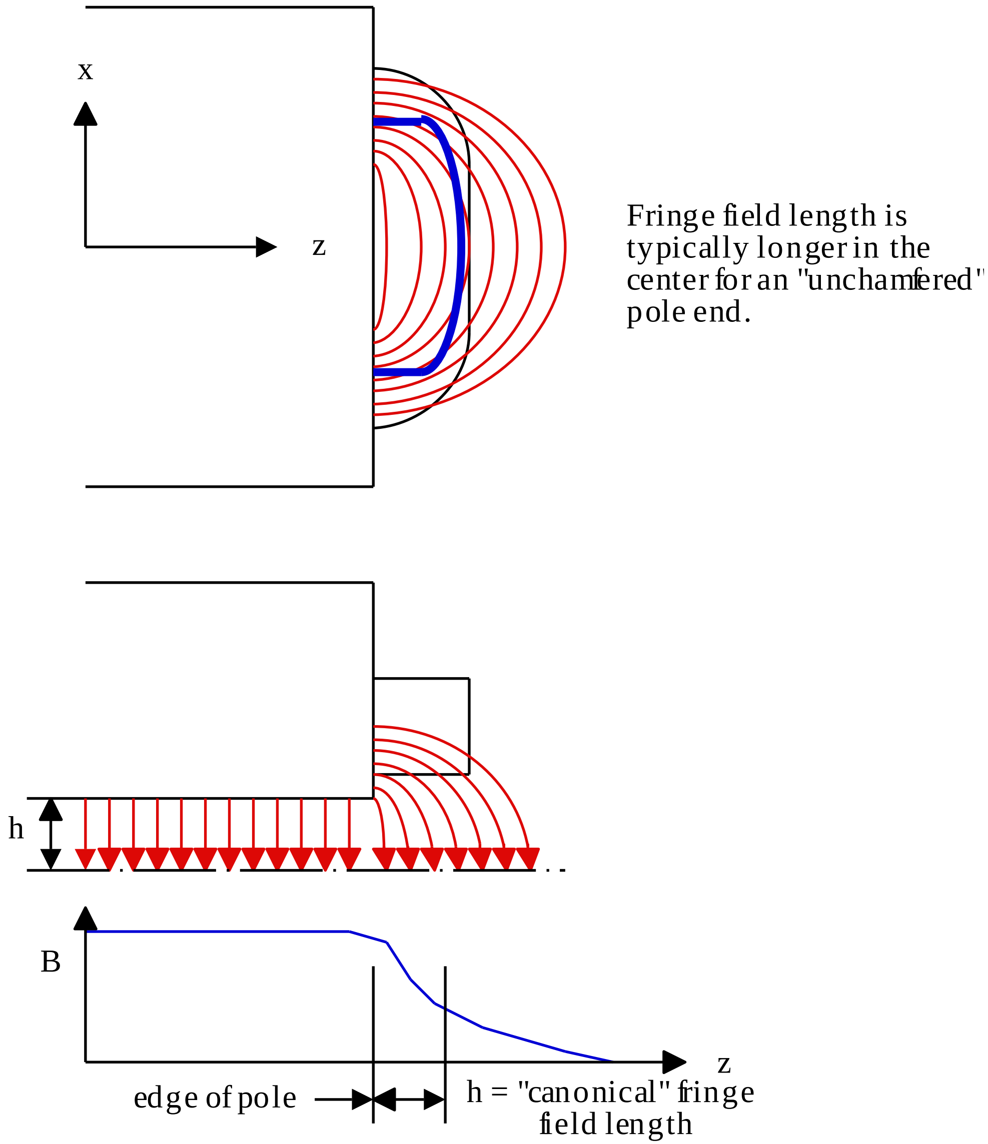
Fringe Fields and Effective Lengths

- Often, canonical rules of thumb are adopted in order to estimate the effective length of magnets.
  - Dipole fringe field length = 1 half gap at each end
  - Quadrupole fringe field length = 1/2 pole radius at each end.
  - Sextupole fringe field length = 1/3 pole radius at each end.

Three Dimensional Fringe Fields

- The shape of the three dimensional fringe field contributes to the integrated multipole error of a magnet.

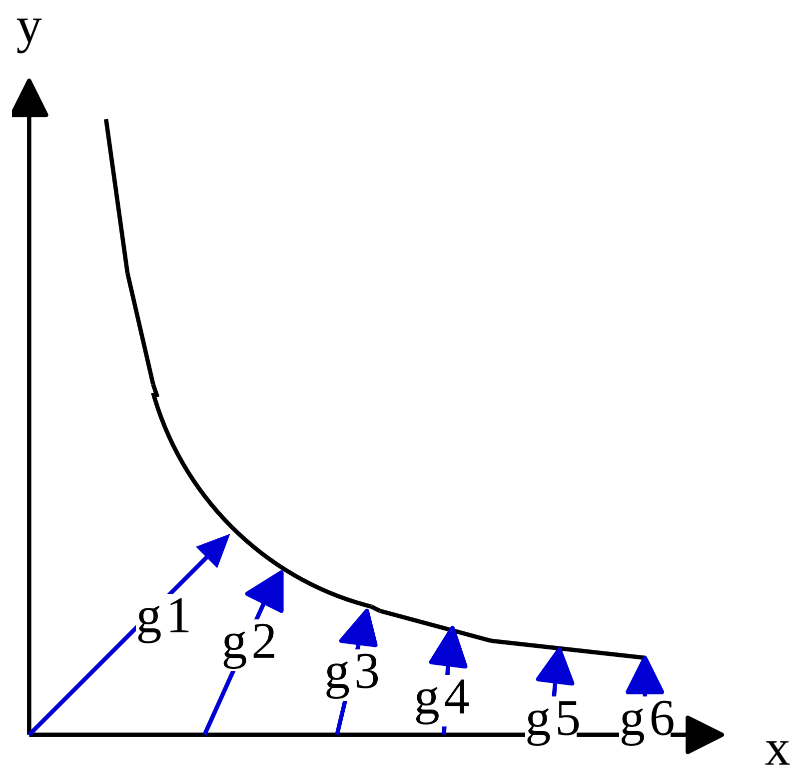
- Dipole Fringe Field
  - Typically, the fringe field is longer at the center of the magnet and drops off near the edges.
  - This distribution is approximately quadratic and the integrated multipole field looks like a sextupole field.



- The photograph shows a removable insert with a machined chamfer installed on the SPEAR3 prototype *gradient* magnet.
- The shape of the chamfer depth was determined empirically and was approximately parabolic. It was designed to reduce the integrated sextupole field.
- The chamfer shape was machined onto the end packs of subsequently manufactured production magnets.

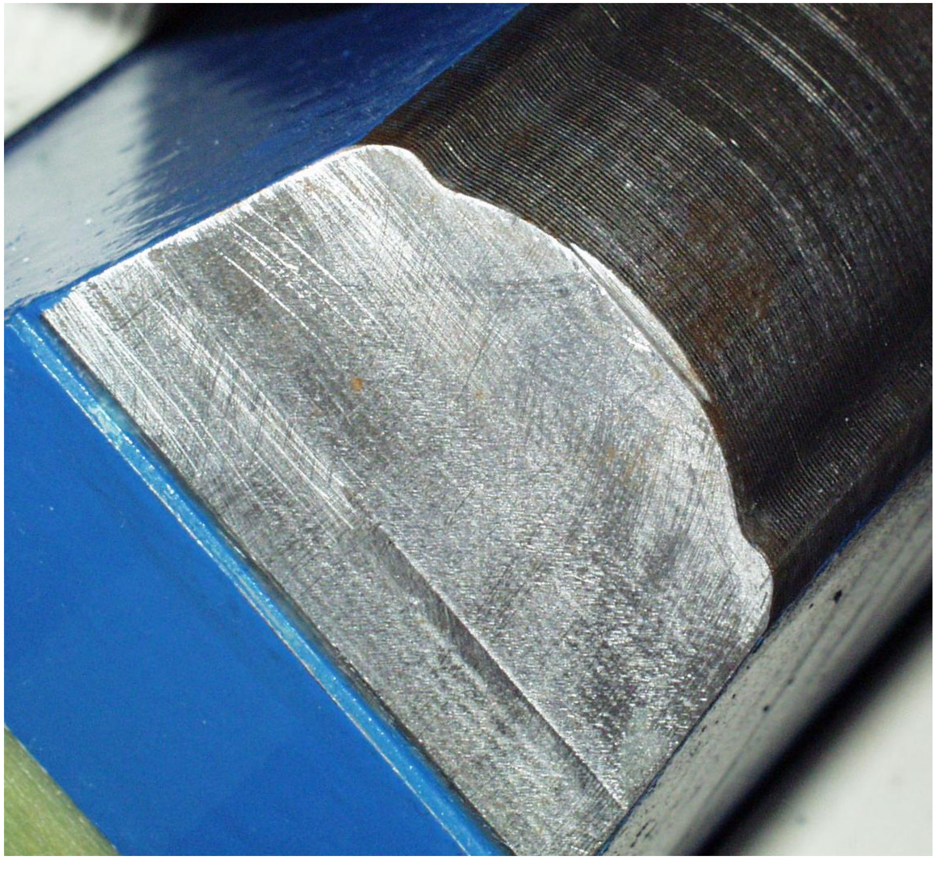
Quadrupole 3-Dimensional Fringe Field

- The quadrupole "gap" is largest at its center. The gap decreases as the distance from its center (g1>g2>g3>g4>g5>g6). Since the fringe field is roughly proportional to the magnet gap, it is longest near the magnet pole center.



Quadrupole Chamfer

- The quadrupole pole chamfer is a straight angled cut, which shortens the pole at its center. The angle is cut such that the pole is longer near its edge.
- Again, this cut was determined empirically by trial an error, minimizing the n=6 integrated multipole error.



Class Closure

- Although electromagnets with iron yokes is a small subset in the field, which includes superconducting magnets, magnets using permanent magnet material, fast pulsed magnets, the area covered in these lectures have ranged over a large spectrum of subjects.
- Fundamental mathematics derived from Maxwell’s equations have been reviewed.
  - Properties of magnetic fields are derived from the understanding of these mathematics.
  - The electronics required to characterize magnetic measurements rely on these mathematics.
  - Theories which identify and quantifies the effects on magnetic fields of mechanical and electrical errors which are inevitable in the manufacture of magnets are discussed.
- Fabrication and assembly processes and principles which, if followed, assure good magnet performance in the synchrotron environment are reviewed.
- Principles which allow the computation of magnetic forces and electrical properties of magnets are reviewed.
- Previous versions of this class were provided by hardcopies of the lecture notes.
  - Although these lecture notes provide a good (if cryptic) reference for future work of the student in this broad field, the opportunity to write a book compiling and summarizing the past classes has been a huge, but rewarding effort.
  - The book format allowed the author to supplement the essential details of the principles of magnet design and fabrication with words which provide the color and texture and fills out the details left out in lecture notes.

- It is hoped that the course, along with book supplied with this course, will provide the student with the tools to perform a complete analysis and design of magnets which will be used in the future for the next generation of particle accelerators.