# Lecture 9: Smith Chart/ S-Parameters

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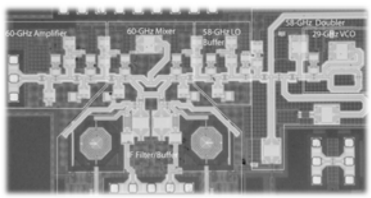

**EE142 – Fall 2010**

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**University of California, Berkeley**

## Announcements

- HW3 was due at 3:40pm today
  - You have up to tomorrow 3:30pm for 30% penalty. After that the solutions are posted and there will be no credit.
- Monday's Discussion section: figuring out the time for review OHs. Starts next week.
- HW4 due next Thursday, posted today

## Outline

- Last Lecture: Achieving Power Gain
  - Power gain metrics
  - Optimizing power gain
  - Matching networks
- This Lecture: Smith Chart and S-Parameters
  - Quick notes about matching networks
  - Smith Chart basics
  - Scattering Parameters

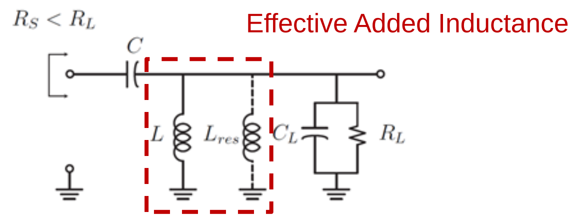


## Matching Network Design

1. Calculate the boosting factor $m = \frac{R_{hi}}{R_{lo}}$
2. Compute the required circuit Q by $(1 + Q^2) = m$, or $Q = \sqrt{m-1}$.
3. Pick the required reactance from the Q. If you're boosting the resistance, e.g. $R_S > R_L$, then $X_S = Q \cdot R_L$. If you're dropping the resistance, $X_p = R_L / Q$
4. Compute the effective resonating reactance. If $R_S > R_L$, calculate $X'_s = X_s(1 + Q^{-2})$ and set the shunt reactance in order to resonate, $X_p = -X'_s$. If $R_S < R_L$, then calculate $X'_p = X_p/(1+Q^{-2})$ and set the series reactance in order to resonate, $X_s = -X'_p$.
5. For a given frequency of operation, pick the value of L and C to satisfy these equations.

## Complex Source/ Load

- First “absorb” the extra reactance/ susceptance
- We can then move forward according to previous guidelines

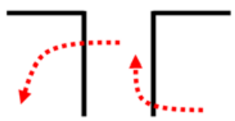



- There might be multiple ways of achieving matching, each will have different properties in terms of BW (Q), DC connection for biasing, High-pass vs Low-Pass,…



## Multi-Stage Matching Networks

1. Cascaded L-Match
   - Wide bandwidth
   - Only in one direction



- T-Match
  - First transform high then low
  - BW is lower than single L-Match

- Pi-Match
  - First low then high
  - BW is lower than single L-match

# TRANSMISSION LINES

## A QUICK OVERVIEW



## Transmission Lines

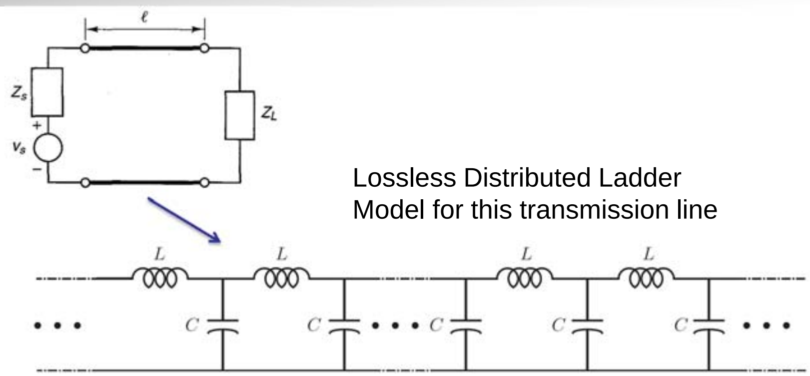
- We are departing from our understandings of lumped element circuits
- Circuit theory concepts (KVL and KCL) do not *directly* apply, we need to take into account the distributed nature of the elements
  - Shorted quarter-wave line
  - KCL on a transmission line
- Main issue is with the delay in the circuit, signals cannot travel faster than speed of light. Once circuits become larger this will become a significant effect.
- We will use our circuit techniques to understand the behavior of a transmission line
  - Remember HW 1

## T-Lines

- Transmission lines are NOT the main focus of this lecture (or course) and are extensively covered in EE117. We will have a brief introduction to help us understand some of the other concepts (Smith Chart and S-Parameters).
  - Please refer to Ch.9 of Prof. Niknejad's book (or Pozar, Gonzalez, Collin, etc)



## Infinite Ladder Network




Lossless Distributed Ladder Model for this transmission line

- From HW1, infinite ladder network with $Z_{series}=jwL$ and $Y_{shunt}=jwC$ leads to:

$$Z_{in} = \frac{j\omega L}{2} \pm \sqrt{-\frac{(\omega L)^2}{4} + \frac{L}{C}}$$

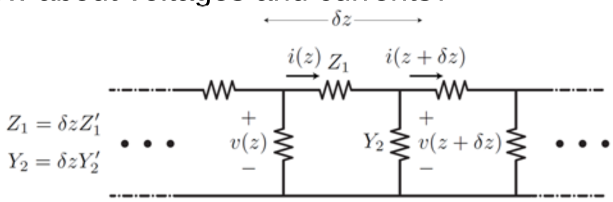
- For a "distributed" model in which the L and C segments are infinitesimal in size:

$$Z_{in} \approx \sqrt{\frac{L}{C}}$$

This is *resistive* value (real) !

## Solving for Voltages and Currents

- We now know the input impedance of the infinite line in terms of the L and C parameters (per unit length values).
- We also know that if we terminate the line with $Z_0$ we will still see the same impedance even if the line is finite.
- How about voltages and currents?




$$v(z+\delta z) = v(z) - i(z)Z_1'\delta z$$

$$i(z) = i(z+\delta z) + \delta z Y_2' v(z+\delta z)$$

$\Rightarrow$

$$\frac{dv}{dz} = -Z_1' i(z)$$

$$\frac{di}{dz} = -Y_2' v(z)$$

$\Rightarrow$

$$\frac{d^2v}{dz^2} = \gamma^2 v(z)$$

$$\frac{d^2i}{dz^2} = \gamma^2 i(z)$$

$$\gamma^2 = Z'_1 Y'_2$$



## Deriving Voltages and Currents

- Interested in steady state solutions we solve the DEs:

$$v(z) = V^+ e^{-\gamma z} + V^- e^{\gamma z}$$

$$i(z) = I^+ e^{-\gamma z} + I^- e^{\gamma z}$$

Take the derivative and using

$$\frac{dv}{dz} = -Z_1' i(z)$$

$$\frac{di}{dz} = -Y_2' v(z)$$

$\Rightarrow$

$$-V^+ \gamma e^{-\gamma z} + V^- \gamma e^{\gamma z} = -Z_1' \left(I^+ e^{-\gamma z} + I^- e^{\gamma z}\right)$$

$$-I^+ \gamma e^{-\gamma z} + I^- \gamma e^{\gamma z} = -Y_2' \left(V^+ e^{-\gamma z} + V^- e^{\gamma z}\right)$$

z=0 yields:

$$-V^+\gamma + V^-\gamma = -Z_1'(I^+ + I^-)$$

$$-I^+\gamma + I^-\gamma = -Y_2'(V^+ + V^-)$$

$$\gamma^2 = Z'_1 Y'_2$$

$$Z_0 = \sqrt{Z'_1 / Y'_2}$$

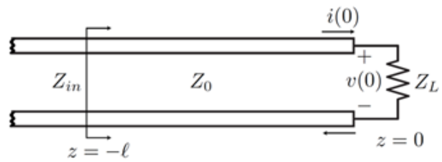
$\Rightarrow$

$$v(z) = V^+ e^{-\gamma z} + V^- e^{\gamma z}$$

$$i(z) = \frac{V^+}{Z_0} e^{-\gamma z} - \frac{V^-}{Z_0} e^{\gamma z}$$

Lossless T-line $Z_0 = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}}$ $\quad \gamma = j\sqrt{LC}\omega$

## Terminated Transmission Line

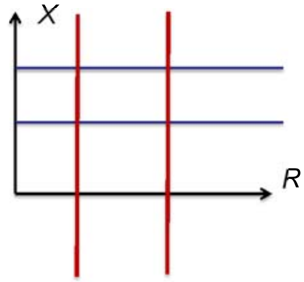





# SMITH CHART

## Don't we have all we need?

- Smith Chart provides a visual tool for designing and analyzing **amplifiers, matching networks and transmission lines**. It is a convenient way of presenting **parameter variations with frequency**.
- You'll also see this is particularly useful for amplifier design in *potentially unstable* region (K<1)
- Start by trying to "plot" impedance values:




But we want to present a very large range of impedances (open to short). This form may not be very useful!



## Bilinear Transform

- We have seen this issue before (Laplace transform to Z-transform). A bilinear transform provided frequency warping there, can we use the same method here?
- Smith Chart plots the "reflection coefficient (Γ)" which is related to the impedance by:

$$\Gamma = |\Gamma| < \theta = \frac{Z - Z_0}{Z + Z_0} = U + jV$$

- Here $Z_0$ is the characteristic impedance of *the* transmission line or just some reference impedance for the Smith Chart.
- The normalized impedance is often used:

$$z = \frac{Z}{Z_o} = \frac{R + jX}{Z_o} = r + jx \qquad \Gamma = \frac{z - 1}{z + 1}$$

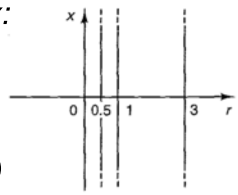
## A closer look at Smith Chart

$$U + jV = \frac{r - 1 + jx}{r + 1 + jx} \Rightarrow U = \frac{r^2 - 1 + x^2}{(r+1)^2 + x^2} \quad V = \frac{2x}{(r+1)^2 + x^2}$$

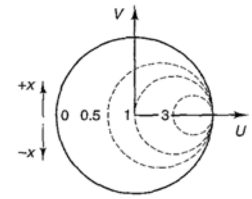
- Now if we eliminate *x:*

$$\left(U - \frac{r}{r+1}\right)^2 + V^2 = \left(\frac{1}{r+1}\right)^2$$

Circles with center at (r/r+1,0)
with radius 1/r+1



Constant resistance lines in the $z = r + jx$ plane

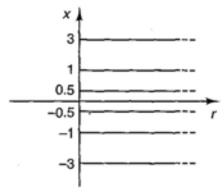


Γ plane

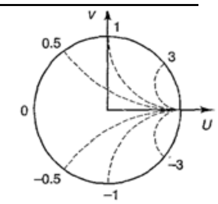
- Eliminating *r* :

$$(U-1)^2 + (V - \frac{1}{x})^2 = \left(\frac{1}{x}\right)^2$$

Circles with center at (1,1/x)
with radius 1/x



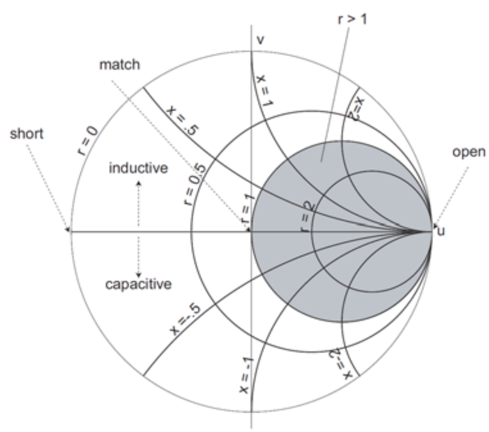
Constant reactance lines (for $r \geq 0$) in the $z = r + jx$ plane



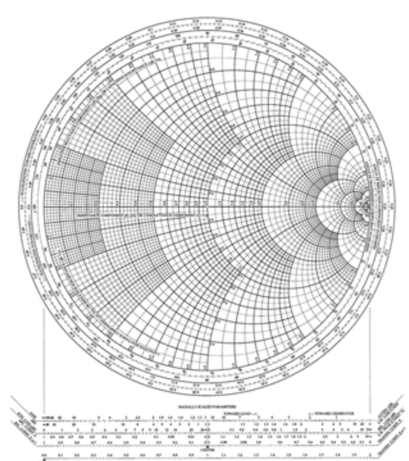
Γ plane



## Smith Chart




Refer to Niknejad, Ch.9



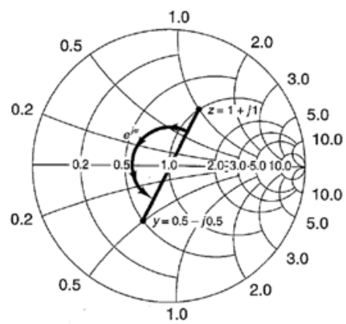
Phillip H. Smith, Murray Hill, NJ, 1977

## The Admittance Chart

$$Y = \frac{1}{Z} \;\; or \;\; y = \frac{Y}{Y_0} = \frac{1}{z} \;\; where \;\; Y_0 = \frac{1}{Z_0}$$

$$\Gamma = |\Gamma| < \theta = \frac{z-1}{z+1} = -\frac{y-1}{y+1} = -\Gamma' = \Gamma' < 180$$

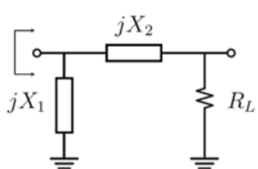
- So to go from impedance point to an admittance point you just need to mirror the point around the center (or 180 degrees rotate)

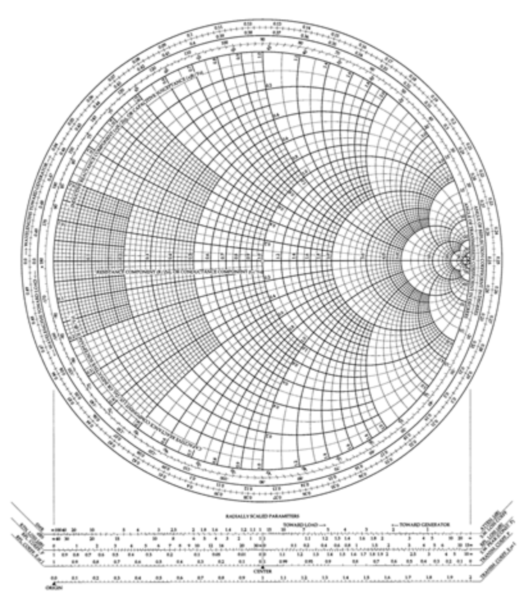



Gonzalez, Prentice Hall, 1984

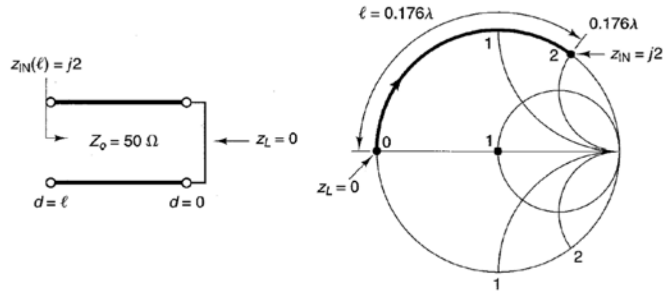


## Compound Impedances on a Smith Chart



## Transmission Lines




- Start from load, rotate clock-wise towards “generator”

EE142-Fall 2010 Gonzalez, Prentice Hall, 1984 21

# SCATTERING PARAMETERS

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## Scattering Parameters

- Y, Z, H, G, ABCD parameters difficult to measure at HF
  - Very difficult to obtain broadband short or open at high frequencies
    - Remember parasitic elements and resonances
  - Difficult to measure voltages and currents at high frequency due to the impedance of equipment
  - Some microwave devices will be unstable under short/open loads
- Therefore, we use scattering parameters to define input and output characteristics. The actual voltages and currents are separated into scattered components (definitions will be given)



## Definitions for a One-Port

## Two-Port S-Parameters