HELPMAN

LECTURE NOTES ON INTERNATIONAL TRADE

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This document contains lecture notes from ECON 2530a - International Trade, taught by Professor Elhanan Helpman at Harvard University, Fall 2015. Any error found in this document is presumably ours.

Contents

1 Trade, Welfare, Comparative Advantage

1.1 Introduction

- World trade has witnessed a remarkable growth since World War II. The ratio of trade to GDP has increased.
  - This is not a continuous process. Interwar period halted 'first globalization' during 1870-1914
  - Trade in manufactures and services has grown particularly fast.
  - There exists sizable differences in exports to GDP ratios across income groups: 24 % for low income countries, 37 % for middle income, 42 % for high-income countries.
- Some of the interesting facts on the trade pattern are as follows
  - About half of world trade is among the developed countries. Only 12 % among the developing countries
  - There exist large shares of intra-industry trade
  - Countries tend to trade with close neighbors
  - Fragmentation of production has been a major factor in the growth of trade and FDI. About 1/3 of world trade is intra-firm.
  - Only a small fraction of firms export.

1.2 Trade and Welfare

- We first start with a neoclassical model of international trade and derive its welfare implications. The following three assumptions serve as the main pillar of the model.
  - Convex technologies and preferences
  - Perfect competition
  - Representative individual in every country.

Theorem 1. *Every country gains from trade*

Proof. Let X, C, p, u be output,consumption, price and utility in a free-trade equilibrium while we use superscript A for corresponding autarky variables. The core inequality we have to prove is the following:

$$e(p, u^A) \leq p \cdot C^A = p \cdot X^A \leq p \cdot X = e(p, u)$$

where $e(p, u) := \min_C \{p \cdot C : u(C) \geq u^A\}$ denotes the minimum expenditure function. The first inequality follows from the definition of $e(p, u^A)$ and the first equality from the notion of autarky. The second equality follows from GDP maximization implied by competition[1] and the last equality from trade balance[2] □

[1] Taehoon: Why does competition imply GDP maximization? What would be the underlying mechanism?

[2] Taehoon: Does every country gain from trade even with a persistent trade imbalance? (e.g. Global Imbalance during the last decades)

- The implication of the theorem can be illustrated in a two-good economy. Notice how the convexity assumption plays an important role here. [3]
- However, the welfare implication of the model hinges on the fact that all individuals have the same utility and technology.
  - Suppose a case where one of two individuals has the endowment for good 1 only and the other individual has the endowment for good 2 only. Then whatever price a free-trade equilibrium achieves, one of them (whose relative price goes down) must be worse off.
- Two questions arise. (i) Can we always make everyone better off through taxation and subsidy? (ii) Can we design such a taxation scheme that does not elicit personal information?
- As for the first question, one easy way is to give lump-sum transfers by the amount that achieves the original income.
  - Let individual h receive a net transfer t^h such that $I^h + t^h = p \cdot C^{hA}$ where I^h is his income in the trade equilibrium.
  - Summing over all individual yields

    $$p \cdot X + \sum_h t^h = p \cdot C^A = p \cdot X^A$$

    Therefore

    $$\sum_h t^h = p \cdot X^A - p \cdot X \leq 0$$

    which means that the government has a budget surplus. The surplus can be distributed to make individuals better off.
  - This kind of lump-sum transfer, however, needs to elicit personal information from individuals so the incentive-comparability issues may arise (e.g. agents do not report truthfully)
- Dixit and Norman (1986) proposed an anonymous tax and subsidy scheme that can achieve Pareto gains from trade.

[3] Helpman: The more the relative price changes after trade liberalization, the more a country gains in general

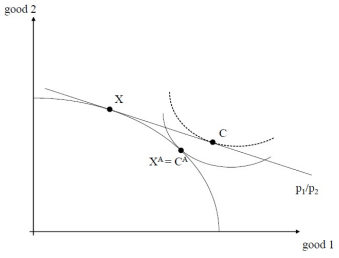


Figure: A two-good example

- The idea is to allow good and factor prices for producers to move to the free trade levels but hold good and factor prices for consumers fixed at their autarky levels.
- Let us denote the autarky prices by (p^a, w^a) and the equilibrium prices with free trade by (p, w). We let firms face (p, w) and set the taxes (vector) on goods at $(p^a - p)$ and the subsidies on factors $(w^a - w)$

1.3 Comparative Advantage

- The main theorem of this section is the following. Given a vector of autarky prices, the theorem gives a testable prediction for the effect of trade liberalization and comparative advantage.

  Theorem 2. *Let $M^k \equiv C^k - X^k$ denote the vector of net imports of country k. The following conditions hold in a free-trade equilibrium.*

  - $p^{Ak} \cdot M^k \geq 0$ *for all* k.
  - $p \cdot M^k = 0$ *for all* k.
  - $\sum_k M^k = 0$

  Proof. Condition 2 and 3 simply reflect trade balance and market clearing. As for condition 1, we need to prove (dropping superscripts k)

  $$p^A M = p^A \cdot C - p^A \cdot X \geq p^A \cdot C^A - p^A X = p^A \cdot X^A - p^A X \geq 0$$

  The first inequality holds because gains from trade imply $p^A \cdot C^A \geq p^A \cdot C$ and the last inequality results from the fact that X^A maximizes GDP in autarky. □

- The basic law of comparative advantage is often written as

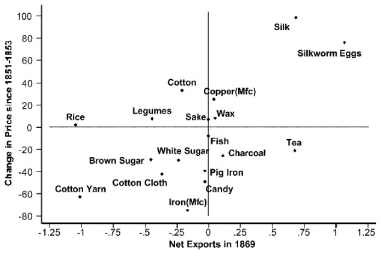
  $$(p^{Ak} - p) \cdot M^k \geq 0 \text{ for all } k$$

  According to this law, there is a positive association across products between the price difference and net imports 'on average'.

  - In a two-country, two-good model, the association can be represented as follows. $M_i^k > 0$ if and only if $p_i^{Ak} > p_i^{A(-k)}$. Or $M_1^H > 0$ if and only if $\frac{p_1^{AH}}{p_2^{AH}} > \frac{p_1^{AF}}{p_2^{AF}}$

- Bernhofen and Brown (2004) provide an empirical validation of this law using historical Japanese data in the late 19th century.

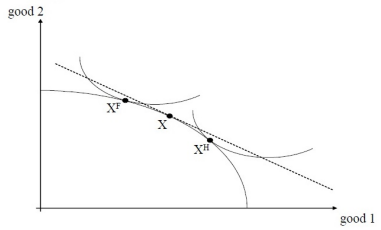
  - Japan was in autarky for more than 200 years, until 1859, when it opened up to trade
  - Their main identifying assumption is that one can use autarky prices from 1951-53 to proxy the counterfactual autarky prices in 1868-75.

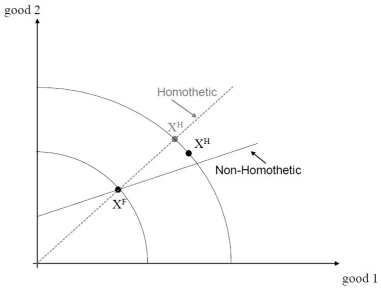


  - They confirmed a positive relation between the price change and the net export. (They use $p - p^{Ak}$ and net export instead of $p^{Ak} - p$ and net import)

- The basic law of comparative advantage highlighted the role of differences in autarky prices in the determination of trade patters and trade volumes
  - with $p^{Ak} = p^A$ for all k, we have no trade
- We can identify three fundamental sources of autarky price differences across countries.
  - Differences in tastes
  - Differences in technologies
  - Differences in endowments
  - Caveat: since most trade seems to flow between similar countries, the above models may not be useful. Still, neoclassical trade theories provide an essential benchmark.
- First, preferences may shape comparative advantage if (i) preferences themselves differ across countries or (ii) preferences are identical worldwide but non-homothetic.
  - An example of (i) is illustrated on the right side. Both countries share the same endowments and technologies, but country H shows a relative preference for good 1, and consequently p_1^A / p_2^A is higher there.
  - As illustrated in the figure in a trading equilibrium country H will import good 1 and export good 2.
  - (ii) Now consider an example with non-homothetic preferences. Preferences are such that the income elasticity is larger than one for good 1. Assume H is richer in terms of income per capita. Then its demand function is tilted towards good 1 and again p_1^A / p_2^A is higher there.
  - As before, in a trading equilibrium country H imports good 1 and exports good 2.
  - A good example of such a preference is the Stone-Geary type, which has a linear Engel Curve

$$U = \alpha \log c_1 + \beta \log(c_2 - b)$$





- Although the literature has largely downplayed the role of preferences since one can generate any pattern with arbitrary differences in preferences, there exist interesting contributions highlighting the role of non-homotheticities.

  - For example, Hunter (1991) suggests that they an explain 20 percent of trade flows. Recent works use more detailed micro data.
  - Fajgelbaum, Grossman and Helpman (2011) use a discrete choice model together with vertical and horizontal product differentiation to highlight the role of income distribution differences across countries an impetus for trade.

- Second, Ricardo highlighted the role of technological differences in shaping the pattern of trade with a famous numerical example.
  - Consider a world with 2 countries (H and F), two goods and one factor of production, labor.
  - Technology is summarized by four unit input requirements a_i^k for $k = H, F, i = 1, 2$.
  - **(Result)** Country H exports good 1 if $\frac{a_1^H}{a_2^H} < \frac{a_1^F}{a_2^F}$ because the fraction reflects the relative autarky price between good 1 and good 2. That is, $\frac{a_1^H}{a_2^H} = \frac{w^{AH} a_1^H}{w^{AH} a_2^H} = \frac{p_1^{AH}}{p_2^{AH}}$ where w is a unit cost of labor, p_i is a price for good i.

- Dornbuch et al (1977) extended Ricardian model to a case of two-country and a continuum of goods.

1.4 (3) Literature Review

An Empirical Assessment of the Comparative Advantage Gains from Trade: Evidence from Japan (Bernhofen and Brown 2005)

Main Idea	This paper provides an empirical assessment of the comparative advantage gains. In theory, the gain from trade can be measured only with the use of autarky prices, which rarely exist in the real world. The authors take notice of Japan in the nineteen century. It was one of the few cases where a completely autarky state opened up international trade. To measure the welfare gain, they use the Slutsky compensation measure i.e. $$\Delta W_{1850s} = \mathbf{p}^a_{1850s} \cdot \mathbf{c}^f_{1850s} - \mathbf{p}^a_{1850s} \cdot \mathbf{c}^a_{1850s} \quad (*)$$ Here, p is a vector of prices, c is a vector of consumption and superscript f, a denote free trade and autarky states respectively. Because $\mathbf{c}^a_{1850s}$ does not exist in the data, what they do is to get an upper bound for ΔW_{1850s}. It can be easily seen by some algebra that the following upper bound holds. $$\frac{\mathbf{p}^a_{1850s} \cdot \mathbf{c}^f_{1850s} - \mathbf{p}^a_{1850s} \cdot \mathbf{c}^a_{1850s}}{GDP_{1850s}} \leq \frac{\mathbf{p}^a_{1850s} \mathbf{T}_{1850s}}{GDP_{1850s}}$$ where $\mathbf{T}_{1850s}$ is the counter-factual net import vector. ($\mathbf{T}_{1868-1875}$ is used as a proxy) They obtain the result that that the right hand side is 8 to 9 percent and suggest evidence that the welfare gain seems to be close to this bound.
Data	Data from Japanese history research papers. (I skip the detail since we will probably never use them in our research)
Professor's Remark	N/A
Comments	(Taehoon) I feel like the estimate is disappointingly small, taking into account that the trade liberalization made a huge impact on Japan's economy back then. Can we get the welfare implications of free trade in a dynamic sense? like how they affect the future growth rate. I suppose there should be some lines of research on this.

A Direct Test of the Theory of Comparative Advantage: The Case of Japan (Bernhofen and Brown 2004)

Main Idea	The basic law of comparative advantage predicts that the inner product of autarky price vector and the net export vector should be negative. This paper seeks to provide empirical evidence for this conjecture by using the trade liberalization of Japan in the nineteenth century. Under the assumption that the counter-factual autarky price in 1868-1875 did not change much from the actual price levels in 1850s, they calculated eight years' of inner-products $$p^A_{1850s} T_i, \quad i = 1868\ 1875 \text{ each year}$$ where p^A_{1850s} is the autarky price in 1850s and T_i is the net export in year of i. They confirmed the negative relationship for all the eight years respectively.
Data	Data from Japanese history research papers. (I skip the detail since we will probably never use them in our research)
Professor's Remark	N/A
Comments	(Taehoon) What about trade balance in a free-trade equilibrium? This is another prediction proposed by the basic law of comparative advantage but they didn't checked it for Japan's case. Actually, come to think of it, the mere existence of the current trade imbalance may be the simplest counter example that the comparative advantage does not hold completely in practice.

2 Classical Trade Theories

Every country gains from trade in a static model presented in the previous chapter. We also arrived at a general result that **countries export goods that have relative prices going up, and import goods that have relative prices going down after opening up trade**. However, the question remains: what makes prices different in autarky? The potential answers are: (1) difference in tastes (downplayed in the literature), (2) difference in technologies, and (3) difference in endowments.

In this chapter, we survey two classical models: the Ricardian model, which focuses on technological differences, and the Heckscher-Ohlin model, which focuses on endowments differences.

2.1 Ricardian Model and Technological Differences

General setup:

- Two countries, one factor of production (labor).
- Labor is mobile between sectors within a country, but not across country.

Therefore, any good produced at home must pay the same for labor

2.1.1 *Two-good model*

Production of good i requires unit requirement of labor a_i at home and a_i^* abroad. We assume that $\frac{a_1^*}{a_1} > \frac{a_2^*}{a_2}$, i.e. country 1 has *comparative advantage* in producing good 1.

Let the prices of two goods be p_1 and p_2 respectively, and assume $\frac{p_1}{p_2} = p$. Wage is equal to marginal product of labor, so $w = \frac{p_1}{a_1} = \frac{p_2}{a_2}$, due to wage equalization across industries. Under our assumption, we immediately have $p^A = \frac{a_1}{a_2} < p^{*A} = \frac{a_1^*}{a_2^*}$.

In autarky, due to wage equalization across sector, the sector with comparative advantage has higher marginal producty of labor, hence its price must be lower.

Now, let two countries engage in trade. The only case when trade balance can be satisfied is when the new relative price $p \in (p^A, p^{*A})$ (otherwise, both countries would export the same good, or import the same good).

With this new relative price, country 1 exports good 1, and country 2 exports good 2.

2.1.2 *Continuum of goods Model (Dornbusch, Fischer, and Samuelson (1977))*

Assume now that there is a continuum of good $z \in [0,1]$. Same as before, let $a(z)$ and $a^*(z)$ be the unit labor requirement of good z at home and foreign country, respectively. WLOG, denote $A(z) = \frac{a^*(z)}{a(z)}$, and assume that $A'(z) < 0$. Denote $p(z)$ the price of good z under trade.

$A(z)$: index of comparative advantage of home over foreign. Home has most comparative advantage in producing good 0, and least in good 1.

Supply: Which good is produced at home? A good is produced at home if and only if

$$a(z)w \leq a^*(z)w^* \Leftrightarrow z \leq A^{-1}\left(\frac{w}{w^*}\right) \equiv A^{-1}(\omega) \equiv \tilde{z}(\omega)$$

In autarky, the price of good z at home is $p^A(z) = wa(z)$, and abroad is $p^{*A}(z) = wa^*(z)$.

Hence, home produces goods $[0, \tilde{z}(\omega)]$ and foreign produces goods $[\tilde{z}(\omega), 1]$.

Relative home/foreign wage is sufficient statistic to determine which good is produced where.

Demand: Assume identical Cobb-Douglas preference for both countries. So both of them have fixed budget share, which is also the world's budget share. Denote $b(z) \geq 0$ the share of world income spent on good z, and $\int_0^1 b(z)dz = 1$. Then, the fraction of world income spent on good produced at home is $\theta(\omega) = \int_0^{\tilde{z}} b(z)dz > 0$.

Equilibrium: This works through balance of trade: ω adjusts to equalize home's income and expenditure (and vice versa, foreign):

$$wL = \theta(\tilde{z}(\omega))(wL + w^*L^*) \Leftrightarrow \omega L = \theta(\omega)(\omega L + L^*) \Leftrightarrow \boxed{\omega = \frac{\theta(\omega)}{1-\theta(\omega)}\frac{L^*}{L}}$$

There is a unique relative wage ω^* that achieves this equilibrium.

LHS increase in ω, LHS decreases in ω.

2.1.3 *Comparative Statics & Extensions of D-F-S Model*

- **Population growth:** Suppose foreign population increases: $L^* \uparrow$. To clear market, we must also have $\omega \uparrow$. So home loses industries ($\tilde{z} \downarrow$), but per capita income rises. Foreign gains industries, but is worse-off.

Increase in foreign population creates a labor surplus in the foreign country, and labor demand in home country, which creates a trade surplus for home country. Therefore, relative wage must rise to eliminate that trade surplus.

- **Technical Progress:** A uniform decrease in $a^*(z)$ reduces comparative advantage, lowering relative wage (home resident worse off) and transfering industries to foreign $\tilde{z} \uparrow$.

2.1.4 *Other remarks about D-F-S*

- Welfare effect on home was purely through term of trade effect.
- D-F-S predicts a simplified version of the gravity equation:

$$\text{Trade volume} = 2\frac{Y^H Y^F}{Y^W}$$

Denote indirect utility $v[p, R(p,V)]$ where V is vector of factors, and R is the revenue function. Then, differentiating wrt p gives $dv/v_I = -\sum_i M_i dp_i = -\sum_i (p_i M_i)\hat{p}_i$. So, we have a positive dv iif weighted sum (by export shares) of export prices exceeds weighted sum (by import shares) of import prices.
Any model with complete specialization, homothetic preferences, and no trade barriers delivers the gravity equation prediction.

- D-F-S also analyzes economies with melting iceberg transport costs. The specialization pattern is then [home exports][non-traded][foreign exports].

- Costinot (2010) extends D-F-S to many countries using the log-modularity approach.

2.2 Heckscher-Ohlin Model and Endowment Differences

2.2.1 *Preliminaries*

- A country can produce two goods with technologies $y_i = f_i(L_i, K_i)$, $i = 1, 2$. **Important assumptions:** $f_i(\cdot, \cdot)$ are increasing in each argument, concave, and homogeneous of degree one in the in the inputs (L_i, K_i).

Concavity of production function implies that production possibilities set is **convex**
$f(L, K)$ is homogeneous of degree 1 implies that $f_L(L, K)$ is homogeneous of degree zero: $f_L(L, K) = f_L(L/K, 1)$

- Total endowments: $L_1 + L_2 \leq L$ and $K_1 + K_2 \leq K$.
- **Goods market**: Denote p to be the vector of prices. We have **GDP function**:

$$G(p_1, p_2, L, K) = \max_{y_1} p_1 y_1 + p_2 h(y_1, L, K)$$

GDP maximization and Envelope Theorem imply that

$$\boxed{\frac{p_1}{p_2} = -\frac{\partial y_2}{\partial y_1}} \qquad \boxed{\frac{\partial G}{\partial p_i} = y_i}$$

$y_2 = h(y_1, L, K)$ is the PPF

Vu: Taehoon, perfect competition does lead to GDP maximization: with perfect competition, the economy produces at the tangent point between the PPF and the price vector. Say, if the economy is at the point that is over producing good 2 and underproducing good 1 compared to the GDP-maximizing amount, then good 1 is overvalued at this price, so firms that are producing good 2 would keep switching to produce good 1 until we reach the optimal level.

- **Factors market:** Given a vector of factor prices, define the unit cost functions:

$$c_i(w, r) = \min_{L_i, K_i} \{wL_i + rK_i | f_i(L_i, K_i) \geq 1\}$$

Denote a_{iL} and a_{iK} the optimal labor and capital choice for good i, respectively.

The Envelope theorem implies:

$$\boxed{\frac{\partial c_i}{\partial w} = a_{iL}} \qquad \boxed{\frac{\partial c_i}{\partial r} = a_{iK}}$$

minimum cost to produce 1 unit of good i. Due to CRS assumption, unit cost = marginal cost = average cost

- **Equilibrium** depends on two conditions:
  - Free-entry: This implies zero profit. That means $p_i = c_i(w, r)$.
  - Full employment: This means no slack resource.

$$\begin{bmatrix} a_{1L} & a_{2L} \\ a_{1K} & a_{2K} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} L \\ K \end{bmatrix}$$

In matrix notation: $Ay = V$

A: matrix of efficient factor choice for each industry derived from international factor prices (w, r), y: optimal output derived from international price p, V: vector of factor endowments.

- **Factor prices determination**: $p_i = c_i(w, r)$ and $A(w, r)y(p) = V$ gives us four equations and four unknown w, r, y_1, y_2.
- **Lemma (Factor Price Insensitivity):** If both goods are produced and factor intensity reversals (FIRs) do not occur, then each (p_1, p_2) maps one-to-one to a unique factor price vector (w, r).

Read next section for the discussion of factor price reversal and factor price equalization.

- **Factor Price Equalization Theorem (Samuelson 1949):** If two countries are engaged in free trade, having identical technologies but different factor endowments. If both countries produce both goods, and FIRs do not occur, then the factor prices (w, r) are equalized across countries.

This result does not hold in the case of one good. In the case of one good, country that is abundant in labor would have lower wage. To see this, recall that $w = pf_L(L, K)$, and since $f_{LL} < 0$, increasing L lowers w.

In the case of two goods, the labor-abundant country can produce more and export the labor-intensive good. In that way, it can fully employ its labor while still paying the same wages as a capital-abundant country.

2.2.2 *Heckscher-Ohlin-Samuelson Model*

Imagine there are two countries H and F, each has the same set of preliminaries as above. In autarky, the relative demand of two goods given a vector price (p_1, p_2) in each country is the same (assuming same population, otherwise, would scale proportionally). Assume also that good 1 is labor intensive: $a_{K1}(w)/a_{L1}(w) < a_{K2}(w)/a_{L2}(w)$ (Note: the price (p_1, p_2) maps to a unique (w, r) under the factor price insensitivity lemma when we have no FIR and both goods are produced. Normalize $r = 1$. WLOG, one good will be labor intensive, the other will be capital intensive.)

The country with relative abundance in labor can produce relatively more labor-intensive good, and vice versa. (In other words, different PPF shapes).

It is easy to see from the figure above that the country abundance in labor needs to have higher relative price for the labor-intensive good in autarky. From our previous result, when these two countries engage in trade, the country with labor abundance will export the labor-intensive good.

Heckscher-Ohlin Theorem: The country relatively rich in labor exports labor-intensive products, while the country relatively rich in capital exports capital-intensive products.

2.3 Discussion of Factor Price Equalization

3 *Increasing Returns and Monopolistic Competition*

Neoclassical trade theory exhibits a few uncomfortable features:

- stresses countries' asymmetries, while a bulk (70%) of trade is **intra-industry**; (Ricardo: technology differences; HO: endowment differences)
- hard to generate predictions about bilateral trade flows;
- misses important aspects of trade liberalization (within industry reallocation);
- hard to think about firms (intrafirm trade; multinational activity).

In this chapter, we will address these caveats by two modifications:

- increasing returns to scale.
- imperfect competition and product differentiation.

3.1 External Economies of Scale

Note that we *cannot* have increasing returns under perfect competition, unless that increasing returns effect is *external* to firms.

If profit of firm $\Pi(p, w) > 0$, other firms will enter, which implies profits is always zero. Same goes for a firm trying to double its plant size; profit is still zero. Therefore we cannot have increasing returns under perfect competition.

To formulate external effect, assume we have the cost function

$$c_i(w, \zeta) \equiv \min_{v_i}(w \cdot v_i | f_i(v_i, \zeta) \geq 1)$$

where ζ is a vector of external effects (can be domestic or foreign). Standard assumption: $\zeta = X_i$, the industry **total** output, and that $\partial c / \partial \zeta < 0$.

Infinitessimal firms have measure o and do not internalize their effects on ζ. Given any ζ, firms have constant return to scale production $f_i(v_i, \zeta)$.

Equilibrium conditions in autarky and in trade are same as before: zero profit, and full employment; however, now we have ζ^A and ζ^T in every condition.

Autarky equilibrium in country j:

$$p_i^{Aj} = c_i(w^{Aj}, \zeta^{Aj}) \qquad \text{and} \qquad \sum_{i \in I} a_{li}^j(w^{Aj}, \zeta_i^{Aj}) = V_l^j$$

Trade equilibrium:

$$P_i \leq c_i^j(w^j, \zeta) \qquad \text{equality if } i \text{ is produced in } j, \text{ and} \qquad \sum_{i \in I} a_{li}^j(w^j, \zeta) X_i^j = V_l^i$$

Implications of external effects:

- Replication of integrated equilibrium requires that an industry with external effects to be produced in **one** country.

- FPE set may look differently (see figure below), may not include 45 degree line, and no longer convex.

Since FPE may not include 45 degree line, we may need country dissimilarities in factor composition for factor price equalization.

- equilibrium is not unique (even if FPE holds).

- countries with identical relative factor endowments may still feature positive trade flows.

- gains from trade are not ensured, unless every country's GDP is higher under ζ^T than under ζ^A.

- HO theorem may fail even in the $2x2x2$ case. Countries that are relatively more endowed in capital may not even have the capital-intensive industry.

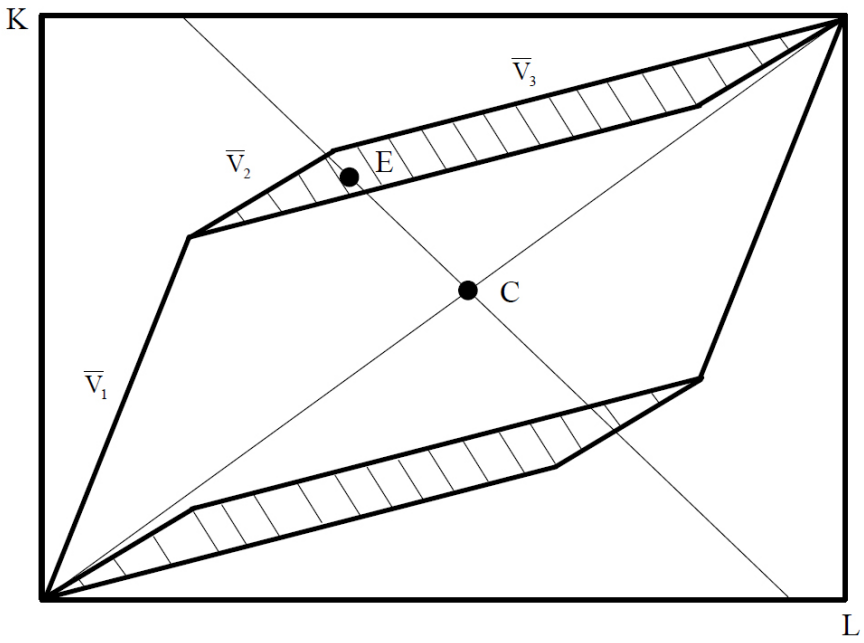
See Helpman (1985), pg 62.

KRUGMAN (1995)'S COMMENTS on external effects: it is unsatisfying for three reasons:

1. How would you recognize these external effects if you saw them?

2. Yields a bewildering variety of equilibria. Therefore, it leaves the modeler with a taxonomy rather than a clear set of insights.

3. External effects work modify or distort pattern of specialization away from that implied by resource abundances. Given the dominance in comparative advantage thinking, most international economists think of this effects as being minor.



Note that the Vanek equation still holds in this case. Antweiler and Trefler (2002) used this to estimate external economies of scale.

3.2 Monopolistic Competition

Another way to model trade in the presence of increasing returns is **monopolistic competition**. The idea is that there are a limited numbers of "products" (industries), each of which can be divided into many differentiated "varieties."

At the firm level, each variety is produced and priced to maximize profits, taking as given the variety choice and pricing strategy of the other producers in the industry. When people have a demand structure that demonstrates "love for varieties," each country will produce different varieties of the same product, while every variety is demanded in both countries. This easily generates intra-industry trade.

3.2.1 *Product Differentiation*

A consumer who chooses goods from I industries has preference

$$U = U(u_1(\cdot), ..., u_I(\cdot))$$

where each u_i depends on the choice of quantity of each variety

$$u_i(x(\omega)) = \left[\int_0^{n_i} x(\omega)^{\alpha_i} d\omega\right]^{1/\alpha_i}, \quad \alpha_i \in (0,1)$$

Note that $\sigma_i = \frac{1}{1-\alpha_i}$ represents the elasticity of substitution between

It is easy to derive σ from definition: $\sigma = \partial \ln(x(k)/x(l))/\partial \ln MRS_{k,l}$. Since $U_k/U_l = (x(k)/x(l))^{\alpha_i - 1}$, we have $\ln(x(l)/x(k)) = \sigma \ln(U_k/U_l)$ where $\sigma = 1/(1-\alpha)$

varieties of i. Given wealth E_i devoted to sector i, demand for variety $\omega \in [0, n_i]$ is

$$\boxed{x_i(\omega) = \frac{E_i}{P_i^{1-\sigma_i}} p(\omega)^{-\sigma_i}}$$

where $P = \left[\int_0^{n_i} p^{1-\sigma_i} dw\right]^{1/(1-\sigma_i)}$ is the ideal price (i.e. minimum cost of obtaining one unit of utility).

3.2.2 *Krugman (1980)*

Assume that there is only one sector. Following the previous analysis, demand for variety ω is

$$x(\omega) = \frac{E}{P^{1-\sigma}} p(\omega)^{-\sigma}$$

Facing this demand curve, a monopolistic firm chooses price to maximize total profit

$$\Pi(\omega) = \left[p(\omega) - \frac{w}{\psi}\right] x(\omega) - wf$$

where wf is the fixed cost of starting business, $1/\psi$ is the amount of labor required to produce one unit of ω (hence ψ is productivity per unit factor), and w is the wage. The optimal price is therefore

$$\boxed{p(\omega) = \underbrace{\frac{\sigma}{\sigma - 1}}_{\text{constant markup}} \frac{w}{\psi}} \tag{3.1}$$

As $\sigma \to \infty$, i.e. $\sigma/(\sigma-1) \to 1$, varieties become perfect substitutes, so this problem reduces to the case of perfect competition. That is, monopolistic firm's price now is exactly w/ψ, the marginal cost.

Free-entry (or zero-profit) condition now becomes $p(\omega)x(\omega) = \sigma w f$, or

$$\boxed{x(\omega) = (\sigma - 1) f \psi} \tag{3.2}$$

Note that this pricing and quantity choice only depends on wage, technology (ψ), and fixed cost f. Therefore, when computing trade equilibrium or autarky equilibrium, we still have these prices and quantities for each variety.

Autarky equilibrium: Labor market clearing requires

$$\underbrace{n}_{\text{num. of varieties}} \cdot \underbrace{(f + x/\psi)}_{\text{labor cost per variety}} = L$$

which implies

$$\boxed{n = L/(\sigma f)} \tag{3.3}$$

Number of varieties increase linearly with L. More people gives more labor, hence more varieties are produced as a result of extra labor. This is the "love for variety"

Trade equilibrium: Suppose two countries H, F only differ in populations L^H, L^F. We still have $x(\omega) = (\sigma - 1) f \psi$, labor market clearing still implies $n^k = L^k/(\sigma f)$ for $k = H, F$. Goods market clearing implies all

varieties to be demanded by the same amount (since they were produced by the same amount). Recall that

$$x(\omega) = \frac{E}{P}\left(\frac{p(w)}{P}\right)^{-\sigma}$$

this implies $p(w)$ are the same for all varieties in the world, but $p(\omega) = \sigma/(\sigma-1)\cdot w/\psi$. This means that wage is equalized across the two countries:

$$w_H = w_F$$

Home consumers then consume a fraction $L^H/(L^H + L^F)$ of the world's production of **all** varieties. Therefore, this generates intra-industry trade.

3.2.3 *Integrated Equilibrium by Krugman and Helpman (1985)*

Helpman and Krugman (1985) shows how to embed IRS, monopolistic competition, and product differentiation into a standard multi-sector, multi-factor model.

Key equations that describe equilibrium:

1. Free-entry drives price down to average cost:

$$p_i = c_i(\bar{w}, \bar{x}_i) \equiv \frac{C_i(\bar{w}, \bar{x}_i)}{\bar{x}_i}$$

2. Optimal pricing by the monopolist yields

$$MR_i(\bar{p}, \bar{n}) = C_{ix}(\bar{w}, \bar{x}_i) \equiv MC(\bar{w}, \bar{x}_i)$$

3. Factor market clearing requires

$$\sum_{i\in I} a_{ji}(\bar{w}, \bar{x}_i)\cdot \bar{X}_i = \bar{V}_j, \qquad \forall\, j$$

4. Goods market clearing requires

$$\alpha_i(\bar{p}, \bar{n}) = \frac{\bar{p}_i \bar{X}_i}{\sum_{i'\in I}(\bar{p}_{i'}\bar{X}_{i'})}$$

Final remarks:

1. Replication of the integrated equilibrium requires that every variety is produced only in one country. (but with continuum of variety, this places no further constraint on the FPE set)
2. The pattern of trade continues to be indeterminate, but the Vanek equations continue to hold within the FPE set.
3. What have we gained? We can say something about **bilateral volumes of trade**, and the **composition of trade** (intraindustry vs. interindustry).

3.3 Incorporating Transportation Cost

Suppose that goods selling abroad are subject to the Krugman-type 'iceberg' transportation cost τ.

i.e. it would take τ/ψ units of labor to produce a product instead of $1/\psi$ as before due to loss during transport ($\tau > 1$).

This tweak of the model generates unequal factor prices across country; particularly, different wages in two different countries. In this set up, we can prove that **bigger countries have higher wages**.

See derivation of results in the chapter appendix.

3.4 Home Market Effect

Transportation cost as in the previous section generates wage differences across countries, but not 'home market effect': country k's share of varieties is exactly its share of world population.

3.5 Appendix: Derivation of Formulae

3.5.1 *Results of Transportation Cost (Krugman (1980))*

Domestic producer of variety ω has to decide on prices and quantities $x^{HH}(\omega)$ and $x^{HF}(\omega)$ - the amounts to be sold domestic and abroad respectively - to maximize profit

$$\pi(\omega) = (p^{HH} - \frac{w^H}{\psi})x^{HH} + (p^{HF} - \tau\frac{w^H}{\psi})x^{HF} - fw^H$$

Since demand from abroad still has constant price elasticity of demand, prices of variety ω to be sold at home and foreign are

$$\boxed{p^{HH} = \frac{\sigma}{\sigma-1}\frac{w^H}{\psi}} \qquad \boxed{p^{HF} = \frac{\sigma}{\sigma-1}\frac{\tau w^H}{\psi}}$$

4 *Vertical Product Differentiation*

International trade flows reveal systematic patterns of vertical specialization.

- **(Export)** When rich and poor countries export goods in the same product category, the richer countries sell goods with higher unit values. (e.g. Schott 2004, Hallak and Schott 2011)
- **(Import)** Also, when a country imports goods in a product category from several sources, the higher-quality goods are imported disproportionately from the higher income countries. (e.g. Bils and Klenow 2001, Hallak 2006).

A number of analytic frameworks have been proposed to explain the patterns of vertical specialization. Some of the supply-side determinants that have been researched are as follows

- **(Hecksher-Olin)** Markusen (1986) and Berstrand (1990) argue that the country with higher per capita income exports the luxury good because that good happens to be capital intensive.
- **(Ricardian)** In Flam and Helpman (1987), Stokey (1991), Murphy and Shleifer (1997) and Matsuyama (2000), the pattern of trade follows from an assumption that richer countries have relative technological superiority in producing higher-quality goods.
- **(Melitz style)** Baldwin and Harrigan (2011) and Johnson (2013) seek to explain that more productive firms export higher-priced products since they have greater incetive to undertake quality-enhancing investments. Richer countries are home to a disproportionate share of the high-productivity firms.

Among various models, this lecture sheds light particularly on a demand-based explanation that nonhomothetic preferences over goods of different qualities play an important role. (Fajgelbaum, Grossman and Helpman 2011)

- This approach is reminiscent of that used by Linder (1961) in a sense that firms in any country produce goods suited to the predominant

tastes of their local consumers and sell them worldwide to others who share these tastes.

4.0.2 *Fajgelbaum, Grossman and Helpman (2011)*

- The key insight of the model is that income distribution becomes a source of comparative advantages.
- **(Demand Side)** We use a non-homothetic demand structure.
  - Agent h picks one unit of differentiated good with quality q_i, where $i \in \Omega$ and brand $j \in Q_i$. Remaining income is spent on homogeneous good z. So the utility is

$$u_{i,j}^h = zq_i + \varepsilon_{i,j}^h = (y - p_{i,j})q_i + \varepsilon_{i,j}^h \tag{4.1}$$

    featuring complementarity between the quantity of the homogeneous good and the quality of the differentiated product.
  - Each consumer has an idiosyncratic component vector $\varepsilon^h = \left\{\varepsilon_{i,j}^h\right\}$ that is drawn from the nested logit.

$$F_\varepsilon(\varepsilon) = e^{-\sum_{i\in\Omega}[\sum_{j\ in Q_i} e^{-\varepsilon_{i,j}/\theta_i}]^\theta}$$

  - Individual with income y^h and taste parameters ε^h chooses the quality and variety that yield the highest utility (4.1) among all available options
  - The calculations imply that the fraction of people with income y who choose j and q is

$$\rho_j(y) = \rho_{j|q} \cdot \rho_q(y)$$

    where

$$\rho_{j|q} = \frac{e^{-p_j q/\theta_q}}{\sum_{l\in J_q} e^{-qp_l/\theta_q}} \tag{4.2}$$

    is the fraction of consumers who by j among those who purchase q and

$$\rho_q(y) = \frac{[\sum_{j\in J_q} e^{(y-p_j)q/\theta_q}]^{\theta_q}}{\sum_{w\in Q}[\sum_{j\in J_w} e^{(y-p_j)w/\theta_w}]^{\theta_w}} \tag{4.3}$$

    is the fraction of consumers with income y who opt for quality q.
- **(Supply Side)** The supply-side uses a standard IRS technology that is quality-specific.
  - The profit is represented by

$$\pi_{i,j} = Nd_{i,j}(p_{i,j} - c_i) - f_i \tag{4.4}$$

where $d_{i,j}$ is the demand per capita in the economy as follows

$$d_{i,j} = \int_{y_{min}}^{\infty} \rho_{i,j}(y) dF_y(y) \tag{4.5}$$

So the pricing is determined by

$$p_{i,j} = p_i = c_i + \frac{\theta_i}{q_i} \tag{4.6}$$

where higher θ_i implies dissimilarity among varieties (more market power) and higher q_i implies more sensitivity to price changes (less market power)

5 Firm Heterogeneity

5.1 Motivations

- The Helpman-Krugman type model has a universal exporting firm: every brand is produced by a single firm domestically and exports to every country.
- This is not consistent with firm level data:
  - Only a small fraction of firms export.
  - Exporters are larger, pay higher wage, and more productive firms.
  - Exporters sell most of their products domestically.

Share of exports of manufactures

Country	Year	Top 1% of firms	Top 10% of firms
U.S.A.	2002	81	96
Belgium	2003	48	84
France	2003	44	84
Germany	2003	59	90
Norway	2003	53	91
U.K.	2003	42	80

5.2 Theory (Melitz, 2003)

- Recent empirical research has substantiated the existence of large and persistent productivity differences among firms in industry.
  - Some of these studies have shown that these productivity differences are strongly correlated with the export status.
  - Other studies have highlighted the large level of resource reallocation that occur after the exposure to trade.
- This paper lays a theoretical framework that introduces such heterogeneity in standard models of international trade.

(1) Demand

- The preferences of a representative consumer are given by a C.E.S. utility function over a continuum of goods indexed by ω:

$$U = \left[\int_{\omega\in\Omega} q(w)^{\rho} d\omega\right]^{1/\rho}$$

where the measure of Ω represents the mass of available goods.

  - These goods are substitutes, implying $0 < \rho < 1$ and an elasticity of substitution between two goods $\sigma = 1/(1-\rho) > 1$

- As in Dixit and Stiglitz (1977), consumer behavior can be modeled by considering the aggregate consumption $Q \equiv U$ and aggregate price

$$P = \left[\int_{\omega\in\Omega} p(\omega)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}} \tag{5.1}$$

- These aggregates can then be used to derive the optimal consumption and expenditure decisions

$$q(\omega) = Q\left[\frac{p(w)}{P}\right]^{-\sigma}, \qquad r(\omega) = R\left[\frac{p(\omega)}{P}\right]^{1-\sigma} \tag{5.2}$$

where R = PQ = $\int_{\omega\in\Omega} r(\omega) d\omega$.

(2) Production

- There is a continuum of firms that are heterogeneous in terms of productivity φ.
  - Production requires only labor, which is inelastically supplied at its aggregate level L, an index of the economy size.
  - Labor used is a linear function of output q

$$l = f + q/\varphi$$

where f is the fixed overhead cost and $1/\varphi > 0$ is a constant marginal cost.

- Every firm faces a residual demand curve with constant elasticity σ and thus has the same mark-up to maximize the profit.

$$p(\varphi) = \frac{w}{\rho\varphi} = \frac{1}{\rho\varphi} \tag{5.3}$$

where the common wage rate w is normalized to one.

- Firm profit is then

$$\pi(\varphi) = r(\varphi) - l(\varphi) = \frac{r(\varphi)}{\sigma} - f$$

where $r(\varphi)$ is firm revenue and $r(\varphi)/\sigma$ is variable profit.

- $r(\varphi)$ and $\pi(\varphi)$ also depend on the aggregate price and revenue

$$r(\varphi) = R(P\rho\varphi)^{\sigma-1}, \qquad \pi(\varphi) = \frac{R}{\sigma}(P\rho\varphi)^{\sigma-1} - f \tag{5.4}$$

  On the other hand, the ratios of any two firms' outputs and revenues only depend on the ratio of their productivity levels.

$$\frac{q(\varphi_1)}{q(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma}, \qquad \frac{r(\varphi_1)}{r(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma-1}, \tag{5.5}$$

  So more productive firms will be bigger and earn higher profits

  (3) Aggregation

- An equilibrium of firm dynamics will be characterized by a mass M and a distribution $\mu(\varphi)$ of productivity levels.
- It turns out that $\mu(\varphi)$ can be summarized by a weighted average of the firm productivity level $\tilde{\varphi}$
  - Note that, in equilibrium, the aggregate price P is given by

$$P = \left[\int_0^\infty p(\varphi)^{1-\sigma} M\mu(\varphi)d\varphi\right]^{\frac{1}{1-\sigma}}$$

  - Using the pricing rule $p(\varphi) = \frac{w}{\rho\varphi}$, this can be written $P = M^{1/(1-\sigma)}$ where

$$\tilde{\varphi} = \left[\int_0^\infty \varphi^{\sigma-1}\mu(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}} \tag{5.6}$$

- Then all aggregate variables are represented by $\tilde{\varphi}$

$$P = M^{\frac{1}{1-\sigma}} p(\tilde{\varphi}), \qquad R = PQ = Mr(\tilde{\varphi})$$
$$Q = M^{1/\rho} q(\tilde{\varphi}), \qquad \Pi = M\pi(\tilde{\varphi})$$

  where $R = \int_0^\infty r(\varphi)M\mu(\varphi)d\varphi$ and $\Pi = \int_0^\infty \pi(\varphi)M\mu(\varphi)d\varphi$

- Thus an industry comprised of M and $\mu(\varphi)$ induces the same aggregate outcome as an industry with M representative firms sharing the same productivity $\varphi = \tilde{\varphi}$

  (4) Firm Entry and Exit

- The model studies steady state equilibria where firms enter and exit depending on their productivities and prospective incomes.
- There is a large pool of prospective entrants into the industry
  - To enter, firms first make an initial investment $f_e > 0$ which is a fixed cost measured in units of labor

  - Firms then draw their initial productivity parameter φ from a common distribution $g(\varphi)$. Its cumulative distribution is $G(\varphi)$

- Upon entry, a firm with a low productivity draw may immediately exit and not produce
  - If the firm does produce, it then faces a constant probability δ in every period of a bad shock that would force it to exit.
  - Though simplified, the model retains an essential pattern: new entrants have, on average, lower productivity and a higher probability of exit than incumbents.

- Assuming that there is no time discounting, each firm's value function is given by

$$v(\varphi) = \max\left\{0, \sum_{t=0}^{\infty}(1-\delta)^t \pi(\varphi)\right\} = max\left\{0, \frac{1}{\delta}\pi(\varphi)\right\}$$

  since each firm's productivity does not change over time.
  - Thus $\varphi^* = inf\{\varphi : v(\varphi)\}$ identifies the lowest productivity level of producing firm.
  - Any entering firm drawing a productivity level $\varphi < \varphi^*$ will immediately exit.

- Since the exit process is uncorrelated with productivity, $\mu(\varphi)$ is determined by the initial productivity draw from g.

$$\mu(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\varphi^*)} & \varphi \geq \varphi^*, \\ 0 & \text{otherwise} \end{cases} \tag{5.7}$$

  and $p_{in} \equiv 1 - G(\varphi^*)$ is the ex-ante probability of successful entry.

- This defines the aggregate productivity level $\tilde{\varphi}$ as a function of the cutoff level ϕ^*

$$\tilde{\varphi}(\varphi^*) = \left[\frac{1}{1-G(\varphi^*)}\int_{\varphi^*}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi\right]^{\frac{1}{\sigma-1}} \tag{5.8}$$

- **(Zero Cutoff Profit Condition)** Since the average productivity level $\tilde{\varphi}$ is completely determined by the cutoff productivity level φ^*,

$$\tilde{r} = r(\tilde{\varphi}) = \left[\frac{\tilde{\varphi}(\varphi^*)}{\varphi^*}\right]^{\sigma-1} r(\varphi^*)$$

$$\tilde{\pi} = \pi(\tilde{\varphi}) = \left[\frac{\tilde{\varphi}(\varphi^*)}{\varphi^*}\right]^{\sigma-1} \frac{r(\varphi^*)}{\sigma} - f$$

  The zero profit condition then implies that

$$\pi(\varphi^*) = 0 \Leftrightarrow r(\phi^*) = \sigma f \Leftrightarrow \tilde{\pi} = fk(\varphi^*) \tag{5.9}$$

  where $k(\phi^*) = [\tilde{\varphi}(\varphi^*)/\varphi^*]^{\sigma-1} - 1$

- **(Free Entry)** The expected profit should be good enough to cover the investment cost f_e. Let $\tilde{v}$ be the present value of the average profit

$$\tilde{v} = \sum_{t=0}^{\infty}(1-\delta)^t \tilde{\pi} = (1/\delta)\tilde{\pi}$$

Define v_e to be the net value of entry

$$v_e = p_{in}\tilde{v} - f_e = \frac{1 - G(\varphi^*}{\delta}\tilde{\pi} - f_e \tag{5.10}$$

which should be zero by the free entry assumption.

(5) Equilibrium in a closed economy

- The FE and ZCP conditions represent two different relations linking the average profit level $\tilde{\pi}$ with cutoff productivity level φ^*.

$$\tilde{\pi} = fk(\varphi^*), \qquad \tilde{\pi} = \frac{\delta f_e}{1 - G(\varphi^*)} \tag{5.11}$$

The existence and uniqueness of the equilibrium are represented graphically in the figure.

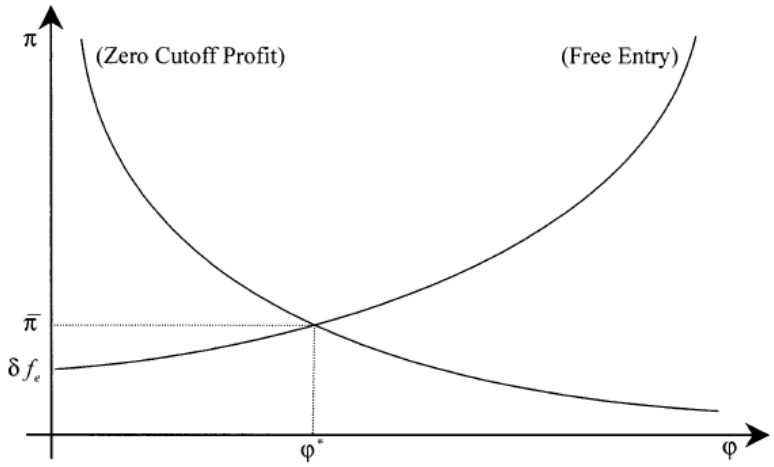


FIGURE 1.—Determination of the equilibrium cutoff φ^* and average profit $\tilde{\pi}$.

(6) Equilibrium in the open economy

- All $n+1$ countries are identical in size so they share the same wage and same aggregate variables.
- Pricing rules are different between domestic and foreign markets.
  - In domestic market, the pricing rule is given by $p_d(\varphi) = 1/\rho\varphi$
  - Firms who export incur the increased marginal cost τ so $p_x(\varphi) = \tau/\rho\varphi$

- Thus revenues earned from domestic sales and export sales to any given country are $r_d(\varphi) = R(P\rho\varphi)^{\sigma-1}$ and $r_x(\varphi) = \tau^{1-\sigma} r_d(\varphi)$ where R and P are the aggregate indexes in every country.
- The combined revenue of a firm, $r(\varphi)$, depends on its export status

$$r(\varphi) = \begin{cases} r_d(\varphi) & \text{if it does not export} \\ r_d(\varphi) + n r_x(\varphi) = (1 + n\tau^{1-\sigma}) r_d(\varphi) & \text{if it exports to all} \end{cases}$$

Since the export cost is equal across country, a firm either exports to all countries or never export.

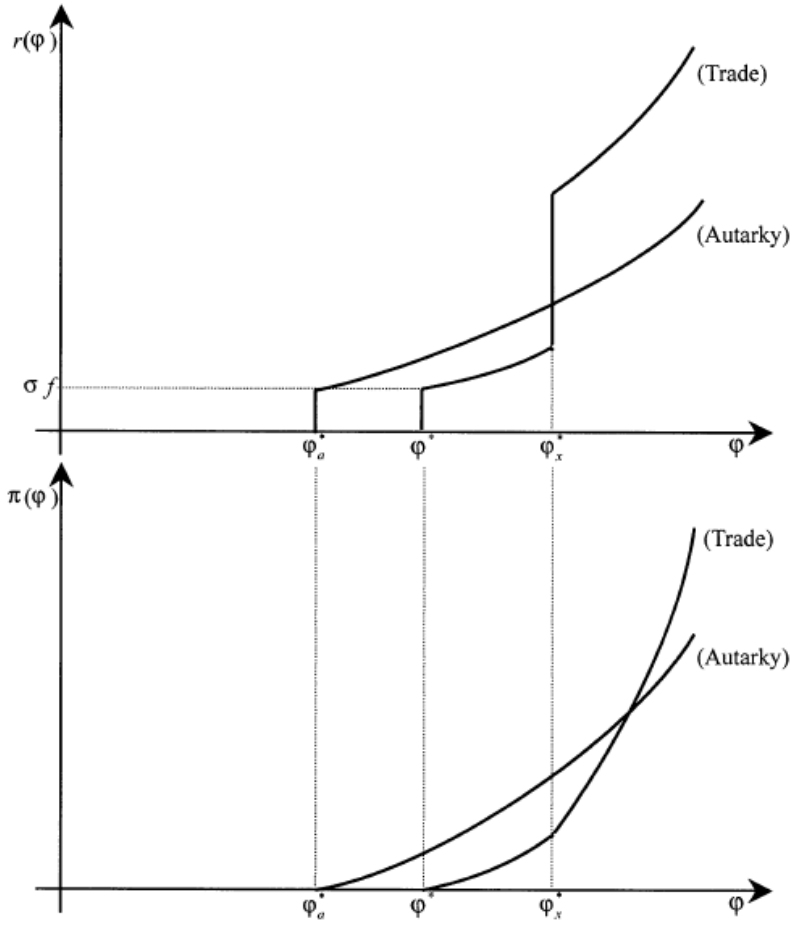


FIGURE 2.—The reallocation of market shares and profits.

(7) Impact of Trade

- Let φ_a^* and $\tilde{\varphi}_a$ denote the cutoff and average productivity levels in autarky.

  - Similarly let $r_a > 0$ and $\pi_a(\varphi) \geq 0$ denote the firm's revenue and profit in autarky. M_a is the number of firms in autarky.

- **(Result 1)** The exposure to trade induces an increase in the cutoff productivity level i.e. $\varphi^* > \varphi_a^*$

  - This is due to the ZCP curve shifting up.
  - The least productive firms between φ_a^* and φ^* can no longer earn positive profits in the new equilibrium
  - Another selection process is that only firms with above φ_x^* enter the export markets.
  - The selection effects in both domestic and export markets reallocate market shares towards more efficient firms

- **(Result 2)** The equilibrium number of firms is smaller than that of autarky i.e. $M < M_a$

  - This is associated with an increase in average revenue (Note: M1, M2)
  - Even so, consumers in the country still 'typically' enjoy greater product variety, $M_t = (1 + np_x)M > M_a$.
  - When the export costs are high, the variety may decrease. However, the aggregate productive gain dominates, resulting in a welfare gain.

- **(Result 3)** All firms incur a loss in domestic sales in the open economy and an exporting firm makes up for its loss with export sales.

$$r_d(\varphi) < r_a(\varphi) < r_d(\varphi) + nr_x(\varphi), \forall \varphi \geq \varphi^*$$

  - A firm who exports increases its share of industry revenues while a firm who does not loses market share.

- **(Result 4)** While $\Delta\pi(\varphi) = \pi(\varphi) - \pi_a(\varphi)$ increases with φ, only a subset of the more productive firms who export gain from trade.

  - Clearly, firms that do not export incur a profit loss due to revenue shrinkage.
  - Exporting firms face both revenue gains and the additional export cost.

- The exposure to trade thus generates a type of Darwinian evolution within an industry

5.2.1 *Summary of results*

Melitz (2003) provides a framework that features:

- In the closed economy, there is a productivity cutoff for production φ^*.
- In the open economy, there is a cutoff for production φ_d^* and a cutoff for export φ_x^*, with $\varphi_d^* < \varphi_x^*$. This fits the 'selection into exports' in the data.

  provided $f_x \tau^{\sigma-1} > f$
- Opening up the economy raises productivity: $\varphi^* < \varphi_d^*$. Firms that are between this range exit when the economy opens up.
- Intuition: Operate through **domestic factor market**. Open up to trade $\rightarrow$ higher demand for labor $\rightarrow$ higher real wage $w/P \rightarrow$ least productive firms cannot afford to produce.
- Welfare: New source of gain from trade: increased productivity.

5.3 Firm heterogeneity: Estimation

5.3.1 *Arkolakis, Costinot, Rodriguez-Clare (2013)*

- Welfare implication for a large class of models[1] is given by two sufficient statistics:

  $$\hat{W} = \hat{\lambda}^{1/\epsilon}$$

  where λ is the share of spending on domestic good (which is one minus share of spending on import), and ϵ is the elasticity of import with respect to the variable trade cost.

  [1] including Eaton and Kortum (2002), Krugman (1980), variations of Melitz (2003)

  $\hat{x} \equiv dx/x$ is the percentage change of a variable.
- Quick calculation on the US: $\lambda \approx 0.97$, and following Anderson and van Wincoop (2004), $\epsilon \in [-5, -10]$. This gives welfare gains from trade from 0.7% to 1.4%.
- Critics of Melitz and Redding (2014): ACR formula is correct, but their conclusion is wrong. To compare between alternative models, must calibrate parameters so as to yield the same cost structure, productivity distribution, and other economic variables.
- Extension to multi-sectors: With multiple sectors and Cobb-Douglas sectoral expenditure shares η^s:

  $$\hat{W} = \Pi_{s=1}^{S} \left(\frac{\hat{\lambda}_{jj}^s}{\hat{L}_j^s} \right)^{\frac{\eta^s}{\epsilon^s}}$$

  So, labor allocation now affects welfare gains. For example, if there are two sectors: homogeneous good and differentiated good, then a labor reallocation from the former to the latter raises welfare.

5.3.2 *Extensions of Melitz (2003)*

- Bernard, Redding and Schott (2007) integrate factor proportions into the Melitz framework.
- Bustos (2011) extends it to incorporate technology upgrading and provides evidence supporting its implications.
- Manova (2011) extends it to account for the impact of ÂĚnancial development on comparative advantage.
- Sampson (2014) introduces matching with heterogenous workers and analyzes wage inequality.
- Helpman, Melitz and Rubinstein (2008) extend it to a setting with asymmetric countries and develop an econometric approach for estimating trade ÂĞows. This methodology:
  - provides a generalization of the gravity equation;[2]
  - accounts for zero trade flows across some country pairs;
  - separates the intensive from the extensive margin of trade;
  - allows asymmetric responses to trade resistance measures.

  Result of HMR (2008):
  - standard gravity estimates overestimate the effect of distance on trade flows (τ^{ij}) since they ascribe to the intensive margin an effect that really works through selection into exporting;
  - the bias stemming from firm heterogeneity is more important than the Heckman selection bias;
  - there are large differences in w^{ij}, and this helps explains imbalanced trade flows.

[2] Anderson-van Wincoop's version of the gravity equation is

$$X^{ij} = A\frac{Y^i Y^j}{{\tau^{ij}}^{1-\sigma}}$$

where $A \equiv (\tilde{P}^i \tilde{P}^j)^{1-\sigma}/Y^w$

6 Labor Market Frictions

6.1 Motivation

Three prominent features of product and labor markets:

- Variation in labor composition across firms.
- Variation within firms for labor of the same observed characteristics.
- unemployment rate varies across countries.

It becomes apparent that Stolper-Samuelson effects fail to provide adequte explanation for inequality trends around the globe. We will focus on the following question:

- what are the impacts of one country's labor market frictions on its trade partners?
- how does the removal of trade impediments impact countries with different labor market frictions?
- how does an economy with labor market frictions adjust to foreign trade?
- what is the impact of trade on inequality and unemployment?

We will therefore focus on residual inequality, i.e. inequality among workers with similar observable characteristics, as wage variation across such workers account for the majority of wage variation within a firm.

6.2 Helpman & Itskhoki (2010)

Two countries, two sectors: homogeneous good and differentiated good sector. One input: homogeneous labor.There is search and matching in sectoral labor markets; wage bargaining; unemployment. Homogeneous sector: identical firms under CRS; competition in product market; serves as numeraire. Differentiated sector: brands of a diÂd'erentiated product

produced by heterogeneous firms; monopolistic competition in product market. Fixed and variable trade costs.

Households' preferences:

$$U = q_0 + \frac{1}{\xi}Q^{\xi}, \qquad Q = \left[\int_{\omega\in\Omega} q(\omega)^{\beta} d\omega\right]^{1/\beta}$$

Production: $y_0 = h_0$ in the homogeneous sector, $y = \theta h$ in the differentiated sector.

6.2.1 *Homogeneous sector*

Matching function:

$$H_0 = m_0 V_0^{\chi} N_0^{1-\chi}$$

Denote x_0 as the probability of finding a job (also, **market tightness**): $x_0 \equiv H_0/N_0 = m_0(V_0/N_0)^{\chi}$, then the probability of a firm finding worker is

$$\frac{H_0}{V_0} = m_0^{1+\alpha} x_0^{-\alpha}, \qquad \alpha \equiv \frac{1-\chi}{\chi}$$

Cost of posting vacancy is v_0 per worker, so entry cost for each firm (which requires 1 worker) is v_0. Bargaining makes the firm and worker splits the surplus of 1 evenly: $w_0 = \pi_0 = 1/2$.

Free entry $\to$ expected profit = entry cost, which gives

$$x_0 = a_0^{-1/\alpha}, \qquad a_0 \equiv \frac{2v_0}{m_0^{1+\alpha}}$$

a_0 here summarizes market friction in this sector, and is a function of **vacancy posting cost** and **efficiency of matching** technology.

Expected income of worker searching for job in the homogeneous sector is

$$\omega_0 = x_0 w_0 = \frac{1}{2} a_0^{-1/\alpha}$$

6.2.2 *Differentiated good sector*

Let x denotes market tightness in this sector. Stole-Zweibel type bargainning implies that $w = b$. So expected income in the differentiated sector is xb.[1] This must be the same as in the homogeneous sector (work through workers reallocation):

$$xb = \omega_0$$

[1] Remember b is the hiring cost

Market tightness in the differentiated factor depends on market tightness in the homogeneous sector

The number of successful matches is $H = xV^{\chi}N^{1-\chi}$, so $b = V/H$, and

$$b = \frac{1}{2} a x^{\alpha}, \qquad a \equiv \frac{2v}{m^{1+\alpha}}$$

IN SUMMARY, the hiring cost in the differentiated sector of country j is

$$x_j = x_{0j}\left(\frac{a_{0j}}{a_j}\right)^{1/(1+\alpha)} = \left(\frac{1}{a_{0j}^{1/\alpha} a_j}\right)^{1/(1+\alpha)}$$

$$b_j = w_j = \frac{1}{2}\left(\frac{a_j}{a_{0j}}\right)^{1/(1+\alpha)}$$

6.2.3 *Equilibrium*

Firm has an entry cost f_e (required to draw productivity θ). Firms with $\theta > \theta_x > \theta_d$ produce and sell both domestically and export; firms with $\theta \in [\theta_d, \theta_x]$ only produce for domestic market; firms with $\theta < \theta_d$ exit.

There will be a zero cutoff profit condition in equilibrium.

Unemployment rate for country j is

$$u_j = \frac{N_{0j}}{L}(1 - x_{0j}) + \frac{N_j}{L}(1 - x_j)$$

6.2.4 *Impacts of Trade*

Assuming there are two identical countries A and B, only different in labor market frictions. Particularly, assume country B has relatively less labor market friction in the differentiated product sector:

$$\frac{a_A}{a_{0A}} > \frac{a_B}{a_{0B}}$$

Immediately, we have that country B has a lower hiring cost in the differentiated sector ($b_B < b_A$) and has a relatively higher market tightness in the differentiated sector ($x_A/x_{0A} < x_B/x_{0B}$). The results of this analysis is:

- **Trade**: (i) A larger fraction of differentiated-sector firms export in country B,[2] (ii) country B exports differentiated products on net and imports the homogeneous goods, and (iii) the share of intra-industry trade is lower when b_A/b_B is higher (similar is better).
- **Unemployment**: In a symmetric world economy: (i) reductions in labor market frictions in the differentiated sectors at the same rate in both countries reduce aggregate unemployment if and only if $a < a_0[1 + (\beta - \xi)/\beta\xi]^{1+\alpha}$; (ii) reductions in trade impediments raise aggregate unemployment if and only if $a > a_0$.
- **Welfare**: (i) A reduction in labor market frictions in country j's differentiated sector raises its welfare and reduces the welfare of its trade partner. (ii) A simultaneous proportional reduction in labor market frictions in the differentiated sectors of both countries raises welfare in both of them. (iii) A reduction of trade impediments raises welfare in both countries $\Rightarrow$ both countries gain from trade.

[2] Differentiated-sector firms in B spend less on finding workers

7 Trade and Inequality

In the last chapter, we have looked at the effect of friction in labor market on gains from trade. However, the previous model is static, which can be justified as the steady state of a dynamic model.

It is natural then to ask "to what extent does labor market friction affect re-allocation across firms, depressing productivity, and lowering gains from trade in transition from one steady state to another?" This question is explored in Helpman and Itskhoki (2015), by combining the Melitz model with a homogeneous sector and a differentiated sector, together with the DMP-style search frictions in the labor market.

In Helpman and Itskhoki (2015), firms in the differentiated sector pay the same wage. In the data, however, there is a great variation in wage across workers with the same observed characteristics within **the same sector**. Data from Brazil has shown that this within component is driven by wage dispersion across **firms** (within the same sector), and wage dispersion between firms is related to firm employment size and trade participation. Therefore, we must examine **residual inequality**, i.e. wage differentials for unobserved worker heterogeneity. This question will be explored in Helpman, Itskhoki, and Redding (2010) (theory) and Helpman, Itskhoki, Muendler, and Redding (2015) (empirical).

7.1 Firm, Trade, and Labor Market Dynamics (Helpman and Itskhoki, 2015)

7.1.1 *Environment*

- **Households:** Quasi-linear utility in the homogeneous good

$$u(q_0, Q) = q_0 + \frac{1}{\xi} Q^{\xi}, \quad \text{where } Q \equiv \left[\int_{\omega \in \Omega} q(\omega)^{\beta} d\omega \right]^{1/\beta}$$

$\xi \in (0, \beta)$ ensures that homogeneous good and differentiated goods are never perfect substitutes.

  Measure L of workers, either employed in 1 of two sectors, or unemployed. Unemployed workers can search frictionlessly in either sector. Unemployed workers receive benefit b_u during each period of being unemployed, financed by a lump sum tax on households.

- **Non-traded homogeneous good sector:** competitive, a match between a firm and a worker produces Δ numeraire (homogeneous good) in time length Δ ($\Delta \to 0$).

  Let b_0 be the expected cost of attracting a worker, then with Cobb-Douglas matching function,

$$b_0 = a_0 x_0^{\alpha} \tag{7.1}$$

  where a_0 is a derived parameter increasing in cost of vacancy and decreasing in productivity of matching technology; and x_0 is the job-finding rate. Nash bargaining[1] implies that

$$[2(r + s_0) + x_0]b_0 = 1 - b_u \tag{7.2}$$

  Equations (7.1) and (7.2) together solve for (x_0, b_0), the market tightness and cost of hiring (constant over time).

x_0 is commonly known as "market tightness"

[1] read the paper's appendix for careful derivation

- **Traded (differentiated) sector:** Firms pay sunked cost f_e to enter and draw productivity θ from a Pareto distribution $G(\theta) = 1 - \theta^{-k}$. Fixed cost of export f_e and ice-berg variable trade cost $\tau \geq 1$.

  The firm Nash bargains with workers in the Stole-Zwiebel (2006) style: bargining bilaterally, taking into account that if a worker leaves, they have to rebargain with the rest of the workers. The outcome wage scheduled $w(h)$ is that workers are compensated for their flow value of unemployed (opportunity cost $rJ^U = rJ_0^U = b_u + x_0 b_0$) plus a share of the firm's profit:

$$w(h) = \frac{\beta}{1+\beta}\frac{R(h)}{h} + \frac{1}{2}rJ^U$$

  The flow operating profit of a firm is therefore

$$\varphi(h, \iota; \theta) = \frac{1}{1+\beta}\frac{R(h)}{h} - \frac{1}{2}rJ^U - f_d - \iota f_x$$

  where ι is an indicator variable for exporting.

  A key assumption in this model is that hiring is costly, but firing is costless. Hence, the cost of changing employment from h to h' is

$$C(h, h') = b \max\{h' - (1 - \sigma\Delta)h), 0\}$$

  where σ is the (exogenous) rate of separation with workers.

7.1.2 *Long-run Equilibrium*

We can write down the steady state value of an entrant with productivity θ and zero employment:

$$J^V(\theta) = \frac{1}{r+\delta}\max_{\iota \in \{0,1\}}\left\{\frac{1-\beta}{1+\beta}\Phi\Theta(\iota;\theta) - f_d - \iota f_x\right\}$$

We can define the production cutoff θ_d s.t. $J^V(\theta_d) = 0$ and export cutoff θ_x such that $\iota(\theta) \equiv \mathbb{1}_{\{\theta \geq \theta_x\}}$. These two long-run cutoff conditions can be rewritten as

This shows that when f_x is large enough, $\theta_x > \theta_d$.

$$\frac{1-\beta}{1+\beta}\Phi Q^{-\frac{\beta-\xi}{1-\beta}}\theta_d^{\epsilon-1} = f_d \tag{7.3}$$

$$\frac{\theta_x}{\theta_d} = \tau\left(\frac{f_x}{f_d}\right)^{1/(\epsilon-1)} \tag{7.4}$$

The steady state employment in the differentiated sector is

$$H = \Phi^{\frac{1-\beta}{\beta}}Q^{\xi}$$

and the steady state number of firms in the differentiated sector is

$$M = \frac{1}{k(r+\delta)f_e}\frac{\beta}{1+\beta}Q^{\xi}$$

There are 2 results, as presented in Proposition 1 of Helpman& Itskhoki (2015):

(1) Assume a symmetric world of identical country. A bilateral reduction of τ creates a proportional steady increase in all aggregate variables Q (differentiated output), H (diffentiated sector employment), and M (number of firms):

$$\left(\frac{Q'}{Q}\right)^{\xi} = \frac{H'}{H} = \frac{M'}{M} = \left(\frac{\theta_d'}{\theta_d}\right)^{\frac{\beta\xi}{\beta-\xi}}$$

(2) Long-run welfare gains from trade do not depend on the extent of labor market frictions: Define welfare gains from trade as:

$$GT' \equiv \underbrace{\frac{(I'-I)}{\frac{1-\xi}{\xi}Q^{\xi}}}_{\text{Change in market income}} + \underbrace{\frac{\frac{1-\xi}{\xi}[(Q')^{\xi}-Q^{\xi}]]}{\frac{1-\xi}{\xi}Q^{\xi}}}_{\text{Gains in consumer surplus}}$$

Since $I' = I = wxL/(x+s)$, there is no change in market income. Therefore, the **only long run welfare gain from trade is the increase in consumer surplus**.

7.1.3 *Dynamic Gains from Trade*

Proposition 2 in Helpman and Itskhoki (2015) posits that the gains in consumer surplus from trade, i.e. the unique source of long-run gains from trade (from Proposition 1), are achieved **instantaneously**.[2]

Dynamics of firm's employment is more complicated, but well-summarized by figure 7.1

[2] Precisely, if Q' is the new long-run level of differentiated outputs, then $Q_t \geq Q'$ for all t after trade liberalization. Strict inequality happens when there is no firm entry, in which case, we may have **overshooting**.

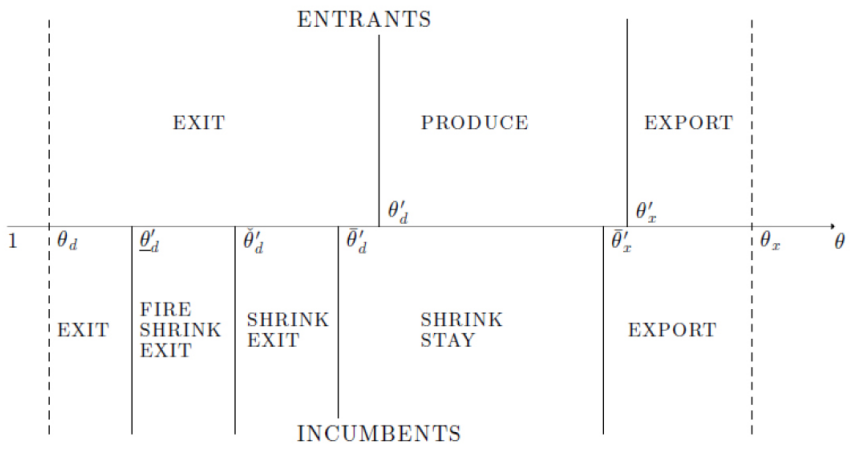


Figure 7.1: Summary of Firm's Dynamics

A key point of the model is that **incumbents and entrants are not the same**: they differ in the initial stock of labors. Because there is a sunk cost of hiring (stemming from labor market frictions), having already a labor stock is an advantage.

Entrants:

- Employment of low-productivity non-exporting entrants shrinks after trade liberalization due to increased foreign competition[3]
- Employment of high-productivity entrants that choose to exports expand.

[3] Trade liberalized $\rightarrow$ more productive firms can select into exports $\rightarrow$ higher domestic labor demand $\rightarrow w/P$ increases (like Melitz model)$\rightarrow$ less productive firms shrink.

Incumbents:

- New production and export cutoffs for incumbents are strictly less than for entrants when there is labor market friction: $\bar{\theta}'_x < \theta'_x$ and $\bar{\theta}'_d < \theta'_d$. This is because these firms have already incurred the sunk cost **before** the unanticipated reduction of trade cost τ.[4]
- All firms with $\theta \in [\theta_d, \bar{\theta}'_d)$ all exit in the long-run, but their dynamic behavior in transitions are different (more productive firms do not have to fire people right away, just have to shrink slowly and exit; some have to fire workers on impact, then shrink and exit; some have to close off right away).

[4] in a sense, incumbents firms can be allowed to be less productive because the have incurred the sunk costs beforehand

7.1.4 *Good Jobs, Bad Jobs*

In a model with heterogeneous firms, not all jobs fare equally well after trade liberalization, even though **in steady state** all workers receive the same wage.

- Good jobs: workers employed by new entrants and exporting incumbents that expand after trade liberalization. These workers receive the same amount of numeraire (homogeneous goods) before and after the trade liberalization, but price index is lower, so they have higher real wage.
- Bad jobs: workers employed by non-exporting incumbents that need to shrink and potentially exit. The key idea is these firms **employed too many people** in the new competitive environment. As employment shrinks, wage recovers and gets to the same level as the good jobs in the long run.

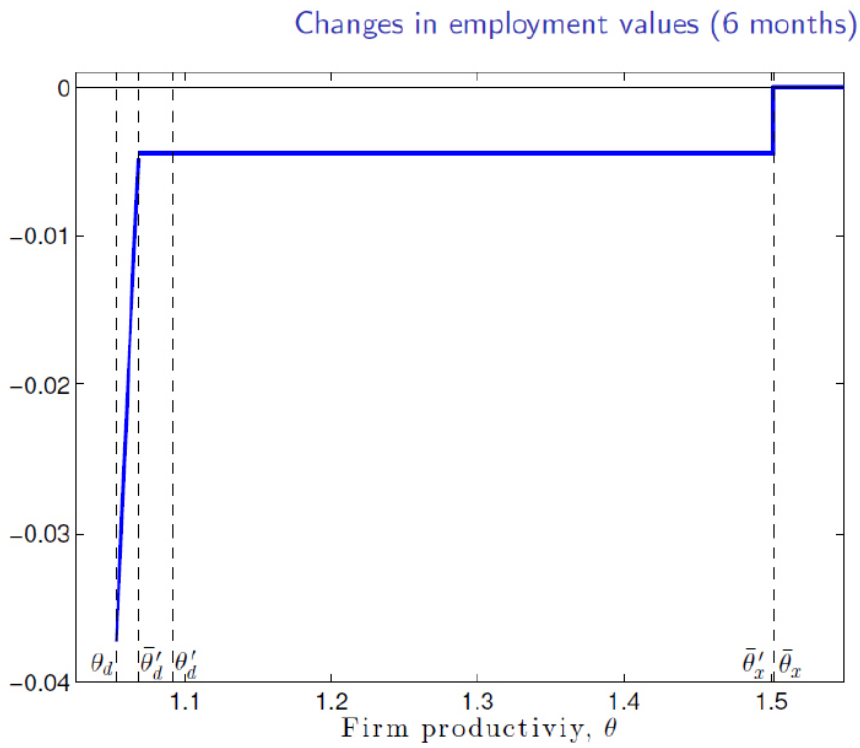


Figure 7.2: Good jobs, bad jobs

Bibliography