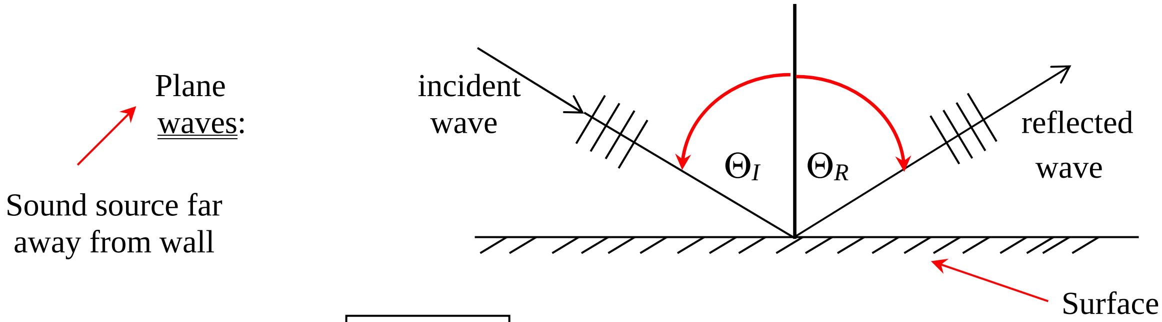
Lecture Notes III (Continued – Part 2)

Reflection of Sound Waves: — Sound waves bounce (*i.e.* reflect) off of walls – just like light waves (*i.e. EM* waves) bouncing off of/reflecting from a mirror:



The Law of Reflection: $\boxed{\Theta_I = \Theta_R}$ Angle of Incidence = Angle of Reflection.

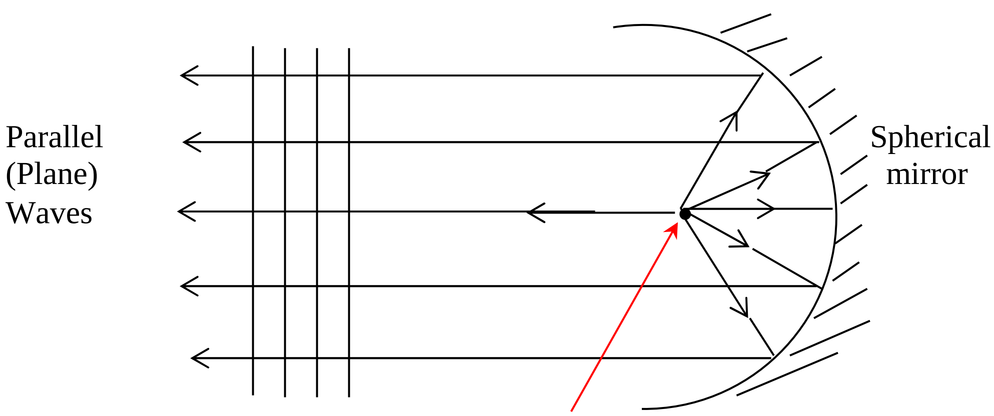
The law of reflection for sound waves is same as that for light waves, *e.g.* light reflecting off of a mirror and/or a refracting interface. The law of reflection (in either case) physically arises from (microscopic) conservation of energy and momentum at the interface/reflecting mirror!

Sound Waves Can Be Focused Just Like Light!!!

In one dimension, define the sound source location, S_{source}. Define the receiver/observer location, $S_{observer}$. The focal length of a (*concave*) spherical mirror, $f = +R/2$, where R = radius of curvature of spherical mirror.{For a *convex* spherical mirror, $f = -R/2$}. All distances are measured with respect to the *apex* of the mirror.

Then: $$\boxed{\frac{1}{S_{source}} + \frac{1}{S_{observer}} = \frac{1}{f} = \frac{2}{R}}$$ "Acoustic Mirror Equation"

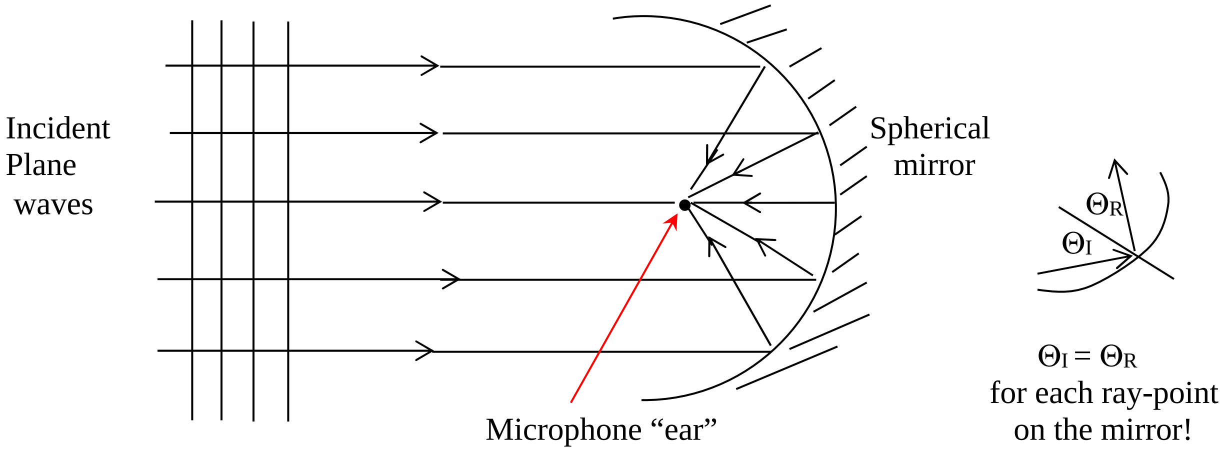
Thus, if the sound source is located at the focal point of spherical mirror, $S_{source} = f = R/2$, then the sound emerges from the acoustic mirror as parallel rays (*i.e.* as plane waves) – just as in the optics case (see figure below)! The observer's location is at $S_{observer} = \infty$.



Sound source located at focal point of mirror, $f = +R/2$.

Time-Reversed Situation:

If the sound source is located infinitely far from the focal point of spherical mirror, *i.e.* $S_{source} = \infty$, the sound from the source impinges on the spherical mirror as plane waves, and the sound converges to the focal point of the acoustic mirror – just as in the optics case! The observer location is at the focal point of the mirror, $S_{observer} = f = R/2$.



Thus, a spherical acoustic mirror can be used as a *supersensitive* "ear" – focusing (i.e. converging) the incoming sound plane waves to a single point - at the focus of the mirror! Using two such acoustic mirrors facing each other and *e.g.* separated by a large distance $d = 100$ *ft*, two people, each standing at the focus of one mirror (and facing it) whispering to each other can very clearly hear each other – this two-mirror configuration is known as a *whispering gallery*…

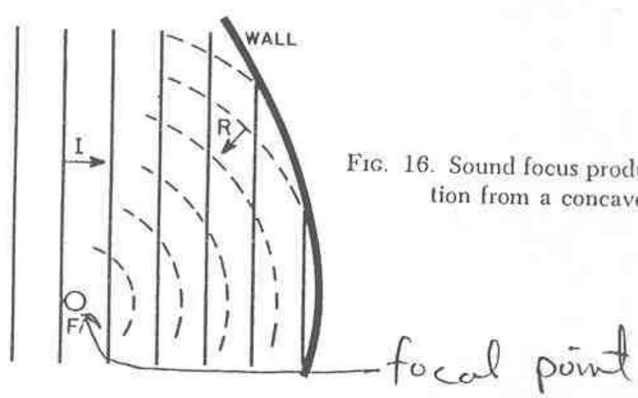


FIG. 16. Sound focus produced by reflection from a concave wall.

n.b. The sign conventions needed/used in acoustic mirror equation (above) are the same as that as used in optics with the optical mirror equation for optical image formation with an optical mirror.

Sound waves behave very similarly/analogously to that of light waves/*EM* waves (photons)!

Refraction (*i.e.* Bending) of Sound Waves ("Dispersion")

Refraction of sound waves arises from temperature/pressure/density gradient(s) in air.

Listen to the phase shift/flanging effect of jet airplane engine's when jet is in the air. This sound effect arises from interference effect from mixing (*i.e.* superposition) of sound amplitudes from same sound source, but due to (slightly) different paths taken by sound from the sound source (jet) to observer/listener, resulting in (slightly) different path lengths of the sound in air, thus having (slightly) different propagation times from sound source to observer!

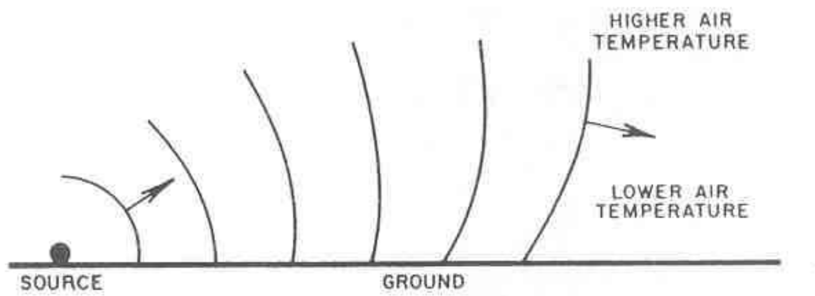


FIG. 17. Refraction of sound waves.

Refraction (bending) of sound "rays" in air arises due to density dependence of the speed of sound. From the Ideal Gas Law $PV = NRT$, the speed of sound propagation in air also depends on the temperature/pressure of air.

Interference of Sound (& Light) Waves:

Many individual sound (& light) waves propagating in a medium can exist simultaneously at the same point, x and at the same time, t in that medium.

Linear Superposition (*i.e.* Addition) of Sound Waves:

When two (or more) sound waves spatially/temporally overlap each other, in general they will ***interfere*** with each other. We must then add *e.g.* individual over-pressure (or displacement) amplitudes together to obtain the total over-pressure (or displacement) amplitude:

e.g. 2 waves: $p_{tot}(z,t) = p_1(z,t) + p_2(z,t)$

e.g. *N* waves: $p_{tot}(z,t) = p_1(z,t) + p_2(z,t) + ... + p_N(z,t) = \sum_{i=1}^{N} p_i(z,t)$

Sound (& light) waves can interfere $\left\{ \begin{matrix} \text{constructively} \\ \text{destructively} \end{matrix} \right\}$ or somewhere in-between these two!

2-wave ***constructive*** interference: $p_{tot}(z,t) \cong 2p_1(z,t)$.

2-wave ***destructive*** interference: $p_{tot}(z,t) \cong 0 \qquad \{p_2(z,t) \cong -p_1(z,t)\}$

The mathematical addition of individual amplitudes must be done carefully, in order to preserve (relative) phase information. We discuss in detail how this is accomplished, below.

Interference of Sound Waves:

Two sound sources – at same frequency – there will be points in space where the overall sound level is high ($p_{tot}(z,t)$ is large – constructive interference) and other places where over all sound level is ~ zero ($p_{tot}(z,t) \sim 0$ – destructive interference).

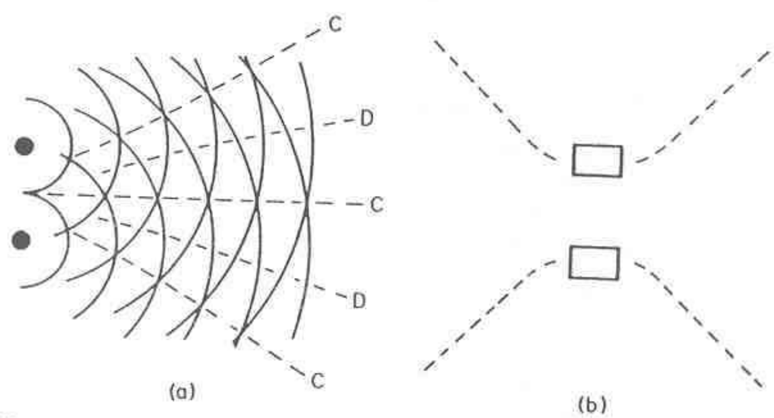
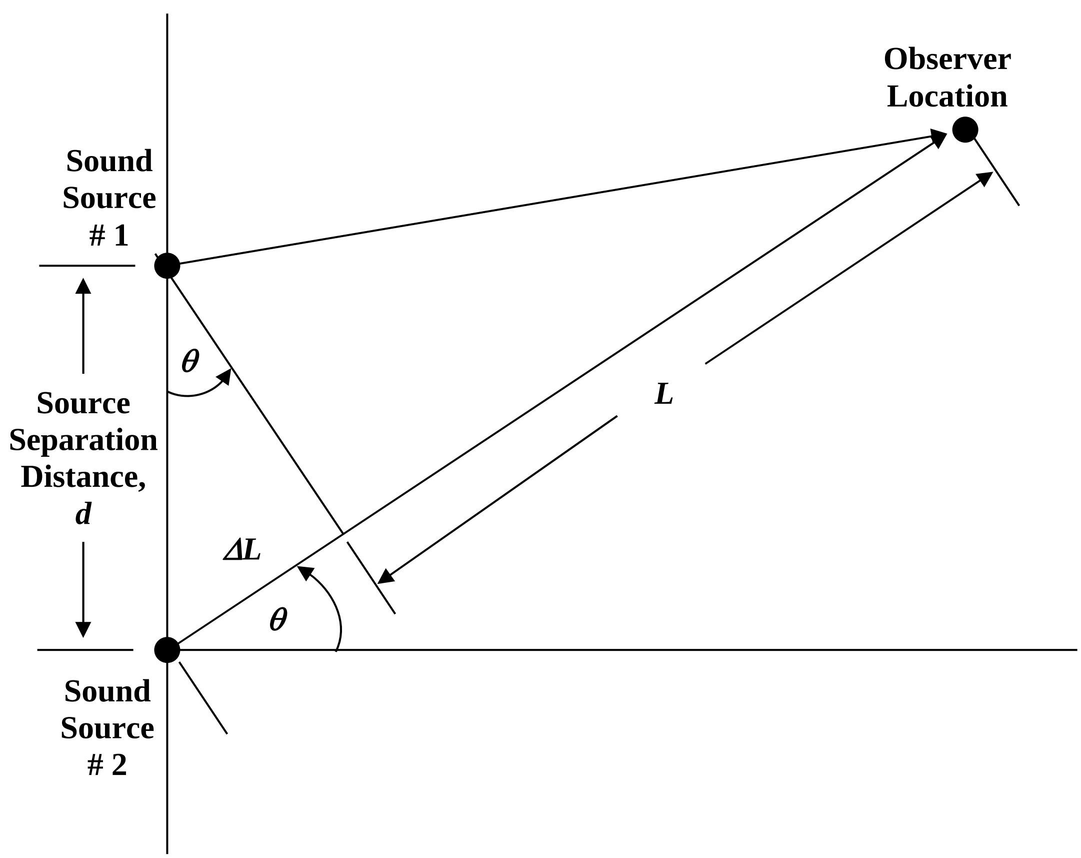


FIG. 19. (a) Interference of waves from two identical sources. (b) Destructive interference of sound waves from two prongs of a tuning fork.

Consider two point sources of sound waves that emit precisely/exactly the same sound – *i.e.* having the same frequency, same amplitude and phase (*e.g.* a pair of stereo loud-speakers), as shown in the figure below:



If the distance of the observer/listener from both of the two sound sources is ***large*** compared to the sound source separation distance, *i.e.* $L >> d$, {the so-called "far-field" limit} then $\tan\theta \approx \sin\theta$ and hence $\Delta L \approx d\sin\theta$. The relative phase difference between the two amplitudes at the observer/listener location is: $\delta = k\,\Delta L = 2\pi\Delta L/\lambda \approx 2\pi d\sin\theta/\lambda$ (in radians).

At the observer/listener location, suppose the individual over-pressure amplitudes at a given instant in time are given by:

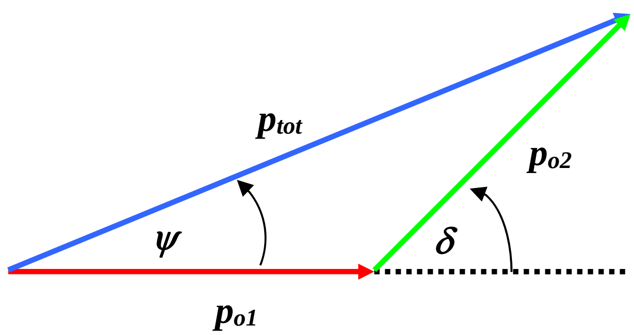
$$p_1(L,t) = p_{o_1}\cos(\omega t - kL)$$

$$p_2(L,t) = p_{o_2}\cos[\omega t - k(L+\Delta L)] = p_{o_2}\cos(\omega t - kL - \delta)$$

$$p_{tot}(L,t) = p_1(L,t) + p_2(L,t) = p_{o_1}\cos(\omega t - kL) + p_{o_2}\cos(\omega t - kL - \delta)$$

The phase-sensitive interference relation between the two individual over-pressure amplitudes and the resultant/total over-pressure amplitude heard by the observer/listener can be represented graphically using a so-called ***phasor diagram***, as shown in the figure below. The phasor diagram adds the two vector amplitudes together to form the resultant/overall/net vector amplitude.

The phasor diagram, by convention, orients the over-pressure amplitude associated with the first sound source $p_1(z,t)$ on the horizontal axis. The base of the over-pressure amplitude associated with the second sound source, $p_2(z,t)$ is placed at the tip of the first, and angled away from the x-axis by the relative phase difference angle, δ. The resultant/total/net displacement amplitude, $y_{tot}(t)$ is the vector drawn from the base of the first displacement amplitude to the tip of the second displacement amplitude, as shown in the figure below:



Note that the phasor triangle obeys the trigonometrical *law of cosines* relation:

$$c^2 = a^2 + b^2 - 2ab\cos(\pi - \delta) = a^2 + b^2 + 2ab\cos\delta$$

{the latter relation on the RHS of this equation was obtained using the trigonometric identity: $\cos(A - B) = \cos A\cos B + \sin A\sin B$).

The *magnitude* (*i.e.* length) of the resultant/total/net over-pressure amplitude, p_{tot} is given by:

$$p_{tot}^2 = p_{o_1}^2 + p_{o_2}^2 + 2p_{o_1}p_{o_2}\cos\delta \quad \text{or:} \quad p_{tot} = \sqrt{p_{o_1}^2 + p_{o_2}^2 + 2p_{o_1}p_{o_2}\cos\delta}$$

Maxima (*i.e*. total constructive interference, $p_{tot}^2 = p_{o_1}^2 + p_{o_2}^2$ occur when cos $\delta = +1$, *i.e*. when: $\delta = 0, \pm 2\pi, \pm 4\pi, \pm 6\pi, \ldots = 2n\pi$, $n = 0, \pm 1, \pm 2, \pm 3, \ldots$ corresponding to
$\Delta L/\lambda = d \sin\theta/\lambda = 0, \pm 1, \pm 2, \pm 3, \ldots = 2n/2 = n$, i.e.
$\Delta L \;\; = d \sin\theta \;\;\; = 0\lambda, \pm 1\lambda, \pm 2\lambda, \pm 3\lambda, \ldots = 2n\lambda/2 = n\lambda$.

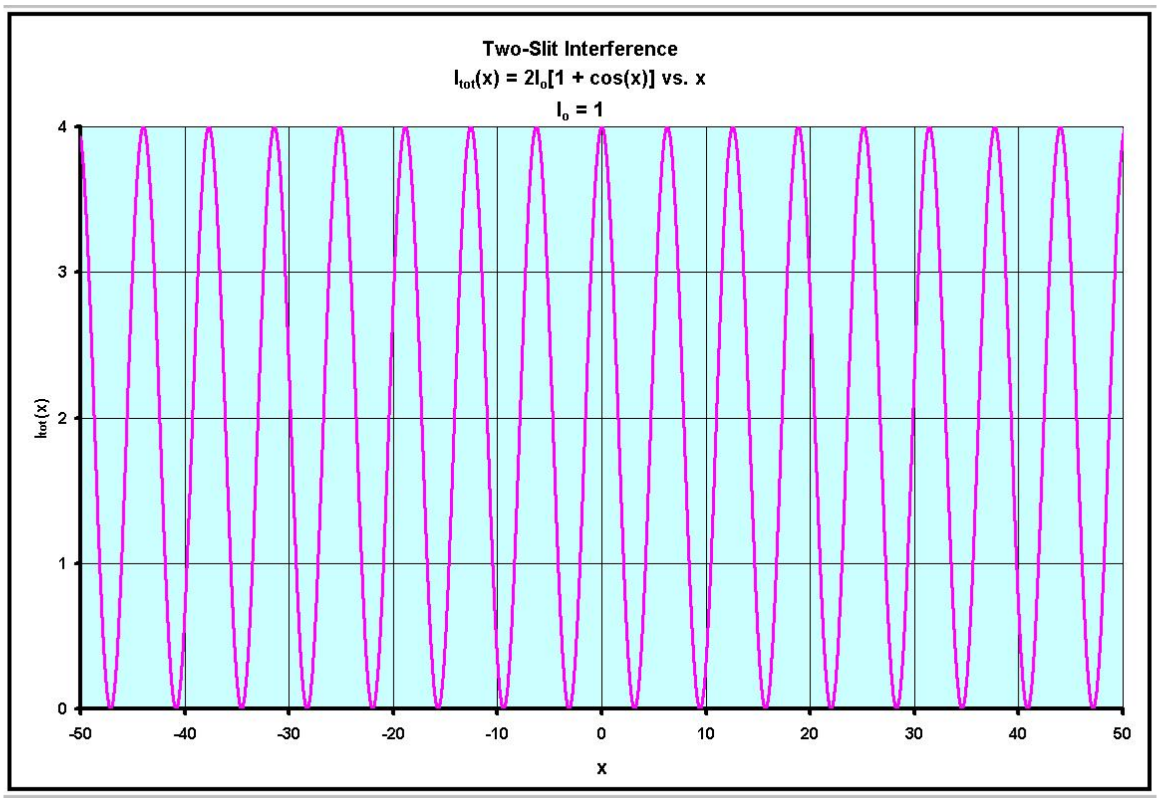
Minima (*i.e*. total destructive interference, $p_{tot}^2 = p_{o_1}^2 - p_{o_2}^2$ occur when cos $\delta = -1$, *i.e*. when $\delta = \pm\pi, \pm 3\pi, \pm 5\pi, \pm 7\pi, \ldots = (2n+1)\pi$, $n = 0, \pm 1, \pm 2, \pm 3, \ldots$ corresponding to
$\Delta L/\lambda = d \sin\theta/\lambda = \pm 1/2 , \pm 3/2, \pm 5/2, \ldots = (2n+1)/2$, i.e.
$\Delta L \;\; = d \sin\theta \;\;\; = \pm\lambda/2, \pm 3\lambda/2, \pm 5\lambda/2, \ldots = (2n+1)\lambda./2$.

As drawn in the above figure, this phasor diagram represents a “snapshot” in time – *i.e*. at some particular time *t*. As time *t* progresses, the entire phasor triangle precesses (i.e. rotates with angular frequency ω about its origin (the base point of over-pressure amplitude # 1)) in a counter-clockwise direction.

Note that the acoustical interference of two sound sources with each other – *e.g*. two loudspeakers – is the analog of Young’s two-slit interference experiment in optics!

Note also that the sound intensity, I ($Watts/m^2$) is proportional to the (modulus) square of the over-pressure amplitude – *i.e*. $I(z,t) \sim p^2(z,t)$. Thus, we can rewrite the above formula in terms of sound intensities: $I_{tot} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\delta$.

The sound intensity distribution $I_{tot}(x)$ *vs*. x (= δ) = transverse distance (at $z = L$) is shown in below for equal intensities from two sound sources, $I_1 = I_2 = I_o = 1$, when $I_{tot}(x) = 2I_o[1+\cos(x)]$.



Interference Effects from Multiple Sound Sources – Phased Arrays:

Obviously, the phasor diagram can be extended to multiple (*i.e.* N) identical sound sources, each with sound intensity I_o arranged in a line (*e.g.* the x-axis), each separated by a lateral distance d from each other, and interfering with each other – analogous to multiple, or N-slit interference in optics!

The intensity distribution for N-slit interference on a transverse screen is given by:

$$I_{tot}(x) = I_o \left\{ \frac{\sin^2\left(\frac{N\delta}{2}\right)}{\sin^2\left(\frac{\delta}{2}\right)} \right\}$$

Minima – *i.e.* intensity zeroes (complete destructive interference) occur when the numerator factor $N\delta/2 = \pm\pi, \pm 2\pi, \pm 3\pi, \ldots = n\pi$, $n = \pm 1, \pm 2, \pm 3, \ldots$ *except* when the denominator factor simultaneously has $\delta/2 = \pm\pi, \pm 2\pi, \pm 3\pi, \ldots = n\pi$, $n = \pm 1, \pm 2, \pm 3, \ldots$ then we have a ***global maximum*** of the intensity, where $I_{tot} = N^2 I_o$.

The limiting case is where the number of sources/slits, $N \to \infty$ – *i.e.* a *continuum* of closely-spaced/immediately adjacent/contiguous, infinitesimally small sound sources, all in phase with each other, as in the 2-sound source case described immediately above. This limiting case describes the phenomena of *diffraction* of sound waves (or light waves) *e.g.* through a constricting aperture!

The phasor method can also be used for obtaining the intensity distribution associated *e.g.* with a 2-dimensional phased array of sound sources since, as in the case for light/EM waves, sound interference effects along one axis (*e.g.* x) do ***not*** interfere with those along a different axis (*e.g.* y). For example, the overall intensity distribution for a 2-dimensional rectangular array of N_x and N_y sound sources in the far-field limit is given by the product expression:

$$I_{tot}(x,y) = I_o \left\{ \frac{\sin^2\left(\frac{N_x\delta_x}{2}\right)}{\sin^2\left(\frac{\delta_x}{2}\right)} \right\} \left\{ \frac{\sin^2\left(\frac{N_y\delta_y}{2}\right)}{\sin^2\left(\frac{\delta_y}{2}\right)} \right\}$$

where in the far-field limit: $\delta_x = 2\pi d_x \sin\theta_x/\lambda$ and $\delta_y = 2\pi d_y \sin\theta_y/\lambda$ (in radians).

Additional info & plots on 1-D and 2-D N-slit "far-field" interference is available on the Physics 406 Software webpage at the following URL:

http://courses.physics.illinois.edu/phys406/406pom_sw.html

<u>Diffraction (i.e. Spreading) of (Light &) Sound Waves Through Constricting Apertures:</u>

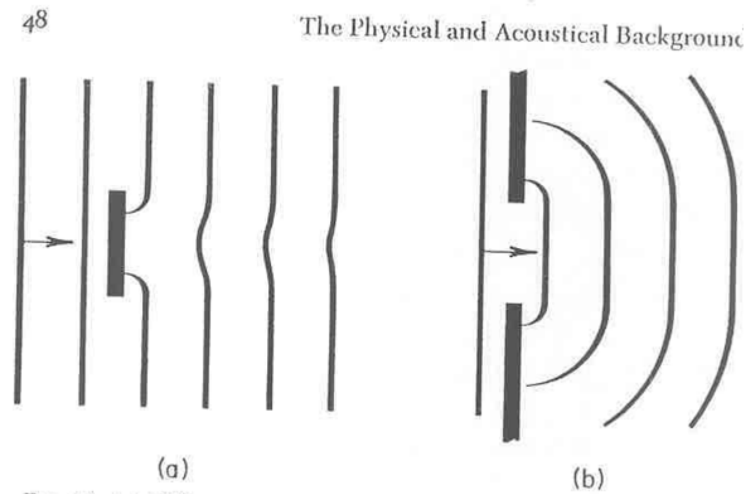
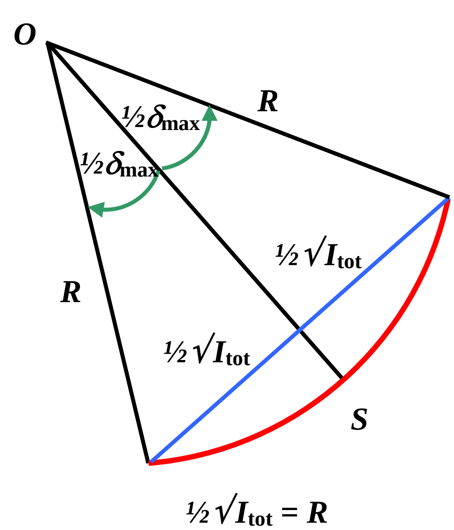


FIG. 18. (a) Diffraction of waves around an obstacle. (b) Diffraction of waves through an aperture.

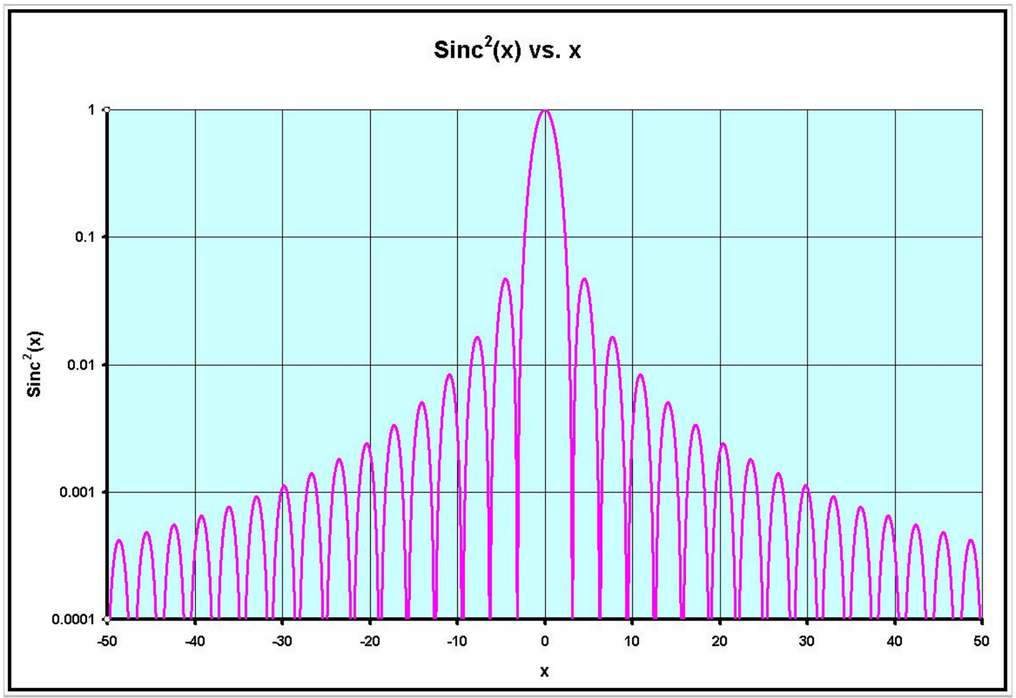
For sound (or light) incident as plane waves on a single, narrow slit/aperture of lateral width, a, the intensity $I(\theta)$, <u>far</u> from the aperture (the so-called Fraunhofer limit), making an angle θ with the initial direction of propagation of the sound (or light) waves, is given by:

$$I(\theta) = I_o \left\{ \frac{\sin^2\left(\frac{\pi a \sin\theta}{\lambda}\right)}{\left(\frac{\pi a \sin\theta}{\lambda}\right)^2} \right\} = I_o Sinc^2\left(\frac{\pi a \sin\theta}{\lambda}\right)$$

where $I_o = I(\theta = 0)$ and $\mathrm{Sinc}(x) \equiv \sin x / x$. This formula results from considering the interference arising from a succession of contiguous, infinitesimally small slits of lateral width, δa adding up to the total lateral width, a of the physical aperture. The phasor diagram for this situation is an arc – *i.e.* a segment of a circle of radius R, as shown in the figure below. Note that the arc length formula, $S = R\theta$ and the formula for the chord of a circle are used in deriving the above relation.



The Sinc^2 function – Sinc$^2(x)$ *vs*. x, where $x = \frac{1}{2}\delta_{\max} = (\pi a \sin\theta/\lambda)$, relevant for diffraction of sound (or light) through a narrow slit/aperture of lateral width a is shown in the figure below, as a ***semi-log*** plot. The global maximum of the intensity/power is in the central lobe, near $|x| \sim 0$.



Diffraction minima occur when $x = \frac{1}{2}\delta_{\max} = (\pi a \sin\theta/\lambda) = \pm\pi, \pm 2\pi, \pm 3\pi, \ldots = \pm m\pi$, $m = 1, 2, 3, \ldots$.

Diffraction Through a Circular Aperture of Radius, *R*:

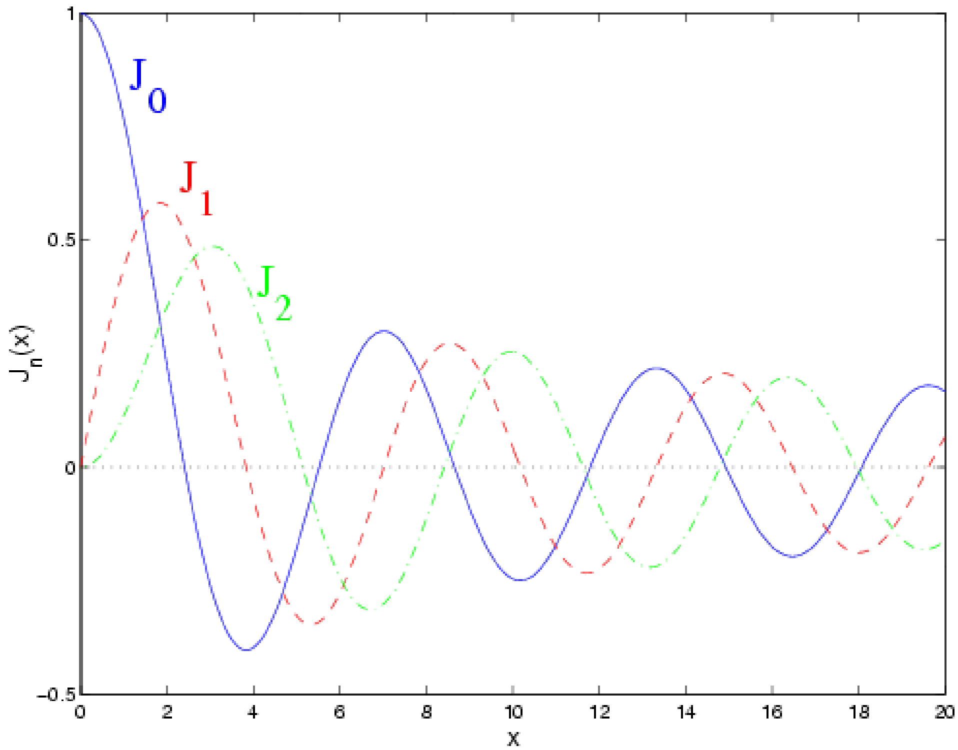
A more realistic situation for diffraction of sound is that of diffraction through a circular aperture. Diffraction occurs in *all* sound-generating transducers, such as loudspeakers. For a circular loudspeaker of radius R (*n.b.* also mounted on an infinite baffle) the angular intensity distribution $I(\theta)$ resulting from the sound diffracting from the aperture of the loudspeaker is given by:

$$I(\theta) = I_o\left[\frac{2J_1(\rho)}{\rho}\right]^2 = I_o\left[\frac{2J_1(kR\sin\theta)}{kR\sin\theta}\right]^2$$

where θ is the polar angle from the axis of the loudspeaker, $\rho \equiv kR\sin\theta$ and $J_1(\rho)$ is the ordinary Bessel function of order 1. Bessel functions frequently arise in situations where circular/cylindrical symmetry is involved. The Bessel function of order n, $J_n(x)$ can be expressed as a power series expansion in x:

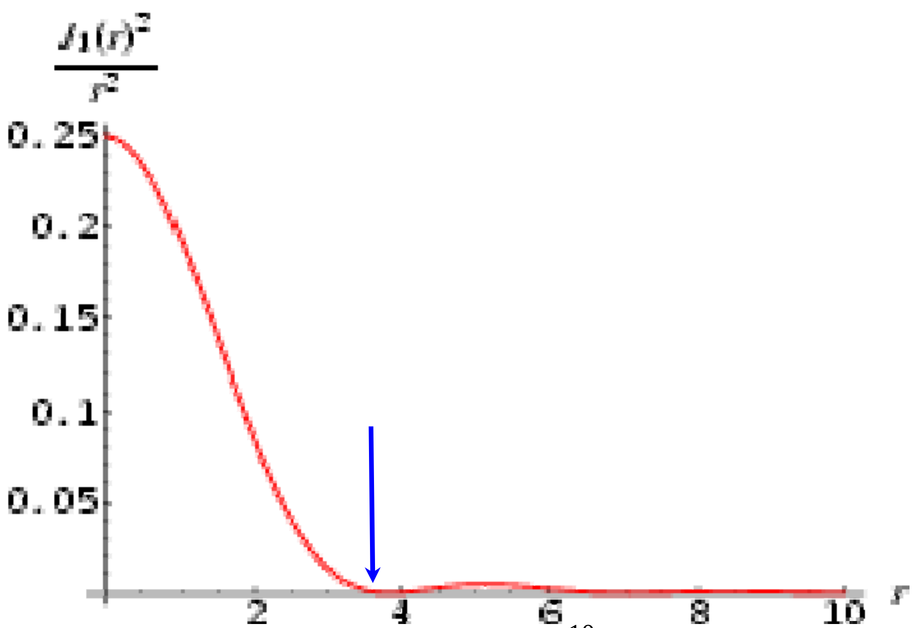
$$J_n(x) = \frac{x^n}{2^n\Gamma(n+1)}\left\{1 - \frac{x^2}{2(2n+2)} + \frac{x^4}{2\bullet 4(2n+2)(2n+4)} - \ldots\right\} = \sum_{k=0}^{\infty}\frac{(-1)^k (x/2)^{n+2k}}{k!\,\Gamma(n+k+1)}$$

A plot of $J_n(x)$ vs. x for the three lowest-order Bessel functions (n = 0, 1 & 2) is shown in the figure below.



The first few zeros of the first order Bessel function, $J_1(x)$ occur at x = 0.0000, 3.8317, 7.0156, 10.1735, 13.3237, 16.4706, 19.6159, … These are obviously not simply-related to one another – numerical computational and/or graphical techniques are usually used to determine them…

A plot of normalized intensity $J_1^2(r)/r^2$ vs. r is shown in the figure below.



The *first* intensity *zero* (*i.e*. a diffraction intensity *minimum*) associated with the diffraction of a plane wave through a circular aperture of radius *R* occurs at:

$$\sin\theta = \frac{3.8317}{kR} = \frac{3.8317\lambda}{2\pi R} = \frac{1.2197\lambda}{D} \simeq \frac{1.22\lambda}{D}$$

where θ is the angle from the symmetry axis (*e.g. z*-axis) of the circular aperture.

The situation for acoustic diffraction for sound waves of wavelength λ diffracting through a circular aperture of radius *R* is the same as that for light/EM waves of wavelength λ diffracting through a circular aperture of radius *R*. In the latter case, the *bright central annular region* is known as the so-called *Airy Disk*. Most of the intensity/power (~ 98.3%) is contained within this central region.

Acoustic Diffraction *and* Interference:

In the real world, *both* diffraction and interference effects operate simultaneously. For example, a stereo system consisting of two loudspeakers, each of radius *R* separated by a transverse distance *a* will have an overall intensity pattern, $I_{tot}(\theta)$ arising from the *product* of the intensity pattern associated with *interference* effects arising from the two speakers, *modulated* by the intensity pattern associated with sound *diffraction* effects associated with a *single* loudspeaker, since the latter is a phenomenon common to/operative on *both* loudspeakers. Thus, the overall intensity pattern e.g. associated with a pair of stereo loudspeakers is given by:

$$I_{tot}(\theta) = I_{interference}(\theta) \bullet I_{diffraction}(\theta)$$

Additional info & plots on 1-D and 2-D diffraction and diffraction & interference are available on the Physics 406 Software webpage at the following URL:

http://courses.physics.illinois.edu/phys406/406pom_sw.html

<u>Beats Phenomenon</u>:

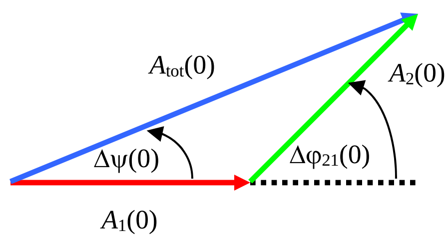
Beats is the phenomenon of interference with 2 (or more) signals of *approximately* the same, but *NOT* identical frequency, *i.e.* $f_1 \sim f_2$.

Linearly superpose (*i.e.* add) two "generic" signals with amplitudes $A_1(t)$ and $A_2(t)$, and which have similar/comparable frequencies, $\omega_2(t) = 2\pi f_2(t) \sim \omega_1(t) = 2\pi f_1(t)$, with instantaneous phase of the second signal relative to the first of $\Delta\varphi_{21}(t)$:

$$A_1(t) = A_{10}\cos(\omega_1(t)t) \qquad A_2(t) = A_{20}\cos(\omega_2(t)t + \Delta\phi_{21}(t))$$
$$A_{tot}(t) = A_1(t) + A_2(t) = A_{10}\cos(\omega_1(t)t) + A_{20}\cos(\omega_2(t)t + \Delta\phi_{21}(t))$$

Note that at the amplitude level, there is nothing explicitly overt and/or obvious in the above mathematical expression for the overall/total/resultant amplitude, $A_{tot}(t)$ that *easily* explains the phenomenon of beats associated with adding together two signals that have comparable amplitudes and frequencies. From the above formula, clearly the total waveform simply consists of two individual waveforms, one with slightly different frequency than the other.

However, let us consider the (instantaneous) phasor relationship between the individual amplitudes for the two signals, $A_1(t)$ and $A_2(t)$ respectively. Their relative initial phase difference at time t = 0 is $\Delta\varphi_{21}(t=0)$ and the resultant/total amplitude, $A_{tot}(t=0)$ is shown in the figure below, for time, t = 0:



From the *law of cosines*, the magnitude of the total amplitude, $A_{tot}(t)$ at an arbitrary time t is obtained from the following:

$$A_{tot}^2(t) = A_1^2(t) + A_2^2(t) - 2A_1(t)A_2(t)\cos\left[\pi - \left((\omega_2(t) - \omega_1(t))t + \Delta\phi_{21}(t)\right)\right]$$
$$A_{tot}^2(t) = A_1^2(t) + A_2^2(t) + 2A_1(t)A_2(t)\cos\left((\omega_2(t) - \omega_1(t))t + \Delta\phi_{21}(t)\right)$$

Thus:

$$A_{tot}(t) = \sqrt{A_1^2(t) + A_2^2(t) + 2A_1(t)A_2(t)\cos\left((\omega_1(t) - \omega_2(t))t + \Delta\phi_{21}(t)\right)}$$
$$= \sqrt{\begin{array}{l} A_{10}^2\cos^2(\omega_1(t)t) + A_{20}^2\cos^2(\omega_2(t)t) \\ +2A_{10}A_{20}\cos(\omega_1(t)t)\cos(\omega_2(t)t + \Delta\phi_{21}(t))\cos\left((\omega_2(t) - \omega_1(t))t + \Delta\phi_{21}(t)\right) \end{array}}$$

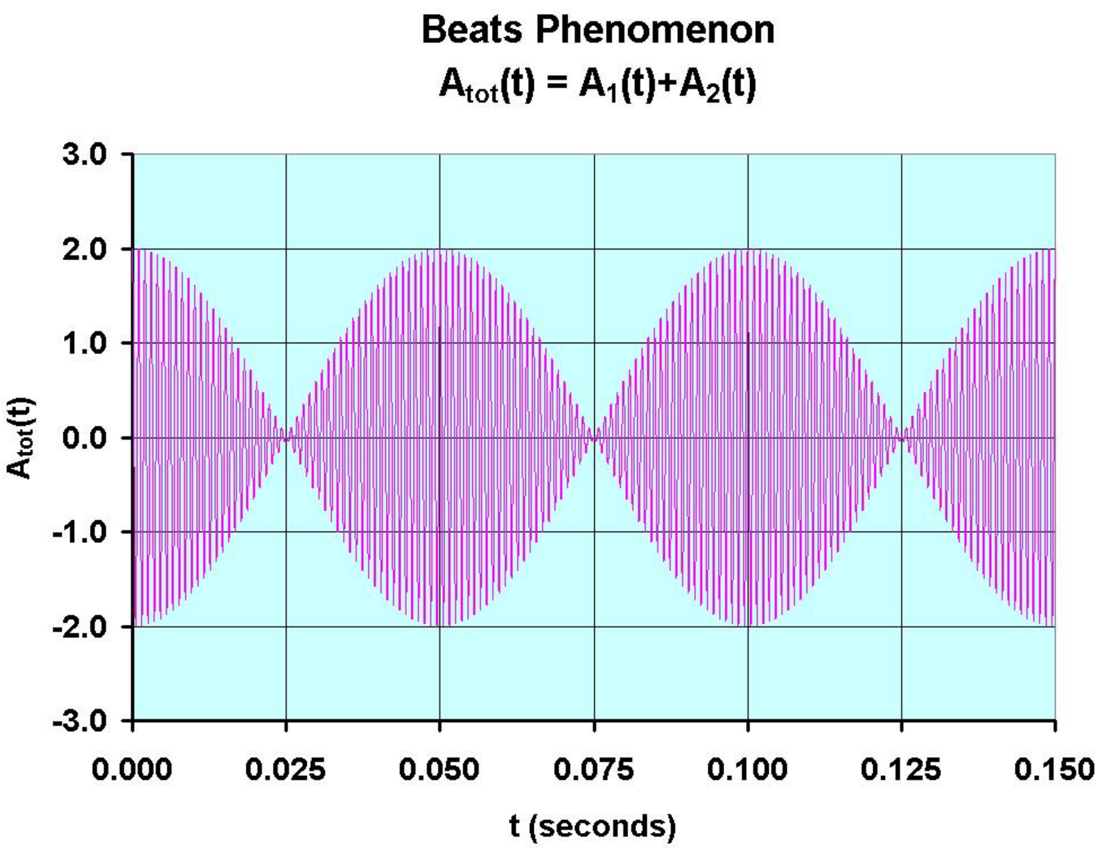
For equal amplitudes $A_{10} = A_{20} = A_0$, zero relative initial phase $\Delta\varphi_{21} = 0$ and constant (i.e. time-independent) frequencies, ω_2 and ω_1, this expression simplifies to:

$$A_{tot}(t) = A_0\sqrt{\cos^2\omega_1 t + \cos^2\omega_2 t + 2\cos\omega_1 t\cos\omega_2 t\cos((\omega_2 - \omega_1)t)}$$

The phase of the total amplitude $A_{tot}(t)$ relative to that of the first amplitude $A_1(t)$, at an arbitrary time t is $\Delta\psi(t)$ and is obtained from the projections of the total amplitude phasor $A_{tot}(t)$ onto the y- and x- axes of the 2-D phasor plane:

$$\tan\Delta\psi = \frac{A_2(t)\cos\left(\omega_2(t)t + \Delta\varphi_{21}(t)\right)\sin\Delta\varphi_{21}(t)}{A_1(t)\cos\left(\omega_1(t)t\right) + A_2(t)\cos\left(\omega_2(t)t + \Delta\varphi_{21}(t)\right)\cos\Delta\varphi_{21}(t)}$$

The total amplitude $A_{tot}(t) = A_1(t) + A_2(t)$ vs. time t is shown in the figure below, for time-independent/constant frequencies of $f_1 = 1000$ *Hz* and $f_2 = 980$ *Hz*, equal amplitudes of unit strength $A_{10} = A_{20} = 1.0$ and zero relative initial phase $\Delta\varphi_{21} = 0.0$

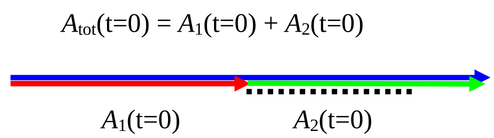


Clearly, the beats phenomenon can be seen in the above waveform of total amplitude $A_{tot}(t) = A_1(t) + A_2(t)$ vs. time t. When $A_{tot}(t) = 0$, we have complete destructive interference of the two individual amplitudes – *i.e*. the 2nd amplitude is 180^o out of phase relative to the first. When $A_{tot}(t) = 2$, we have complete constructive interference of the two amplitudes – the two individual amplitudes are exactly in phase with each other. More on this, below…

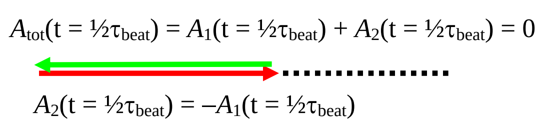
From the above graph, it is also obvious that the beat period $\tau_{beat} = 1/f_{beat} = 0.050$ sec = 1/20th sec, corresponding to a beat frequency of $f_{beat} = 1/\tau_{beat} = 20$ *Hz*, which is simply the (absolute value of the) frequency difference $f_{beat} \equiv |f_1 - f_2|$ between $f_1 = 1000$ *Hz* and $f_2 = 980$ *Hz*. Thus, the beat period $\tau_{beat} = 1/f_{beat} = 1/|f_1 - f_2|$. When $f_1 = f_2$, the beat period becomes infinitely long, and no beats are heard.

Human beings perceive/hear beats in a rather peculiar manner – when the two (or more) individual frequencies are quite close together, i.e. $f_1 \sim f_2$, and in fact so close together such that their frequency difference, $\Delta f = f_{beat} = |\, f_1 - f_2|$ is less than the so-called ***critical band*** for human hearing (typically ~ 90 Hz for frequencies in the human voice range (~ 100 *Hz* – 1 *KHz*)). We humans don't perceive the individual frequencies as separate, rather, we perceive/hear only a single frequency, as the (~ log-weighted intensity) average of the frequencies present. For two signals with equal amplitudes/equal sound intensities having $f_1 \sim f_2$ with $\Delta f = f_{beat} = |\, f_1 - f_2|$ less than the critical band, the perceived average frequency is simply $< f > = ½(f_1 + f_2)$. What we humans hear as beats in this situation is so-called amplitude modulation of a sound wave consisting of a single average frequency $< f > = ½(f_1 + f_2)$, much like someone rhythmically turning the volume control of an amplifier up and down at a frequency of $\Delta f = f_{beat} = |\, f_1 - f_2|$, or equivalently, a beat period of $\tau_{beat} = 1/f_{beat} = 1/|\, f_1 - f_2|$.

In terms of the phasor diagram, as time progresses, the individual amplitudes $A_1(t)$ and $A_2(t)$ actually precess at (angular) rates of $\omega_1 = 2\pi f_1$ and $\omega_2 = 2\pi f_2$ radians per second respectively, completing one revolution in the phasor diagram, for each cycle/each period of $\tau_1 = 2\pi/\omega_1 = 1/f_1$ and $\tau_2 = 2\pi/\omega_2 = 1/f_2$, respectively. If at time $t = 0$ the two phasors are precisely in phase with each other (i.e. with initial relative phase $\Delta\varphi_{21} = 0.0$), then the resultant/total amplitude $A_{tot}(t = 0) = A_1(t = 0) + A_2(t = 0)$ will be as shown in the figure below.



As time progresses, if $\omega_1 \neq \omega_2$, (phasor 1 with angular frequency $\omega_1 = 2\pi f_1 = 2*1000\pi = 2000\pi$ radians/sec and $\omega_2 = 2\pi f_2 = 2*980\pi = 1960\pi$ radians/sec in our example above) phasor 1, with higher angular frequency will precess more rapidly than phasor 2 (by the difference in angular frequencies, $\Delta\omega = (\omega_1 - \omega_2) = (2000\pi - 1960\pi) = 40\pi$ radians/second). Thus, as time increases, phasor 1 will ***lead*** phasor 2; eventually (at time $t = ½\tau_{beat} = 0.025 = 1/40^{th}$ sec in our above example) phasor 2 will be exactly $\Delta\varphi = \pi$ radians, or 180 degrees behind in phase relative to phasor 1. Phasor 1 at time $t = ½\tau_{beat} = 0.025$ sec $= 1/40^{th}$ sec will be oriented exactly as it was at time $t = 0.0$ (having precessed exactly $N_1 = \omega_1 t/2\pi = 2\pi f_1 t/2\pi = f_1 t = 25.0$ revolutions in this time period), however phasor 2 will be pointing in the opposite direction at this instant in time (having precessed only $N_2 = \omega_2 t/2\pi = 2\pi f_2 t/2\pi = f_2 t = 24.5$ revolutions in this same time period), and thus the total amplitude $A_{tot}(t = ½\tau_{beat}) = A_1(t = ½\tau_{beat}) + A_2(t = ½\tau_{beat})$ will be precisely zero (if the magnitudes of the two individual amplitudes are precisely equal to each other), or minimal (if the magnitudes of the two individual amplitudes are not precisely equal to each other), as shown in the figure below.



As time progresses further, phasor 2 will continue to lag farther and farther behind, and eventually (at time $t = \tau_{beat} = 0.050$ sec $= 1/20^{th}$ sec in our above example) phasor 2, having precessed through $N_2 = 49.0$ revolutions will now be exactly $\Delta\varphi = 2\pi$ radians, or 360 degrees (or one full revolution) behind in phase relative to phasor 1 (which has precessed through $N_1 = 50.0$ full revolutions), thus, the net/overall result is the same as being exactly in phase with phasor 1! At this point in time, $A_{tot}(t = \tau_{beat}) = A_1(t = \tau_{beat}) + A_2(t = \tau_{beat}) = 2A_1(t = \tau_{beat}) = 2A_1(t = \tau_{beat})$, and the phasor diagram looks precisely like that at time $t = 0$.

Thus, it should (hopefully) now be clear to the reader that the phenomenon of beats is manifestly that of time-dependent alternating constructive/destructive interference between two periodic signals of comparable frequency, at the amplitude level. This is by no means a trivial point, as often the beats phenomenon is discussed in physics textbooks in the context of intensity, $I_{tot}(t) = |A_{tot}(t)|^2 = |A_1(t) + A_2(t)|^2$. From the above discussion, the *physics origin* of the beats phenomenon has absolutely *nothing* to do with *intensity* of the overall/ resultant signal.

The primary reason that the phenomenon of beats is discussed more often in terms of intensity, rather than amplitude is that the physics is perhaps easier to understand from the intensity perspective – at least mathematically, things appear more obvious, physically:

$$I_{tot}(t) = \left|A_{tot}(t)\right|^2 = \left|A_1(t) + A_2(t)\right|^2$$
$$I_{tot}(t) = A_{10}^2 \cos^2\left(\omega_1(t)t\right) + A_{20}^2 \cos^2\left(\omega_2(t)t + \Delta\phi_{21}(t)\right) + 2A_{10}A_{20}\cos\left(\omega_1(t)t\right)\cos\left(\omega_2(t)t + \Delta\phi_{21}(t)\right)$$

Let us define: $\vartheta_1(t) \equiv \omega_1(t)t$ and: $\vartheta_2(t) \equiv \left(\omega_2(t)t + \Delta\varphi_{21}(t)\right)$

And then let us use the mathematical identity:

$$\cos\vartheta_1 \cos\vartheta_2 \equiv \tfrac{1}{2}\left[\cos\left(\vartheta_2 + \vartheta_1\right) + \cos\left(\vartheta_2 - \vartheta_1\right)\right]$$

Thus:

$$I_{tot}(t) = A_{10}^2 \cos^2\left(\omega_1(t)t\right) + A_{20}^2 \cos^2\left(\omega_2(t)t + \Delta\phi_{21}(t)\right)$$
$$+A_{10}A_{20}\left[\cos\left((\omega_2(t) + \omega_1(t))t + \Delta\phi_{21}(t)\right) + \cos\left((\omega_2 - \omega_1)t - \Delta\phi_{21}(t)\right)\right]$$

The let us define:

$$\Omega_{21}(t) \equiv \left(\omega_2(t) + \omega_1(t)\right) \qquad \Delta\omega_{21}(t) \equiv \left|\omega_2(t) - \omega_1(t)\right|$$

We then obtain:

$$I_{tot}(t) = A_{10}^2 \cos^2\left(\omega_1(t)t\right) + A_{20}^2 \cos^2\left(\omega_2(t)t + \Delta\phi_{21}(t)\right) + A_{10}A_{20}\left[\cos\left(\Omega_{21}(t)t + \Delta\phi_{21}(t)\right) + \cos\left(\Delta\omega_{21}(t)t - \Delta\phi_{21}(t)\right)\right]$$

Using the above mathematical identity again, we see that:

$$\cos^2\vartheta = \cos\vartheta\cos\vartheta \equiv \tfrac{1}{2}\left[\cos 0 + \cos 2\vartheta\right] = \tfrac{1}{2}\left[1 + \cos 2\vartheta\right]$$

and thus we obtain an additional relation, one which is not usually presented and/or discussed in many physics textbooks, but one which is very interesting:

$$I_{tot}(t) = \tfrac{1}{2}A_{10}^2\left[1 + \cos^2 2\left(\omega_1(t)t\right)\right] + \tfrac{1}{2}A_{20}^2\left[1 + \cos^2 2\left(\omega_2(t)t + \Delta\varphi_{21}(t)\right)\right]$$
$$+ A_{10}A_{20}\left[\cos\left(\Omega_{21}(t)t + \Delta\varphi_{21}(t)\right) + \cos\left(\Delta\omega_{21}(t)t - \Delta\varphi_{21}(t)\right)\right]$$

This latter formula shows that there are:

a.) DC (*i.e*. zero frequency, $f = 0\ Hz$) components (*i.e*. constant terms) present, associated with/arising from both of the individual amplitudes A_{10} and A_{20}.
b.) 2nd harmonic components present with $2f_1$ and $2f_2$, as well as:
c.) a component associated with the *sum* of the two frequencies, $\Omega_{21} = f_1 + f_2$, and:
d.) a component associated with the *difference* of the two frequencies, $\Delta f_{21} = f_1 - f_1$.
This is a remarkably similar result to that associated *e.g*. with the output response from a system having a *quadratic* *non-linear* *response* to a pure/single-frequency sine-wave input! (Please see/read the Physics 406 Lecture Notes on Distortion for more details...)

So, simply stated, beats is a phenomenon where *e.g*. two waves from separate sound sources with slightly different frequencies are combined/allowed to *mix*. The resultant total/overall wave exhibits *interference* between the two waves of slightly different frequencies. We hear this interference effect as an amplitude modulation of the overall envelope of the waveform.

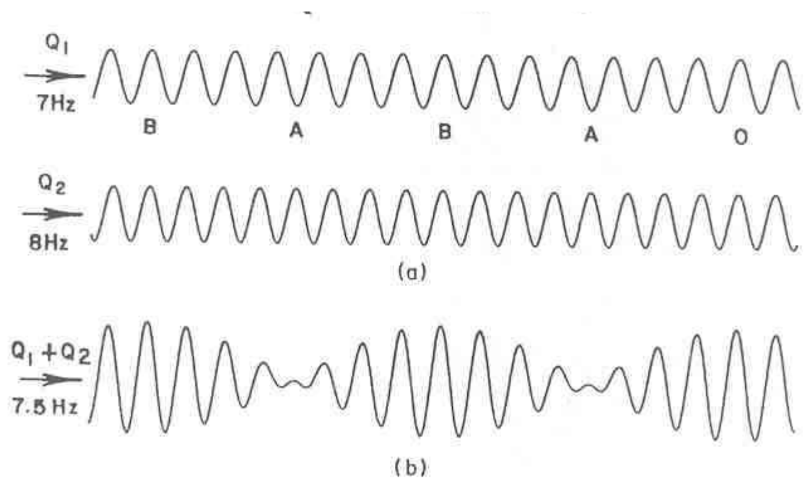


FIG. 20. Production of beats. (a) Waves from two separate sources of slightly different frequency. (b) Resultant wave formed by the superposition of the two separate waves.

We, as human beings hear/perceive the two frequencies as an *average* frequency, $< f_{avg} > = ½(f_1+f_2)$, if the two signals have equal amplitude/intensities, and we hear/perceive the difference frequency as a beat frequency – an amplitude modulation effect of the overall waveform, with $\Delta f = f_{beat} = | f_1 - f_2|$ (= absolute value of the frequency difference). Fundamentally, beats is manifestly an *interference phenomenon* associated with two (or more) waves of *nearly* the same, but not identical frequency.

Note that the phenomenon of beats is *not* associated solely with acoustical physics – for example, in two *entangled* beams of light/EM waves of nearly the same frequency will also exhibit the property known as *optical beats* – this is the principal of operation of *LIDAR* (*LI*ght *D*etection *A*nd *R*anging), as well as Doppler *RADAR* (*RA*dio *D*ectection *A*nd *R*anging). In LIDAR, a laser beam is split into two separate beams, e.g. using a beam splitter – a *reference* beam and a *probe* beam – the latter of which reflects off of a moving object, resulting in a Doppler-shifted frequency (see below), which upon mixing with the reference beam, results in optical beats. Thus, e.g. police use *LIDAR* devices for (very accurately) monitoring the speed of vehicles on interstate highways…

The principal of operation of *LIDAR* and/or Doppler *RADAR* can also be used in an acoustical application, if the probe beam is used to illuminate *e.g*. a small, light-weight aluminized mylar mirror mounted on the cone of a loudspeaker. Then the frequency of the probe beam is Doppler-shifted by the motion of the vibrating cone of the loudspeaker. When recombined with the reference beam, optical beats occurs, and if the ***envelope*** of the overall resultant/total/combined light intensity is detected e.g. using a photodiode, then electrically amplifying the signal output from the photodiode, and output to ***another*** (*i.e*. 2^{nd}) loudspeaker, the sound output from the ***original*** loudspeaker can be heard in the ***second*** loudspeaker!

The Doppler Effect – Frequency Shifts Due To Motional Effects:

Waves and Wave Propagation 45

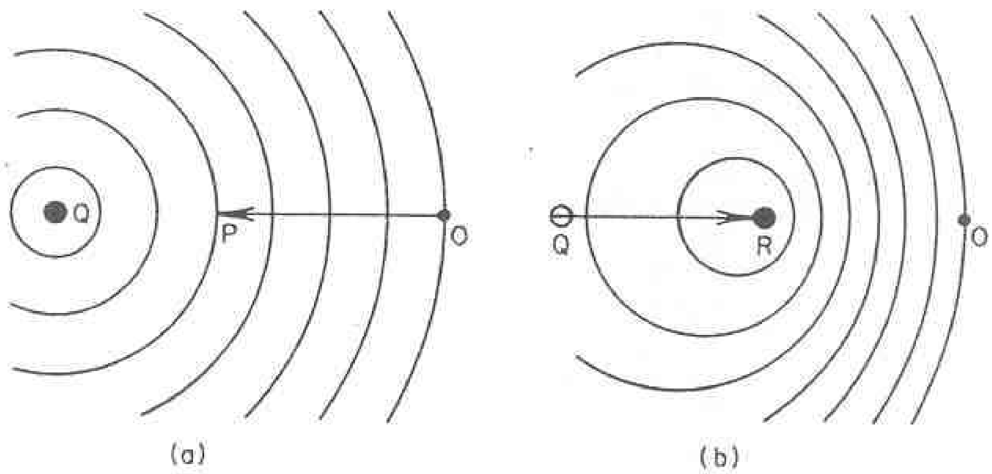


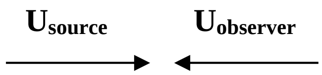
FIG. 14. Doppler effect. (a) Observer moving toward the sound source. (b) Source moving toward the observer.

Simplest Case: Relative Motion of Sound Source and Observer in *1-Dimension*:

Let us first define:
a.) Ground speed = speed of an object ***relative*** to ground (ground is assumed stationary).
b.) Moving sound source has ground speed, U_{source}.
c.) Moving observer has ground speed, $U_{observer}$.
d.) Speed of propagation of sound in medium (e.g. air) has ground speed, V_{medium}.
e.) Sound source emits sound with frequency, f_{source} *in sound source reference frame*.
f.) Moving observer hears/perceives frequency, $f_{observer} \neq f_{source}$ *in his/her reference frame*!

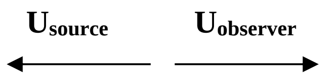
There are four possible/different/distinct cases/situations for the Doppler effect:

a.) Sound source and observer are both moving in *opposite* directions – but *approaching* each other. The *relative motion* of sound source and observer is *toward* each other:



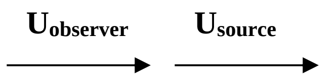
$$f_{observer} = \left(\frac{V_{medium} + U_{observer}}{V_{medium} - U_{source}} \right) f_{source}$$

b.) Sound source and observer are both moving in *opposite* directions – but *receding* from each other. The *relative motion* of sound source and observer is *away from* each other:



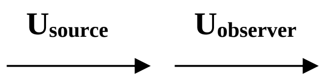
$$f_{observer} = \left(\frac{V_{medium} - U_{observer}}{V_{medium} + U_{source}} \right) f_{source}$$

c.) Sound source and observer are both moving in *same* direction, but the source is *ahead* of the observer:



$$f_{observer} = \left(\frac{V_{medium} + U_{observer}}{V_{medium} + U_{source}} \right) f_{source}$$

d.) Sound source and observer both moving in *same* direction, but the source is *behind* the observer:



$$f_{observer} = \left(\frac{V_{medium} - U_{observer}}{V_{medium} - U_{source}} \right) f_{source}$$

A frequency shift $\Delta f = f_{observer} - f_{source}$ occurs when the sound source and/or observer are in motion with respect to ground reference frame!

The frequency heard/perceived by observer is *higher* if the *relative motion* of the sound source and observer is *toward* each other: $f_{observer} > f_{source}$, thus $\Delta f = f_{observer} - f_{source} > 0$.

The frequency heard/perceived by observer is *lower* if the *relative motion* of the sound source and observer is *away* from each other: $f_{observer} < f_{source}$, thus $\Delta f = f_{observer} - f_{source} < 0$.

For each of above four cases, can get limiting/special cases, e.g. when ground speed of observer, $U_{observer} = 0$ and/or when ground speed of sound source, $U_{source} = 0$.

If there exists a *wind*, then the *component of wind velocity vector projected onto the line of relative motion between sound source and observer must be added (or subtracted) from ground speed of propagation of sound,* V_{sound}. The presence/existence of wind has *no* effect if it is transverse (i.e. perpendicular) to the line defined by the relative motion between the sound source and observer.

Formally, the Doppler effect is actually a 3-D *vector* problem – involving the 3-D velocity vectors of all three items – i.e. the 3-D velocity vectors associated with the sound source, observer and the wind (if present). The above four 1-D formulae are correct only for the *projections* of these velocity vectors onto the line defined by the relative 1-D motion between sound source and observer.

<u>An Example of the Musical Use of the Doppler Effect - The Leslie Speaker Cabinet</u>:

The Leslie speaker cabinet, developed by Don Leslie in ~ 1940 – most frequently used in conjunction with the venerable Hammond B3 organ (but which also can be used with guitar, bass, vocals, harmonica, …) is a 2-way, 2-speed (fast/slow) <u>rotating</u> speaker system (with passive cross-over network) - highs (f_{hi} > 800 Hz) come out of a rotating horn, lows (f_{low} < 800 Hz) emanate from a (fixed, non-rotating) 15" woofer with rotating rotor (black cloth-covered cylinder below the 15" woofer), as shown in the 3 pix below of the back/inside of a Leslie cabinet:



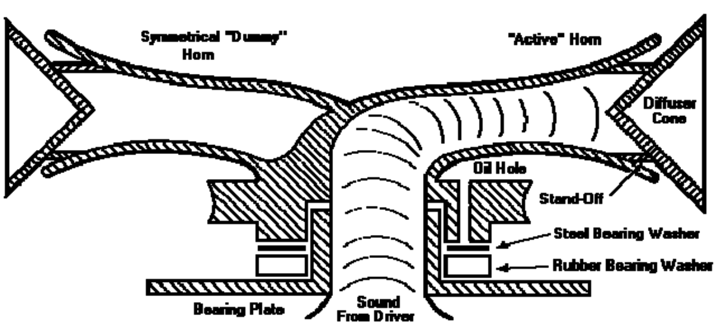


Figure 1. Plan-view of the Leslie Treble Rotor

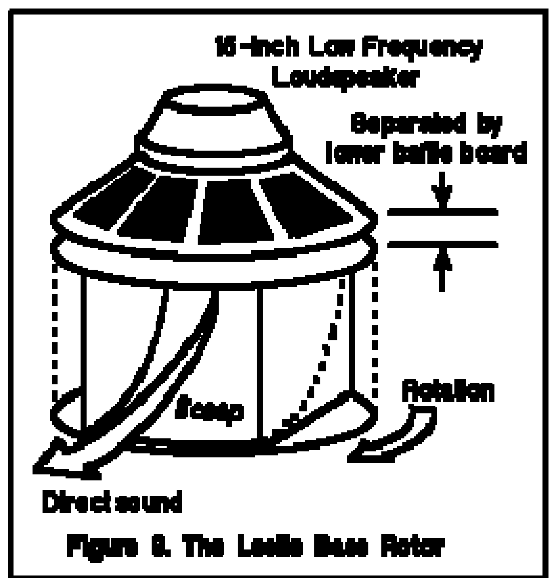
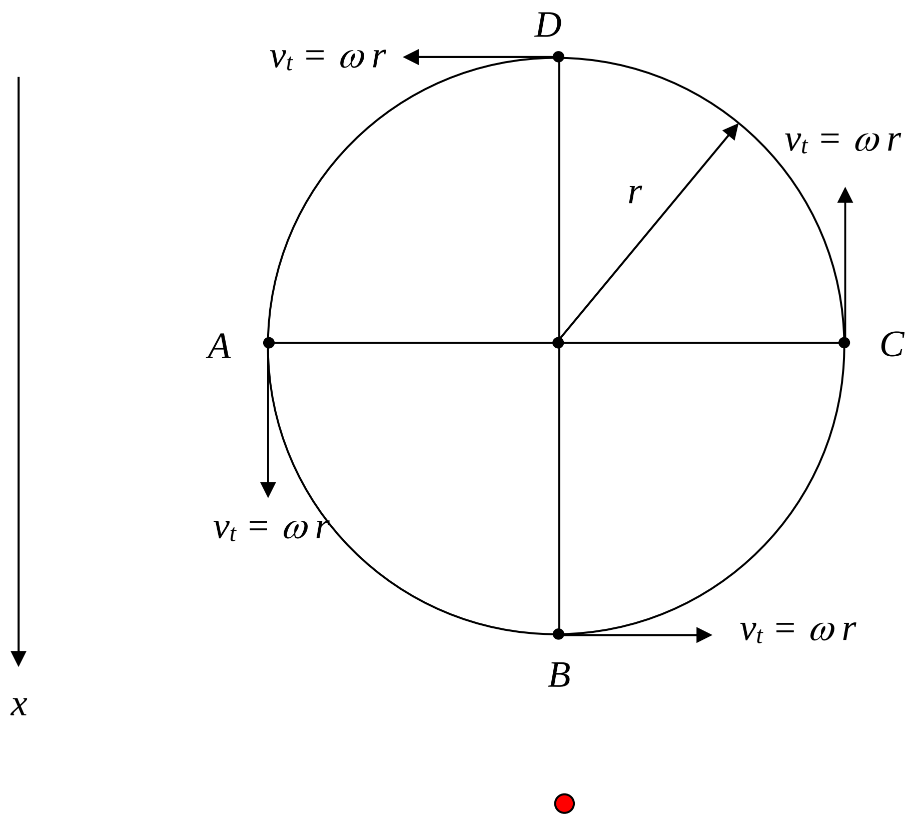


Figure 8. The Leslie Bass Rotor

The single-opening/mouth of the rotating high-frequency horn and the single-opening/mouth of the rotating rotor for the woofer act/behave as (independently) <u>rotating</u> sound sources, rotating at angular frequencies ω_{hi} and ω_{lo}, respectively. Since the tangential velocity of a rotating object of radial size r is given by $\vec{v}_t = \vec{\omega}\times\vec{r}$, and $v_t = |\vec{v}_t| = |\vec{\omega}\times\vec{r}| = \omega r \sin\Theta = \omega r$ (since $\Theta = 90^o$ = angle between the $\vec{\omega}$ and $\vec{r}$ vectors – *i.e.* they are perpendicular to each other), the respective hi/lo frequency rotor tangential speeds are thus $v_{hi} = \omega_{hi} r_{hi}$ and $v_{lo} = \omega_{lo} r_{lo}$.

Note that for a fixed radius rotor, the tangential speed v_t is linearly proportional to the angular frequency ω. Design-wise, for a fixed angular frequency ω the tangential speed v_t is linearly proportional to the rotor radius *r* – hence using a larger diameter rotor will give a larger Doppler effect than a smaller diameter one at a given/fixed angular frequency.



Observer position (far from rotating sound source)

When a rotating sound source of finite radial size *r* is oriented such that it is instantaneously moving directly towards or directly away from a (distant) observer (sound source points *A* and *C*, respectively in the above diagram), the Doppler shift formula a.) and b.) as given above apply at those instants:

At point *A* (source moving directly towards a distant, stationary observer):

$$f_{observer}^{A} = \left(\frac{V_{medium} + U_{observer}}{V_{medium} - U_{source}}\right) f_{source} = \left(\frac{V_{air} + 0}{V_{air} - v_{t\,source}}\right) f_{source} = \left(\frac{V_{air}}{V_{air} - \omega_{source} r}\right) f_{source} > f_{source}$$

At point *C* (source moving directly away from a distant, stationary observer):

$$f_{observer}^{C} = \left(\frac{V_{medium} + U_{observer}}{V_{medium} - U_{source}}\right) f_{source} = \left(\frac{V_{air} + 0}{V_{air} + v_{t\,source}}\right) f_{source} = \left(\frac{V_{air}}{V_{air} + \omega_{source} r}\right) f_{source} < f_{source}$$

At the points *B* and *D*, the orientation of the instantaneous tangential velocity vector of the rotating sound source $\vec{v}_t$ is perpendicular to the sound source – distant observer direction, thus at these instants in time, the observer hears no Doppler shift (up or down in pitch), i.e. $f_{observer}^{B} = f_{observer}^{D} = f_{source}$. Thus, as time progresses, the (distant) observer hears a sinusoidal variation of the frequency over the range: $f_{observer}^{C} \le f_{source} \le f_{observer}^{A}$.

For one full revolution of the Leslie speaker rotor, the *x*-component of the tangential speed of the rotating sound source (refer to above figure) as a function of time is: $\boxed{v_x(t) = v_t \cos\omega t = \omega r \cos\omega t}$ thus the frequency heard by an observer/listener (red dot in the above figure) is:

$$\boxed{f_{observer}(t) = \left(\frac{V_{air}}{V_{air} + v_t(t)}\right) f_{source} = \left(\frac{V_{air}}{V_{air} + \omega r \cos\omega t}\right) f_{source}}$$

A rhythmic, or periodic/sinusoidal variation in frequency (= "pitch" in musical parlance) is known as *vibrato*.

The sound from a Leslie cabinet used inside a room, or an auditorium is actually ***far*** more rich and complex than just that as described above! The reason(s) for this are:

a.) The sound radiated from the each of the rotating speakers of the 2-way Leslie speaker cabinet also reflects off of the walls, floor and ceiling in a myriad of ways – single and multiple reflections, and with correspondingly differing path lengths (hence differing propagation delay times) which depend on the details of the geometry of the room, the Leslie speaker cabinet location and observer/listener location in the room. The indirect, reflected sounds seemingly coming from everywhere in the room will thus have their own specific Doppler shifts in frequency, as dictated by the law of reflection from the wall/floor/ceiling surfaces of the room, which are also heard by the observer, in addition to the vibrato sound coming directly from the Leslie cabinet.

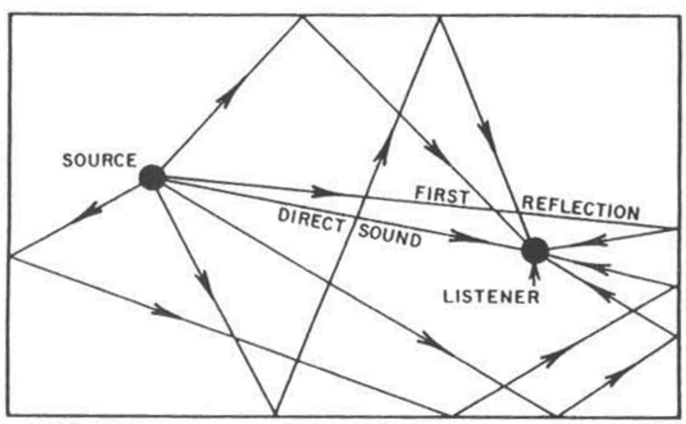


FIG. 1. Multiple reflections from the walls of a room of a single impulse produced by a sound source.

At the listener's position, the Doppler-shifted direct sounds will interfere in a complex manner with the indirect sounds, sometimes constructively/destructively – resulting in a beats-type/amplitude modulation/tremolo effect, especially at lower frequencies, for which diffraction effects from the mouth of the rotating sound source are larger than for higher frequencies {since the Airy disk is within an angular region of $\theta \le \sin^{-1}\left(1.22\lambda/D\right)$ from the instantaneous axis of the mouth of the Leslie speaker rotor}. However for mid-range and higher frequencies, the mixing of direct and indirect sounds from a Leslie operated in a room often results predominantly in more of a flanged/phase-shifted vibrato-type sounding effect.

b.) The musical sound(s) coming from a Leslie cabinet are almost never just a single/pure-tone frequency, but are complex musical sounds – *i.e.* all types of chords, which consist of multiple frequencies, and harmonics thereof. Thus, the direct and indirect sounds associated with a whole hierarchy of Doppler-shifted harmonics associated with these chords are heard.

Thus, the overall sound of a Leslie heard by a listener in a room is an extremely lush-sounding 3-dimensional, texturally shimmering "chorale" type of sound effect, made famous in many rock-and-roll songs over the years, by many talented/gifted Hammond B3 organ players (e.g. Booker T. Jones/Booker T. & The MG's, Matthew Fisher/Procol Harum, "A Whiter Shade of Pale", Billy Preston, Keith Emerson/Nice/ELP, Al Kooper, Greg Allman/Allman Brothers Band, Benmont Tench/Tom Petty & The Heartbreakers, Garth Hudson/The Band, Jon Lord/Deep Purple, Paul Shafer, …), many guitarists (e.g. George Harrison/The Beatles, "Let It Be", and "Lucy in the Sky With Diamonds", Eric Clapton/The Cream, "Badge"), it has also been used e.g. for harp (harmonica players) as well as vocals in various songs. More information on the Leslie cabinet is discussed e.g. in the book "The Hammond Organ – Beauty in the B", by Mark Vail, 2nd Ed., Backbeat Books, 2002. Many websites for Leslies also exist on the internet.

The Leslie sound can be emulated (to a certain degree) e.g. via various types of analog and/or digital electronic FX circuits, however none of these truly fully captures the totality/complexity of the Leslie sound (although they are certainly more convenient to bring to/from a gig, as well as hook up and use in live performances). The most famous of these FX boxes is the Univox Uni-Vibe, as used e.g. by Jimi Hendrix (Band of Gypsys), David Gilmour (Pink Floyd) and e.g. Robin Trower in various of their songs on albums from the late 1960's/early 1970's.

<u>The Univox Uni-Vibe</u>. The pedal controls the speed of the vibrato/chorus effect(s).

Acoustic Energy & Acoustic Power in Sound Waves

In order to create an acoustic "disturbance' in a medium (gas, liquid, solid..), must input/expend energy. The energy supplied to create the acoustic wave travels with the wave as it propagates.

Process whereby acoustic energy is carried away from the sound source is called RADIATION.

In order to make a sound source radiate, *e.g.* a constant over-pressure amplitude sound wave requires a certain amount of energy input per unit time into the sound source – *i.e.* power, $P(t)$:

$$\text{Power, } P(t) = \frac{\Delta E(t)}{\Delta t} = \frac{\partial E(t)}{\partial t} = \text{time rate of change in energy} = \textit{Joules/second} = \textit{Watts}$$

The sound source then radiates sound energy; the acoustic power in the sound wave is expressed in acoustic Watts.

TABLE II
Measured greatest power outputs of some musical instruments

	POWER OUTPUT, WATTS
Large orchestra	67
Bass drum	25
Snare drum	12
Cymbals	9.5
Trombone	6.4
Piano	0.44
Trumpet	0.31
Tuba	0.20
Double bass	0.16
Flute	0.055
French horn	0.053
Clarinet	0.050

Electrical (and/or mechanical) power input to the sound source = power supplied (Watts)

Note that the efficiency for conversion of *e.g.* electrical power into acoustical power, *e.g.* using a loudspeaker is not very high:

$$\text{Efficiency} \equiv \frac{\text{acoustic power}}{\text{power supplied}} \sim 1\text{-}2\% \Leftarrow \text{typical efficiency for loudspeakers!}$$

Thus, *e.g.* for a 100 Watt (rms) guitar amplifier, the power rating of the amplifier actually refers to the ***electrical*** power driving the loudspeaker(s) of the amp; the actual ***acoustic*** power radiated by loudspeakers of the 100 Watt (rms) guitar amplifier is typically only ~ 1-2 acoustic Watts (rms)!

Relation Between Sound Intensity, *I* and Radiated Acoustic Power, *P*

Sound intensity $I \equiv$ acoustic power P radiated from sound source *per unit area* A.

Sound *e.g.* from a "point" sound source is radiated over the surface area of (an imaginary) sphere of radius R centered on the sound source. The sound intensity I at a radial distance $r = R$ from the sound source is thus:

$$I(r=R) = P/A_{Sphere} = P/4\pi R^2 \quad \text{(SI units: } Watts/m^2\text{)}$$

Waves and Wave Propagation

Sound energy is conserved, and propagates radially outward in all directions from the point sound source.

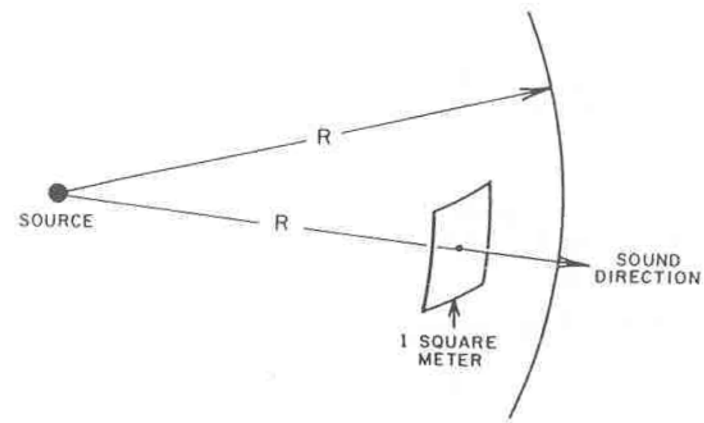


FIG. 21. Sound intensity at a distance from a small source.

Sound Absorption:

Sound energy can also be *absorbed* in propagating through a medium, and/or upon *reflection* from a surface.

The transmitted and/or reflected sound intensity is in general less than incident sound intensity:

$$I_{Incident} = I_{Transmitted} + I_{Reflected} + I_{Absorbed}$$ (from conservation of energy)

Define: $a \equiv$ sound absorption coefficient $\equiv I_{Absorbed}/I_{Incident}$

$$0 \le a \le 1$$

$a = 0$: no sound absorbed
$a = 1$: sound completely absorbed

The amount of sound absorption in a given material depends on the detailed nature of the material and also the frequency, *i.e.* in general the absorption coefficient $a = a(f)$.
We will discuss this further in subsequent lecture(s), *e.g.* on auditorium/room acoustics.

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