POWER ELECTRONICS

Lecture Note

5

AC-AC Converter: AC Voltage Controller

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Contents of this Lecture:

► Basic AC Voltage Controller circuit
► AC Phase Control Chopper
► AC Voltage controller characteristics

AC Choppers

A power electronic ac–ac converter, in generic form, accepts electric power from one system and converts it for delivery to another ac system with waveforms of different amplitude, frequency, and phase. They may be single-phase or three-phase types depending on their power ratings.

AC-AC converter can be categorized into Three topologies (see Fig.1):
➢ AC-AC Voltage Controller Converter (AC Choppers)
➢ AC Cycloconverter
➢ Matrix Converter
❑ Indirect Matrix Converter (AC-DC-AC Converter)
❑ Direct Matrix Converter

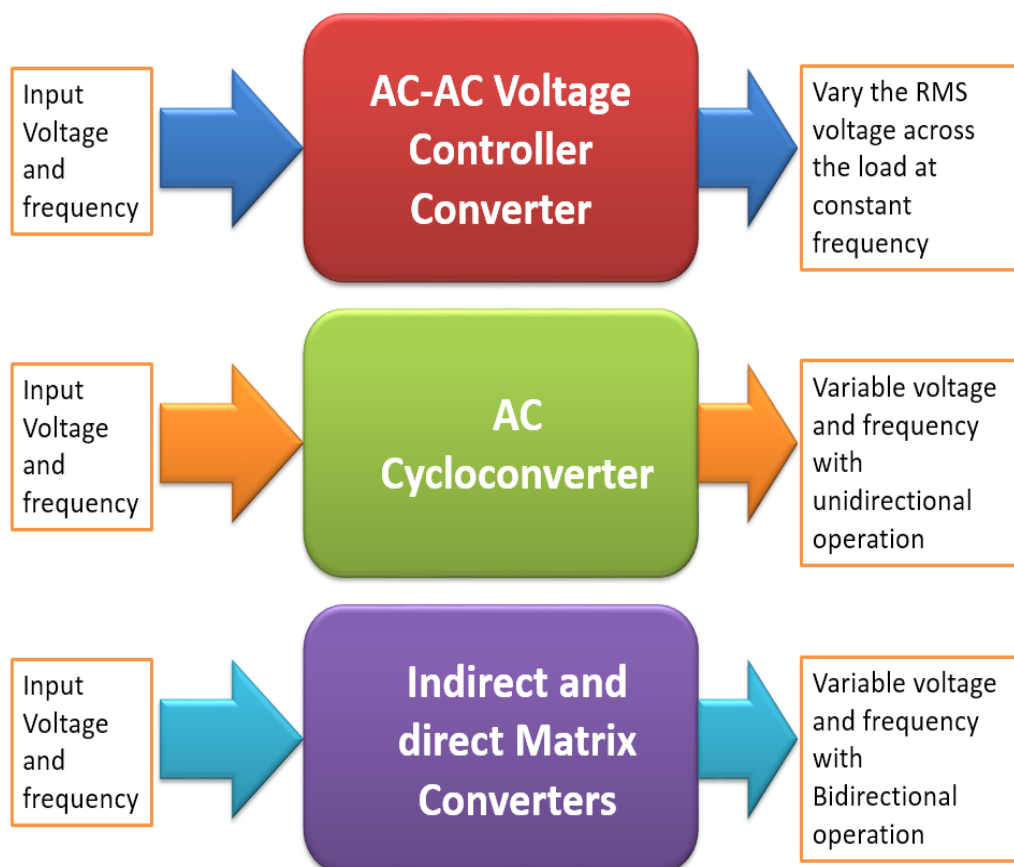| Input Voltage and frequency | → | **AC-AC Voltage Controller Converter** | → | Vary the RMS voltage across the load at constant frequency |
| Input Voltage and frequency | → | **AC Cycloconverter** | → | Variable voltage and frequency with unidirectional operation |
| Input Voltage and frequency | → | **Indirect and direct Matrix Converters** | → | Variable voltage and frequency with Bidirectional operation |

Fig.1

The circuit diagrams for the above converters can be depicted below. However, many traditional and advance AC-AC converters are developed to improve their performance based on their applications.
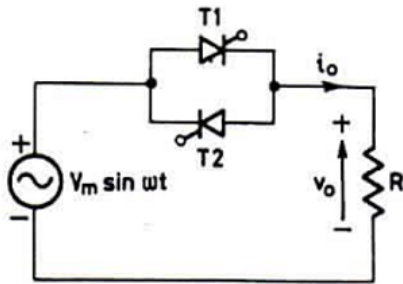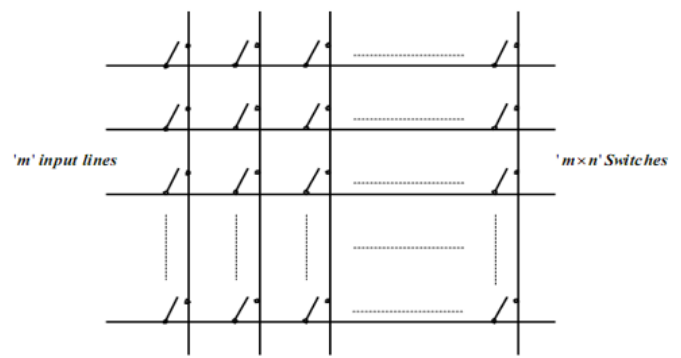

AC-AC Voltage Controller Converter
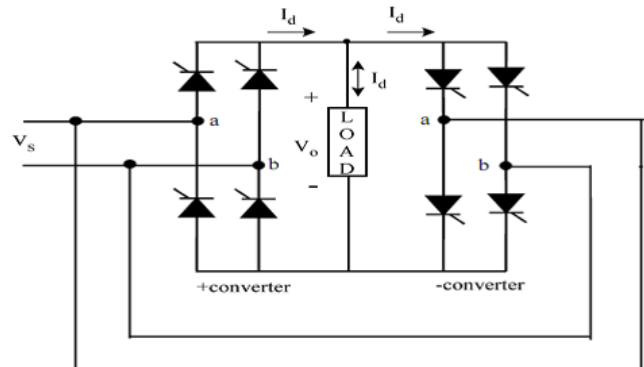
Cycloconverter
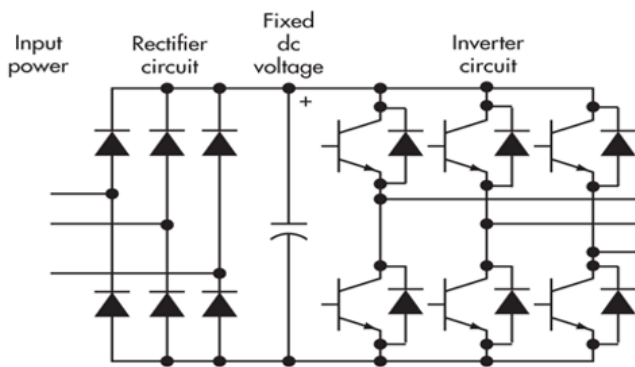
Indirect Matrix Converter

direct Matrix Converter

In this PE course the first, second, and third converter systems will be studied in details. The first topology, which is AC voltage controller or called AC choppers will be presented in this lecture.

AC Choppers

AC voltage controllers (ac line voltage controllers) are employed to vary the RMS value of the alternating voltage applied to a load circuit by introducing Thyristors between the load and a constant voltage ac source. The RMS value of alternating voltage applied to a load circuit is controlled by controlling the triggering angle of the Thyristors/Triac in the ac voltage controller circuits.

There are two different types of thyristor control used in practice to control the ac power flow:

☐ On-Off control
☐ Phase control

Applications of Ac Choppers

- Lighting / Illumination control in ac power circuits.
- Induction heating.
- Transformer tap changing or On load tap changers
- Speed control of induction motors

Basic Circuit can be used as AC Chopper

Triac Light Dimmer

A light dimmer regulates power flow to a resistive load, such as an incandescent light bulb, in an efficient way by allowing only a portion of the 50/60 Hz current to pass through. The firing angle is controlled by a potentiometer, an RC circuit, and a Diac.
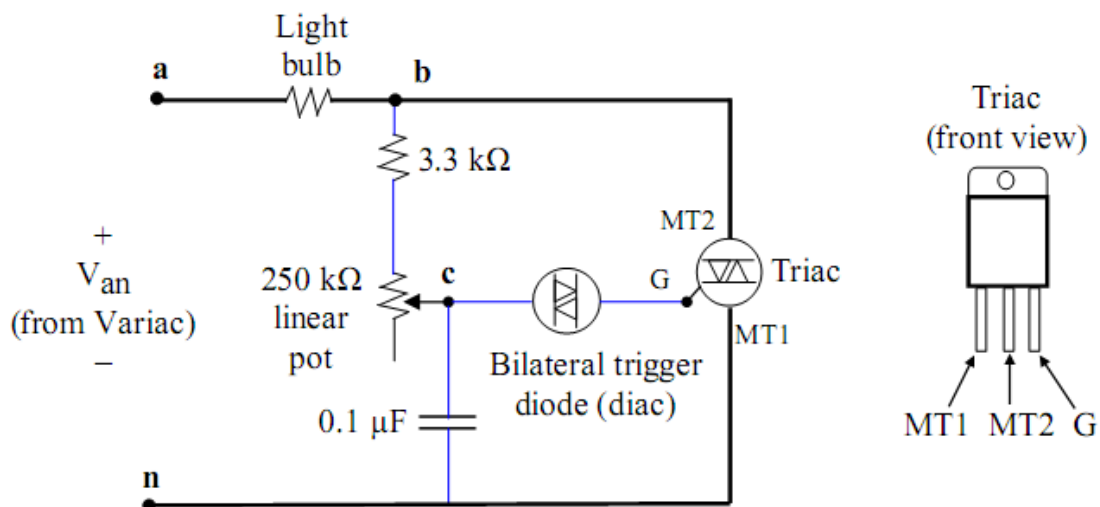

During each half-cycle, when the voltage across the capacitor (either positive or negative) exceeds the breakover voltage of the Diac and "fires" the Triac, current then flows through the load.
The RC time constant of the series 3.3 kΩ + 250 kΩ linear potentiometer and 0.1 μF capacitor determines the phase delay and magnitude of the sinusoidal capacitor voltage with respect to the source voltage.
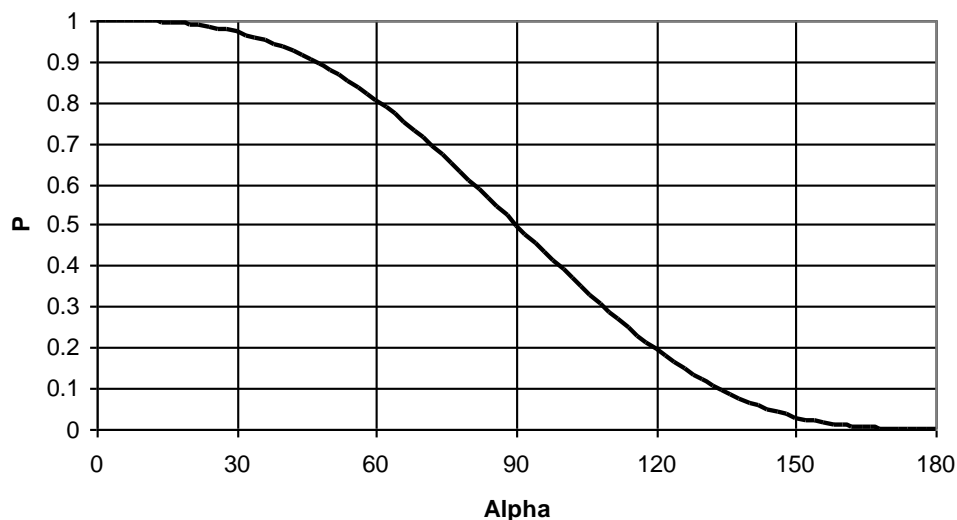Figure below shows the Vc for different pot resistance value:
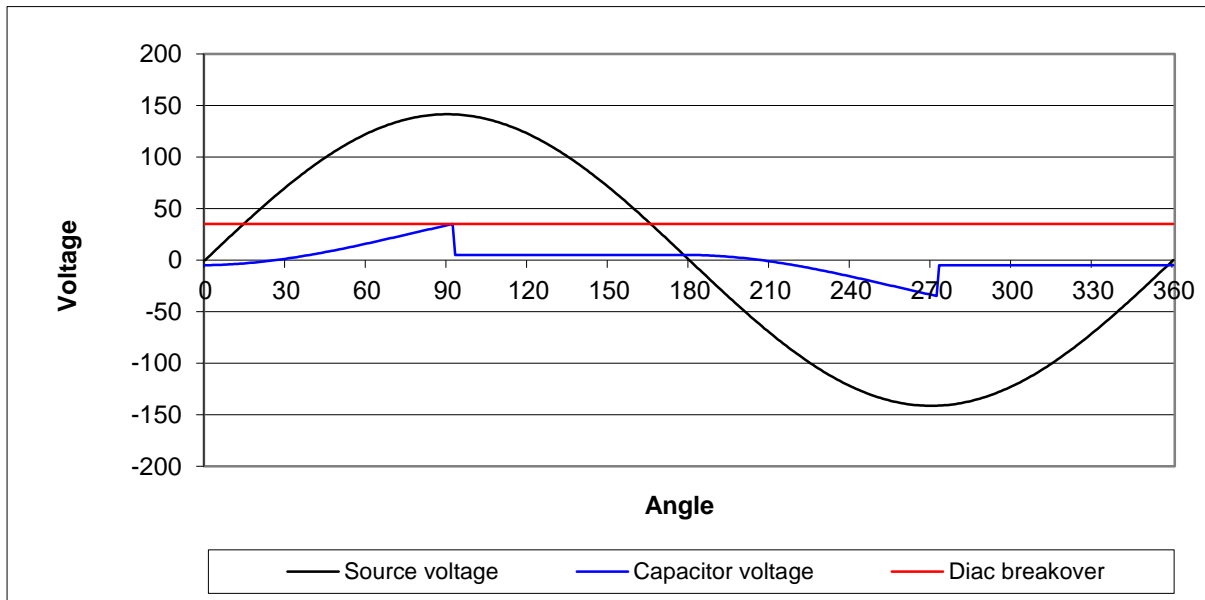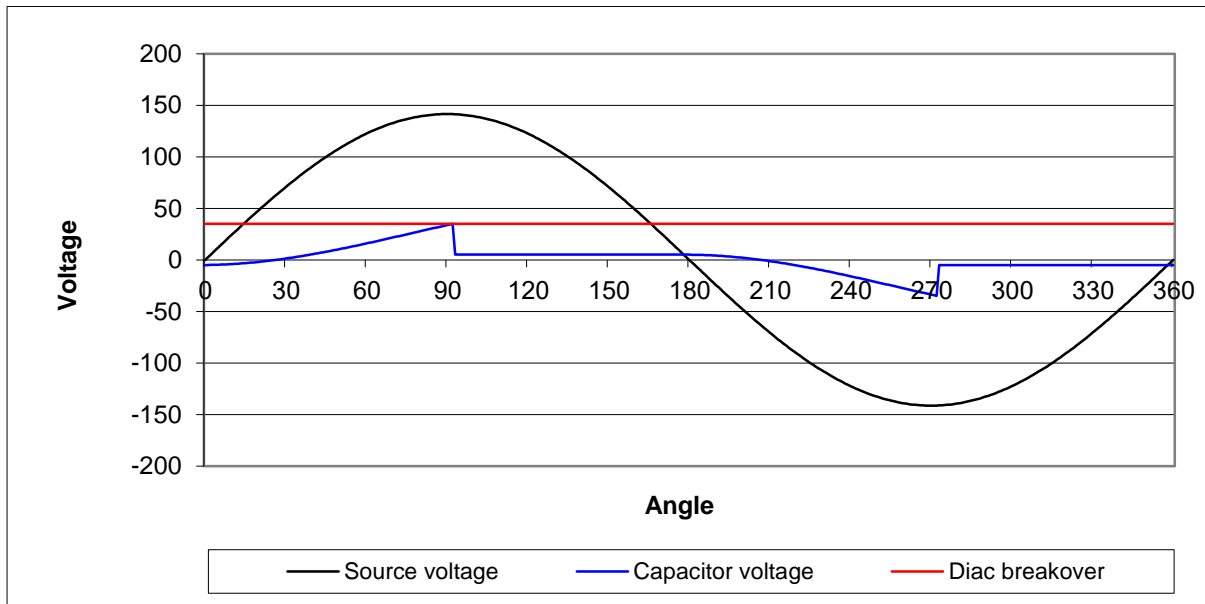
Voltage/current waveforms for a resistive load are shown in Figure below for firing angles α = 30º, 90º, and 150º.
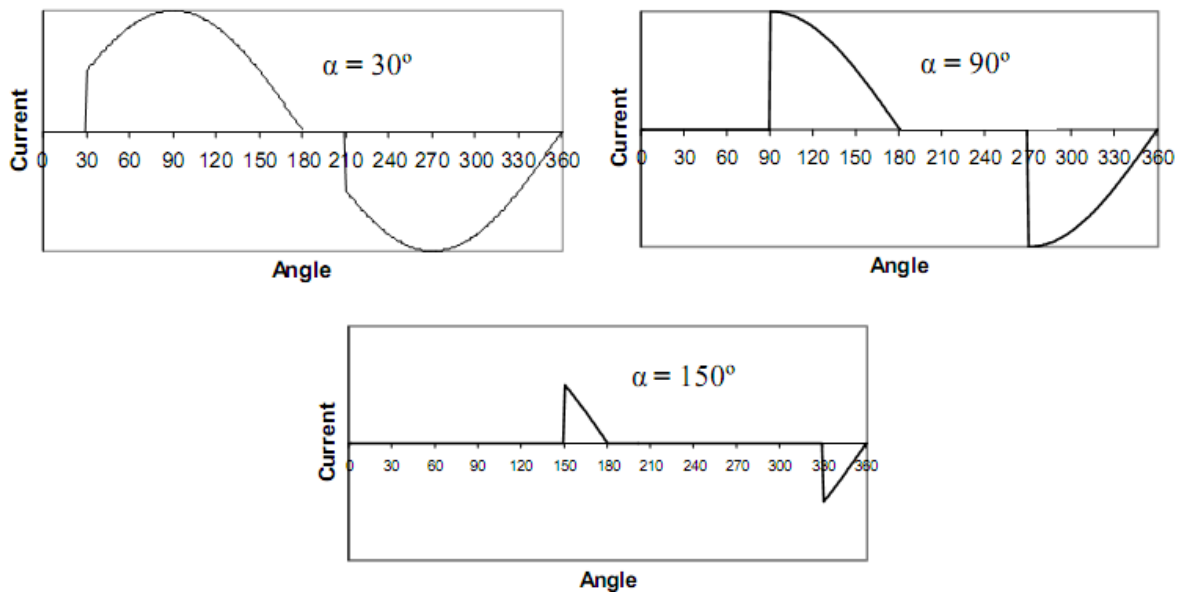
The RMS output voltage can found using the following equation:

$$V_{ab,rms}^2 = V_{an,rms}^2 \left[1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi} \right]$$

The power delivered to resistive load versus firing angle plotted below:

On-off AC voltage regulator

In On-Off control technique Thyristors are used as switches to connect the load circuit to the ac supply (source) for a few cycles of the input ac supply and then to disconnect it for few input cycles. The Thyristors thus act as a high speed contactor (or high speed ac switch).

Waveforms pertaining to integral cycle control.
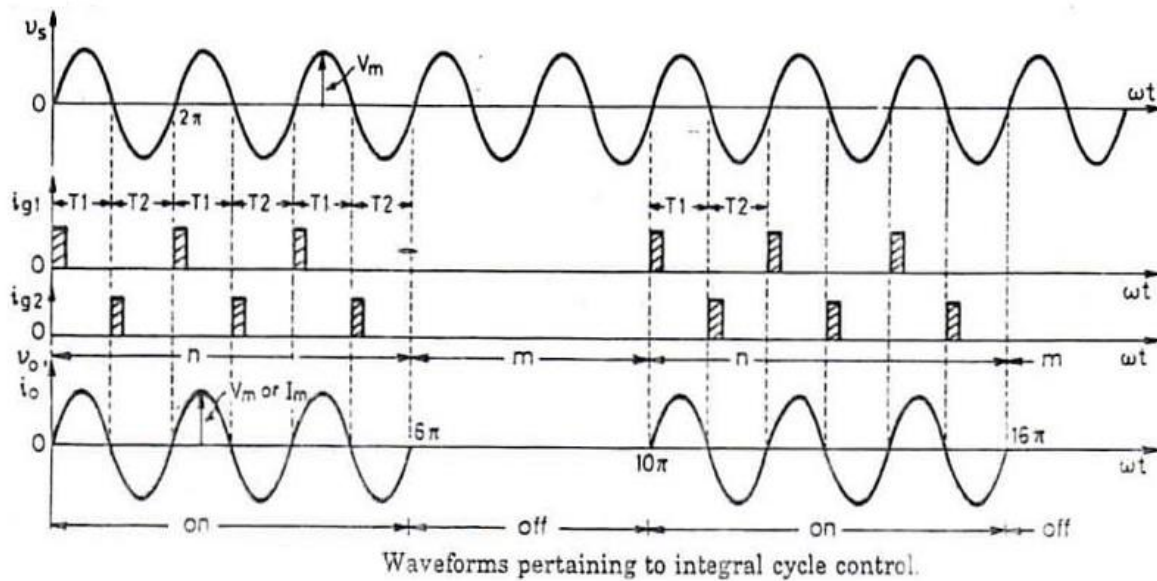
By varying the number of n and m cycles, the power delivered can be regulated

For n on-cycles and m off-cycles, the periodicity $= (n+m) \, 2\pi$ radians,

$$\therefore \quad V_{or} = \left[\frac{n}{2\pi \, (n+m)} \int_0^{2\pi} V_m^2 \sin^2 \omega t \cdot d \, (\omega t) \right]^{1/2}$$

$$= \left[\frac{n \, V_m^2}{4\pi \, (n+m)} \int_0^{2\pi} (1 - \cos 2\omega t) \, d \, (\omega t) \right]^{1/2}$$

$$V_{or} = \frac{V_m}{\sqrt{2}} \cdot \sqrt{\frac{n}{n+m}} = V_s \sqrt{\frac{n}{n+m}} = V_s \sqrt{k}$$

Rms load current, $\qquad I_{or} = \dfrac{V_{or}}{R}$

Power delivered to load $\quad = \dfrac{V_{or}^2}{R} = \dfrac{V_s^2}{R} \left(\dfrac{n}{n+m} \right) = \dfrac{k \cdot V_s^2}{R}$

Rms value of input current, I_s = rms value of load current, I_{or}

Input VA = V_s (rms value of source current)

$$= V_s \cdot I_s = V_s \cdot I_{or} = V_s \cdot \frac{V_{or}}{R}$$

Input VA × pf = power delivered to load

$$\therefore \text{Input pf} \quad = \frac{V_{or}^2}{R} \cdot \frac{R}{V_s \cdot V_{or}} = \frac{V_{or}}{V_s} = \sqrt{\frac{n}{n+m}} = \sqrt{k}$$

As each thyristor conducts for π radians during each cycle of n on-cycles, the average value of thyristor current is given by

$$= \frac{n}{2\pi\,(n+m)} \int_0^\pi I_m \cdot \sin \omega t \cdot d\,(\omega t)$$

$$= \frac{I_m}{\pi} \cdot \frac{n}{m+n} = \frac{k \cdot I_m}{\pi}$$

Similarly, rms value of thyristor current is

$$I_{TR} = \left[\frac{n}{2\pi\,(n+m)} \int_0^\pi I_m^2 \sin^2 \omega t \cdot d\,(\omega t) \right]^{1/2}$$

$$= \frac{I_m}{2} \sqrt{\frac{n}{n+m}} = \frac{I_m \sqrt{k}}{2}$$

Example

A single-phase voltage controller has input voltage of 230 V, 50 Hz and a load of R = 15 Ω. For 6 cycles on and 4 cycles off, determine (a) rms output voltage, (b) input pf and (c) average and rms thyristor currents.

Solution. (a) rms value of output voltage is

$$V_{or} = V_s \sqrt{\frac{n}{n+m}} = 230 \sqrt{\frac{6}{6+4}} = 178.157 \text{ V}$$

(b) input pf $= \sqrt{k} = \sqrt{\frac{n}{n+m}} = \sqrt{\frac{6}{6+4}} = 0.7746 \text{ lag.}$

Also power delivered to load $= I_{or}^2\,R = \dfrac{V_{or}^2}{R} = \dfrac{178.157^2}{15} = 2116 \text{ W}$

Input VA $= 230 \times \dfrac{230\sqrt{6}}{15} = 2731.74 \text{ VA}$

∴ Input pf $= \dfrac{2116}{2731.74} = 0.7746 \text{ lag}$

(c) Peak thyristor current, $I_m = \dfrac{230\sqrt{2}}{15} = 21.681 \text{ A}$

average value of thyristor current,

$$I_{TA} = \frac{k\,I_m}{\pi} = \frac{0.6 \times 21.681}{\pi} = 4.1407 \text{ A}$$

rms value of thyristor current,

$$I_{TR} = \frac{I_m \cdot \sqrt{k}}{2} = \frac{21.681 \times \sqrt{0.6}}{2} = 8.397 \text{ A}$$

AC Phase Control Choppers

In phase control the Thyristors are used as switches to connect the load circuit to the input ac supply, for a part of every input cycle. That is the ac supply voltage is chopped using Thyristors/Triac during a part of each input cycle. The switch is turned on for a part of every half cycle, so that input supply voltage appears across the load and then turned off during the remaining part of input half cycle to disconnect the ac supply from the load.

By controlling the phase angle or the trigger angle 'α' (delay angle), the output RMS voltage across the load can be controlled.

The trigger delay angle 'α' is defined as the phase angle (the value of ωt) at which the thyristor turns on and the load current begins to flow.
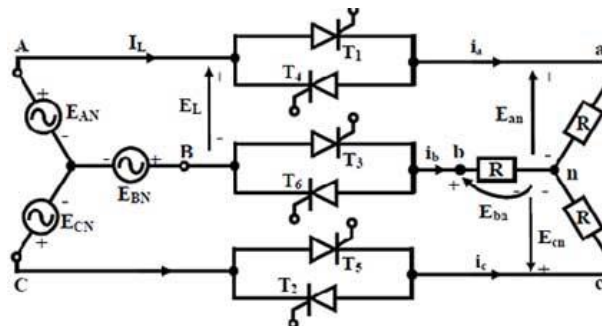
Type of Ac Voltage Controllers

The ac voltage controllers are classified into two types based on the type of input ac supply applied to the circuit.

☐ Single Phase AC Controllers.
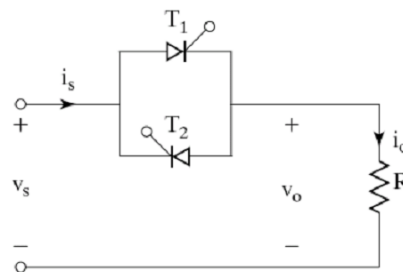
☐ Three Phase AC Controllers.

Each type of controller may be sub divided into

☐ Uni-directional or half wave ac controller.

☐ Bi-directional or full wave ac controller.

Principle of AC Phase Control

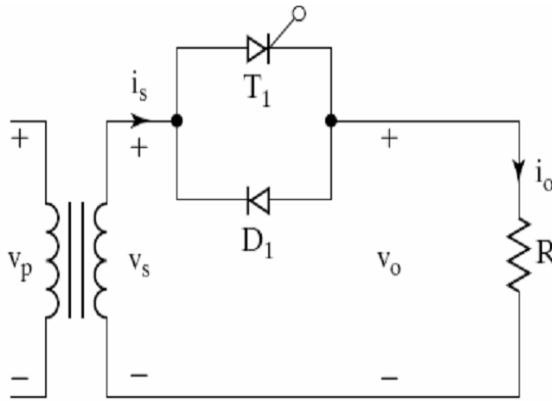## Half wave AC phase controller (Unidirectional Controller)

Fig.: Halfwave AC phase controller (Unidirectional Controller)

$v_o = v_L = 0$; for $\omega t = 0$ to α

$v_o = v_L = V_m \sin \omega t$; for $\omega t = \alpha$ to 2π.

$i_o = i_L = \dfrac{v_o}{R_L} = \dfrac{V_m \sin \omega t}{R_L}$; for $\omega t = \alpha$ to 2π.

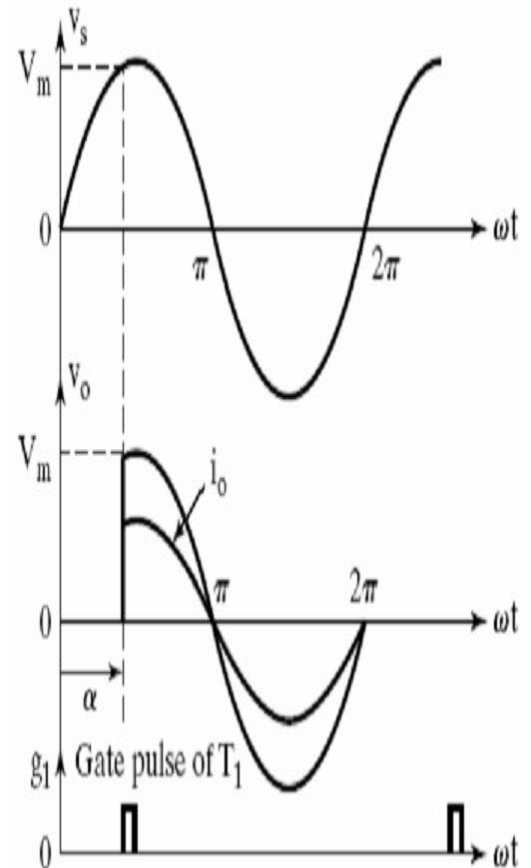$i_o = i_L = 0$; for $\omega t = 0$ to α.

RMS Output Voltage

$$V_{O(RMS)} = \sqrt{\frac{1}{2\pi}\left[\int_{\alpha}^{2\pi} V_m^2 \sin^2 \omega t \cdot d(\omega t)\right]} \qquad V_{O(RMS)} = \sqrt{\frac{V_m^2}{2\pi}\left[\int_{\alpha}^{2\pi}\left(\frac{1-\cos 2\omega t}{2}\right).d(\omega t)\right]}$$

$$\boxed{V_{O(RMS)} = V_S \sqrt{\frac{1}{2\pi}\left[(2\pi - \alpha)+\frac{\sin 2\alpha}{2}\right]}}$$

Where, $V_{i(RMS)} = V_S = \dfrac{V_m}{\sqrt{2}}$ = RMS value of input supply voltage (across the transformer secondary winding).

A typical control characteristic of single phase half-wave phase controlled ac voltage controller is as shown below We can observe from the control characteristics and that he range of RMS output voltage control is from 100% of V_S to 70.7% of V_S when we vary the α from zero to 180 degrees. **Thus the half wave ac controller has the drawback of limited range RMS output voltage control.**
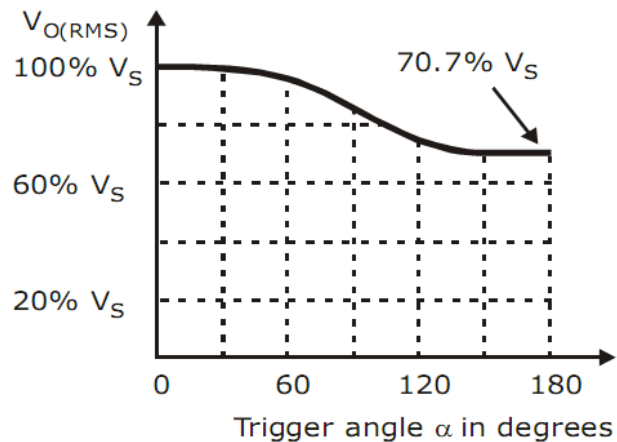
Example 1: Derive an expression for DC voltage in a single-phase AC voltage controller.

Solution:

Based on the output voltage waveforms as shown aside the V_{odc} can be found as derived below
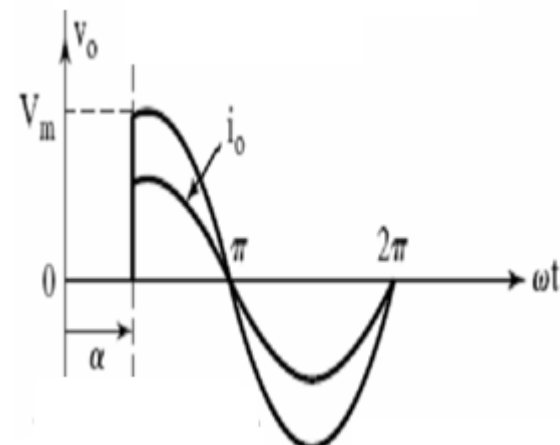

$$V_{O(dc)} = \frac{1}{2\pi} \int_{\alpha}^{2\pi} V_m \sin \omega t . d(\omega t)$$

$$V_{O(dc)} = \frac{V_m}{2\pi} \left[-\cos \omega t \Big/ \begin{smallmatrix} 2\pi \\ \\ \alpha \end{smallmatrix} \right]$$

$$V_{dc} = \frac{V_m}{2\pi} [\cos \alpha - 1] \quad ; \quad V_m = \sqrt{2} V_S$$

Hence $V_{dc} = \frac{\sqrt{2} V_S}{2\pi} (\cos \alpha - 1)$

When 'α' is varied from 0 to π. V_{dc} varies from 0 to $\frac{-V_m}{\pi}$

Disadvantages of single phase half wave ac voltage controller

- The output load voltage has a DC component which can result in the problem of core saturation of the input supply transformer.
- ac power flow to the load can be controlled only in one half cycle.
- Half wave ac voltage controller gives limited range of RMS output voltage control.

Single Phase Full Wave Ac Voltage Controller
With R-Load
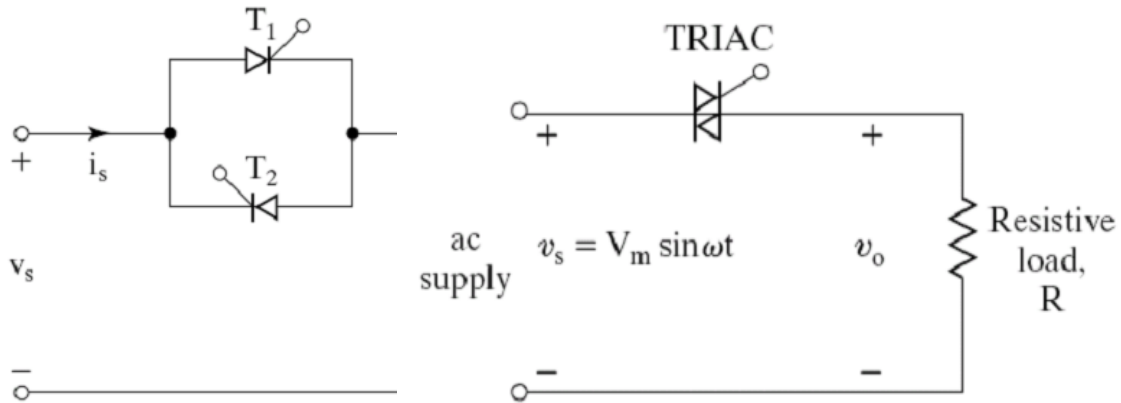
Back-back SCR or instead Triac can be used for bidirectional full-wave AC chopper.

The main waveforms are shown below:

for $\omega t = \alpha$ to π and $\omega t = (\pi + \alpha)$ to 2π $v_O = v_L = V_m \sin \omega t$;

$$i_O = \frac{v_O}{R_L} = \frac{V_m \sin \omega t}{R_L} = I_m \sin \omega t$$
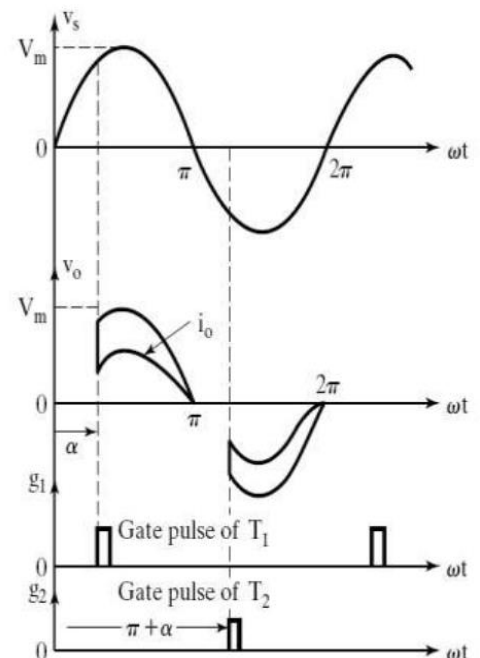
RMS Output Voltage

$$V_{L(RMS)}^2 = \frac{1}{2\pi} \left[\int_{\alpha}^{\pi} (V_m \sin \omega t)^2 d(\omega t) + \int_{\pi+\alpha}^{2\pi} (V_m \sin \omega t)^2 d(\omega t) \right]$$

After simplification:

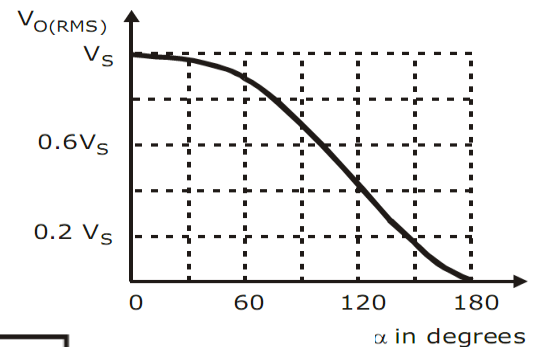$$\boxed{V_{L(RMS)} = V_S \sqrt{\frac{1}{\pi} \left[(\pi - \alpha) + \frac{\sin 2\alpha}{2} \right]}}$$

The maximum voltage appears across the load at α=0⁰ lead to:

$$V_{L(RMS)}\Big|_{\alpha=0} = \frac{V_m}{\sqrt{2}} = V_{i(RMS)} = V_S$$

A typical control characteristic of single phase full-wave phase controlled ac voltage controller is as shown below

We can notice from the figure, that we get a full range output voltage control by using a single phase full wave ac voltage controller.

Need For Isolation

In the single phase full wave ac voltage controller circuit using two SCRs in parallel, the gating circuits (gate trigger pulse generating circuits) of Thyristors must be isolated. Figure shows a pulse transformer with two separate windings to provide isolation between the gating signals.
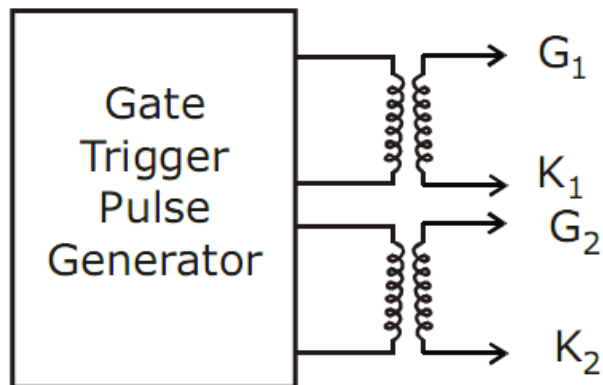
Performance Parameters

- **RMS Output Voltage** $V_{O(RMS)} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{\pi}\left[(\pi-\alpha) + \frac{\sin 2\alpha}{2}\right]}$; $\frac{V_m}{\sqrt{2}} = V_S$ = RMS input

  supply voltage.

- $I_{O(RMS)} = \frac{V_{O(RMS)}}{R_L}$ = RMS value of load current.

- $I_S = I_{O(RMS)}$ = RMS value of input supply current.
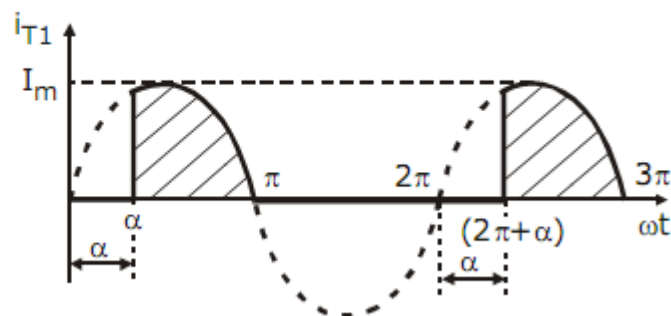- **Output load power**

$$P_O = I_{O(RMS)}^2 \times R_L$$

- **Input Power Factor**

$$PF = \frac{P_O}{V_S \times I_S} = \frac{I_{O(RMS)}^2 \times R_L}{V_S \times I_{O(RMS)}} = \frac{I_{O(RMS)} \times R_L}{V_S}$$

$$\boxed{PF = \frac{V_{O(RMS)}}{V_S} = \sqrt{\frac{1}{\pi}\left[(\pi - \alpha) + \frac{\sin 2\alpha}{2}\right]}}$$

- **Average Thyristor Current,**

$$I_{T(Avg)} = \frac{1}{2\pi}\int_{\alpha}^{\pi} i_T \, d(\omega t) = \frac{1}{2\pi}\int_{\alpha}^{\pi} I_m \sin \omega t . d(\omega t)$$

$$\boxed{I_{T(Avg)} = \frac{I_m}{2\pi}\left[-\cos \pi + \cos \alpha\right] = \frac{I_m}{2\pi}\left[1 + \cos \alpha\right]}$$

- **Maximum Average Thyristor Current, for $\alpha = 0$,**

$$I_{T(Avg)} = \frac{I_m}{\pi}$$

- **RMS Thyristor Current**

$$I_{T(RMS)} = \sqrt{\frac{1}{2\pi}\left[\int_{\alpha}^{\pi} I_m^2 \sin^2 \omega t . d(\omega t)\right]}$$

$$I_{T(RMS)} = \frac{I_m}{\sqrt{2}}\sqrt{\frac{1}{2\pi}\left[(\pi - \alpha) + \frac{\sin 2\alpha}{2}\right]}$$

- **Maximum RMS Thyristor Current, for $\alpha = 0$,**

$$I_{T(RMS)} = \frac{I_m}{2}$$

**Note:** In the case of a single phase full wave ac voltage controller circuit using a Triac with

resistive load, the average thyristor current =0. Because the Triac conducts in both the half cycles and the thyristor current is alternating and we obtain a symmetrical thyristor current waveform which gives an average value of zero on integration.

Single Phase Full Wave Ac Voltage Controller
With RL-Load

In practice most of the loads are of RL type. For example if we consider a single phase full wave ac voltage controller controlling the speed of a single phase ac induction motor, the load which is the induction motor winding is an RL type of load, where R represents the motor winding resistance and L represents the motor winding inductance.

Due to the inductance in the load, the load current i_o flowing through T_1 would not fall to zero at $\omega t = \pi$ when the input supply voltage starts to become negative. T_1 will continue to conduct the load current until all the inductive energy stored in the load inductor L is completely utilized and the load current through T_1 falls to zero at $\omega t = \beta$, β where is referred to as the **Extinction angle**
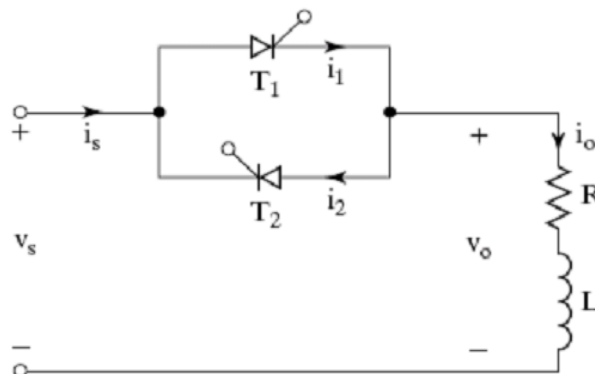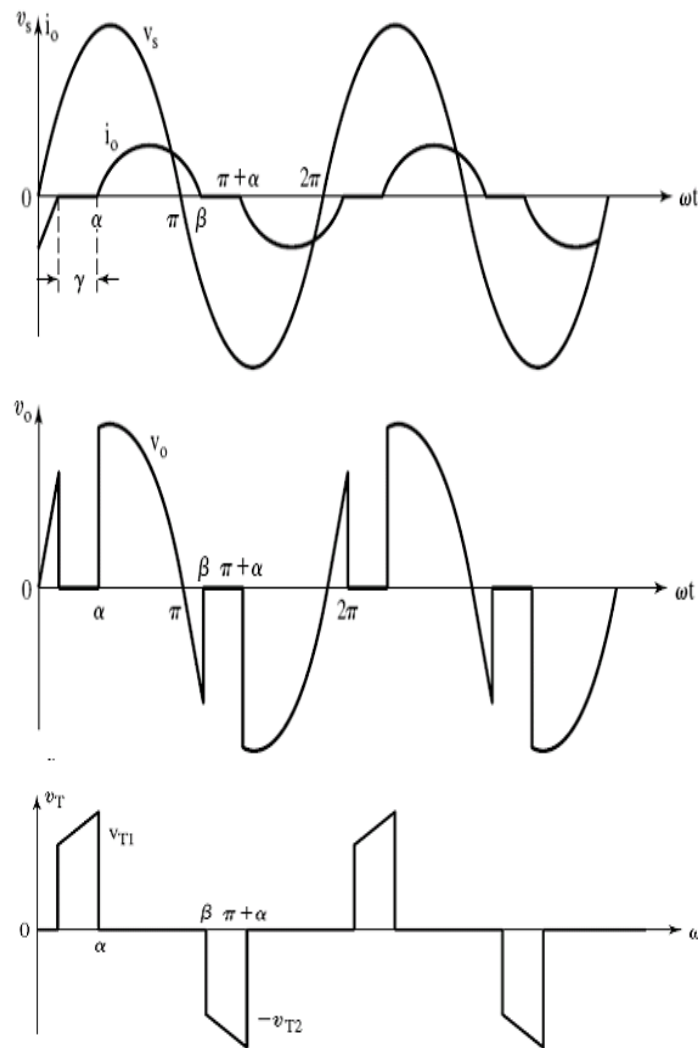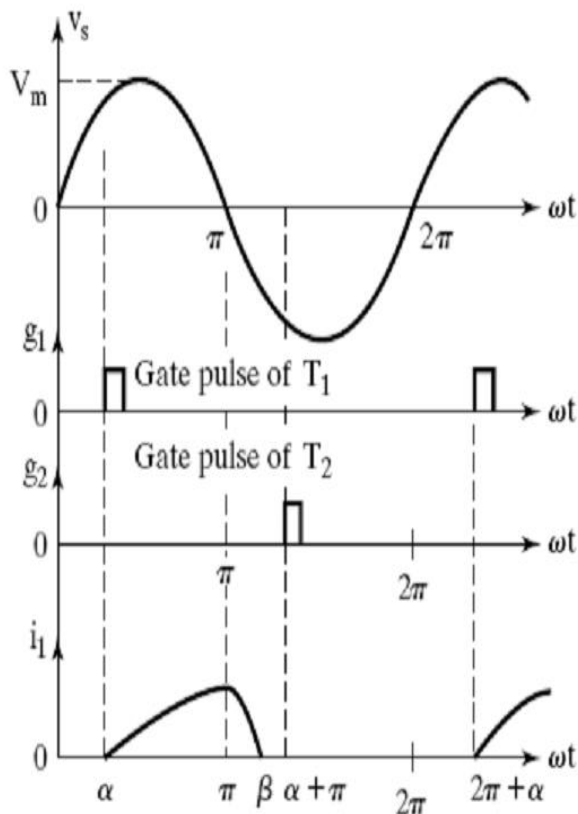
T_1 conducts from $\omega t = \alpha$ to $\omega t = \beta$, T_1 then conduction angle $\delta = (\beta - \alpha)$. δ depends on α and the load impedance angle ϕ.

Discontinuous load current operation occurs for $\alpha > \phi$ and $\beta < \pi + \alpha$
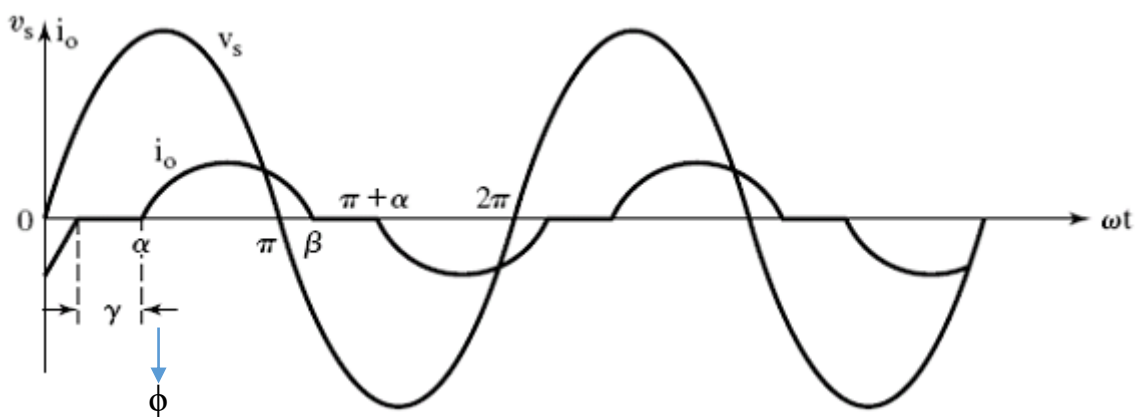
i.e., $(\beta - \alpha) < \pi$, conduction angle $< \pi$.

❑ This circuit can be used to regulate the RMS voltage across the terminals of an ac motor (induction motor). It can be used to control the temperature of a furnace by varying the RMS output voltage.

❑ For very large load inductance 'L' the SCR may fail to commutate and the load voltage will be a full sine wave.

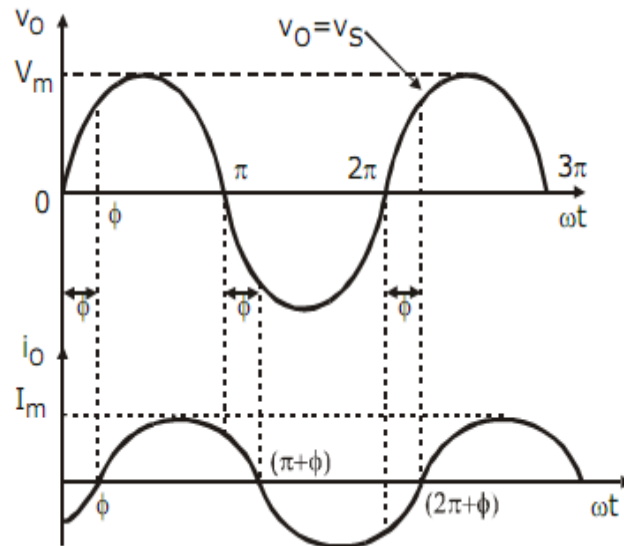A continuous load current and the output voltage waveform appears as a continuous sine wave identical to the input supply voltage waveform for trigger angle α≤ϕ. We lose the control on the output voltage and thus we obtain:

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}} = V_S \; ; \; \text{for } \alpha \le \phi$$

RMS Output Voltage

for α>ϕ the load current and load voltage waveforms become discontinuous.

$$V_{O(RMS)} = \left[\frac{1}{\pi} \int_{\alpha}^{\beta} V_m^2 \sin^2 \omega t . d(\omega t) \right]^{\frac{1}{2}} \quad V_{O(RMS)} = \left[\frac{V_m^2}{2\pi} \left\{ \int_{\alpha}^{\beta} d(\omega t) - \int_{\alpha}^{\beta} \cos 2\omega t . d(\omega t) \right\} \right]^{\frac{1}{2}}$$

$$\boxed{V_{O(RMS)} = \frac{V_m}{\sqrt{2}} \left[\frac{1}{\pi} \left\{ (\beta - \alpha) + \frac{\sin 2\alpha}{2} - \frac{\sin 2\beta}{2} \right\} \right]^{\frac{1}{2}}}$$

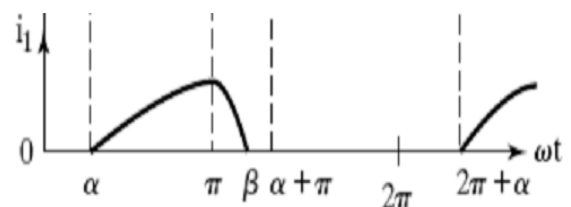## Output Current for (Inductive Load)

during T_1 conduction:

$$L \left(\frac{di_o}{dt} \right) + Ri_O = V_m \sin \omega t$$

The solution of the above differential equation gives the general expression for the output load current which is of the form

$$i_O = \frac{V_m}{Z} \sin(\omega t - \phi) + A_1 e^{\frac{-t}{\tau}} \; ; \quad Z = \sqrt{R^2 + (\omega L)^2} = \text{Load impedance.}$$

$$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) = \text{Load impedance angle (power factor angle of load).}$$

$$\tau = \frac{L}{R} = \text{Load circuit time constant.}$$

At $\omega t = \alpha$, $i_o = 0$ Amp, hence,

$$i_O = 0 = \frac{V_m}{Z}\sin(\alpha - \phi) + A_1 e^{\frac{-R}{L}t} \qquad\qquad A_1 e^{\frac{-R}{L}t} = \frac{-V_m}{Z}\sin(\alpha - \phi)$$

$$A_1 = \frac{1}{e^{\frac{-R}{L}t}}\left[\frac{-V_m}{Z}\sin(\alpha - \phi)\right]$$

By substituting $\omega t = \alpha$:
$$\boxed{A_1 = e^{\frac{R(\alpha)}{\omega L}}\left[\frac{-V_m}{Z}\sin(\alpha - \phi)\right]}$$ then,

$$i_O = \frac{V_m}{Z}\sin(\omega t - \phi) + e^{\frac{-R}{L}t}e^{\frac{R(\alpha)}{\omega L}}\left[\frac{-V_m}{Z}\sin(\alpha - \phi)\right] \; ;$$ which results in the instantaneous output current equal to:

$$\boxed{i_O = \frac{V_m}{Z}\left[\sin(\omega t - \phi) - \sin(\alpha - \phi)e^{\frac{-R}{\omega L}(\omega t - \alpha)}\right] \; ; \quad \text{Where } \alpha \le \omega t \le \beta.}$$

The above expression also represents the thyristor current i_{T1}, during the conduction time interval of thyristor T_1 from $\omega t = \alpha$ to β.

Calculate Extinction Angle β

At $\omega t = \beta$, $i_o = 0$ Amp, hence,

$$i_O = 0 = \frac{V_m}{Z}\left[\sin(\beta - \phi) - \sin(\alpha - \phi)e^{\frac{-R}{\omega L}(\beta - \alpha)}\right] \qquad \text{As } \frac{V_m}{Z} \ne 0 \text{ we can write}$$

$$\boxed{\sin(\beta - \phi) = \sin(\alpha - \phi)e^{\frac{-R}{\omega L}(\beta - \alpha)}}$$

❑ β can be determined from this transcendental equation by using *the iterative method of soluti*on (trial and error method).

- For δ<π, for β< π+ α the load current waveform appears as a discontinuous current waveform.
- When α is decreased and made equal to the load impedance angle α=φ, we obtain from the expression for

$$\sin(\beta-\phi)=0 \; ; \; \text{Therefore} \; (\beta-\phi)=\pi \; \text{radians.}$$

Extinction angle $\qquad \beta=(\pi+\phi)=(\pi+\alpha) \; ; \; \text{for the case when} \; \alpha=\phi$

Conduction angle $\qquad \delta=(\beta-\alpha)=\pi \; \text{radians}=180^0 \; ; \; \text{for the case when} \; \alpha=\phi$

Each thyristor conducts for 180^0 (π radians)

T_1 conducts from $\omega t=\phi$ to $(\pi+\phi)$

T_2 conducts from $(\pi+\phi)$ to $(2\pi+\phi)$

Hence we obtain a continuous load current and the output voltage waveform appears as a continuous sine wave identical to the input supply voltage waveform.

Performance Parameters

RMS Output Voltage

$$V_{O(RMS)} = \frac{V_m}{\sqrt{2}} \sqrt{\frac{1}{\pi} \left[(\beta-\alpha) + \frac{\sin 2\alpha}{2} - \frac{\sin 2\beta}{2} \right]}$$

Load Current

$$i_O = i_{T_1} = \frac{V_m}{Z} \left[\sin(\omega t - \phi) - \sin(\alpha-\phi) e^{\frac{-R}{\omega L}(\omega t - \alpha)} \right] \; ; \quad \text{for } \alpha \le \omega t \le \beta$$

The Average Thyristor Current

$$I_{T(Avg)} = \frac{1}{2\pi} \left[\int_{\alpha}^{\beta} i_{T_1} d(\omega t) \right]$$

$$I_{T(Avg)} = \frac{1}{2\pi} \left[\int_{\alpha}^{\beta} \frac{V_m}{Z} \left[\sin(\omega t - \phi) - \sin(\alpha-\phi) e^{\frac{-R}{\omega L}(\omega t - \alpha)} \right] d(\omega t) \right]$$

$$I_{T(Avg)} = \frac{V_m}{2\pi Z} \left[\int_\alpha^\beta \sin(\omega t - \phi).d(\omega t) - \int_\alpha^\beta \sin(\alpha - \phi) e^{\frac{-R}{\omega L}(\omega t - \alpha)} d(\omega t) \right]$$

Maximum value of $I_{T(Avg)}$ occur at $\alpha = 0$. The thyristors should be rated for

maximum $I_{T(Avg)} = \left(\dfrac{I_m}{\pi} \right)$, where $I_m = \dfrac{V_m}{Z}$.

RMS Thyristor Current $I_{T(RMS)}$

$$I_{T(RMS)} = \sqrt{\left[\frac{1}{2\pi} \int_\alpha^\beta i_{T_1}^2 d(\omega t) \right]}$$

Maximum value of $I_{T(RMS)}$ occurs at $\alpha = 0$. Thyristors should be rated for maximum

$$I_{T(RMS)} = \left(\frac{I_m}{2} \right)$$

When a Triac is used in a single phase full wave ac voltage controller with RL type of

load, then $I_{T(Avg)} = 0$ and maximum $I_{T(RMS)} = \dfrac{I_m}{\sqrt{2}}$

Example 2: A single phase full wave ac voltage controller supplies an RL load. The input supply voltage is 230V, RMS at 50Hz. The load has L = 10mH, R = 10Ω, the delay angle of SCR 1 and 2 are equal, where $\alpha_1 = \alpha_2 = \pi/3$
Determine
a. Conduction angle of the thyristor .
b. RMS output voltage.
c. The input power factor.
Comment on the type of operation.

Solution

$$V_s = 230V, \quad f = 50Hz, L = 10mH, \ R = 10\Omega, \quad \alpha = 60^0, \quad \alpha = \alpha_1 = \alpha_2 = \frac{\pi}{3}$$

$$V_m = \sqrt{2}V_s = \sqrt{2} \times 230 = 325.2691193 \ V$$

$$Z = \text{Load Impedance} = \sqrt{R^2 + (\omega L)^2} = \sqrt{(10)^2 + (\omega L)^2}$$

$$\omega L = (2\pi fL) = (2\pi \times 50 \times 10 \times 10^{-3}) = \pi = 3.14159\Omega$$

$$Z = \sqrt{(10)^2 + (3.14159)^2} = \sqrt{109.8696} = 10.4818\Omega$$

$$I_m = \frac{V_m}{Z} = \frac{\sqrt{2} \times 230}{10.4818} = 31.03179 \ A$$

Load Impedance Angle $\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$

$$\phi = \tan^{-1}\left(\frac{\pi}{10}\right) = \tan^{-1}(0.314159) = 17.44059^0$$

Trigger Angle $\alpha > \phi$. Hence the type of operation will be discontinuous load current operation, we get

$$\beta < (\pi + \alpha)$$

$$\beta < (180 + 60) \ ; \ \beta < 240^0$$

Therefore the range of β is from 180 degrees to 240 degrees. $\left(180^0 < \beta < 240^0\right)$

Extinction Angle β is calculated by using the equation

$$\sin(\beta - \phi) = \sin(\alpha - \phi) e^{\frac{-R}{\omega L}(\beta - \alpha)}$$

In the exponential term the value of α and β should be substituted in radians. Hence

$$\sin(\beta - \phi) = \sin(\alpha - \phi) e^{\frac{-R}{\omega L}(\beta_{Rad} - \alpha_{Rad})} \ ; \ \alpha_{Rad} = \left(\frac{\pi}{3}\right)$$

$$(\alpha - \phi) = (60 - 17.44059) = 42.5594^0$$

$$\sin(\beta - 17.44)^0 = \sin(42.5594^0) e^{\frac{-10}{\pi}(\beta - \alpha)}$$

$$\sin(\beta - 17.44)^0 = 0.676354 e^{-3.183(\beta - \alpha)} \quad 180^0 \to \pi \text{ radians, } \beta_{Rad} = \frac{\beta^0 \times \pi}{180^0}$$

Assuming $\beta = 190^0$;

$$\beta_{Rad} = \frac{\beta^0 \times \pi}{180^0} = \frac{190^0 \times \pi}{180} = 3.3161$$

L.H.S: $\sin(190 - 17.44)^0 = \sin(172.56) = 0.129487$

R.H.S: $0.676354 \times e^{-3.183\left(3.3161 - \frac{\pi}{3}\right)} = 4.94 \times 10^{-4}$

Assuming $\beta = 183^0$;

$$\beta_{Rad} = \frac{\beta^0 \times \pi}{180^0} = \frac{183^0 \times \pi}{180} = 3.19395$$

$$(\beta - \alpha) = \left(3.19395 - \frac{\pi}{3}\right) = 2.14675$$

L.H.S: $\sin(\beta - \phi) = \sin(183 - 17.44) = \sin 165.56^0 = 0.24936$

R.H.S: $0.676354e^{-3.183(2.14675)} = 7.2876 \times 10^{-4}$

Assuming $\beta \approx 180^0$

$$\beta_{Rad} = \frac{\beta^0 \times \pi}{180^0} = \frac{180^0 \times \pi}{180} = \pi$$

$$(\beta - \alpha) = \left(\pi - \frac{\pi}{3}\right) = \left(\frac{2\pi}{3}\right)$$

L.H.S: $\sin(\beta - \phi) = \sin(180 - 17.44) = 0.2997$

R.H.S: $0.676354e^{-3.183\left(\pi - \frac{\pi}{3}\right)} = 8.6092 \times 10^{-4}$

Assuming $\beta = 197^0$

$$\beta_{Rad} = \frac{\beta^0 \times \pi}{180^0} = \frac{197^0 \times \pi}{180} = 3.43829$$

L.H.S: $\sin(\beta - \phi) = \sin(197 - 17.44) = 7.69 = 7.67937 \times 10^{-3}$

R.H.S: $0.676354e^{-3.183\left(3.43829 - \frac{\pi}{3}\right)} = 4.950386476 \times 10^{-4}$

Assuming $\beta = 197.42^0$

$$\beta_{Rad} = \frac{\beta^0 \times \pi}{180^0} = \frac{197.42 \times \pi}{180} = 3.4456$$

L.H.S: $\sin(\beta - \phi) = \sin(197.42 - 17.44) = 3.4906 \times 10^{-4}$

R.H.S: $0.676354e^{-3.183\left(3.4456 - \frac{\pi}{3}\right)} = 3.2709 \times 10^{-4}$

a)

Conduction Angle $\delta = (\beta - \alpha) = (197.42^0 - 60^0) = 137.42^0$

b)

$$V_{O(RMS)} = V_S \sqrt{\frac{1}{\pi} \left[(\beta - \alpha) + \frac{\sin 2\alpha}{2} - \frac{\sin 2\beta}{2} \right]}$$

$$V_{O(RMS)} = 230 \sqrt{\frac{1}{\pi} \left[(2.39843) + 0.4330 - 0.285640 \right]} = 207.0445 \text{ V}$$

c)

$$PF = \frac{P_O}{V_S \times I_S} \qquad I_{O(RMS)} = \frac{V_{O(RMS)}}{Z} = \frac{207.0445}{10.4818} = 19.7527 \text{ A}$$

$$P_O = I_{O(RMS)}^2 \times R_L = (19.7527)^2 \times 10 = 3901.716 \text{ W}$$

$$V_S = 230V, \qquad I_S = I_{O(RMS)} = 19.7527$$

$$PF = \frac{P_O}{V_S \times I_S} = \frac{3901.716}{230 \times 19.7527} = 0.8588$$

Exercise1: A single phase full wave controller has an input voltage of 120 V (RMS) and a load
resistance of 6 ohm. The firing angle of thyristor is 2π . Find
    a) RMS output voltage (84.85 Volts)
    b) Power output (1200 watts)
    c) Input power factor (0.707 lag)
    d) Average and RMS thyristor current. (4.5 A, 10 Amps)

Exercise2: A single phase half wave ac regulator using one SCR in anti-parallel with a diode feeds 1 kW, 230 V heater. Find load power for a firing angle of $\pi/4$
Ans: 954.56 Watts

Exercise3: A single phase voltage controller is employed for controlling the power flow from 220 V, 50 Hz source into a load circuit consisting of R = 4 Ω and L = 6 mH. Calculate the following

a. Control range of firing angle $56.3^0 < \alpha < 180^0$
b. Maximum value of RMS load current 30.5085 Amps
c. Maximum power and power factor 3723.077 W, 0.5547
d. Maximum value of average and RMS thyristor current. 13.7336 Amps, 21.57277 Amps

Self-assessments:

a) What phase angle control is as applied to single phase controllers? Highlight the   advantages and disadvantages of such a method of control. Draw all the wave forms.

b) What are unidirectional controllers? Explain the operation of the same with the help of waveforms and obtain the expression for the RMS value of the output voltage.   What are the advantage and disadvantages of unidirectional controllers?