DEPARTMENT OF ELECTRICAL & ELECTRONICS ENGINEERING

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LECTURE NOTES

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UNIT-I

Measuring Instruments

UNIT-I

1.1. METHODS OF MEASUREMENTS AND MEASURING INSTRUMENTS

The measurement methods can be analog or digital methods, deflection or null methods, active or passive methods, direct or indirect methods and absolute or secondary methods. Measurement generally involves an instrument as a physical means of determining an unknown quantity or a variable called the *parameter*. The instrument is a means for determining the value or magnitude of the measurand. The instruments can also be divided into separate classes according to several criteria as, analog or digital instruments, deflection or null type instruments, power operated (active) or self generating (passive) instruments, contacting or non-contacting instruments, mechanical or electrical instruments and monitoring or control instruments.

- Signals which vary continuously with the change in the measurand are *analog* signals and the devices producing them are *analog instruments*. The deflection type dynamometer type wattmeter is a good example of an analog instrument. As the input value changes, the moving system or pointer exhibits a smooth continuous motion. The signals which vary in discrete steps and have only finite number of values in any given range are *digital* signals and the associated devices are *digital instruments*. A digital instrument has an output varying in discrete steps. An electronic counter is an example of a digital instrument.
- If the quantity is to be directly measured, then *deflection* methods are used. For e.g., ammeter, voltmeter, etc. acting as meters indicating the value of the measurand by the deflection of a pointer over a graduated and calibrated scale. Alternatively, if the value is measured based on the null balance conditions, then it is a *null* method. Null methods are used only to detect the null condition of a measurand through a given path or circuit. AC/DC Bridge measurements for measurement of resistance, inductance, capacitance, frequency, etc. are null methods. They involve balance detection by using null detectors, such as, Galvanometer, Vibration Galvanometers and Head Phones. Null instruments are more accurate than the deflection instruments.
- If the output of the instrument is entirely produced by the measurand, then it is an *active instrument*. These are the *power operated instruments* requiring some source of auxiliary power for their operation such as compressed air, electricity and hydraulic supply. On the other hand, if the measurand modulates the magnitude of some external power source, then it is a *passive instrument.* Passive instruments are *self generating instruments* where the energy requirements are met entirely from the input signal.
- The *direct* methods involve measuring the measurand by comparison against its own standard. They are very common for measurement of physical quantities such as length, mass and time. They are less sensitive and inaccurate since they involve human operators. Thus direct methods are not usually preferred. On the other hand, *indirect methods* use measuring systems, which are the systems having a transducer to convert the measurand into its analogous form. This converted signal is processed, fed to the end devices to obtain the results.
- *Absolute* methods give the magnitude of the quantity under measurement in terms of the physical constants of the instrument. They do not require calibration. They are used only for calibration of other instruments. For e.g., Tangent Galvanometer, Rayleigh's current balance and potentiometers. *Secondary* methods are so constructed that the desired quantity is measured only by observing the

output of the instrument, which needs to be calibrated. Thus, they measure the quantity in terms of their deflection, for which they are already calibrated. These are the ones which are the most commonly used. For e.g., Voltmeters, Thermometer, Pressure gauge, etc. Secondary methods work on either Analog mode or Digital mode and hence lead to analog or digital methods.

- *Contacting type instruments* are those which are kept in the measuring medium itself. For e.g., clinical thermometer. A *non-contacting* or *proximity type instrument* measures the desired input even though it is not in close contact with the measuring medium. For e.g., optical pyrometer measuring the temperature of a blast furnace, variable reluctance tachometer measuring the speed of a rotating body, etc.
- *Mechanical instruments* are very reliable for static conditions. Their parts are very bulky, rigid and have a heavy mass. Hence they cannot respond rapidly to measurements of dynamic and transient conditions. Besides, many of them are the potential sources of noise. On the other hand, *electrical instruments* are very rapid in response. However, their operating mechanism normally depends on a mechanical meter movement as an indicating device.
- *Monitoring instruments* are useful for monitoring functions. If their output is in a form that can be directly included as part of an automatic control system, then they become *control instruments.*

1.2. Requirements of Instruments

A measuring instrument should possess some of the following important characteristic features:

- It should have a very high instrument *efficiency* which is the ratio of the quantity being measured to the power utilized by the instrument at full scale.
- It should have a high *sensitivity* which is the ratio of the magnitude of the output signal to the magnitude of the quantity being measured. The inverse of this ratio is the Inverse Sensitivity or Deflection Factor.
- The output of the instrument should be linearly proportional to the input. In such cases, the scale will be uniform and hence easier to calibrate.
- It should have a very low *threshold* which is the minimum value of below which the change in output cannot be detected by the instrument.
- It should have lowest *dead time* which is the minimum time required for the instrument to respond to a change in the quantity being measured.
- It should virtually have no *dead zone* where dead zone is the largest change of the input quantity for which there is no output of the instrument. Dead zone is caused due to friction, hysterisis, backlash, etc.
- It should have perfect *reproducibility* (precision) which is specified in terms of the scale readings over a given period of time. This is different from repeatability which is defined as the variation of scale readings and is random in nature.
- It should not have any *drift*. For an instrument, perfect reproducibility means that the instrument has no drift. This means that for a given input, the measured values are constant and do not vary with time.
- It should have minimum *noise*. Noise is any signal that does not convey any useful information. Noise is due to extraneous disturbances caused by many sources such as stray fields, shocks and thermal stresses.
- It should be modifiable and properly priced.

All the indicating instruments require three important torques for their operation: deflecting torque-T_d, controlling torque-T_c and damping torque-T_D.

- The *deflecting* torque is responsible for the movement of the pointer in proportion to the value of the measurand. It is provided by the different effects of electric current on which the operation of the given instrument depends.
- The *controlling* torque is responsible for controlling the movements of the pointer. It is very high at the null position of the pointer. When the pointer gets deflected due to the deflecting torque exerted on it, the controlling torque provides the retarding torque and the two torques are equal at the equilibrium position of the 5
- pointer. The controlling torque is provided by spring or gravity control. This also ensures the zero setting of the pointer when the deflecting torque is absent.
- The *damping* torque provides the forces required to damp out the oscillatory movements of the pointer, if any, due to the inertia of the system. The damping torque is provided by eddy current, air friction or fluid friction methods.

1.3. Classification of Instruments

- Various effects of electric current are made use of by electrical instruments for their operation, such as, magnetic effect, heating effect, chemical effect, electrostatic effect and electromagnetic induction effect.
- Since the instruments using *magnetic* effect, such as, ammeter, voltmeter and wattmeter, are simpler, cheaper, and could be commonly adopted on both ac and dc circuits, they are more common.
- The instruments using *heating* effect are not much used due to their cost and comparable inaccuracies.
- The instruments using *chemical* effect, such as, energy meters, are also not very common, due to their cost and complications involved.
- The *electrostatic* effect is made use of in voltmeters, both on ac and dc. Though expensive, the electrostatic voltmeters are very useful for high voltage measurements.
- *Electromagnetic induction* forms the basis for many instruments, though on only AC. The common meters using the electromagnetic induction effect of current are: ammeter, voltmeter, wattmeter and energy meter. Of these, the wattmeter and energy meter are more common, although they are costlier.
- Instruments can also be classified in a more general way as indicating instruments, recording instruments and integrating instruments.
- *Indicating* instruments provide information about the measuring quantity in terms of the deflection of a pointer over a pre-calibrated scale. For e.g., ammeter, voltmeter, wattmeter, etc.
- *Recording* instruments make a record of the unknown quantity, usually on a paper, against time or any other variable. For e.g., strip-chart temperature recorder. The recording instruments are usually used in power houses and substations, where a continuous record of the current, voltage, power or energy is required.
- *Integrating* instruments always record the unknown quantity in an integrated manner indicating the total cumulative value of the measurand at any instant of time. For e.g., energy meters and ampere-hour meters.

- General Theory Permanent Magnet Moving Coil (PMMC) Instruments

- The general theory of moving-coil instruments may be dealt with considering a rectangular coil of turns, free to rotate about a vertical axis. N

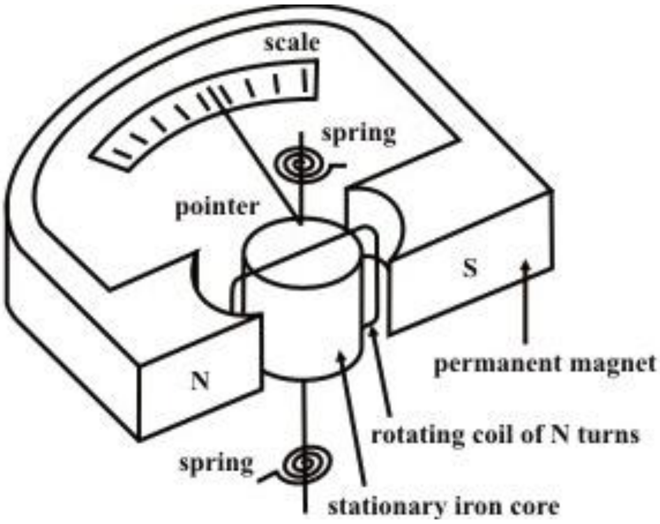


Fig. shows the basic construction of a PMMC instrument. A moving coil instrument consists basically of a permanent magnet to provide a magnetic field and a small lightweight coil is wound on a rectangular soft iron core that is free to rotate around
its vertical axis. When a current is passed through the coil windings, a torque is developed on the coil by the interaction of the magnetic field and the field set up by the current in the coil. The aluminum pointer attached to rotating coil and the pointer moves around the calibrated scale indicates the deflection of the coil.

To reduce parallax error a mirror is usually placed along with the scale. A balance weight is also attached to the pointer to counteract its weight (see Fig.).To use PMMC device as a meter, two problems must be solved. First, a way must be found to return the coil to its original position when there is no current through the coil. Second, a method is needed to indicate the amount of coil movement. The first problem is solved by the use of hairsprings attached to each end of the coil as shown in Fig.

These hairsprings are not only supplying a restoring torque but also provide an electric connection to the rotating coil. With the use of hairsprings, the coil will return to its initial position when no current is flowing though the coil. The springs will also resist the movement of coil when there is current through coil. When the developing force between the magnetic fields (from permanent magnet and electro magnet) is exactly equal to the force of the springs, the coil rotation will stop. The coil set up is supported on jeweled bearings in order to achieve free movement.

Principle of operation

It has been mentioned that the interaction between the induced field and the field produced by the permanent magnet causes a deflecting torque, which results in rotation of the coil. The deflecting torque produced is described below in mathematical form:

Deflecting Torque: If the coil is carrying a current of $i\ amp.$, the force on a coil side = $B\,i\,l\,N$ (newton, N).

$$\therefore \text{Torque due to both coil sides} = (2r)(B\,i\,l\,N)\ (Nm)$$
$$= G\,i\ (Nm)$$

where G is the Galvanometer constant and it is expressed as $G = 2rBlN\ \ (Nm/amp.)$ $= NBA\ (Nm/amp.)$. (note $A = 2rl$ = area of the coil.)

N = no. of turns of the coil.
B = flux density in Wb/m^2 Wb/m^2.
l = length of the vertical side of the coil, m.
$2\,r$ = breadth of the coil, m
i = current in ampere.
$A = 2rl$ = area, m^2

flux density B is constant and the torque is proportional to the coil current and instrument scale is linear.

Controlling Torque: The value of control torque depends on the mechanical design of the control device. For spiral springs and strip suspensions, the controlling torque is directly proportional to the angle of deflection of the coil.

i.e., Control torque $= C\,\theta$

where, θ = deflection angle in radians and C = spring constant Nm/rad.

Damping Torque: It is provided by the induced currents in a metal former or core on which the coil is wound or in the circuit of the coil itself. As the coil moves in the field of the permanent magnet, eddy currents are set up in the metal former or core. The magnetic field produced by the eddy currents opposes the motion of the coil. The pointer will therefore swing more slowly to its proper position and come to rest quickly with very little oscillation. Electromagnetic damping is caused by the induced effects in the moving coil as it rotates in magnetic field, provided the coil forms part of closed electric circuit.

Let the velocity of the coil is $\omega(t) = \dfrac{d\theta}{dt}\,rad./\sec.$, and let the resistance of the

1.4. Construction and Basic principle operation of Moving-iron Instruments :

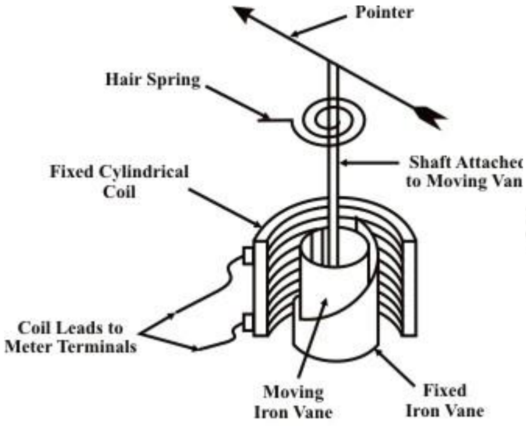
We have mentioned earlier that the instruments are classified according to the principles of operation. Furthermore, each class may be subdivided according to the nature of the movable system and method by which the operating torque is produced. Specifically, the electromagnetic instruments are sub-classes as (i) moving-iron instruments (ii) electro-dynamic or dynamometer instruments, (iii) induction instruments. In this section, we will

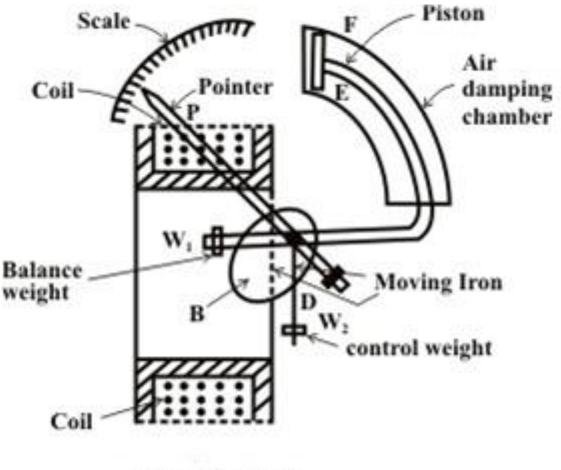
discuss briefly the basic principle of moving-iron instruments that are generally used to measure alternating voltages and currents. In moving –iron instruments the movable system consists of one or more pieces of specially-shaped soft iron, which are so pivoted as to be acted upon by the magnetic field produced by the current in coil. There are two general types of moving-iron instruments namely (i)
Repulsion (or double iron) type (ii) Attraction (or single-iron) type. The brief description of different components of a moving-iron instrument is given below.

- **Moving element:** a small piece of soft iron in the form of a vane or rod
- **Coil:** to produce the magnetic field due to current flowing through it and also to magnetize the iron pieces.
- **In repulsion type**, a **fixed** vane or rod is also used and magnetized with the same polarity.
- **Control torque** is provided by spring or weight (gravity)
- **Damping torque** is normally pneumatic, the damping device consisting of an air chamber and a moving vane attached to the instrument spindle.
- **Deflecting torque** produces a movement on an aluminum pointer over a graduated scale.

1.5. Construction of Moving-iron Instruments

The deflecting torque in any moving-iron instrument is due to forces on a small piece of magnetically ‘soft’ iron that is magnetized by a coil carrying the operating current. In repulsion (Fig.) type moving–iron instrument consists of two cylindrical soft iron vanes mounted within a fixed current-carrying coil. One iron vane is held fixed to the coil frame and other is free to rotate, carrying with it the pointer shaft. Two irons lie in the magnetic field produced by the coil that consists of only few turns if the instrument is an ammeter or of many turns if the instrument is a voltmeter. Current in the coil induces both vanes to become magnetized and repulsion between the similarly magnetized vanes produces a proportional rotation. The deflecting torque is proportional to the square of the current in the coil, making the instrument reading is a true ‘RMS’ quantity Rotation is opposed by a hairspring that produces the restoring torque. Only the fixed coil carries load current, and it is constructed so as to withstand high transient current. Moving iron instruments having scales that are nonlinear and somewhat crowded in the lower range of calibration.





Attraction type

Torque Expressions: Torque expression may be obtained in terms of the inductance of the instrument. Suppose the initial current is I, the instrument inductance L and the deflection θ. Then let I change to $I+dI$, dI being a small change of current; as a result let θ changes to $(\theta+d\theta)$, $and\ L\ to\ (L+dL)$. In order to get an incremental change in current dI there must be an increase in the applied voltage across the coil.

Applied voltage $v=\dfrac{d(LI)}{dt}=I\dfrac{dL}{dt}+L\dfrac{dI}{dt}$

The electric energy supplied to the coil in dt is

$vI\,dt=I^2\,dL+IL\,dI$

Increase in energy stored in the magnetic field $=\frac{1}{2}(I+dI)^2(L+dL)-\frac{1}{2}I^2L$

$$\simeq IL\,dI+\frac{1}{2}I^2dL$$

(neglecting second and higher terms in small quantities)

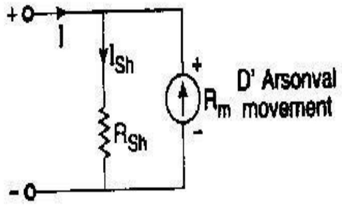
If T is the value of the control torque corresponding to deflection θ, the extra energy stored in the control due to the change $d\theta$ is $T\,d\theta$. Then, the stored increase in stored energy $=\simeq IL\,dI+\frac{1}{2}I^2dL+T\,d\theta$

From principle of the conservation of energy, one can write the following expression
Electric energy drawn from the supply = increase in stored energy + mechanical work done

$$I^2\,dL+IL\,dI=IL\,DI+\frac{1}{2}I^2dL+T\,d\theta$$

$$T\,(torque)=\frac{1}{2}I^2\frac{dL}{d\theta}\quad(Nm)$$

1.6. Extension of Range



Shunts are used for the extension of range of Ammeters. So a good shunt should have the following properties:-

1- The temperature coefficient of shunt should be low

2- Resistance of shunt should not vary with time

3- They should carry current without excessive temperature rise

4- They should have thermal electromotive force with copper

* **'Manganin'** is used for DC shunt and **'Constantan'** as AC shunt.

Ammeter:- PMMC is used as indicating device. The current capacity of PMMC is small. It is impractical to construct a PMMC coil, which can carry a current greater than 100 mA. Therefore a shunt is required for measurement of large currents.

R_m = Internal resistance of movement (coil) in Ω
R_{sh} = Resistance of shunt in Ω
I_m = I_{fs} = Full scale deflection current of movement in Amperes I_{sh} = Shunt current in Amperes
I = Current to be measured in Amperes

Since the shunt resistance is in parallel with the meter movement, the voltage drop across shunt and movement must be same.

$$I_{sh}R_{sh} = I_m R_m$$

$$R_{sh} = {I_m R_m}\big/{I_{sh}}$$

$$I_{sh} = I - I_m$$

$\therefore$ We can write $R_{sh} = {I_m R_m}\big/{(I - I_m)}$

$$\frac{I}{I_m} - 1 = \frac{R_m}{R_{sh}}$$

$$\frac{I}{I_m} = 1 + \frac{R_m}{R_{sh}}$$

$\frac{I}{I_m} = m$ is known as '

of shunt

Resistance of shunt $R_{sh} = {R_m}\big/{(m$

Or $R_{sh} = {R_m}\Big/{\left(\frac{I}{I_m} - 1\right)}$

$$R_{sh1} = {R_m}\big/{(m_1 - 1)}$$

$$R_{sh2} = {R_m}\big/{(m_2 - 1)}$$

$$R_{sh3} = {R_m}\big/{(m_3 - 1)}$$

$$R_{sh4} = {R_m}\big/{(m_4 - 1)}$$

1.7. Multi Range Ammeter:- I

ying powers for current I_1, I_2, I_3, I_4.

$$R_{sh1} = {R_m}\big/{(m_1 - 1)}$$

$$R_{sh2} = {R_m}\big/{(m_2 - 1)}$$

$$R_{sh3} = {R_m}\big/{(m_3 - 1)}$$

$$R_{sh4} = {R_m}\big/{(m_4 - 1)}$$

Voltmeter:-

For measurement of voltage a series resistor or a multiplier is required for extension of range.

I_m = Deflection current of movement
R_m = Internal resistance of movement
R_s = Multiplier resistance
V = Full range voltage of instrument

$$V = I_m(R_s + R_m)$$

$$R_s = \frac{V - I_m R_m}{I_m} = \frac{V}{I_m} - R_m$$

* For more than 500 V multiplier is mounted outside the case.

Multi Range Voltmeter:

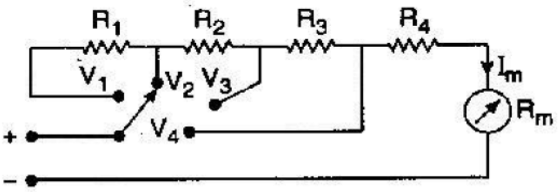
Multi Range Voltmeter:

$$R_{s1} = \frac{V_1}{I_m} - R_m$$

$$R_{s2} = \frac{V_2}{I_m} - R_m$$

$$R_{s3} = \frac{V_3}{I_m} - R_m$$

$$R_{s4} = \frac{V_4}{I_m} - R_m$$



** For average value divide the reading by 1.11. For peak value multiply the voltage by 1.414. To get peak-to-peak ratio multiply the reading by 2.828.*
*** Thermocouple and hot wire instruments are used for measurement of true power and rms value of voltage & current.*

Voltmeter & Ammeter by Moving Coil Instrument:- Same process as applied in PMMC.
Electrodynamic type Voltmeter & Ammeter:- Shunt is connected across the circuit for ammeter and multiplier resistance is connected in series for voltmeter.

UNIT-II

INSTRUMENT TRANSFOMERS AND SPECIAL METERS

2.1. Instrument Transformers:

Instrument Transformers Basics
Why instrument transformers?
In power systems, currents and voltages handled are very large.
Direct measurements are not possible with the existing equipments.
Hence it is required to step down currents and voltages with the help of instrument transformers so that they can be measured with instruments of moderate sizes

Instrument Transformers
Transformers used in conjunction with measuring instruments for measurement purposes are called “Instrument Transformers”.
The instrument used for the measurement of current is called a “Current Transformer” or simply “CT”.
The transformers used for the measurement of voltage are called “Voltage transformer” or “Potential transformer” or simply “PT”.

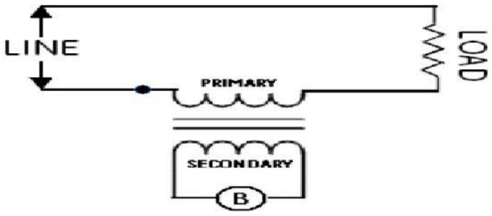


Fig 1. Current Transformer

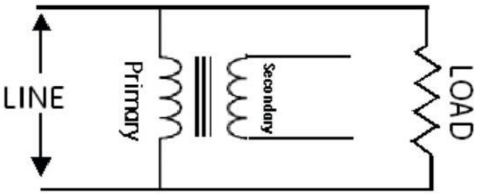


Fig 2. Potential Transformer

Fig 1. indicates the current measurement by a C.T. The current being measured passes through the primary winding and the secondary winding is connected to an ammeter. The C.T. steps down the current to the level of ammeter.
Fig 2. shows the connection of P.T. for voltage measurement. The primary winding is connected to the voltage being measured and the secondary winding to a voltmeter. The P.T. steps down the voltage to the level of voltmeter.
Merits of Instrument Transformers:

1. Instruments of moderate size are used for metering i.e. 5A for current and 100 to 120 volts for voltage measurements.
2. Instrument and meters can be standardized so that there is saving in costs. Replacement of damaged instruments is easy.
3. Single range instruments can be used to cover large current or voltage ranges, when used with suitable multi range instrument transformers.
4. The metering circuit is isolated from the high voltage power circuits. Hence isolation is not a problem and the safety is assured for the operators
5. There is low power consumption in metering circuit.
6. Several instruments can be operated from a single instrument

$$R = \frac{|\text{Primary phasor}|}{|\text{secondary phasor}|}$$

$$\frac{\text{Primary winding current}}{\text{Secondary winding current}} \quad \text{for a C.T.}$$

$$\frac{\text{Primary winding voltage}}{\text{secondary winding voltage}} \quad \text{for a P.T.}$$

Nominal Ratio: It is the ratio of rated primary winding current (voltage) to the rated secondary winding current (voltage).

$$K_n = \frac{\text{rated primary winding current}}{\text{rated secondary winding current}} \qquad \text{for a C.T.}$$

$$= \frac{\text{rated primary winding voltage}}{\text{rated secondary winding voltage}} \qquad \text{for a P.T.}$$

Turns ratio: This is defined as below

$$n = \frac{\text{number of turns of secondary winding}}{\text{number of turns of primary winding}} \quad \text{for a C.T.}$$

$$= \frac{\text{number of turns of primary winding}}{\text{number of turns of secondary winding}} \quad \text{for a P.T.}$$

Burden of an Instrument Transformer:

The rated burden is the volt ampere loading which is permissible without errors exceeding the particular class of accuracy.

Total secondary winding burden

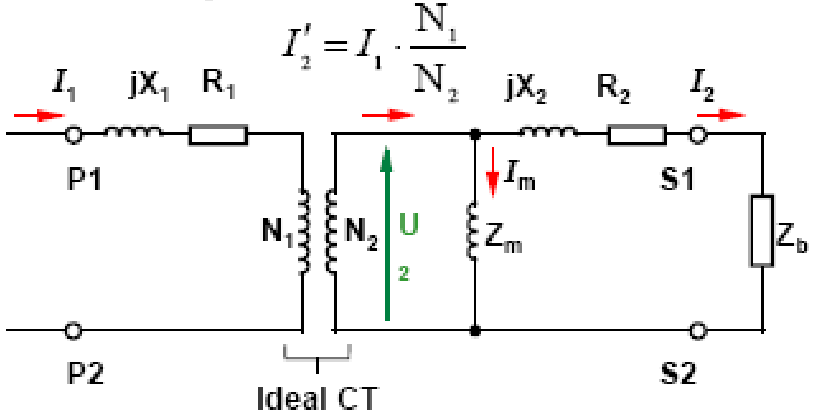
$$= \frac{(\text{secondary winding induced voltage})^2}{(\text{impedance of secondary winding circuit including impedance of secondary winding})}$$

$$= (\text{secondary winding current})^2 \times (\text{impedance of secondary winding circuit including secondary winding})$$

$$\text{secondary winding burden due to load} = \frac{(\text{secondary winding terminal voltage})^2}{(\text{impendance of load on secondary winding})}$$

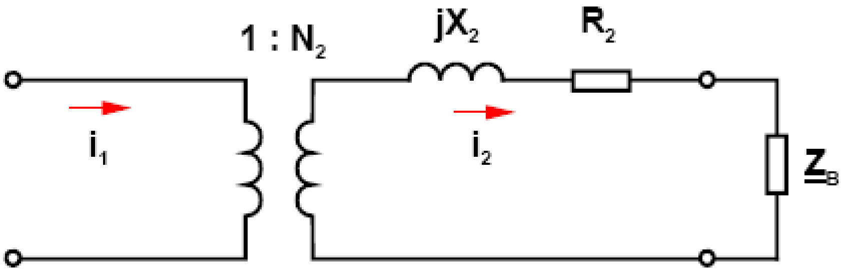
$$= (\text{secondary winding current})^2 \times (\text{impedance of load in the secondary winding circuit})$$

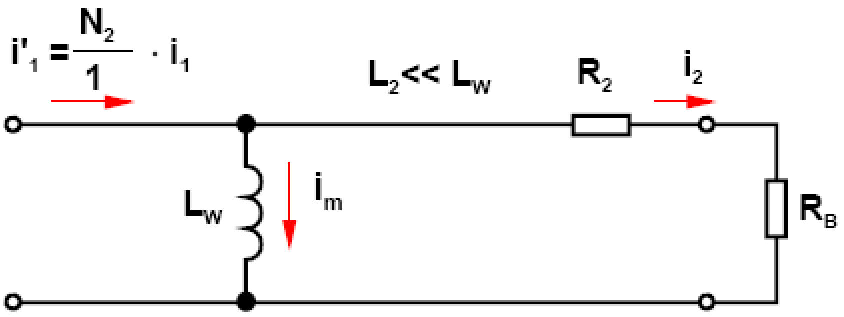
2.2. Current Transformer equivalent circuit:



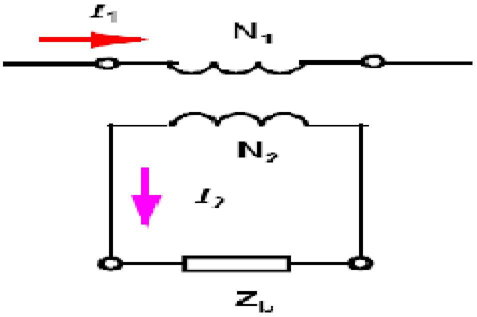
X1 = Primary leakage reactance
R1 = Primary winding resistance
X2 = Secondary leakage reactance
Z0 = Magnetizing impedance
R2 = Secondary winding resistance
Zb = Secondary load
Note: Normally the leakage fluxes X1 and X2 can be neglected

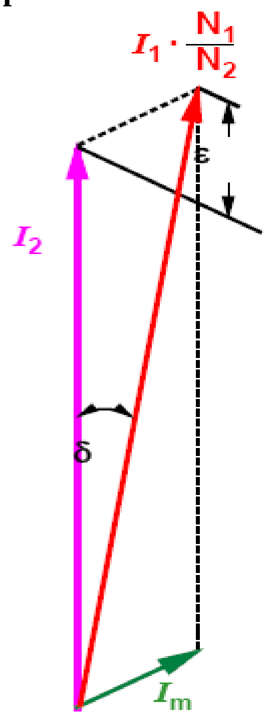
2.3. Current transformer, simplified equivalent circuit:





2.4. Current transformer: Phase displacement and current ratio error :





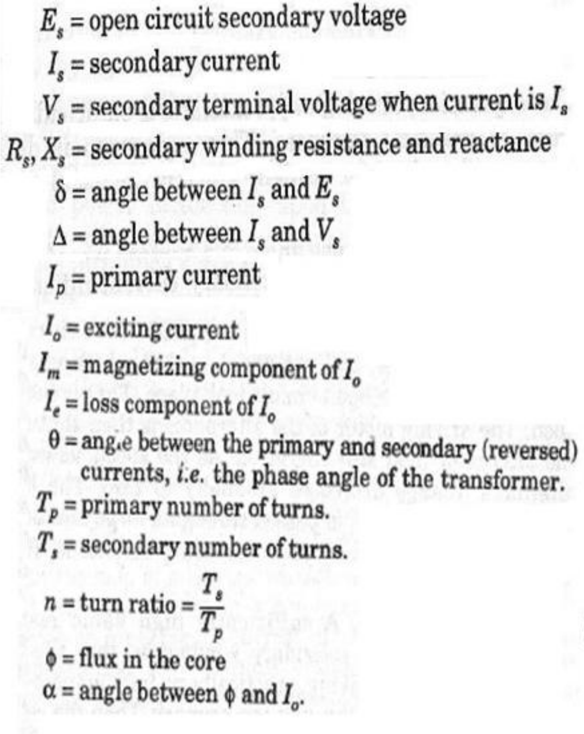
2.5. Construction of CT

Construction of Current Transformer:

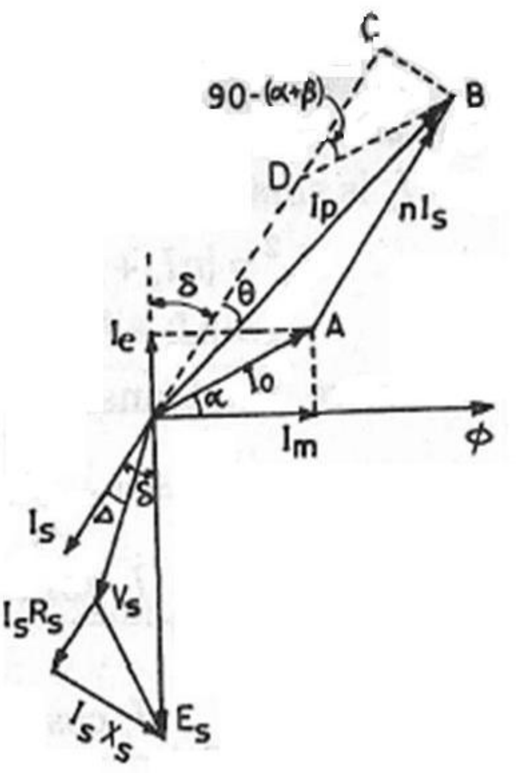
Current transformers are constructed in various ways. In one method there are two separate windings on a magnetic steel core. The primary winding consists of a few turns of heavy wire capable of carrying the full load current while the secondary winding consist of many turns of smaller wire with a current carrying capacity of between 5/20 amperes, dependent on the design. This is called the wound type due to its wound primary coil.

2.6. Wound Type

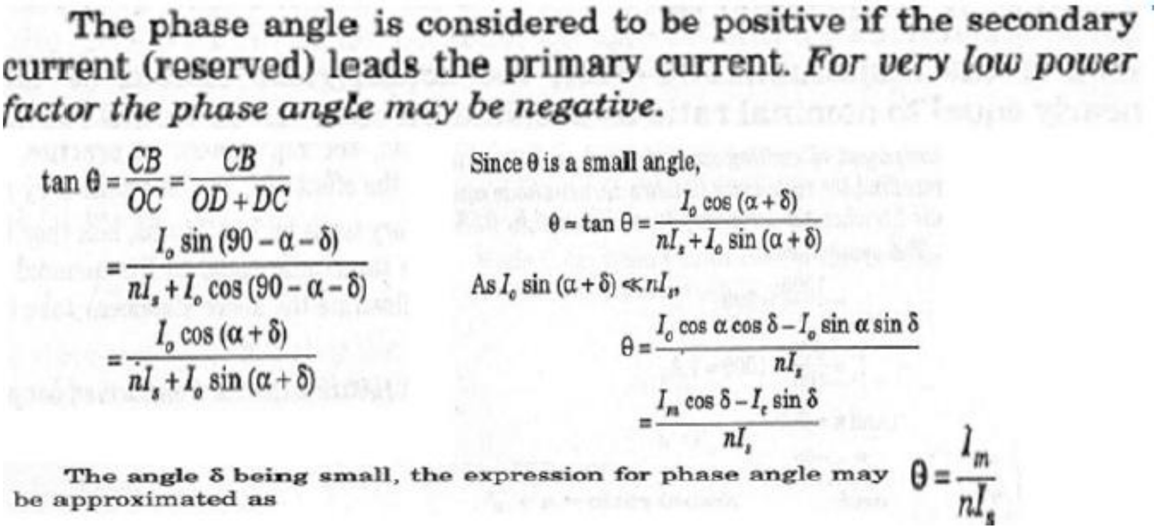
Phasor Diagram



E_s = open circuit secondary voltage
I_s = secondary current
V_s = secondary terminal voltage when current is I_s
R_s, X_s = secondary winding resistance and reactance
δ = angle between I_s and E_s
Δ = angle between I_s and V_s
I_p = primary current
I_o = exciting current
I_m = magnetizing component of I_o
I_e = loss component of I_o
θ = angle between the primary and secondary (reversed) currents, *i.e.* the phase angle of the transformer.
T_p = primary number of turns.
T_s = secondary number of turns.
n = turn ratio = $\frac{T_s}{T_p}$
ϕ = flux in the core
α = angle between ϕ and I_o.



Angle by which the reversed I_2 differs in phase from the I_1vector
Ideally the I_2should lag the $I_1$by $180^0$and hence the phase angle is ZERO
In practice this angle is $< 180^0$due to magnetizing and loss component of the I_1



The phase angle is considered to be positive if the secondary current (reserved) leads the primary current. *For very low power factor the phase angle may be negative.*

$$\tan\theta = \frac{CB}{OC} = \frac{CB}{OD+DC}$$
$$= \frac{I_o \sin(90-\alpha-\delta)}{nI_s + I_o \cos(90-\alpha-\delta)}$$
$$= \frac{I_o \cos(\alpha+\delta)}{nI_s + I_o \sin(\alpha+\delta)}$$

Since θ is a small angle,

$$\theta = \tan\theta = \frac{I_o \cos(\alpha+\delta)}{nI_s + I_o \sin(\alpha+\delta)}$$

As $I_o \sin(\alpha+\delta) \ll nI_s$,

$$\theta = \frac{I_o \cos\alpha \cos\delta - I_o \sin\alpha \sin\delta}{nI_s}$$
$$= \frac{I_m \cos\delta - I_e \sin\delta}{nI_s}$$

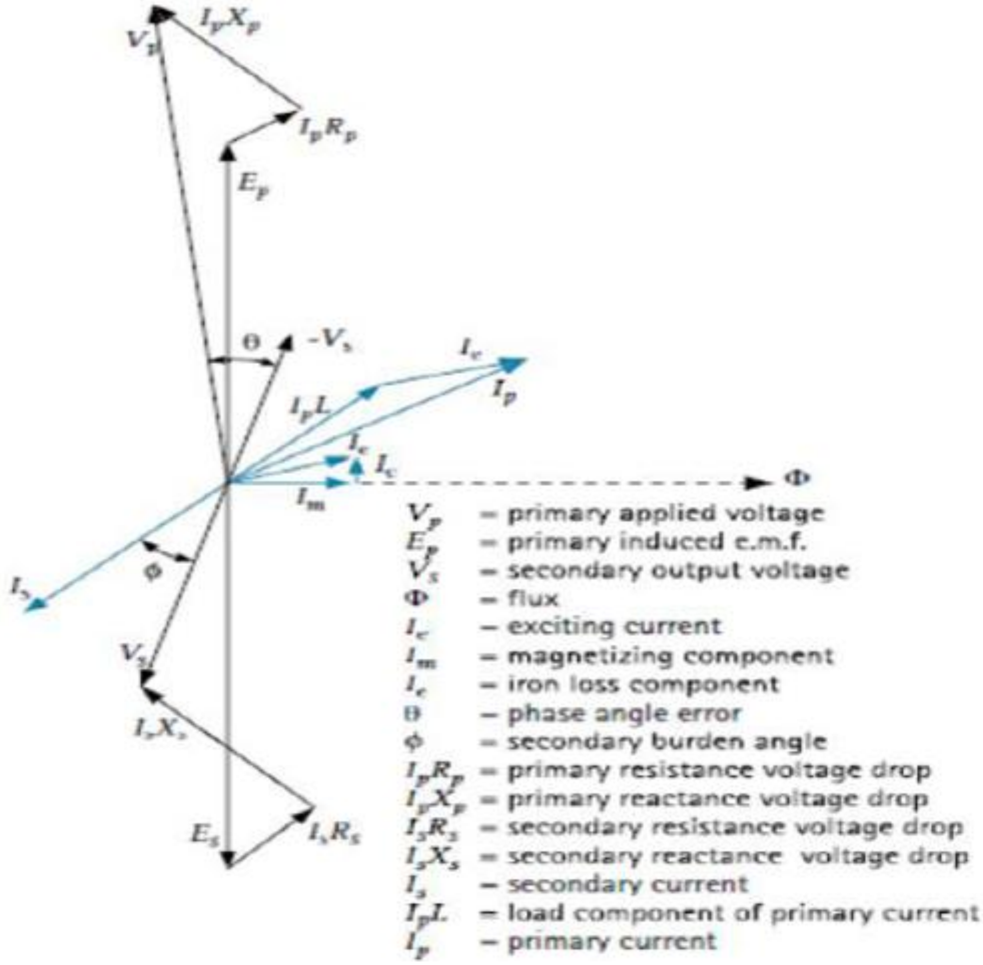
The angle δ being small, the expression for phase angle may be approximated as $\theta = \frac{I_m}{nI_s}$

Potential Transformer Basics

Potential transformers are normally connected across two lines of the circuit in which the voltage is to be measured. Normally they will be connected L-L (line-to-line) or L-G (line-to-ground). A typical connection is as follows:

2.8. Phasor Diagram of Potential Transformer:

The theory of a potential transformer is the same as that of a power transformer. The main difference is that the power loading of a P.T. is very small and consequently the exciting current is of the same order as the secondary winding current while in a power transformer the exciting current is a very small fraction of secondary winding load current.



- **PTs** are widely used to scale down the line to neutral voltage Y system or the line-to-line voltage of a Delta system to the rated input scale of the meter (typically 110 V).
- Transformers can also be used in electrical instrumentation systems
- Due to t/fs ability to step up or step down voltage and current, and the electrical isolation they provide, they can serve as a way of connecting electrical instrumentation to high-voltage, high current power systems.
- They are used in the transmission lines for the purpose of voltage measurement, power metering, and the protection of the lines.
- The **PT** (like the **CT**) is used for the substation service

UNIT III
Measurement of Power and Energy

UNIT-III

POWER MEASUREMENTS

3.1. INTRODUCTION

Electric power is the rate of doing work. It is expressed in Watts. The higher units of power used in practice include kilowatts, megawatts, etc. $P_{Watts} = VI\ COS\ \varphi$, i.e., a power of one watt is said to be expended when a source of one volt passes a current of one ampere through a load resistance/ impedance of one ohm at unity power factor.

The power measurements are made with the help of a wattmeter. Wattmeter is an indicating deflecting type of instrument used in laboratories for measurement of power in various ranges. A wattmeter consists of two coils as shown in the schematic representative figure 3.1

Current coil (CC): connected in series with circuit and carries the load current. It is designed such that it is wound with 2 to 3 turns of thick wire and hence it has a very low resistance.

Voltage or Pressure or Potential coil (PC): connected across the load circuit and hence carries a current proportional to the load current. The total load voltage appears across the PC. It is designed such that it is wound with several turns of thin wire and hence it has a very high resistance.

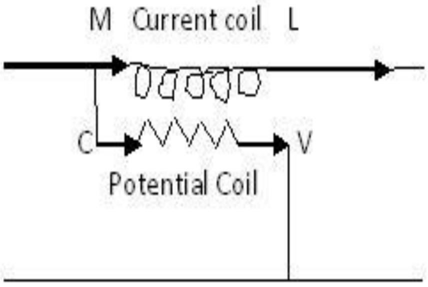


Fig. 3.1 Wattmeter Connections

The wattmeter can be a UPF meter or LPF meter depending on the type of the load connected in the measuring circuit. For power measurements in AC circuits, the *wattmeter* is widely adopted. In principle and construction, it is a combination of those applicable for an ammeter and a voltmeter.

The electrical power can be of three forms:

Real power or simply, the power is the power consumed by the resistive loads on the system. It is expressed in watts (W). This is also referred as true power, absolute power, average power, or wattage.

Reactive power is the power consumed by the reactive loads on the system. It is expressed in reactive volt-amperes (VAr).

Apparent power is the vector sum of the above two power components. It is expressed in volt-amperes (VA).

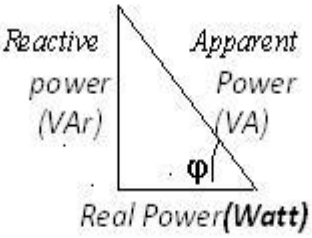


Fig.3.2 The Power Triangle

Thus, it is observed from the power triangle shown in figure 3.2, that more is the deviation of power factor from its unity value, more is the deviation of real power from the apparent power. Also, we have

$$VA^2 = W^2 + VAr^2 \tag{3.1}$$

And power factor, $\cos \phi = (\text{Watts} / \text{VA})$ (3.2)

3.2 SINGLE PHASE REAL POWER MEASUREMENTS

3.2.1 Electrodynamometer Wattmeter

An electrodynamometer wattmeter consists of two fixed coils, F_A and F_B and a moving coil M as shown in figure 3.3. The fixed coils are connected in series with the load and hence carry the load current. These fixed coils form the *current coil* of the wattmeter. The moving coil is connected across the load and hence carries a current proportional to the voltage across the load. A highly non-inductive resistance R is put in series with the moving coil to limit the current to a small value. The moving coil forms the *potential coil* of the wattmeter.

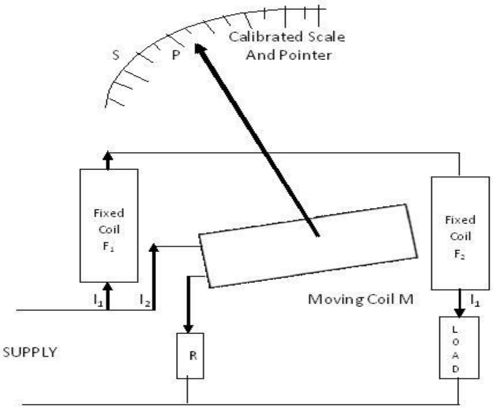


Fig. 3.3 Electrodynamometer Wattmeter

The fixed coils are wound with heavy wire of minimum number of turns. The fixed coils embrace the moving coil. Spring control is used for movement and damping is by air. The deflecting torque is proportional to the product of the currents in the two coils. Theses watt meters can be used for both DC and AC measurements. Since the deflection is proportional to the average power and the spring control torque is proportional to the deflection, the scale is uniform. The meter is free from waveform errors. However, they are more expensive.

3.2.2. Expression for the deflection torque:

Let i_C, i_P : Current in the fixed and moving coils respectively, M : Mutual inductance between the two coils,

θ : Steady final deflection of the instrument,

K: Spring constant,

V, I : RMS values of voltage and current in the measuring circuit and

R_P : Pressure coil resistance.

Instantaneous voltage across pressure coil, $v = \sqrt{2}\, V \sin wt$

Instantaneous current in the pressure coil, $i_P = \sqrt{2}\, V/R_P \sin wt = \sqrt{2}\, I_P \sin wt$

Instantaneous current in the current coil, $i_C = \sqrt{2}\, I \sin (wt-\phi)$

Instantaneous torque is given by: $T_i = i_C\; i_P\, (dM / d\theta)$

$$= [\sqrt{2}\, I \sin (wt-\phi)]\; [\sqrt{2}\, I_P \sin wt]\, (dM / d\theta) \tag{3.3}$$

T

Average deflecting torque, $T_d = (1/T) \int_{0} T_i \; d\,wt$

$$= (1/T) \int_{0}^{T} I_P \, I \, [\, \cos \phi - \cos (2wt - \phi)\,] \, (d M / d \theta) \, d\,wt$$

$$= (V I / R_P) \cos \phi \; (d M / d \theta) \qquad (3.4)$$

Since the controlling torque, $T_c = K\theta$, we have at balance of the moving pointer, $T_d = T_c$, so that,

$$\theta = [\, V I \cos \phi / (K R_P) \,] \, (d M / d \theta)$$

$$= (K' \; d M / d \theta) \, P \qquad (3.5)$$

Where $K' = K R_P$ and P is the power consumption. Thus the deflection of the wattmeter is found to be the direct indication of the power being consumed in the load circuit.

3.2.3 Low Power Factor Wattmeter

If an ordinary electrodynamometer wattmeter is used for measurement of power in low power factor circuits, (PF<0.5), then the measurements would be difficult and inaccurate since:

- The deflecting torque exerted on the moving system will be very small and
- Errors are introduced due to pressure coil inductance (which is large at LPF)

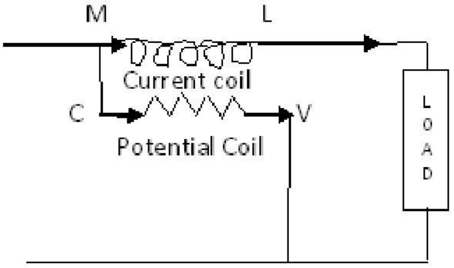
Thus, in a LPF wattmeter, special features are incorporated in a general electro-dynamometer wattmeter circuit to make it suitable for use in LPF circuits as under:

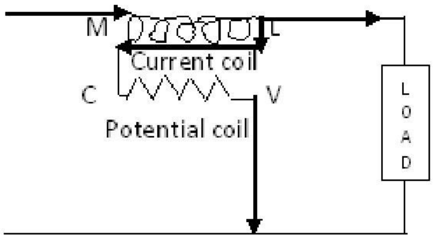
(a) Pressure coil current:

The pressure coil circuit is designed to have a low value of resistance so that the current through the pressure coil is increased to provide an increased operating torque.

(b) Compensation for pressure coil current:

On account of low power factor, the power is small and the current is high. In this context, there are two possible connections of the potential coil of a wattmeter as shown in figure 4.4. The connection (a) can not be used, since owing to the high load current, there would be a high power loss in the current coil and hence the wattmeter reading would be with a large error. If the *connection (b)* is used, then the power loss in the pressure coil circuit is also included in the meter readings.





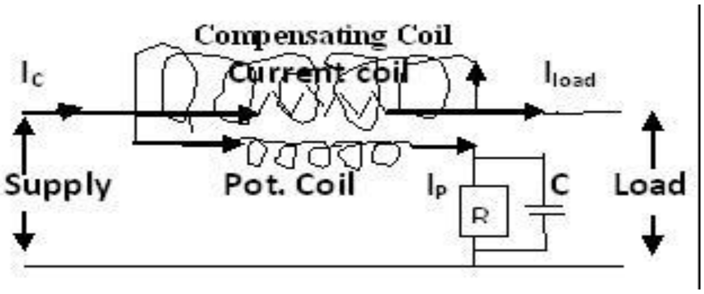
Thus it is necessary to compensate for the pressure coil current in a low power factor wattmeter. For this, a compensating coil is used in the instrument to compensate for the power loss in the pressure coil circuit as shown in figure 3.5.

(c) Compensation for pressure coil inductance:

At low power factor, the error caused by the pressure coil inductance is very large. Hence, this has to be compensated, by connecting a capacitor C across a portion of the series resistance in the pressure coil circuit as shown in figure 3.5.

(d) Realizing a small control torque:

Low power factor wattmeters are designed to have a very small control torque so that they can provide full scale deflection (f.s.d.) for power factor values as low as 10%. Thus, the complete circuit of a low power factor wattmeter is as shown in figure 3.5.



3.3 REACTIVE POWER MEASUREMENTS

A single wattmeter can also be used for three phase reactive power measurements. For example, the connection of a single wattmeter for 3-phase reactive power measurement in a balanced three phase circuit is as shown in figure 4.6.

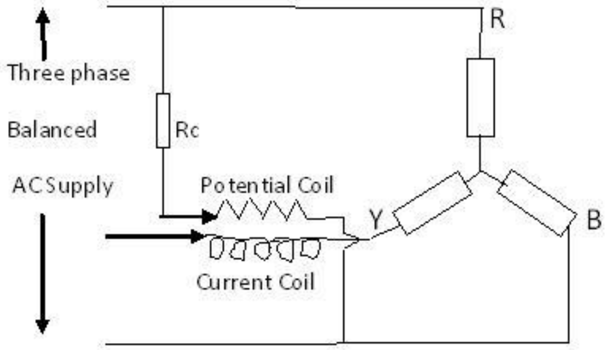
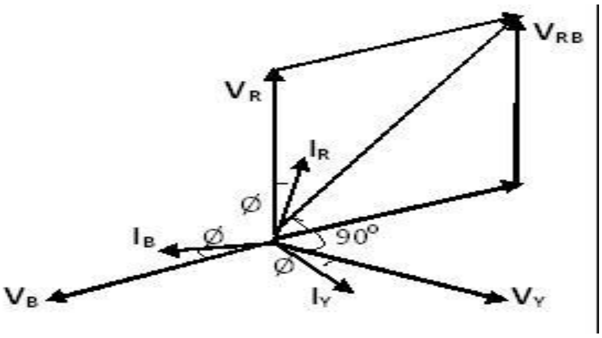
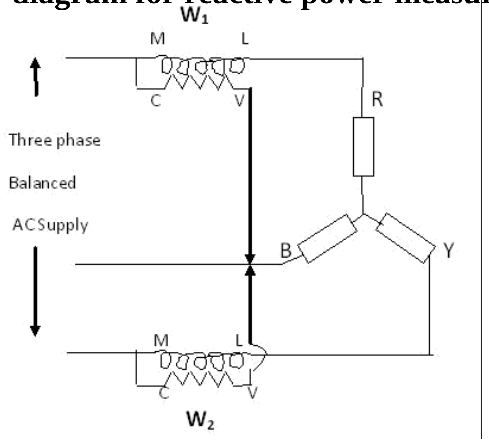


Fig. 4.6 Reactive power measurement circuit

The current coil of the wattmeter is inserted in one line and the potential coil is connected across the other two lines. Thus, the voltage applied to the voltage coil is $V_{RB} = V_R - V_B$, where, V_R and V_B are the phase voltage values of lines R and B respectively, as illustrated by the phasor diagram of figure 4.7.



Phasor diagram for reactive power measurements



The reading of the wattmeter, $\mathbf{W_{3ph}}$ for the connection shown in figure 3.6 can be
obtained based on the phasor diagram of figure 34.7, as follows:

Wattmeter reading, **Wph** = Iy VRB
= $I_y V_L$ cos (90+Ø)
= - √3 $V_{ph} I_{ph}$ sin Ø
= - √3 (Reactive power per phase) (3.6)

Thus, the three phase power, $\mathbf{W_{3ph}}$ is given by,
W3ph = (VArs/phase)
= 3 [W_{ph} /- √3]
= - √*3 (wattmeter reading)* (3.7)

3.4 THREE PHASE REAL POWER MEASUREMENTS

The three phase real power is given by,

P_{3ph}= 3 $V_{ph} I_{ph}$ cos Ø or
P_{3ph}= √3 $V_L I_L$ cos Ø (3.8)

The three phase power can be measured by using either one wattmeter, two wattmeters or three wattmeters in the measuring circuit. Of these, the two wattmeter method is widely used for the obvious advantages of measurements involved in it as discussed below.

3.4.1 Single Wattmeter Method

Here only one wattmeter is used for measurement of three phase power. For circuits with the balanced loads, we have: W_{3ph}=3(*wattmeter reading).* For circuits with the unbalanced loads, we have: W_{3ph}=sum of the three readings obtained separately by connecting wattmeter in each of the three phases. If the neutral point is not available (3 phase 3 wire circuits) then an artificial neutral is created for wattmeter connection purposes. Instead three wattmeters can be connected simultaneously to measure the three phase power. However, this involves more number of meters to be used for measurements and hence is not preferred in practice. Instead, the three phase power can be easily measured by using only two wattmeters, as discussed next.

3.4.2 Two Wattmeter Method

The circuit diagram for two wattmeter method of measurement of three phase real power is as shown in the figure 34.7. The current coil of the wattmeters W_1 and W_2 are inserted respectively in R and Y phases. The potential coils of the two wattmeters are joined together to phase B, the third phase. Thus, the voltage applied to the voltage coil of the meter, W_1 is $V_{RB}= V_R-V_B$, while the voltage applied to the voltage coil of the meter, W_2 is $V_{YB}=V_Y-V_B$, where, V_R, V_B and V_C are the phase voltage values of lines R, Y and B respectively, as illustrated by the phasor diagram of figure 3.8. Thus, the reading of the two wattmeters can be obtained based on the phasor diagram of figure 4.8, as follows:

$$W_1 = IR\ VRB$$

$$= I_L V_L \cos (30 - Ø) \qquad (3.9)$$

$$W_2 = IY\ VYB$$

$$= I_L V_L \cos (30 + Ø) \qquad (3.10)$$

Hence, $$W_1+W_2 = \sqrt{3}\, V_L I_L \cos Ø = P_{3ph} \qquad (3.11)$$

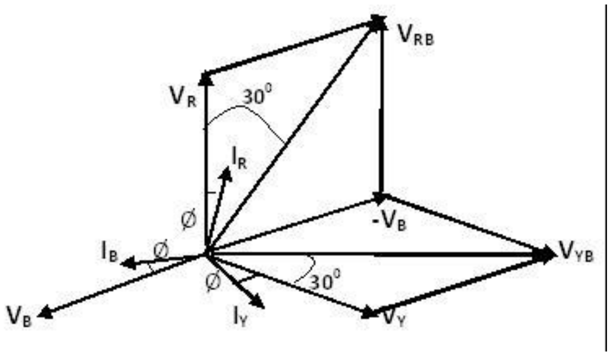
And $$W_1-W_2 = V_L I_L \sin Ø \qquad (3.12)$$

So that then,

$$\mathbf{\tan Ø = \sqrt{3}\, [W_1-W_2]/ [W_1+W_2]} \qquad (3.13)$$

Where Ø is the lagging PF angle of the load. It is to be noted that the equations (3.11) and (3.12) get exchanged if the load is considered to be of leading PF.

Two wattmeter method of 3-phase power measurement



Phasor diagram for real power measurements

The readings of the two wattmeters used for real power measurements in three phase circuits as above vary with the load power factor as described in the table 3.1.

Variation of wattmeter readings with load PF (lag)

PF angle	**PF**	**W_1**	**W_2**	**$W_{3ph}=W_1+W_2$**	**Remarks**
φ **(lag)**	$\cos\varphi$	$V_L I_L \cos(30-\varphi)$	$V_L I_L \cos(30+\varphi)$	$\sqrt{3} V_L I_L \cos\varphi$	Gen. Case (always $W_1 \geq W_2$)
$_{0}$**0**	UPF	$\sqrt{3}/2\ V_L I_L$	$\sqrt{3}/2\ V_L I_L$	$2W_1$ or $2W_2$	$W_1=W_2$
30^0	0.866	$V_L I_L$	$V_L I_L/2$	$1.5W_1$ or $3W_2$	$W_2=W_1/2$
60^0	0.5	$\sqrt{3}/2\ V_L I_L$	ZERO	W_1 alone	W_2 reads zero
$>60^0$	<0.5	W_1	W_2 reads negative	$W_1+(-W_2)$	For taking readings, the PC or CC connection of W_2 should be reversed) (LPF case)

UNIT-IV
Potentiometers

4.1. Potentiometers

- Before the introduction of the moving coil galvanometer, potentiometers were used in measuring voltage, hence the '-meter' part of their name
- Today this method is still important in standards work
- The null-balance principle of measurement is also used in other areas of electronics
- The potentiometer used for measurement is a type of bridge circuit for measuring voltages by comparison between a small fraction of the voltage which could be precisely measured, then balancing the two circuits to get null current flow which could be precisely measured

Types

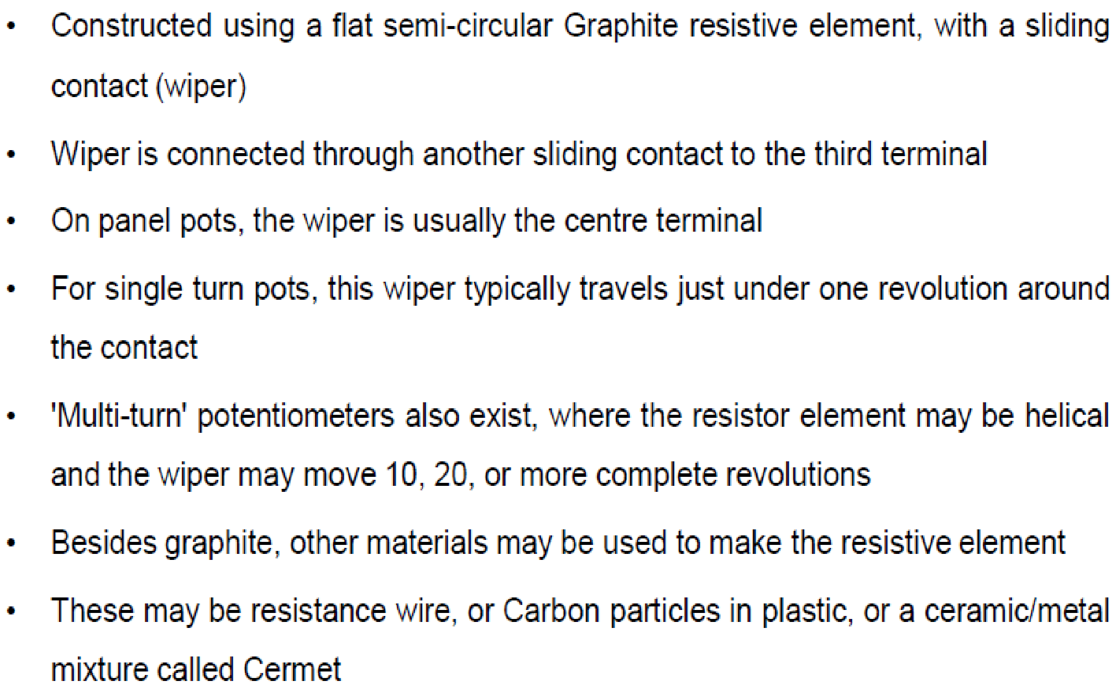
Main Classification

- Constant current potentiometer
- Constant resistance potentiometer
- Microvolt potentiometer
- Thermocouple potentiometer

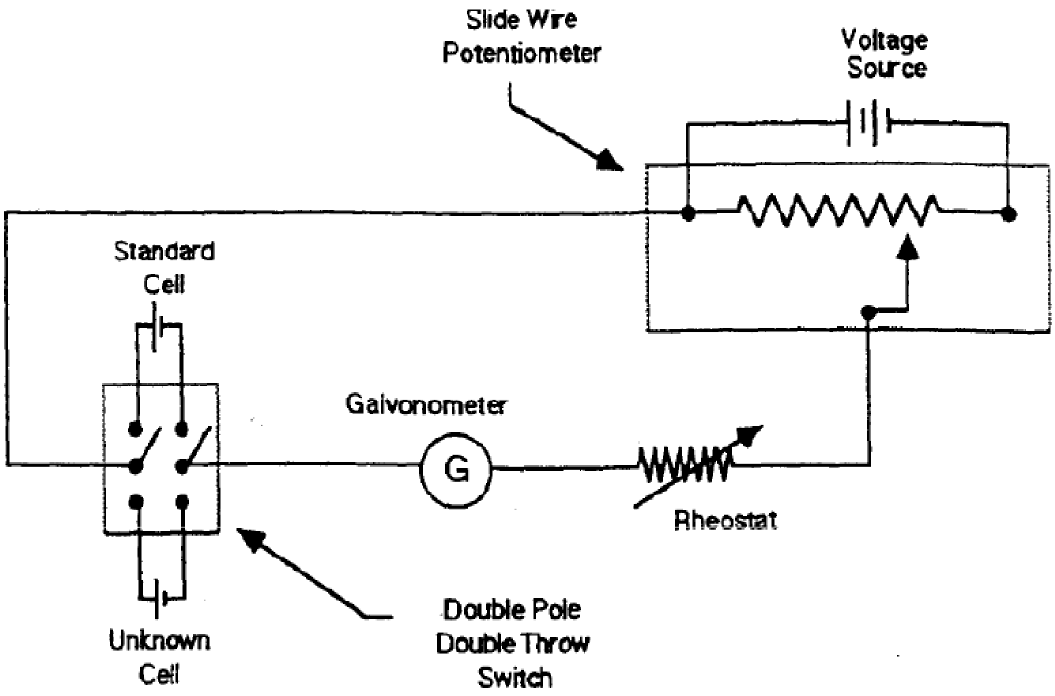
Other

- DC
- AC

4.2. Construction of potentiometers



- Constructed using a flat semi-circular Graphite resistive element, with a sliding contact (wiper)
- Wiper is connected through another sliding contact to the third terminal
- On panel pots, the wiper is usually the centre terminal
- For single turn pots, this wiper typically travels just under one revolution around the contact
- 'Multi-turn' potentiometers also exist, where the resistor element may be helical and the wiper may move 10, 20, or more complete revolutions
- Besides graphite, other materials may be used to make the resistive element
- These may be resistance wire, or Carbon particles in plastic, or a ceramic/metal mixture called Cermet



Slidewire Potentiometer Circuit

- The resistance of the wire is uniform and, therefore, any particular length, L, will have a resistance proportional to that length
- (Ohms Law) - voltage is proportional to the resistance (assuming constant current), the voltage across the length, L, is proportional to L and V = kL, where V is the voltage in volts, L is the variable length of wire in meters and k is the proportionality constant in volts per meter
- If a parallel circuit is connected to the wire with a sliding contact, galvanometer, and standard voltage source (see diagram), the sliding contact may be adjusted to a length of wire in which the voltage drop is just equal to that of the standard cell
- At this point, there is no net current in the parallel circuit and **Emf=kL**
- Since the Emf and L are known, k can be calculated
- Substitution of an unknown cell through a similar procedure will enable one to determine the Emf of that unknown cell

Gall-Tinsley Co-ordinate Type A.C. Potentiometer

This potentiometer consists of two separate potentiometers enclosed in same common case. The circuit diagram of Gall-tinsley coordinate potentiometer is as shown in the Fig.

The in-phase and quadrature potentiometers consist of sliding contacts BB′ and CC′ respectively. The rheostats R and R′ are also provided in the respective potentiometers for the adjustments of current. By using different arrangement, the supplies for the potentiometers are obtained.

A vibration galvanometer V_G is tuned to the supply frequency and it is connected in series with a switch K and electro dynamometer type ammeter.

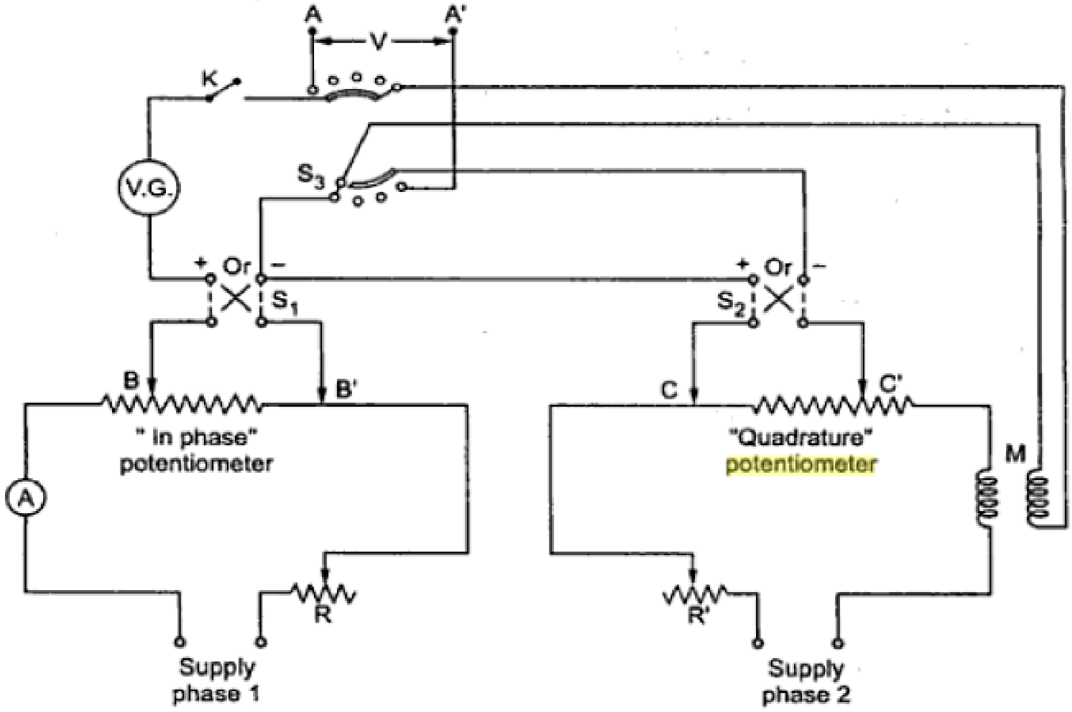


Fig. Connections of Gall-Tinsley potentiometers

S_1 and S_2 are the sign changing switches which are necessary for reversing direction of unknown e.m.f. The unknown voltage is introduced using selector switch S_3 which is having 4 pair of terminals.

Consider Fig. The transformers T_1 and T_2 are step down transformers and they supply about 6 to 8 volts to potentiometers. By using variable resistor and capacitor, the supply for T_2 is obtained. By adjusting R and C, quadrature is obtained.

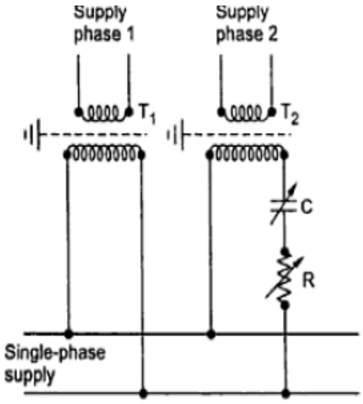


Fig. Phase splitting circuit

Standardisation

First of all d.c. standardisation of potentiometer is done by using standard cell and D'Arsonval type galvanometer. Then without disturbing this setting, a.c. standardisation is done by adjusting slide wire current to give zero deflection. Then previous galvanometer is replaced by vibration galvanometer and also direct current supply is replaced by a.c. supply. Then the rheostat is adjusted till the current in the quadrature potentiometer wire is same as that in the in-phase potentiometer magnitudewise. Also these two currents must be exactly in quadrature.

Measurement of Unknown e.m.f.

The e.m.f. to be measured is connected across the terminals A-A′ using selector switch S_3. The sliding contacts of both the potentiometers are adjusted till the null deflection is obtained in the vibration galvanometer.

Under the balance condition, the in-phase component of the unknown e.m.f. is obtained from in-phase potentiometer while the quadrature component of the unknown e.m.f. is obtained from quadrature potentiometer. If needed the polarity of the test voltage may be reversed by using sign changing switches S_1 and S_2 to balance the potentiometers.

If V_A and V_B are the two potentiometer readings, the magnitude and phase angle of unknown e.m.f. are given by,

$$V = \sqrt{V_A^2 + V_B^2} \quad \text{and}$$

$$\theta = \tan^{-1}\left(\frac{V_B}{V_A}\right)$$

Thus the emf can be expressed in complex form as

$$V = \pm V_A \pm j\,V_B$$

Applications of A.C. Potentiometers

1. Calibration of voltmeter :

The method of calibration with a.c. potentiometers is very much similar to that with d.c. potentiometers. If the working voltage is less than 1.5V, it can be measured directly. If the voltage is very high then a.c. potentiometer must be used along with volt box or two capacitors in series for measurement of medium and high voltages respectively.

2. Calibration of ammeter :

The calibration of ac ammeters may be carried out by using non-inductive standard resistance and successively noting measurements of various alternating current through it. The process of calibrating ammeters using a.c. and d.c. potentiometers is same.

3. Testing of energymeter and wattmeter :

The practical set up for the testing of wattmeter and energy meter with a.c. potentiometers is similar to that of calibration of wattmeter with d.c. potentiometers. Only change in the set up is that a phase shifting transformer is included in potential divider circuit. Using this transformer, the phase of the voltage may be varied with respect to the current. Thus, the energymeter and wattmeter may be tested at different power factors.

4. Measurement of self reactance of coil :

The practical set up for measurement of self reactance of a coil is as shown in the Fig.

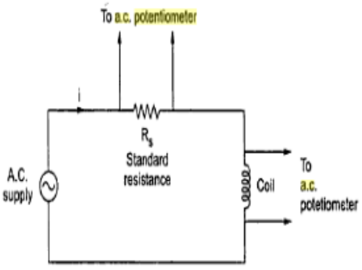


Fig. Measurement of self reactance of coil

A coil whose self reactance is to be measured is connected in series with a standard resistance R_s. The current i in the circuit is supplied by a.c. voltage source. The voltage drop across R_s is first measured using a.c. potentiometer. Let voltage across R_s be v_s. Similarly let the voltage measured across coil be v_c. For these measurements suppose polar ac potentiometer is used. Then ,

Voltage across coil $= v_c = V_c \angle \theta_c$ in polar form

Voltage across $R_s = v_s = V_s \angle \theta_s$

The current through coil can be calculated as,

$$i = \frac{v_s}{R_s} = \frac{V_s \angle \theta_s}{R_s}$$

The impedance of coil can be calculated as,

$$Z = \frac{v_c}{i} = \frac{V_c \angle \theta_c}{\left[\frac{V_s}{R_s} \angle \theta_s\right]} = \frac{R_s V_c}{V_s} \angle \theta_c - \theta_s$$

We can write this impedance Z in rectangular form in real part and imaginary part as resistance and reactance.

The resistive part of impedance is given by,

$$R = Z \cos(\theta_c - \theta_s) = \frac{R_s V_c}{V_s} \cos(\theta_c - \theta_s)$$

The reactive part of impedance is given by,

$$X = Z \sin(\theta_c - \theta_s) = \frac{R_s V_c}{V_s} \sin(\theta_c - \theta_s)$$

Thus above equation represents reactance of the coil.

UNIT-V
Resistance Measurement

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5.1 INTRODUCTION

A resistance is a parameter which opposes the flow of current in a closed electrical network. It is expressed in ohms, milliohms, kilo-ohms, etc. as per the ohmic principle, it is given by;

$R = V/I$; *with temperature remaining constant*. (1)

If temperature is not a constant entity, then the resistance, R_2 at $t_2{}^0C$ is given by;

$R_2 = R_1 [1+ \alpha (t_2-t_1)]$ (2) Where $t= t_1-t_2$, the rise in temperature from $t_1{}^0C$ to $t_2{}^0C$, α is the temperature coefficient of resistance and R_1 is the temperature at $t_1{}^0C$.

Resistance can also be expressed in terms of its physical dimensions as under:

$R = \rho l/A$ (3) Where, l is the length, A is the cross sectional area and ρ is the specific resistance or resistivity of the material of resistor under measurement.

To realize a good resistor, its material should have the following properties:

- Permanency of its value with time.
- Low temperature coefficient of resistance
- High resistivity so that the size is smaller
- Resistance to oxidation, corrosion, moisture effects, etc.
- Low thermo electric emf against copper, etc.

No single material can be expected to satisfy all the above requirements together. Hence, in practice, a material is chosen for a given resistor based on its suitability for a particular application.

5.2 CLASSIFICATION OF RESISTANCES

For the purposes of measurements, the resistances are classified into three major groups based on their numerical range of values as under:

- Low resistance (0 to 1 ohm)
- Medium resistance (1 to 100 kilo-ohm) and
- High resistance (>100 kilo-ohm)

Accordingly, the resistances can be measured by various ways, depending on their range of values, as under:

1. Low resistance (0 to 1 ohm): AV Method, Kelvin Double Bridge, potentiometer, doctor ohmmeter, etc.
2. Medium resistance (1 to 100 kilo-ohm): AV method, wheat stone’s bridge, substitution method, etc.
3. High resistance (>100 kilo-ohm): AV method, Fall of potential method, Megger, loss of charge method, substitution method, bridge method, etc.

5.3. MEASUREMENT OF MEDIUM RESISTANCES

(a) AV Method

Here an ammeter and a voltmeter are used respectively in series and in parallel with the resistor under measurement. Various trial readings are obtained for different current values. Resistances are obtained for each trial as per ohmic-principle, using equation (1).

The average value of all the trails will give the measured value of resistance.

This method is also reefed as the potential drop method or VA method. Here, the meter ranges are to be chosen carefully based on the circuit conditions and the resistance value to be obtained. This method suffers from the ***connection errors.***

(b) Wheatstone's bridge

This bridge circuit has four resistive arms: arm-AB and arm-BC the ration arms with resistances R_1 and R_2, arm-AD with the unknown resistance R_3 and arm CD with a standard known variable resistance R_4, as shown in figure-1 below. The supply is fed across the arm-AC and the arm-BD contains a galvanometer used as a detector connected across it.

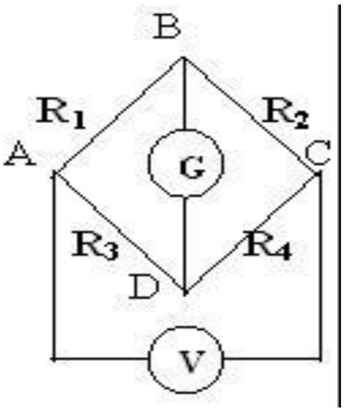


Fig.1 Wheatstone bridge

The bridge is said to be balanced when the galvanometer current is zero. This is called as the position of null reading in the galvanometer, which is used here as a null detector of the bridge under the balanced conditions.

Thus under the balance conditions of the bridge, we have,

$I_g = 0$; $V_{AB} = V_{AD}$ and $V_{BC} = V_{CD}$, so that,

$I_1R_1 = I_2R_3$ and $I_1R_2 = I_2R_4$ (4)

Solving further, we get, $R_1/R_2 = R_3/R_4$ and $\boldsymbol{R_1R_4 = R_2R_3}$ (5)

This is the ***balance equation*** of the bridge.

Thus, unknown resistance, $\mathbf{R_3 = R_1R_4/R_2}$ ohms. (6)

5.4. *Expression for Galvanometer current*

In a WS bridge PQRS as shown in figure 2, below, under the unbalanced conditions of the bridge, the current flowing in the galvanometer is given by:

$I_g = E_{Th}/[R_{Th}+R_g]$ (7)

Where,

$R_{Th} = \{PQ/(P+Q) + RS/(R+S)\}$

$E_{Th} = E\,\{P/(P+Q) - R/(R+S)\}$ (8)

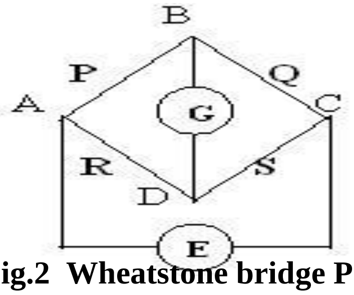


Fig.2 Wheatstone bridge PQRS

5.5. *Errors in WS bridge measurements*

(i) ***Limiting errors***: In a WS bridge PQRS, the percentage limiting error in the measurand resistance, R is equal to the sum of the percentage limiting errors in the bridge arm resistances P, Q and S.

(ii) ***Errors due to heating of elements in the bridge arms:***

$R_t = R_0 [1+ \alpha t]$; $P = I^2R$ Watts; Heat= I^2Rt Joules

The I^2R loss occurring in the resistors of each arm might tend to increase the temperature, which in turn can result in a change in the resistance value, different from the normal value.

(iii) ***Errors due to the effect of the connecting wires and lead resistors***: the connecting lead wire resistance will affect the value of the unknown resistance, especially when it is a low resistance value. Thus, the connecting lead wire resistances have to be accounted for while measuring a low resistance.

(iv) ***Contact resistance errors***: The contact resistance of the leads also affects the value of the measurand resistance, just as in point (iii) above. This resistance value depends on the cleanliness of the contact surfaces and the pressure applied to the circuit.

1.3.3 Limitations of WS bridge

The WS bridge method is used for measurement of resistances that are numerically in the range of a few ohms to several kilo-ohms. The ***upper limit*** is set by the reduction in sensitivity to unbalance caused by the high resistances, as per the equation (7).

1.3.4 Sensitivity of WS bridge

In a WS bridge PQRS as in figure 2, with the resistance S in an arm changed to S+ S, the bridge becomes unbalanced to the extent of the resistance change thus brought about. This is also referred as the unbalanced operation of the WS bridge. Under such circumstances, with $\mathbf{S_v}$ as the voltage sensitivity of the galvanometer, we have,

Galvanometer Deflection, $\theta = \mathbf{S_v}\, e = \mathbf{S_v}\, [ES\ \ R/(R+S)^2]$ (10)

Bridge Senstivity, $S_B = \theta/(\ \ R/R)$ (11)

Rearranging, we get,

$$S_B = \mathbf{S_v} e/\{(R/S)+2+(S/R)\} \qquad (12)$$

Thus, the WS bridge sensitivity is high when **(R/S)=1**. It decreases as the ratio (R/S) becomes either larger or smaller than unity value. This also causes a reduction in accuracy with which the bridge is balanced.

1.4 MEASUREMENT OF LOW RESISTANCES

Of the various methods used for measurement of low resistances, the Kelvin's Double Bridge (KDB) is the most widely used method. Here, the bridge is designed as an improved or modified WS Bridge with the effect of contact and connecting lead wire resistances considered. The circuit arrangement for KDB is as shown in figure 3 below. R is the low resistance to be measured, S is the standard low resistance, r is the very small link resistance between R and S, called the yoke resistance, P,Q,p,q are the non inductive resistances, one set of which, Pp or Qq is variable. The bridge is supplied with a battery E and a regulating resistance R_C. A galvanometer of internal resistance R_g, connected as shown, is used as a null detector.

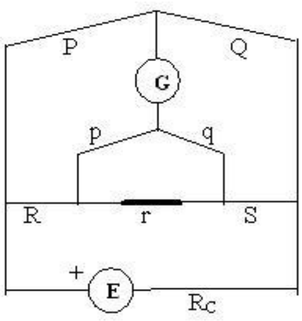
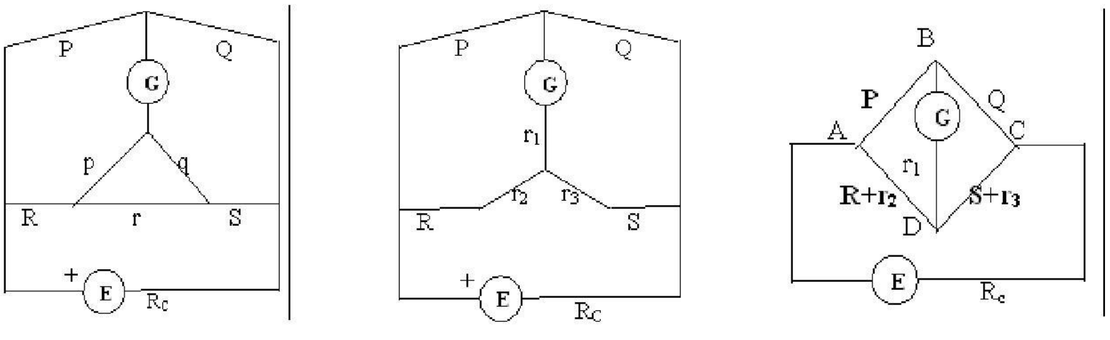


Fig.3 Kelvin's Double Bridge

1.4.1 Balance Equation of KDB

The balance equation of KDB can be derived as under. Using star-delta conversion principle, the KDB circuit of figure 3 can be simplified as shown in figure-4 below.



(a) **(b)** **(c)**

Fig. 4 Simplified circuits of KDB

It is thus observed that the KDB has now become a simplified WS bridge-ABCD as shown in figure 4(c). It is to be noted that the yoke resistance r_1 is ineffective now, since it is in the zero current branch, in series with the galvanometer. Thus, under balance conditions, we have,

$$P\ (S+r_3) = Q\ (R+r_2) \tag{13}$$

Where,

$$r_1 = pq / (p+q+r)$$
$$r_2 = pr / (p+q+r)$$
$$r_3 = qr / (p+q+r) \tag{14}$$

Using (13) and (14), we get,

$$PS + Pqr / (p+q+r) = QR + Qpr / (p+q+r) \tag{15}$$

Simplifying (15), we get the final balance equation of the Kelvin Double Bridge as:

$$R = [P/Q]\ S + [qr / (p+q+r)]\ [(P/Q) – (p/q)] \tag{16}$$

1.4.2 Significance of balance equation of KDB

From the balance equation (16), it can be observed that,

- If the yoke resistance is negligible, (r=0), then R = [P/Q]S, as per the WS bridge balance equation.
- If the ratio arm resistances are equal, i.e., [(P/Q) = (p/q)], then again, R=[P/Q]S, as per WS bridge principle.

1.4.3 Unbalanced KDB

Further, if the KDB is unbalanced due to any change in resistances R or S (by R or S), then the unbalanced galvanometer current during the unbalanced conditions is given by:

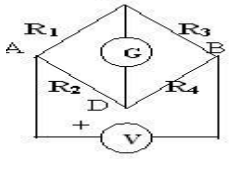
$$I_g = I.\ \ S\ [P/(P+Q)]/\ \{R_g+ pq/(p+q)+ PQ/(P+Q)\} \tag{17}$$

1.4 MEASUREMENT OF HIGH RESISTANCES

Of the various methods used for measurement of high resistances, the fall of potential method is widely used. Also, very high resistances of the order of mega-ohms can be measured by using an instrument called MEGGER, also called as the insulation resistance tester. It is used as a high resistance measuring meter as also a tester for testing the earth resistances.

1.5 SOLVED PROBLEMS

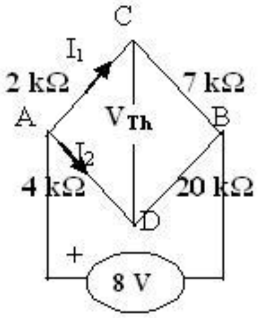
1 Calculate the current through the galvanometer for the Wheatstone bridge circuit shown below in figure P1.



R_1 = 2 k
R_2 = 4 k
R_3 = 7 k
R_4 = 20 k
R_g = 300
V = 8.0 V

Fig. P1

Solution:
The Thevenin's equivalent circuit is used to find I_g. To find E_{Th}, remove R_g as shown in figure P1(a).



$V_{Th} = V_{CD} = I_2 x4\ k \ \ - I_1 x2k$
$I_1 = \ 8 / [\ (2+7)10^3] = 8.888(10^{-4})$ A
$I_2 = \ 8 / [\ (4+20)10^3] = 3.333(10^{-4})$ A
Thus, $V_{Th} = 3.333(10^{-4})\ x4(10^3) - 8.888(10^{-4})\ x2(10^3)$ V

= - *0.4444 V,* *i.e., point C is negative.*

Fig. P1(a)

To find R_{Th}, short the battery as shown in figure P1(b). The points A and B are shorted and can be represented by a single point as shown in figure P1(c).

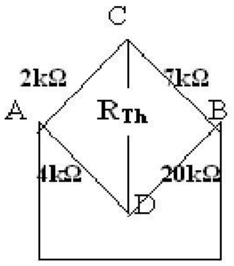


Fig. P1(b)

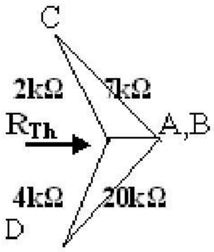
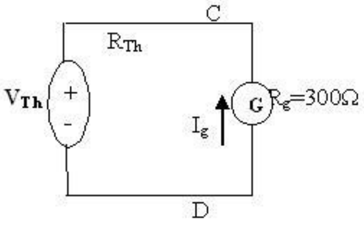


Fig. P1(c)

$R_{Th} = [2(7)/(2+7)]$
$+ [4(20)/(4+20)]$
$= [1.555 + 3.333]$ k
= 4.888 k

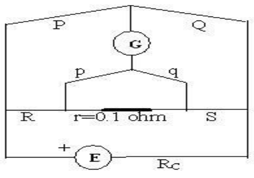


$I_g = V_{Th} / [R_g+R_{Th}]$
$= 0.4444 / [4.888x10^3+300]$
= ***85.65 µA - from D to C as shown in figure P1(d) aside***

Fig. P1(d)

2 A KDB is balanced with the following constants: outer ratio arms: 100 and 1000. Ohms, inner ratio arms: 99.92 and 1000.6 ohms, link resistance: 0.1 ohm, standard resistance: 3.77 milliohms, find the unknown resistance.

Solution:
The KDB circuit is as shown in figure P2.



P = 100 ohms Q
= 1000 ohms p =
99.92 ohms q =
1000.6 ohms r =

0.1 ohm

$S = 3.77(10^{-3)}$ ohms

= 0.00377 ohms

Fig. P2

The unknown resistance R is given by:

R = [P/Q] S + [qr / (p+q+r)] [(P/Q) – (p/q)]

By substituting the specified values, we get,

R = [100/1000]0.00377+[1000.6(0.1)/(99.92+1000.6+0.1)][(100/1000)–(99.92/1000.6)]

= 0.382712 milliohms.

3 The ratio arms of a KDB are 1000 ohms each. The galvanometer has a resistance of 1000 ohms and a sensitivity of 0.003 µA/mm. A current of 10 A is passed by a 2.2 V battery in series with a rheostat. The yoke resistance is negligible. The standard resistance is 0.1 ohm. The galvanometer gave a deflection of 25 mm. by how much the unknown differs in resistance from the standard one.

Solution :

The KDB circuit with the specified values of all the parameters of the bridge is as shown in figure P3.

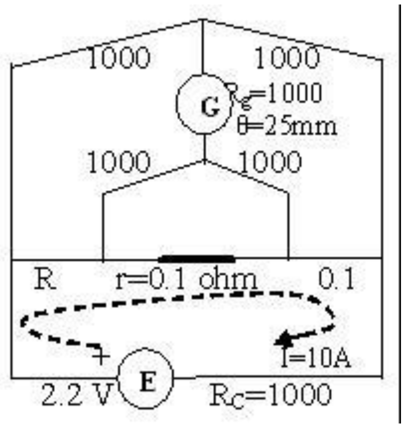


Fig.P3

I = kθ = 0.003µA/mm (25 mm) = 0.075 µA

R = (P/Q)S = [1000/1000] (0.1) = 0.1 ohm.

$I_g = I.\ R\ [P/(P+Q)]/\ \{R_g+ pq/(p+q)+ PQ/(P+Q)\}$

By substituting the specified values, we get,

$0.075(10^{-6}) = 10\ R\ [1/2]/[1000+500+500]$

Thus, solving for R, we ger, ***R = 30 µ***

4 Given that, in a KDB circuit, with the usual notations, P=Q=p=q=1000 ohms, E=100V, R_C=5 ohms, R_g=500 ohms and the bridge is balanced with S=0.001 ohm, what is the approximate value of current through S, at balance? Also find the value of galvanometer current I_g, if S is changed by 1% of its value at balance.

Solution :

Here, it is assumed that the yoke resistance is zero, i.e., r=0, so that then, $R=(P/Q)S = 0.1$ ohm.

(i) $I \approx E/\{R+R_c+r+S\}$

$= 100/ [0.001+5+0+0.001] =$ ***20 A***

(ii) $S = (0.01)\ S = 0.01(0.001) = 10\ \mu$

Thus,

$I_g = \ \ I_g = I.\ \ S\ [P/(P+Q)]/\ \{R_g+ pq/(p+q)+ PQ/(P+Q)\}$ = ***0.0667 µA***

5 The ratio arms of a KDB are 1000 each, $R_g=500$, k=200 mm/µA, R=0.1002 and S=0.1 . A DC of 10 A is passed through R & S from a 2.2 V battery in series with a rheostat. The link resistance is negligible. (i) Find the galvanometer deflection (ii) Find the resistance unbalance to produce the deflection of 1 mm and (iii) Obtain the total internal resistance of the battery circuit.

Solution :

(i) $R = (P/Q)S = 0.1$ ohm

Thus, $R = 0.1002-0.1 = 200\ \mu$

Hence,

$I_g = I.\ \ S\ [P/(P+Q)]/\ \{R_g+ pq/(p+q)+ PQ/(P+Q)\}$ = ***1.6667 µA***

And $\theta_g = k\ \ I_g$ = ***333.33 mm***

(ii) $\theta_g = 1$ mm, $I_g = \theta/k = 1/200 = 0.005\ \mu A$

Thus, $0.005(10^{-6}) = 10\ R(0.5)/[(500+50+50]$

Simplifying and solving, we get, ***R = 0.6 µ***

(iii) $I \approx E/\{R+R_c+r+S\}$

$10 = 2.2 / [0.1002+R_C+0.1+0]$

Solving, we get, ***R_C = 0.0198 ohm.***

6 A KDB is as follows: P=p=10000 , Q=q=1000 , R=0.0997 , S=0.01 , Rg=1000 , r=0.01 and k=0.005 µA/mm. A battery of 2.25 Volts and internal resistance 0.0053 is connected across the bridge circuit. Find the galvanometer deflection.

Solution :

$R = (P/Q)S = 0.1$ ohm (since $P/Q = p/q$)

Thus, $R = R_{true} - R_{measured} = 0.0997 - 0.1 = 300\ \mu$

$I \approx E/\{R+R_c+r+S\}$

$= 2.25 / [0.0997+0.0053+0.01+0.01]$

= ***18 A***

$I_g = I.\ \ S\ [P/(P+Q)]/\ \{R_g+ pq/(p+q)+ PQ/(P+Q)\}$ = ***1.742 µA***

And $\theta_g = \ \ I_g/k = 91.742\ \mu A)/(0.005\ \mu A/mm)$

= ***348.4 mm***.

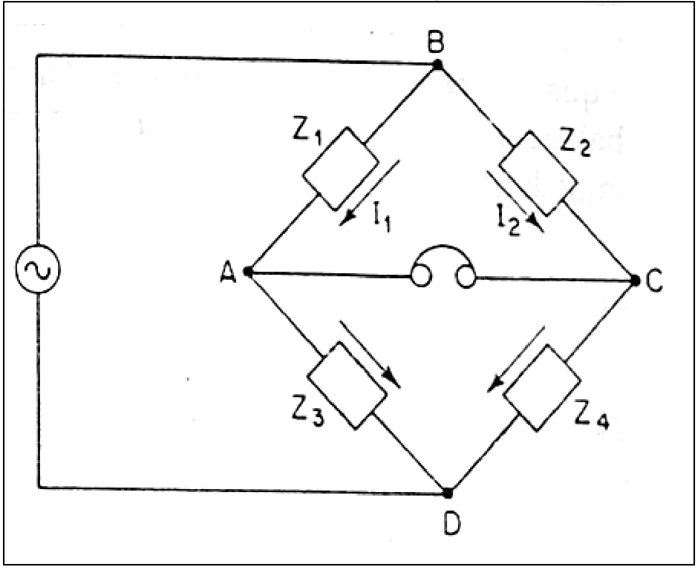
1.6 EXERCISES

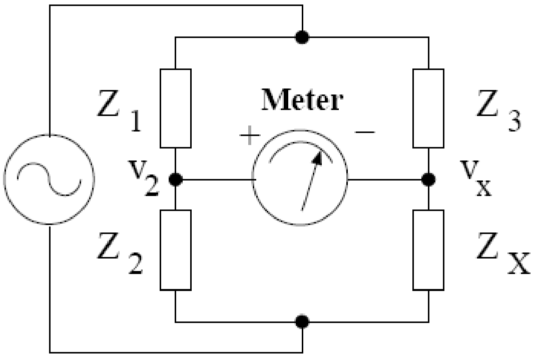
Standard text books listed in the syllabus are to be referred for the worked examples on the topics covered so far for better understanding of the problem solving techniques on resistance measurements.

UNIT-VI
AC Bridges

UNIT-VI

5.1 General Form of the AC Bridge

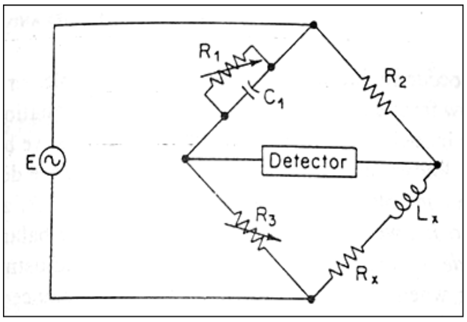


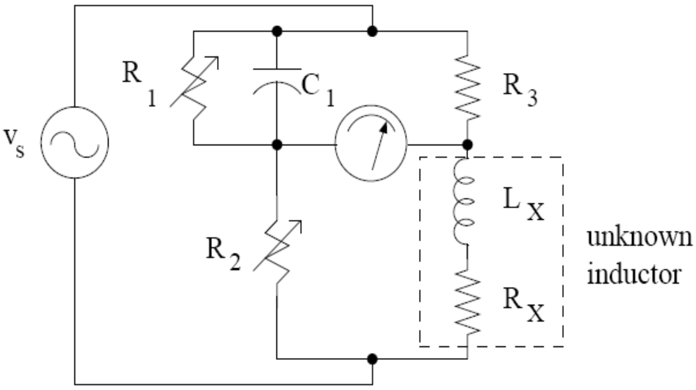


$$Z_1 Z_4 = Z_2 Z_3$$

$$\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3$$

5.2 Maxwell Bridge



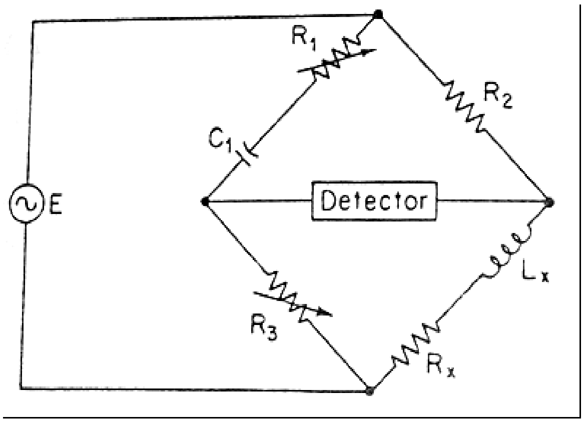


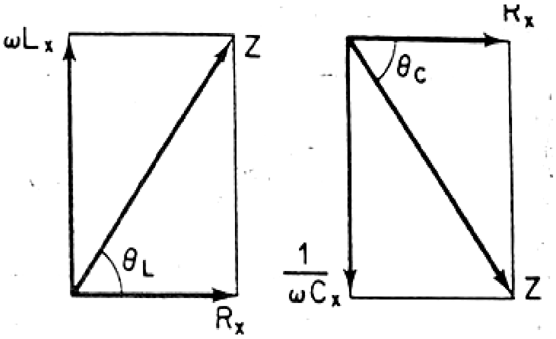
- The Maxwell bridge measures an unknown inductance in terms of a known capacitance.
- The maxwell bridge is limited to the measurement of medium-*Q* coils ($1<Q<10$).

$$R_x = \frac{R_2 R_3}{R_1}$$

$$L_x = R_2 R_3 C_1$$

5.3 Hay Bridge

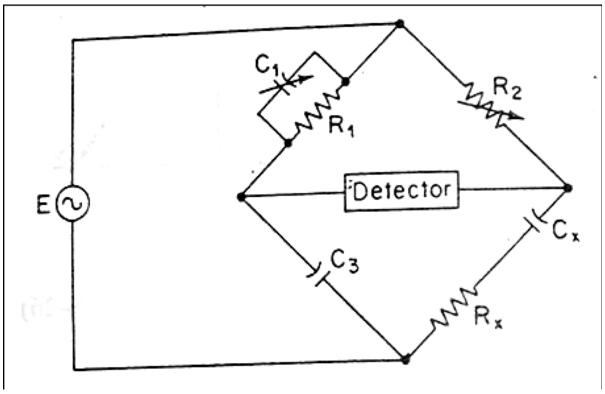




- The Hay circuit is more convenient for measuring high-*Q* coils
- Hay bridge for inductance measurements
- for Q>10 : $L_x = R_2 R_3 C$

5.4 Schering Bridge:

- The Schering bridge, one of the most important bridges, is used extensively for the measurement of capacitors.
- Schering bridge for measurement of capacitance



$$R_x = R_2 \frac{C_1}{C_3}$$

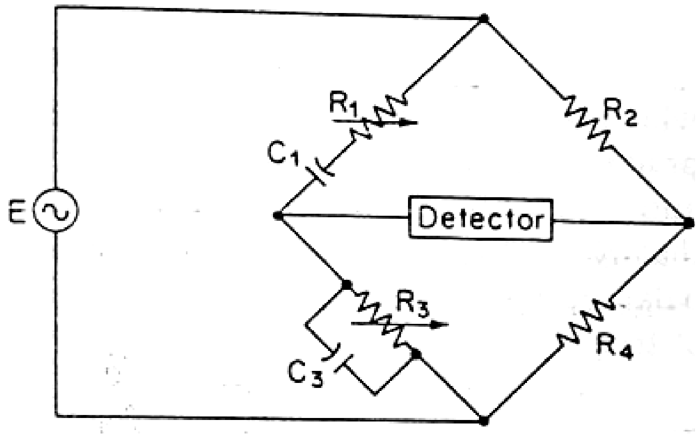
Dissipation factor :

$$C_x = C_3 \frac{R_1}{R_2}$$

$$D = \omega R_1 C_1$$

5.5 Wien Bridge :

Applications :
– Frequency measureent
– Notch filter
– Frequency-determining element

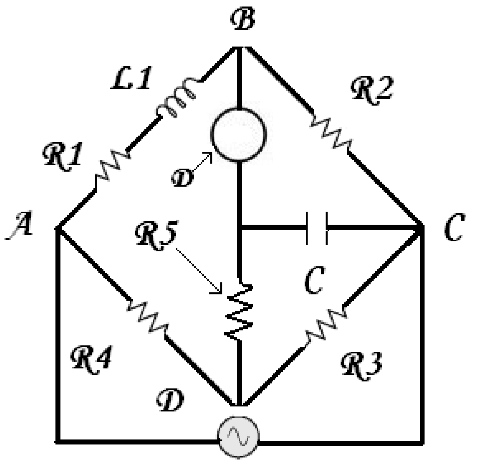


Frequency measurement with the Wien bridge

$$f = \frac{1}{2\pi RC}$$

5.6 Anderson Bridge:

- In the Anderson Bridge the unknown inductance is measured in terms of a known capacitance and resistance.
- this method is capable of precise measurements of inductance over a wide range of values from a few micro-henrys to several henrys and is the best bridge method

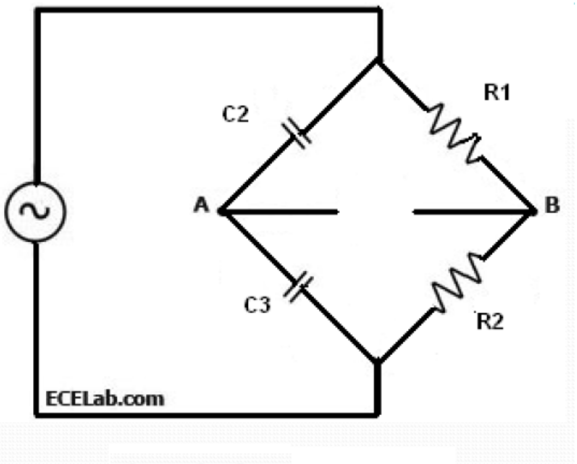


$$L_1 = CR_2\left(R_4 + R_5 + \frac{R_4 R_5}{R_3} \right)$$

5.7 Capacitance Bridge:

We will consider only De Sauty bridge method of comparing two capacitances the bridge has maximum sensitivity when C2 = C3.

- ✓ The simplicity of this method is offset by the impossibility of obtaining a perfect balance if both the capacitors are not free from the dielectric loss.
- ✓ A perfect balance can only be obtained if air capacitors are used. as shown in fig



$$C_2 = C_3 \frac{R_1}{R_2}$$