Python *for* Econometrics

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Welcome to this course and to the world of Python!

Learning objectives of this course:

- Python: The course is about Python programming.
- *for*: You will learn tools and methods.
- Econometrics:
  - Statistics: Numerical programming in Python.
  - applied to: We will use it on examples.
  - Economics: In an economic context.

Knowledge after completing this course:

- You have acquired a basic understanding of programming in general with Python and a special knowledge of working with standard numerical packages.
- You are able to study Python in depth and absorb new knowledge for your scientific work with Python.
- You know the capabilities and further possibilities to use Python in econometrics.

What you should not expect from this course:

- A guide how to install or maintain an application.
- An introduction to programming for beginners.
- An introduction to professional development tools.
- Non-scientific, general purpose programming (beyond the language essentials).
- Few content and less effort...

Course organisation

This course can be seen as an applied lecture:

Lecture:
We try to explain the partly theoretical knowledge on Python by simple, easy to understand examples. You can learn the programming language's subtleties by reading literature.

Exercises:
Digital work sheets in the form of Jupyter notebooks with applied tasks are available for each chapter. For all exercises there are sample solutions available in separate notebooks.

Self-tests:
At the end of each of the five chapters there are typical exam questions.

Written exam:
There will be a final exam. This will be a pure multiple choice exam: 60 questions, 90 minutes.

After the successful participation in the exam you will receive 6 ECTS.

The programming language Python is already established and very well in trend for numerical applications. Some keywords:

- Data science,
- Data wrangling,
- Machine learning,
- Numerical statistics,
- ...

Recommended literature while following this course:

- *Learning Python, 5th Edition* by Mark Lutz,
- *Python Crash Course, 2nd Edition* by Eric Matthes,
- *Python Data Science Handbook* by Jake VanderPlas,
- *Python for Data Analysis, 2nd Edition* by Wes McKinney,
- *Python for Finance, 2nd Edition* by Yves Hilpisch.

We are using *Python 3*. There was a big revision in the migration from Python 2 to version 3 and the new version is no longer backwards compatible to the old version.

Python 3 running [command line]

```
python --version
```

```
## Python 3.9.10
```

The normal execution mode is that the Python interpreter processes the instructions in the background – in other numeric programming languages such as *R* this is known as *batch mode*. It executes program code that is usually located in a source code file.

The interpreter can also be started in an *interactive mode*. It is used for testing and analytic purposes in order to obtain fast results when performing simple applications.

Software: IDEs

For everyday work with Python it would be extremely tedious to make all edits in interactive mode.

There are a number of excellent integrated development environments (IDEs) for Python, with three being emphasized here:

- *Jupyter* (and *IPython*)
- *Spyder* (scientific IDE)
- *PyCharm* (by *IntelliJ*)

Of course, you can also use a simple text editor. However, you would probably miss the comfort of an IDE.

Installing, adding and maintaining Python is not trivial at the beginning. Therefore, as a beginner, you are well advised to download and install the Python distribution *Anaconda*. Bonus: Many standard packages are supplied directly or you can post-install them conveniently.

In this course – in a numeric and analytic context – we use only Jupyter with the IPython kernel.

That is why we have combined

1. all the code from the slides, and
2. all the exercises and solutions

into interactive Jupyter notebooks that you can use online without having to install software locally on your computer. The GWDG has set up a cloud-based *Jupyter-Hub* for you.

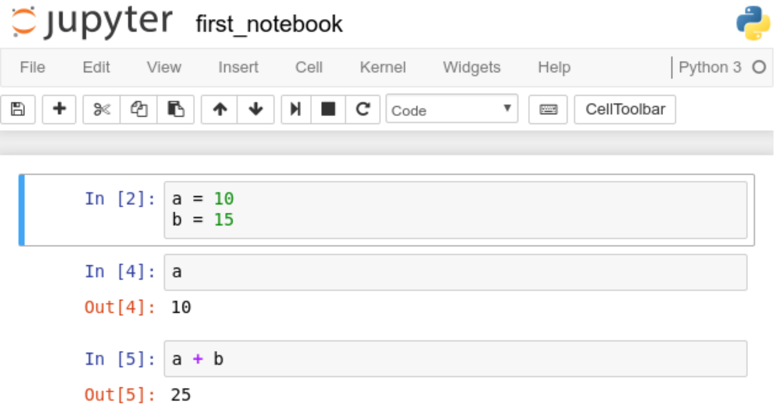
You can access the working environment with your university credentials at

https://jupyter-cloud.gwdg.de/

create a profile and get started right away – even using your smart devices. However, so far you are still asked to upload the course notebooks by yourself or rewrite the code from scratch.

Notebook workflow

A Jupyter notebook is divided into individual, vertically arranged cells, which can be executed separately:



The notebook approach is not novel and comes from the field of computer algebra software.

Actually, an interactive Python interpreter called IPython is started "in the core".

IPython running [command line]

```
ipython --version
```

```
## 8.0.1
```

Roughly speaking, this is a greatly enhanced version of the Python 3 interpreter, which has numerous, convenient advantages over the "normal" interpreter in interactive mode, such as, e.g.,

- printing of return values,
- color highlighting, and
- magic commands.

Finally, we wish you a lot of fun and success with and in this course!

Practice makes perfect!

Contribution and credits:

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GWDG *for the Jupyter-Hub*

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Essential concepts

Essential concepts

- Getting started

Python can be described as

- a dynamic, strongly typed, multi-paradigm and object-oriented programming language,
- for versatile, powerful, elegant and clear programming,
- with a general, high-level, multi-platform application scope,
- which is being used very successfully in the data science sector and very much in trend.

Moreover, Python is relatively easy to learn and its successful language design supports novices to professional developers. Much of Python's success is due to a *high degree of standardization* and a huge community that elaborates and collectively recognizes *conventions and paradigms*.

... of the Python era:

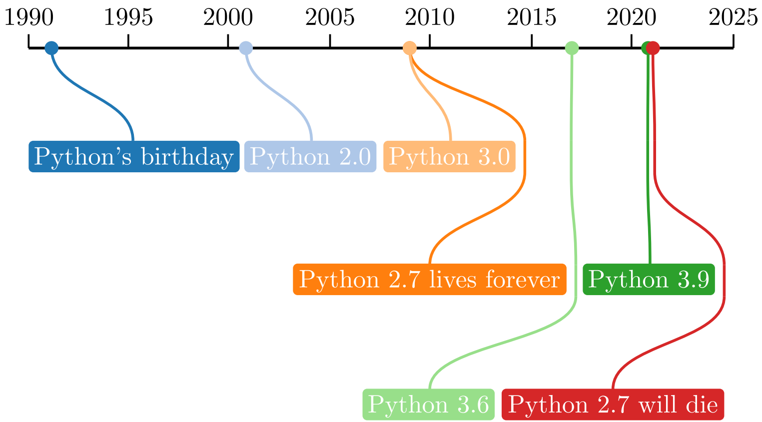
The language was originally developed in 1991 by Guido van Rossum. Its name was based on Monty Python's Flying Circus. Its main identification feature is the novel markup of code blocks – by indentation:

Indentation example

```
password = input("I am your bank. Password please: ")
## I am your bank. Password please: sparkasse
if password == "sparkasse":
    print("You successfully logged in!")
else:
    print("Fail. Will call the police!")

## You successfully logged in!
```

This increases the readability of code and should at the same time encourage the programmer in programming neatly. Since the source code can be written more compactly with Python, an increased efficiency in daily work can be expected.

Overview of the Python development by versions and dates:



In comparison

Comparing the way Python works with common programming languages, we briefly discuss a selection of popular competitors:

C/C++:

- CPython is interpreted, not compiled.
- C/C++ are strongly static, complex languages.

Java:

- CPython is not compiled just-in-time.
- Java has a *C*-type syntax.

MATLAB

- In Python you primarily follow a scalar way of thinking, while in *MATLAB* you write matrix-based programs.
- In the numerical context, the matrix view and syntax are very similar to those of MATLAB.
- MATLAB is partially compiled just-in-time.

Where *CPython* is the reference implementation – the "Original Python", which is implemented in C itself.

R

- In Python you primarily follow a scalar way of thinking, while in *R* you write vector-based programs.
- R has a C-type syntax including additions to novel language concepts.

Stata

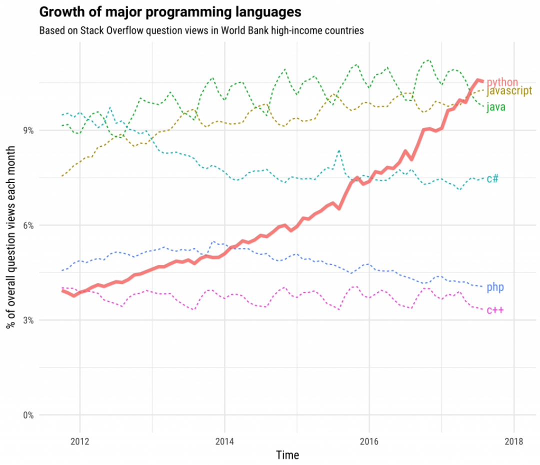
- Any comparison would inadequately describe the differences.

Reference semantics

An extremely important difference between the first two languages, C/C++ and Java, as well as Python itself, and the last three languages is that they follow a call-by-reference semantic, while MATLAB, R and Stata are call-by-copy.

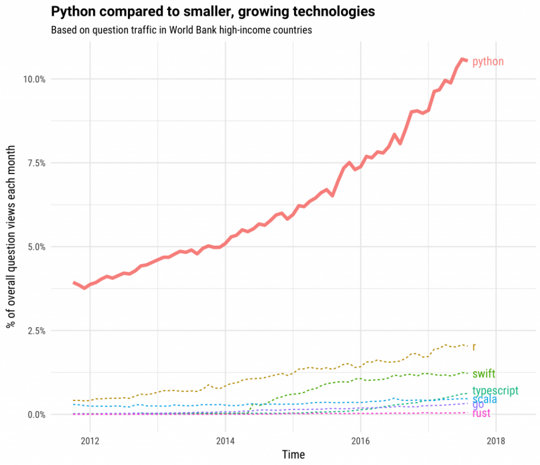
Further specific differences and similarities to MATLAB and R will be addressed in other parts of this course.

Python has become extremely popular:



Source: https://stackoverflow.blog/2017/09/06/incredible-growth-python/

So, you're on the right track – because who wants to bet on the wrong ho*R*se?



Source: https://stackoverflow.blog/2017/09/06/incredible-growth-python/

Areas in which Python is used with great success:

- Scripts,
- Console applications,
- GUI applications,
- Game development,
- Website development, and
- Numerical programming.

Places where Python is used:

Yet another outline

In this course we will successively gain the following insights:

1. General basics of the language.
2. Numerical programming and handling of data sets.
3. Application to economic and analytical questions.

Essential concepts

- Procedural programming

Programs can be implemented very quickly – this is a pretty minimal example. You can write this command to a text file of your choice and run it directly on your system:

Hello there

```
print("Hello there!")

## Hello there!
```

- Only one *function* `print()` (shown here as a *keyword*),
- Function displays *argument* (a string) on screen,
- Arguments are passed to the function in parentheses,
- A string must be wrapped in `" "` or `' '`,
- No semicolon at the end.

Let's add a user input to the program:

Hello you

```
name = input("Please enter your name: ")
## Please enter your name: Angela Merkel
print("Hello " + name + "!")

## Hello Angela Merkel!
```

- The function `input()` is used for interactive text input,
- You can use the equal sign = to assign variables (here: `name`),
- Strings can be joined by the (overloaded) Operator +.

We are now trying to find out on which weekday a person was born (Merkel's birthday is 17-07-1954):

Weekday of birth

```
from datetime import datetime

answer = input("Your birthday (DD-MM-YYYY): ")

## Your birthday (DD-MM-YYYY): 17-07-1954

birthday = datetime.strptime(answer, "%d-%m-%Y")
print("Your birthday was on a " + birthday.strftime("%A") + "!")

## Your birthday was on a Saturday!
```

- It is really easy to import functionality from other *modules*,
- Function `strptime()` is a *method* of *class* `datetime`,
- Both methods, `strptime()` and `strftime()`, are used to convert between strings and date time specifications.

And how many days have passed since then (until Merkel's 4th swearing-in as Federal Chancellor)?

Age in days

```
someday = datetime.strptime("14-03-2018", "%d-%m-%Y")
print("You are " + str((someday - birthday).days) + " days old!")

## You are 23251 days old!
```

- You can create time differences, i.e., the operator `-` is overloaded,
- The difference represents a new *object*, with its own *attributes*, such as `days`,
- When using the overloaded operator `+`, you have to explicitly convert the number of days by means of `str()` into a string.

Time since birth

How many years, weeks and days do you think that is?

Human readable age

```
from dateutil.relativedelta import relativedelta
delta = relativedelta(someday, birthday)
print(f"That’s {delta.years} years, {delta.months} months "
      f"and {delta.days} days!!")

## That's 63 years, 7 months and 25 days!!
```

- You don’t have to keep reinventing the wheel – a wealth of packages and individual modules are freely available,
- A lowercase `f` before `"..."` provides convenient *formatting* – there are other options as well,
- Two strings in sequence are implicitly joined together – `"That" "’s nice"`!

When working with the interactive interpreter, i.e., in a notebook, you can quickly get useful information about Python objects:

Help system

```
help(len)

## Help on built-in function len in module builtins:
##
## len(obj, /)
##     Return the number of items in a container.
```

Alternatively, e.g., for more complex problems, it is best to search directly with your preferred internet search engine.

You can find neat solutions to conventional challenges in literature.

Lexical structure

As with natural language, programming languages have a lexical structure. Source code consists of the smallest possible, indivisible elements, the tokens. In Python you can find the following groups of elements:

- Literals
- Variables
- Operators
- Delimiters
- Keywords
- Comments

These terms give us a rock-solid foundation for exploring the heart of a programming language.

Literals and variables

Basically, we distinguish between *literals* and *variables*:

Assigning variables with literals

```
myint = 7
myfloat = 4.0
myboat = "nice"
mybool = True
myfloat = myboat
```

- In this course, we will work with four different literals: integer (`7`), float (`4.0`), string (`"nice"`) and boolean (`True`),
- Literals are assigned to variables at runtime,
- In Python the data type is derived from the literal and does not have to be described explicitly,
- It is allowed to assign values of different data types to the same variable (name) sequentially,
- If we don't assign a literal to any variables, we forfeit it.

Most *operators* and *delimiters* will be introduced to you during this course. Here is an overview of the operators:

Overview of operators

```
## +         -       *       /      **     //
## %         @       <<      >>     &      |
## ^         ~       ==      !=     <      >
## <=        >=      and     or     not    in
## not in    is      is not
```

An overview of the delimiters follows:

Overview of delimiters

```
## (         )       [       ]      {      }
## ,         :       .       =      ;      ->
## +=        -=      *=      /=     **=    //=
## %=        @=      <<=     >>=    &=     |=
## ^=        '       "       \      @      SPACE
```

All regular *arithmetic operations* involving numbers are possible:

Pocket calculator

```
10 + 5
100 - 20
8 / 2
4 * (10 + 20)
2**3
```

```
## 15
## 80
## 4.0
## 120
## 8
```

- The result of dividing two integers is a floating point number,
- The conventional rules apply: Parentheses first, then multiplication and division, etc.,
- The operator `**` is used for exponentiation.

In order to demonstrate the use of *logical operators* (and formatted strings and `for`-loops), we create a handy table summarizing some important results from *boolean algebra*:

Logical table

```
# Create table head
print("a    b    a and b    a or b    not a\n"
      "--------------------------------")

# Loop through the rows
for a in [False, True]:
    for b in [False, True]:
        print(f"{a:1} {b:3} {a and b:6} {a or b:8} {not a:7}")
```

```
## a    b    a and b    a or b    not a
## --------------------------------
## 0    0       0         0        1
## 0    1       0         1        1
## 1    0       0         1        0
## 1    1       1         1        0
```

The programmer explains the structure of his/her program to the interpreter via a restricted set of short commands, the *keywords*:

Overview of keywords

```
## and       as    assert  break   class     continue
## def       del   elif    else    except    False
## finally   for   from    global  if        import
## in        is    lambda  None    nonlocal  not
## or        pass  raise   return  True      try
## while     with  yield
```

There are two ways to make *comments*:

Provide some comments

```
# Set variable to something - or nothing?
something = None

"""
I am a docstring!
A multiline string comment hybrid.
I will be useful for describing classes and methods.
"""
```

Python offers the following *basic data types*, which we will use in this course:

Data type	Description
`int()`	Integers
`float()`	Floating point numbers
`str()`	Strings, i.e., unicode (UTF-8) texts
`bool()`	Boolean, i.e., `True` or `False`
`list()`	List, an ordered array of objects
`tuple()`	Tuple, an ordered, unmutable array of objects
`dict()`	Dictionary, an unordered, associative array of objects
`set()`	Set, an unordered array/set of objects
`None()`	Nothing, emptyness, the void..

Each data type has its own methods, that is, functions that are applicable specifically to an object of this type.

You will gradually get to know new and more complex data types or object classes.

A *list* is an ordered array of objects, accessible via an *index*:

Listing tech companies

```
stocks = ["Google", "Amazon", "Facebook", "Apple"]
stocks[1]
stocks.append("Twitter")
stocks.insert(2, "Microsoft")
stocks.sort()
```

```
## ['Google', 'Amazon', 'Facebook', 'Apple']
## Amazon
## ['Google', 'Amazon', 'Facebook', 'Apple', 'Twitter']
## ['Google', 'Amazon', 'Microsoft', 'Facebook', 'Apple', 'Twitter']
## ['Amazon', 'Apple', 'Facebook', 'Google', 'Microsoft', 'Twitter']
```

- The constructor for new lists is `[ ]`,
- The first element has the index 0,
- The data type `list()` possesses its own methods.

Tuples are immutable sequences related to lists that cannot be extended, for example. The drawbacks in flexibility are compensated by the advantages in speed and memory usage:

Selecting elements in sequences

```
lottery = (1, 8, 9, 12, 24, 28)
len(lottery)
lottery[1:3]
lottery[:4]
lottery[-1]
lottery[-2:]
```

```
## (1, 8, 9, 12, 24, 28)
## 6
## (8, 9)
## (1, 8, 9, 12)
## 28
## (24, 28)
```

The same operations are also supported when using lists.

Dictionaries are associative collections of *key-value pairs*. The *key* must be immutable and unique:

Internet slang dictionary

```
slang = {"imho": "in my humble opinion",
         "lol": "laughing out loud",
         "tl;dr": "too long; didn't read"}
slang["lol"]
slang["gl&hl"] = "good luck & have fun"
slang.keys()
slang.values()

## {'imho': 'in...ion', 'lol': 'la...oud', 'tl;dr': 'to...ead'}
## laughing out loud
## good luck & have fun
## dict_keys(['imho', 'lol', 'tl;dr', 'gl&hl'])
## dict_values([... & have fun'])
```

- The constructor for `dict()` is `{ }` with `:`,
- The pairs are unordered, iterable sequences.

Sets

A *set* is an unordered collection of objects without duplicates:

Set operations

```
x = {"o", "n", "y", "t"}
y = {"p", "h", "o", "n"}
x & y
x | y
x - y
```

```
## {'y', 'n', 't', 'o'}
## {'p', 'n', 'h', 'o'}
## {'n', 'o'}
## {'p', 'y', 'o', 't', 'n', 'h'}
## {'y', 't'}
```

- The constructor for `set()` is `{ }`,
- Defines its own operators that overload existing ones.
- Empty set via `set()`, because `{}` already creates `dict()`.

Comparison operators

The `<`, `<=`, `>`, `>=`, `==`, `!=` operators compare the values of two objects and return `True` or `False`.

Op.	`True`, only if the value of the left operand is
`<`	less than the value of the right operand
`<=`	less than or equal to the value of the right operand
`>`	greater than the value of the right operand
`>=`	greater than or equal to the value of the right operand
`==`	equal to the right operand
`!=`	not equal to the right operand

The comparison depends on the datatype of the objects. For example `"7" == 7` will return `False`, while `7.0 == 7` will return `True`.

- Numbers are compared arithmetically.
- Strings are compared lexicographically.
- Tuples and lists are compared lexicographically using comparison of corresponding elements. This behaviour can be altered.

Comparing examples

```
x, y = 5, 8
print("x < y is", x < y)

## x < y is True

print("x > y is", x > y)

## x > y is False

print("x == y is", x == y)

## x == y is False

print("x != y is", x != y)

## x != y is True

print("This is", "Name" == "Name", "and not", "Name" == "name")

## This is True and not False
```

Comparing strings, the case has to be considered.

In Python, comparison operators can also be chained.

Chaining comparison examples

```
x = 5

5 >= x > 4

## True

12 < x < 20

## False

2 < x < 10

## True

2 < x and x < 10  # unchained expression

## True
```

The comparison is performed for both sides and combined by `and`.

There are three logical operators: `not`, `and`, `or`.

Op.	Description
`not x`	Returns `True` only if `x` is `False`
`x and y`	Returns `True` only if `x` and `y` are `True`
`x or y`	Returns `True` only if `x` or `y` or both are `True`

Logical operators examples

```
x, y = 5, 8

(x == 5) and (y == 9)

## False

(x == 5) or (y == 8)

## True

not(x == 4) or (y == 9)

## True
```

In some situations, you need a logical operation that is `True` only when the operands differ (one is `True`, the other is `False`). This task can be solved by using the logical operators `not`, `and`, `or` or simply `!=`.

Exclusive or

```
x, y = 5, 8

((x == 5) and not (y == 8)) or (not (x == 5) and (y == 8))

## False

x = 4
((x == 5) and not (y == 8)) or (not (x == 5) and (y == 8))

## True

(x == 5) != (y == 8)

## True
```

In many other programming languages, an operation "exclusive or" or *xor* is explicitly part of the language, but not in Python.

Binary numbers

Bitwise operators operate on numbers, but instead of treating that number as if it were a single (decimal) value, they operate on the string of bits representation, written in binary. A binary number is a number expressed in the base-2 numeral system, also called binary numeral system, which consists of only two distinct symbols: typically 0 (zero) and 1 (one).

Binary numbers

```
## Decimal: Binary:
##       0:       0
##       1:       1
##       2:      10
##       3:      11
##       4:     100
##       5:     101
##       6:     110
##       7:     111
##       8:    1000
##       9:    1001
##      10:    1010
```

How to convert binary numbers to integers (the unknown keywords and language structures will be introduced soon):

Binary to integer

```
def bintoint(binary):
    binary = binary[::-1]
    num = 0
    for i in range(len(binary)):
        num += int(binary[i]) * 2**i
    return num

bintoint("1101001")

## 105

int("1101001", 2)  # compare with built-in function

## 105
```

How to convert integers to binary numbers:

Integers to binary

```
def inttobin(num):
    binary = ""
    if num != 0:
        while num >= 1:
            if num % 2 == 0:
                binary += "0"
                num = num / 2
            else:
                binary += "1"
                num = (num - 1) / 2
    else:
        binary = "0"
    return binary[::-1]
inttobin(105)

## '1101001'

bin(105)[2:]  # compare with built-in function

## '1101001'
```

Python offers distinct bitwise operators. Some of them will be redefined entirely different by extensions, such as, e. g., vectorization.

Bit. op.	Description
`x >> y`	Returns `x` with the bits shifted to the left by `y` places
`x << y`	Returns `x` with the bits shifted to the right by `y` places
`x & y`	Does a bitwise `and`
`x | y`	Does a bitwise `or`
`~ x`	Returns the complement of `x`
`x ^ y`	Does a bitwise exclusive or

Bitwise operators

```
a, b = 5, 7
c = a & b  # bitwise and
## a: 101
## b: 111
## c: 101
print(c)

## 5
```

Bitwise operators

```
a, b = 5, 7
c = a | b  # bitwise or
```

```
## a: 101
## b: 111
## c: 111
```

```
print(c)
```

```
## 7
```

```
a = 13
b = a << 2  # bitwise shift
```

```
## a: 1101
## b: 110100
```

```
a, b = 35, 37
c = a ^ b  # bitwise exclusive or
```

```
## a: 100011
## b: 100101
## c: 000110
```

Python has only one kind of conditional statement – `if-elif-else`:

Computer data sizes

```
bytes = 100000000 / 8  # e.g. DSL 100000
if bytes >= 1e9:
    print(f"{bytes/1e9:6.2f} GByte")
elif bytes >= 1e6:
    print(f"{bytes/1e6:6.2f} MByte")
elif bytes >= 1e3:
    print(f"{bytes/1e3:6.2f} KByte")
else:
    print(f"{bytes:6.2f} Byte")

##  12.50 MByte
```

Control flow structures may be nested in any order:

Nestings

```
if a > 1:
    if b > 2:
        pass  # a special keyword for empty blocks
```

In Python there exist two conventional *program loops* – `for-in-else`:

Total sum

```
numbers = [7, 3, 4, 5, 6, 15]
y = 0
for i in numbers:
    y += i
print(f"The sum of 'numbers' is {y}.")

## The sum of 'numbers' is 40.
```

Lists or other collections can also be created dynamically:

Powers of 2

```
powers = [2 ** i for i in range(11)]
teacher = ["***", "**", "*"]
grades = {star: len(teacher) - len(star) + 1 for star in teacher}

## [1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024]
## {'***': 1, '**': 2, '*': 3}
```

Loops can skip iterations (`continue`):

Continue the loop

```
for x in ["a", "b", "c"]:
    a = x.upper()
    continue
    print(x)
print(a)

## C
```

Or a loop can be aborted instantly (`break`):

Breaking the habit

```
y = 0
for i in [7, 3, 4, "x", 6, 15]:
    if not isinstance(i, int):
        break
    y += i
print(f"The total sum is {y}.")

## The total sum is 14.
```

For loops where the number of iterations is not known at the beginning, you use `while`-`else`.

Have you already noticed the keyword `else`? Python only executes the branch if it was not terminated by `break`:

Favorite lottery number

```
import random
n = 0
favorite = 7
while n < 100:
    n += 1
    draw = random.randint(1, 49)  # e.g. German lottery
    if draw == favorite:
        print("Got my number! :)")
        break
else:
    print("My favorite did not show up! :(")
print(f"I tried {n} times!")
```

```
## Got my number! :)
## I tried 10 times!
```

Functions are defined using the keyword `def`. The structure of *function signature* and *body* is specified by indentation, too:

Drawing lottery numbers

```
def draw_sample(n, first=1, last=49):
    numbers = list(range(first, last + 1))
    sample = []
    for i in range(n):
        ind = random.randint(0, len(numbers) - 1)
        sample.append(numbers.pop(ind))
    sample.sort()
    return sample

draw_sample(6)
draw_sample(6, 80, 100)
draw_sample(3, first=5)

## [2, 3, 4, 16, 23, 28]
## [82, 84, 94, 95, 99, 100]
## [5, 12, 16]
```

Functions are of type `callable()`, defined as closures, and can be created and used like other objects:

Prime numbers

```
def primes(n):
    numbers = [2]

    def is_prime(num):
        for i in numbers:
            if num % i == 0:
                return False
        return True
    if n == 2:
        return numbers
    for i in range(3, n + 1):
        if is_prime(i):
            numbers.append(i)
    return numbers
primes(50)

## [2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47]
```

Seems weird? We discuss namespaces in the next section.

Essential concepts

▶ Object-orientation

There are three widely known programming paradigms: *procedural*, *functional* and *object-oriented programming* (*OOP*). Python supports them all.

You have learned how to handle predefined data types in Python. Actually, we have already encountered classes and instances, take for example `dict()`.

In this section you will learn the basics of dealing with (your own) classes:

1. References
2. Classes
3. Instances
4. Main principles
5. Garbage collection

OOP is a wide field and challenging for beginners. Don't get discouraged and, if you find deficits in yourself, read the literature.

References

When you assign a variable, a *reference* to an object is set:

Equal but not identical

```
a = ["Star", "Trek"]
b = ["Star", "Trek"]
c = a
a == b
a == c
a is b
a is c
```

```
## ['Star', 'Trek']
## ['Star', 'Trek']
## ['Star', 'Trek']
## True
## True
## False
## True
```

- Two equal but not identical objects are created,
- Variables `a` and `c` link to the same object.

When we introduced lists, we initially did not mention that they are a first-class example of *mutable* objects:

Collecting grades

```
grades = [1.7, 1.3, 2.7, 2.0]
result = grades.append(1.0)
result
grades
finals = grades
finals.remove(2.7)
finals
grades
```

```
## None
## [1.7, 1.3, 2.7, 2.0, 1.0]
## [1.7, 1.3, 2.0, 1.0]
## [1.7, 1.3, 2.0, 1.0]
```

- Modifications can be *in-place* – the object itself is modified.
- Changing an object that is referenced several times could cause (un)intended consequences.

In Python, arguments are *passed by assignment*, i.e., call-by-reference:

Side effects

```
def last_element(x):
    return x.pop(-1)

a = stocks
last_element(a)
a

## ['Amazon', 'Apple', 'Facebook', 'Google', 'Microsoft', 'Twitter']
## Twitter
## ['Amazon', 'Apple', 'Facebook', 'Google', 'Microsoft']
```

- There are side effects,
- Referenced *mutable* objects might be modified,
- Referenced *immutable* objects might be copyied.

We are able to make an exact copy of the object:

Copying

```
def last_element(x):
    y = x.copy()
    return y.pop(-1)

a = stocks
last_element(a)
a

## ['Amazon', 'Apple', 'Facebook', 'Google', 'Microsoft']
## Microsoft
## ['Amazon', 'Apple', 'Facebook', 'Google', 'Microsoft']
```

- We receive a new object,
- The new object is not identical to the old one.

However, keep in mind that, in most cases, a method `copy()` will create *shallow* copys while only *deep copying* will duplicate also the contents of a mutable object with a complex structure:

Cloning fast food

```
fastfood = [["burgers", "hot dogs"], ["pizza", "pasta"]]
italian = fastfood.copy()
italian.pop(0)
american = list(fastfood)
american.pop(1)
american[0] = american[0].copy()
fastfood[0][1] = "chicken wings"
fastfood[1][0] = "risotto"
italian
american
```

```
## [['risotto', 'pasta']]
## [['burgers', 'hot dogs']]
```

Both approaches, `copy()` and `list()`, create new `list` objects containing new references to the original sub-lists. But for a *deep copy*, you have to recursively create duplicates of all its objects.

In Python everything is an object and more complex objects consist of several other objects.

In the OOP, we create objects according to patterns. These kinds of blueprints are called *classes* and are characterized by two categories of elements:

Attributes:
Variables that represent the properties of

- an object, *object attributes*, or
- a class, named *class attributes*.

Methods:
Functions that are defined within a class:

- *(non-static) methods* can access all attributes, while
- *static methods* can only access class attributes.

Every generated object is an *instance* of such a construction plan.

Specifically, we want to create "rectangle object" and define a separate `Rectangle` class for it:

Rectangle class

```
class Rectangle:
    width = 0
    height = 0

    def area(self):
        return self.width * self.height

myrectangle = Rectangle()
myrectangle.width = 10
myrectangle.height = 20
myrectangle.area()

## 200
```

- New classes are defined using the keyword `class`,
- The variable `self` always refers to the instance itself.

Class constructor

We add a *constructor* (method) `__init__()`, that is called to *initialize* an object of `Rectangle`:

Rectangle class with constructor

```
class Rectangle:
    width = 0
    height = 0

    def __init__(self, width, height):
        self.width = width
        self.height = height

    def area(self):
        return self.width * self.height
myrectangle = Rectangle(15, 30)
myrectangle.area()

## 450
```

In our example, we use the constructor to set the attributes. Methods with names matching `__fun__()` have a special, standardized meaning in Python.

One of the most important concepts of OOP is *inheritance*. A class inherits all attributes and methods of its *parent class* and can *add new* or *overwrite* existing ones:

Square inherits Rectangle

```
class Square(Rectangle):
    def __init__(self, length):
        super().__init__(length, length)

    def diagonal(self):
        return (self.width**2 + self.height**2)**0.5
mysquare = Square(15)

print(f"Area: {mysquare.area()}")
print(f"Diagonal length: {mysquare.diagonal():7.4f}")

## Area: 225
## Diagonal length: 21.2132
```

The methods of the parent class, including the constructor, may be referenced by `super()`.

You do not have to worry about memory management in Python. The *garbage collector* will tidy up for you.

If there are no more references to an object, it is automatically disposed of by the garbage collector:

Garbage collection in action

```
class Dog:
    def __del__(self):
        print("Woof! The dogcatcher got me! Entering the void.. :(")
# My old dog on a leash
mydog = Dog()
# A new dog is born
newdog = Dog()
# Using my leash for the new dog
mydog = newdog

## Woof! The dogcatcher got me! Entering the void.. :(
```

The *destructor* `__del__()` is executed as the last act before an object gets deleted.

Everyone involved in programming will encounter errors of various types. These errors can be stressful and annoying but being aware of the basic types of errors that can occur will give you the chance to handle them.

Seeing the line SyntaxError may let you think "oh no, I've done everything wrong", but errors are normal and even experienced programmers face them frequently. Hints on error handling:

- Dissect the error: Find the line in the error message that is specified. Many errors have messages that are not important to the actual error. In Python you often find the important information at the end of the error message.
- Errors are often oversights: In most cases the error massage will give you the line in your code where the error occurred.
- Search the web: If you are not able to fix the errors on your own, copy the error message into a search engine and read through the results. Probably someone else also had this problem and the community already found a solution.

Exceptions versus syntax errors

A Python program terminates immediately as it encounters an error. In Python, errors can be either *syntax errors* or *exceptions*. Syntax errors occur when the parser detects a wrong sequence in the Python code. An arrow indicates the exact position of the syntax error:

Syntax Error

```
## print("Hello Word"))

## File "<stdin>", line 1
##     print("Hello World"))
##                         ^
##     SyntaxError: invalid syntax
```

An exception occurs whenever a syntactically correct Python code results in an error:

Exception

```
a = 0 / 0

## <stdin> in <module>()
## ----> 1 a = 0 / 0
## ZeroDivisionError: division by zero
```

Exceptions appear in different types and the type is printed as a part of the error message. The next example shows three common built-in exceptions:

Frequent exception

```
0 / 0
```

```
## <stdin> in <module>()
## ----> 1 0 / 0
## ZeroDivisionError: division by zero
```

```
3 + a
```

```
## <stdin> in <module>()
## ----> 1 3 + a
## NameError: name 'a' is not defined
```

```
3 + "2"
```

```
## <stdin> in <module>()
## ----> 1 3 + "2"
## TypeError: unsupported operand type(s) for +: 'int' and 'str'
```

A list of all exception classes of the standard library can be found here.

When an exception occurs, the Python interpreter throws an error message and exits. But in most situations, you do not want your whole program to stop.

The `try` block can test a block of code for errors.
The `except` block lets you handle the error.

Try and except

```
try:
    print(abc)
except:
    print("An exception occurred")

## An exception occurred
```

The statement above will raise an error, because the variable `abc` is not defined.

You can define multiple exception blocks. For example, if you want to execute code when you expect a special kind of error to occur:

Multiple exception blocks

```
try:
    print(abc)
except NameError:
    print("Variable abc is not defined")
except:
    print("Something else went wrong")

## Variable abc is not defined

try:
    0 / 0
except NameError:
    print("Variable abc is not defined")
except:
    print("Something else went wrong")

## Something else went wrong
```

Complementary, like for `if-else`, the `else` keyword defines a block of code to be executed if no errors were thrown:

Else exception

```
try:
    print("Hello World")
except:
    print("Something went wrong")
else:
    print("Everything is okay")

## Hello World
## Everything is okay
```

The `finally` block will be executed regardless if the try block raises an error or not. Hence, you can make sure the code is run:

Finally exception

```
try:
    print(abc)
except:
    print("Something went wrong")
finally:
    print("This will always be displayed")

## Something went wrong
## This will always be displayed

try:
    print("Hello World")
except:
    print("Something went wrong")
finally:
    print("This will always be displayed")

## Hello World
## This will always be displayed
```

Built-in exceptions are raised whenever pre-defined interpreter errors occur. In some situations you might want to raise exceptions on your own:

The `raise` keyword is used to raise an exception.

In the following, the interpreter raises an error if the variable `x` is lower than 0:

Raise exception

```
x = -3
if x < 0:
    raise Exception("Sorry, 'x' is lower than 0.")

## <stdin> in <module>()
## ----> 3 raise Exception(Sorry, 'x' is lower than 0.)
## Exception: Sorry, 'x' is lower than 0.
```

LBYL: *Look before you leap.*
EAFP: *It is easier to ask forgiveness than it is to get permission.*

LBYL and EAFP are two techniques to deal (i.e., avoid) with exceptions. In short, in LBYL you first check whether something will succeed and only proceed if it does. EAFP means that you do what you expect and if an exception might occur, you deal with it:

LBYL

```
if x != 0:
    print(10 / x)
```

EAFP

```
try:
    print(10 / x)
except ZeroDivisionError:
    pass
```

So, why use EAFP although it needs more lines of code?

- Often, the code is more readable and straight.
- Explicit is better than implicit (Zen of Python, see below).
- Best performance in case no exception is raised.
- Detailed exception handling. You can not only consider errors, but also different kinds of errors and then proceed differently.

EAFP

```
try:
    print(10 / x)
except ZeroDivisionError:
    print("Zero division")
except NameError:
    print("Variable 'x' is not defined")
```

Python has multiple built-in exceptions which terminate your program when something goes wrong. But you can also create custom exceptions that serve specific purposes.

Your own exception can implemented by defining a new class which derives from the `Exception` class or a subclass:

User-defined exception

```
class ValueTooLargeError(Exception):
    """Raised when the input value is too large"""
    pass
x = 3
try:
    if x > 2:
        raise ValueTooLargeError
except ValueTooLargeError:
    print("The number is too large.")

## The number is too large.
```

We have already come into contact with *namenspaces* in Python many times. These are hierarchically linked layers in which the references to objects are defined. A rough distinction is made between

- the *global* namespace, and
- the *local* namespace.

The global namespace is the *outermost environment* whose references are known by all objects.

On the other hand, locally defined references are only known in a local, i.e., *internal environment*.

Reference names from the local namespace mask the same names in an outer or in the global namespace:

Namespaces

```
def multiplier(x):
    x = 4 * x
    return x
x = "OH"
multiplier("AH")
multiplier(x)
x
```

```
## OH
## AHAHAHAH
## OHOHOHOH
## OH
```

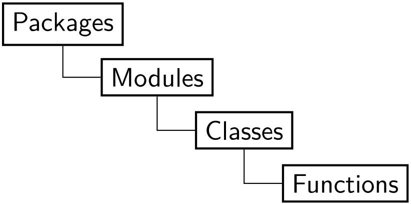
In fact, functions defined in Python are themselves objects that remember and can access their own context where they were created. This concept comes from functional programming and is called *closure*:

Closures

```
def gen_multiplier(a):
    def fun(x):
        return a * x
    return fun

multi1 = gen_multiplier(4)
multi2 = gen_multiplier(5)
multi1
multi1("EH")
multi2("EH")
```

```
## <function gen_multiplier.<locals>.fun at 0x127fc4ee0>
## EHEHEHEH
## EHEHEHEHEH
```

In order to provide, maintain and extend modular functionality with Python, its code containing components can be described hierarchically:



The organization in Python is very straightforward and is based on the local namespaces mentioned before.

When you download and use new *packages*, such as *NumPy* for numerical programming in the next chapter, the packages are loaded and the namespaces initialized.

The development of custom packages is an advanced topic and not essential for a reasonable code structure of small projects, as it is in other programming languages.

Modules provide classes and functions via namespaces. It is Python code that is executed in a local namespace and whose classes and functions you can import. Basically, there are the following alternatives how to *import* from an module:

Import statements

```
import datetime
import datetime as dt
from datetime import date, timedelta
from datetime import *

dt.date.today()
dt.timedelta.days

date.today()
timedelta.days

datetime.now()
```

In the latter case, all classes and functions, but no instances, are imported from the `datetime` namespace.

Build-in modules

A Python installation ships with a *standard library* consisting of *built-in modules*. These modules provide standardized solutions for many problems that occur in everyday programming - “batteries included”. For example, they provide access to system functionality such as file management. The Python Docs give an overview of all build-in modules.

Usage of build-in modules

```
import math
from random import randint

math.pi

## 3.141592653589793

math.factorial(5)

## 120

randint(10, 20)

## 18
```

Often you might want to use extended functionality. Python has a large and active community of users who make their developments publicly available under open source license terms. Packages are containers of modules which can be imported and used within your Python code.

These third-party packages can be installed comfortably by using the (command line) package manager *pip*. The Python Package Index provides an overview of the thousands of packages available. Basic commands for maintaining, for example, the installation of the package "numpy":

- Installing the package: `pip install numpy`
- Upgrading the package: `pip install --upgrade numpy`
- Installing the package locally for the current user: `pip install --user numpy`
- Uninstalling the package: `pip uninstall numpy`

Example: *OpenCV* is a package for image processing in Python. Here you can see how the installation proceeds in a Unix terminal.

```
~$ pip install opencv-python
```

```
Collecting opencv-python
  Downloading https://files.pythonhosted.org/packages/37/49/874d119948a5a084a7eb
e98308214098ef3471d76ab74200f9800efeef15/opencv_python-4.0.0.21-cp36-cp36m-manyl
inux1_x86_64.whl (25.4MB)
    100% |████████████████████████████████| 25.4MB 523kB/s
Requirement already satisfied: numpy>=1.11.3 in /usr/local/lib/python3.6/dist-pa
ckages (from opencv-python) (1.15.4)
Installing collected packages: opencv-python
Successfully installed opencv-python-4.0.0.21
```

Your Python projects will become complex and you will need to maintain the codes properly. Therefore, one can break a large, unwieldy programming task into separate, more manageable modules. Modules can be written in Python itself or in C, but here we keep focussing on the Python language.

Creating modules in Python is very straightforward - a Python module is a file containing Python code, for example:

```
s = "Hello world!"
l = [1, 2, 3, 5, 5]

def add_one(n):
    return n + 1
```

File: **mymodule.py**

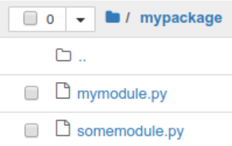
If you import the module **mymodule**, the interpreter looks in the current working directory for a file **mymodule.py**, reads and interprets its contents and makes its namespace available:

Usage of own modules

```
import mymodule
mymodule.s
mymodule.l
mymodule.add_one(5)
```

```
## Hello world!
## [1, 2, 3, 5, 5]
## 6
```

Large projects could require more than one module. Packages allow to structure the modules and their namespaces hierarchically by using the *dot notation*. They are simple folders containing modules and (sub-)packages. Consider the following structure:



The directory **mypackage** contains two modules which we can import separately:

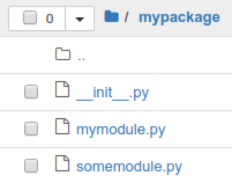
Usage of own package

```python
import mypackage.mymodule
import mypackage.somemodule
mypackage.mymodule.add_one(4)
```

```
## 5
```

Package initialization

If a package directory contains a file **__init__.py**, its code is invoked when the package gets imported. The directory **mypackage**, now, contains the two modules and the initialization file:



The file **__init__.py** can be empty but can also be used for package initialization purposes.

The Zen of Python

```python
import this
```

```
## The Zen of Python, by Tim Peters
##
##
## Beautiful is better than ugly.
## Explicit is better than implicit.
## Simple is better than complex.
## Complex is better than complicated.
## Flat is better than nested.
## Sparse is better than dense.
## Readability counts.
## Special cases aren't special enough to break the rules.
## Although practicality beats purity.
## Errors should never pass silently.
## Unless explicitly silenced.
## In the face of ambiguity, refuse the temptation to guess.
## ...
```

A selection of exciting topics that are among the advanced basics but are not covered in this lecture:

- Dynamic language concepts, such as duck typing,
- Further, complex type classes, such as `ChainMap` or `OrderedDict`,
- Iterators and generators in detail,
- Exception handling, raising exceptions, catching errors,
- Debugging, introspection and annotations.

Numerical programming

2.1 NumPy package

2.2 Array basics

2.3 Linear algebra

Numerical programming

▶ NumPy package

The *Numerical Python* package `NumPy` provides efficient tools for scientific computing and data analysis:

- `np.array()`: Multidimensional array capable of doing fast and efficient computations,
- Built-in mathematical functions on arrays without writing loops,
- Built-in linear algebra functions.

Import NumPy

```
import numpy as np
```

Element-wise addition

```
vec1 = [1, 2, 3, 4, 5, 6, 7, 8, 9]
vec2 = np.array(vec1)
vec1 + vec1

## [1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 2, 3, 4, 5, 6, 7, 8, 9]

vec2 + vec2

## array([ 2,  4,  6,  8, 10, 12, 14, 16, 18])

for i in range(len(vec1)):
    vec1[i] += vec1[i]
vec1

## [2, 4, 6, 8, 10, 12, 14, 16, 18]
```

Matrix multiplication

```
mat1 = [[1, 2, 3], [4, 5, 6], [7, 8, 9]]
mat2 = np.array(mat1)
np.dot(mat2, mat2)

## array([[ 30,  36,  42],
##        [ 66,  81,  96],
##        [102, 126, 150]])

mat3 = np.zeros([3, 3])
for i in range(3):
    for k in range(3):
        for j in range(3):
            mat3[i][k] = mat3[i][k] + mat1[i][j] * mat1[j][k]
mat3

## array([[ 30.,  36.,  42.],
##        [ 66.,  81.,  96.],
##        [102., 126., 150.]])
```

Time comparison

```
import time
mat1 = np.random.rand(50, 50)
mat2 = np.array(mat1)
t = time.time()
mat3 = np.dot(mat2, mat2)
nptime = time.time() - t
mat3 = np.zeros([50, 50])
t = time.time()
for i in range(50):
    for k in range(50):
        for j in range(50):
            mat3[i][k] = mat3[i][k] + mat1[i][j] * mat1[j][k]
pytime = time.time() - t
times = str(pytime / nptime)
print("NumPy is " + times + " times faster!")

## NumPy is 19.49091343854615 times faster!
```

Numerical programming

▶ Array basics

`np.array(list)`: Converts python list into NumPy arrays.
`array.ndim`: Returns Dimension of the array.
`array.shape`: Returns shape of the array as a list.

Creation

```
arr1 = [4, 8, 2]
arr1 = np.array(arr1)
arr2 = np.array([24.3, 0., 8.9, 4.4, 1.65, 45])
arr3 = np.array([[4, 8, 5], [9, 3, 4], [1, 0, 6]])
arr1.ndim

## 1

arr3.shape

## (3, 3)
```

From now on, the name array refers to an `np.array()`.

`np.arange(start, stop, step)`: Creates vector of values from `start` to `stop` with step width `step`.
`np.zeros((rows, columns))`: Creates array with all values set to `0`.
`np.identity(n)`: Creates identity matrix of dimension `n`.

Creation functions

```
np.zeros((4, 3))

## array([[0., 0., 0.],
##        [0., 0., 0.],
##        [0., 0., 0.],
##        [0., 0., 0.]])

np.arange(6)

## array([0, 1, 2, 3, 4, 5])

np.identity(3)

## array([[1., 0., 0.],
##        [0., 1., 0.],
##        [0., 0., 1.]])
```

`np.linspace(start, stop, n)`: Creates vector of `n` evenly divided values from `start` to `stop`.
`np.full((row, column), k)`: Creates array with all values set to `k`.

Array creation

```
np.linspace(0, 80, 5)

## array([ 0., 20., 40., 60., 80.])

np.full((5, 4), 7)

## array([[7, 7, 7, 7],
##        [7, 7, 7, 7],
##        [7, 7, 7, 7],
##        [7, 7, 7, 7],
##        [7, 7, 7, 7]])
```

`np.random.rand(rows, columns)`: Creates array of random floats between zero and one.
`np.random.randint(k, size=(rows, columns))`: Creates array of random integers between `0` and `k-1`.

Array of random numbers

```
np.random.rand(3, 3)

## array([[0.01014591, 0.55955228, 0.48103055],
##        [0.30368877, 0.99078572, 0.61537046],
##        [0.83572553, 0.45976471, 0.63241975]])

np.random.randint(10, size=(5, 4))

## array([[7, 9, 7, 8],
##        [0, 6, 7, 5],
##        [7, 3, 4, 7],
##        [9, 4, 4, 8],
##        [8, 0, 6, 1]])
```

Reference

```
arr3

## array([[4, 8, 5],
##        [9, 3, 4],
##        [1, 0, 6]])

arr = arr3
arr[1, 1] = 777
arr3

## array([[  4,   8,   5],
##        [  9, 777,   4],
##        [  1,   0,   6]])

arr3[1, 1] = 3
```

call-by-reference

`arr = arr3` binds `arr` to the existing `arr3`. They both refer to the same object.

`array.copy()`: Copies an array without reference (call-by-value).

Copy

```
arr3

## array([[4, 8, 5],
##        [9, 3, 4],
##        [1, 0, 6]])

arr = arr3.copy()
arr[1, 1] = 777
arr3

## array([[4, 8, 5],
##        [9, 3, 4],
##        [1, 0, 6]])
```

Reference

```
arr3

## array([[4, 8, 5],
##        [9, 3, 4],
##        [1, 0, 6]])

arr = arr3
arr[1, 1] = 777
arr3

## array([[  4,   8,   5],
##        [  9, 777,   4],
##        [  1,   0,   6]])

arr3[1, 1] = 3
```

Function	Description
array	Convert input array in NumPy array
arange(start,stop,step)	Creates array from given input
ones	Creates array containing only ones
zeros	Creates array containing only zeros
empty	Allocating memory without specific values
eye, identity	Creates N x N identity matrix
linspace	Creates array of evenly divided values
full	Creates array with values set to one number
random.rand	Creates array of random floats
random.randint	Creates array of random int

`array.dtype`: Returns the type of array.
`array.astype(np.type)`: Conducts a manual typecast.

Data types

```
arr1.dtype

## dtype('int64')

arr2.dtype

## dtype('float64')

arr1 = arr1 * 2.5
arr1.dtype

## dtype('float64')

arr1 = (arr1 / 2.5).astype(np.int64)
arr1.dtype

## dtype('int64')
```

Element-wise operations

Calculation operators on NumPy arrays operate element-wise.

Element-wise operations

```
arr3

## array([[4, 8, 5],
##        [9, 3, 4],
##        [1, 0, 6]])

arr3 + arr3

## array([[ 8, 16, 10],
##        [18,  6,  8],
##        [ 2,  0, 12]])

arr3**2

## array([[16, 64, 25],
##        [81,  9, 16],
##        [ 1,  0, 36]])
```

Matrix multiplication

Operator * applied on arrays does not do the matrix multiplication.

Element-wise operations

```
arr3 * arr3

## array([[16, 64, 25],
##        [81,  9, 16],
##        [ 1,  0, 36]])

arr = np.ones((3, 2))
arr

## array([[1., 1.],
##        [1., 1.],
##        [1., 1.]])

arr3 * arr  # not defined for element-wise multiplication

## ValueError: operands could not be broadcast together
```

`array[index]`: Selects the value at position `index` from the data.

Indexing with an integer

```
arr = np.arange(10)
arr

## array([0, 1, 2, 3, 4, 5, 6, 7, 8, 9])

arr[4]

## 4

arr[-1]

## 9
```

`array[start : stop : step]`: Selects a subset of the data.

Slicing in one dimension

```
arr = np.arange(10)
arr

## array([0, 1, 2, 3, 4, 5, 6, 7, 8, 9])

arr[3:7]

## array([3, 4, 5, 6])

arr[1:]

## array([1, 2, 3, 4, 5, 6, 7, 8, 9])
```

Slicing in one dimension with steps

```
arr[:7]

## array([0, 1, 2, 3, 4, 5, 6])

arr[-3:]

## array([7, 8, 9])

arr[::-1]

## array([9, 8, 7, 6, 5, 4, 3, 2, 1, 0])

arr[::2]

## array([0, 2, 4, 6, 8])

arr[:5:-1]

## array([9, 8, 7, 6])
```

Slicing in higher dimensions

In n-dimensional arrays the element at each index is an $(n-1)$-dimensional array.

Indexing rows

```
arr3

## array([[4, 8, 5],
##        [9, 3, 4],
##        [1, 0, 6]])

vec = arr3[1]
vec

## array([9, 3, 4])

arr3[-1]

## array([1, 0, 6])
```

Slicing in two dimensions

```
arr3

## array([[4, 8, 5],
##        [9, 3, 4],
##        [1, 0, 6]])

arr3[0:2, 0:2]

## array([[4, 8],
##        [9, 3]])

arr3[2:, :]

## array([[1, 0, 6]])
```

Slicing

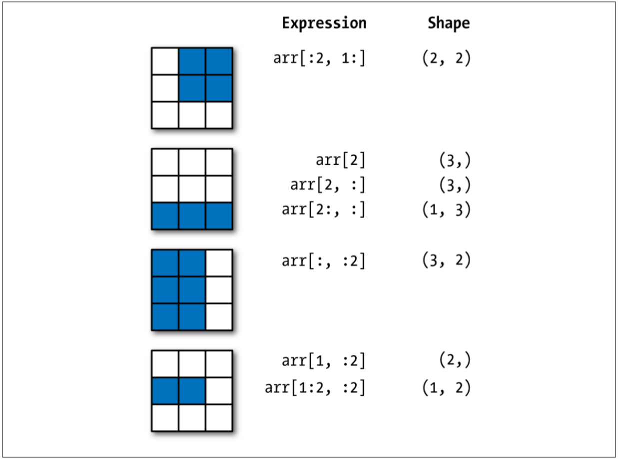


Figure: Python for Data Analysis (2017) on page 99

So far, selecting by index numbers or slicing belongs to *basic indexing* in NumPy. With basic indexing you get NO COPY of your data but a so-called *view* on the existing data set – a different perspective.
A view on an array can be seen as a reference to a rectangular memory area of its values. The view is intended to

- edit a rectangular part of a matrix, e.g., a sub-matrix, a column, or a single value,
- change the shape of the matrix or the arrangement of its elements, e.g., transpose or reshape a matrix,
- change the visual representation of values, e.g., to cast a `float` array into an `int` array,
- map the values in other program areas.

The crucial point here is that for efficiency reasons data arrays in your working memory do not have to be copied again and again for simple index operations, which would require an excessive additional effort writing to the computer memory.

A view is created automatically when you do basic indexing such as slicing:

Create a view by slicing

```
column = arr3[:, 1]
column

## array([8, 3, 0])

column.base

## array([[4, 8, 5],
##        [9, 3, 4],
##        [1, 0, 6]])

column[1] = 100
arr3

## array([[  4,   8,   5],
##        [  9, 100,   4],
##        [  1,   0,   6]])
```

Creating views implicitly

Create a view by slicing

```
elem = column[1:2]
elem.base

## array([[  4,   8,   5],
##        [  9, 100,   4],
##        [  1,   0,   6]])

elem[0] = 3
arr3

## array([[4, 8, 5],
##        [9, 3, 4],
##        [1, 0, 6]])
```

- The middle column is a view of the base array referenced by `arr3`,
- Any changes to the values of a view directly affect the base data,
- A view of a view is another view on the same base matrix.

In addition, an array contains methods and attributes that return a view of its data:

Obtain a view

```
arr3_t = arr3.T
arr3_t

## array([[4, 9, 1],
##        [8, 3, 0],
##        [5, 4, 6]])

arr3_t.flags.owndata

## False

arr3_r = arr3.reshape(1, 9)
arr3_r

## array([[4, 8, 5, 9, 3, 4, 1, 0, 6]])

arr3_t.flags.owndata

## False
```

Obtaining views explicitly

Obtain a view

```
arr3_v = arr3.view()
arr3_v.flags.owndata

## False
```

- The transposed matrix is a predefined view that is available as an attribute,
- Reshaping is also just another way of looking at the same set of data,
- By means of the method `view()` you create a view with an identical representation.

The behavior described above changes with *advanced indexing*, i. e., if at least one component of the index tuple is not a scalar index number or slice. The case of *fancy indexing* is described below:

Advanced and basic indexing

```
arr3

## array([[4, 8, 5],
##        [9, 3, 4],
##        [1, 0, 6]])

arr = arr3[[0, 2], [0, 2]]
arr

## array([4, 6])

arr.base
```

Advanced and basic indexing

```
arr = arr3[0:3:2, 0:3:2]
arr

## array([[4, 5],
##        [1, 6]])

arr.base

## array([[4, 8, 5],
##        [9, 3, 4],
##        [1, 0, 6]])
```

- Contrary to intuition, fancy indexing does not return a (2×2)-matrix, but a vector of the matrix elements $(0, 0)$ and $(2, 2)$. This is a complete copy – a new object and not a view to the original matrix.
- A submatrix (view) with the corner elements of the initial matrix can be obtained with slicing.

A boolean array is a NumPy array with boolean `True` and `False` values. Such an array can be created by applying a comparison operator on NumPy arrays.

Boolean arrays

```
bool_arr = (arr3 < 5)
bool_arr

## array([[ True, False, False],
##        [False,  True,  True],
##        [ True,  True, False]])

bool_arr1 = (arr3 == 0)
bool_arr1

## array([[False, False, False],
##        [False, False, False],
##        [False,  True, False]])
```

The comparison operators on arrays can be combined by means of NumPy redefined bitwise operators.

Boolean arrays and bitwise operators

```
a = np.array([3, 8, 4, 1, 9, 5, 2])
b = np.array([2, 3, 5, 6, 11, 15, 17])
c = (a % 2 == 0) | (b % 3 == 0)  # or
c

## array([False,  True,  True,  True, False,  True,  True])

d = (a > b) ^ (a % 2 == 1)  # exclusive or
d

## array([False,  True, False,  True,  True,  True, False])

c ^ d  # exclusive or

## array([False, False,  True, False,  True, False,  True])
```

Boolean arrays

Logical operations on NumPy arrays work in a similar way compared to bitwise operators.

Boolean arrays can be used to select elements of other NumPy arrays. If `x` is an array and `y` is a boolean array of the same dimension, then `a[b]` selects all the elements of `x`, for which the correspanding value (at the same position) of `y` is `True`.

Indexing with boolean arrays

```
arr3

## array([[4, 8, 5],
##        [9, 3, 4],
##        [1, 0, 6]])

y = arr3 % 2 == 0
y

## array([[ True,  True, False],
##        [False, False,  True],
##        [False,  True,  True]])

arr3[y]

## array([4, 8, 4, 0, 6])
```

Conditional indexing allows you using boolean arrays to select subsets of values and to avoid loops. Applying comparison operator on arrays, every element of the array is tested, if it corresponds to the logical condition. Consider an application setting all even numbers to 5:

Find and replace values in arrays

```
a, b = arr3.copy(), arr3.copy()
for i in range(a.shape[0]):
    for j in range(a.shape[1]):
        if a[i, j] % 2 == 0:
            a[i, j] = 5

b[b % 2 == 0] = 5
b

## array([[5, 5, 5],
##        [9, 3, 5],
##        [1, 5, 5]])

np.allclose(a, b)

## True
```

Find and replace values in arrays, condition: equal

```
arr3

## array([[4, 8, 5],
##        [9, 3, 4],
##        [1, 0, 6]])

arr = arr3.copy()
arr[arr == 4] = 100
arr

## array([[100,   8,   5],
##        [  9,   3, 100],
##        [  1,   0,   6]])
```

- In this example, `arr == 4` creates a boolean array as described before which is then used to index the array `arr`.
- Finally, every element of `arr` which is *marked* `True` according to the boolean index array will be set to `100`.

Step 1a

Integer indexing `array[row index, column index]`: Indexing an n-dimensional array with n integer indices returns the single value at this position.

Best practice Step 1a

```
mat = np.arange(12).reshape((3, 4))
mat

## array([[ 0,  1,  2,  3],
##        [ 4,  5,  6,  7],
##        [ 8,  9, 10, 11]])

mat[2, 2]

## 10

mat[0, -1]

## 3
```

Keep in mind that, in this case only, the results are not arrays but values!

Step 1b

Integer indexing `array[row index]`: In n-dimensional arrays, the element at each index is an $(n-1)$-dimensional array.

Best practice Step 1b

```
mat = np.arange(12).reshape((3, 4))
mat

## array([[ 0,  1,  2,  3],
##        [ 4,  5,  6,  7],
##        [ 8,  9, 10, 11]])

mat[2]

## array([ 8,  9, 10, 11])

mat[0]

## array([0, 1, 2, 3])
```

By specifying the `row index` only, we create arrays which are views.

Step 2a

Slicing `array[start : stop : step]`: Slicing can be used separately for rows and columns.

Best practice Step 2a

```
mat = np.arange(12).reshape((3, 4))
mat

## array([[ 0,  1,  2,  3],
##        [ 4,  5,  6,  7],
##        [ 8,  9, 10, 11]])

mat[0:2]

## array([[0, 1, 2, 3],
##        [4, 5, 6, 7]])

mat[0:2, ::2]

## array([[0, 2],
##        [4, 6]])
```

Step 2b

A frequent task is to get a specific row or column of an array. This can be done easily by slicing.

Best practice Step 2b

```
mat

## array([[ 0,  1,  2,  3],
##        [ 4,  5,  6,  7],
##        [ 8,  9, 10, 11]])

row = mat[1]  # get second row
column = mat[:, 2]  # get third column
row

## array([4, 5, 6, 7])

column

## array([ 2,  6, 10])
```

Slicing with `[:]` means to take every element from the first to the last.

Step 3

Fancy indexing `array[rows list, columns list]`: Return a one dimensional array with the values at the index tuples specified elementwise by the index lists.

Best practice Step 3

```
mat = np.arange(12).reshape((3, 4))
mat

## array([[ 0,  1,  2,  3],
##        [ 4,  5,  6,  7],
##        [ 8,  9, 10, 11]])

mat[[1, 2], [1, 2]]

## array([ 5, 10])

mat[[0, -1], [-1]]

## array([ 3, 11])
```

The index lists might also contain just a single element.

Step 4

Conditional indexing: Applying comparison operators to arrays, the boolean operations are evaluated elementwise in a vectorized fashion.

Best practice Step 4

```
bool_mat = mat > 0
bool_mat

## array([[False,  True,  True,  True],
##        [ True,  True,  True,  True],
##        [ True,  True,  True,  True]])

mat[bool_mat] = 111  # equivalent to mat[mat > 0] = 111
mat

## array([[  0, 111, 111, 111],
##        [111, 111, 111, 111],
##        [111, 111, 111, 111]])
```

Step 5
Replacing values in arrays. Assigning a slice of an array to new values, the shape of slice must be considered.

Best practice Step 5

```
mat[0] = np.array([3, 2, 1])  # Fails because the shapes do not fit

## Error: could not broadcast array from shape (3) into shape (4)

mat[2, 3] = 100
mat[:, 0] = np.array([3, 3, 3])
mat

## array([[  3, 111, 111, 111],
##        [  3, 111, 111, 111],
##        [  3, 111, 111, 100]])

mat[1:3, 1:3] = np.array([[0, 0], [0, 0]])
mat

## array([[  3, 111, 111, 111],
##        [  3,   0,   0, 111],
##        [  3,   0,   0, 100]])
```

`array.reshape((rows, columns))`: Reshapes an existing array.
`array.resize((rows, columns))`: Changes array shape to `rows` x `columns` and fills new values with `0`.

Reshape

```
arr = np.arange(15)
arr.reshape((3, 5))

## array([[ 0,  1,  2,  3,  4],
##        [ 5,  6,  7,  8,  9],
##        [10, 11, 12, 13, 14]])

arr = np.arange(15)
arr.resize((3, 7))
arr

## array([[ 0,  1,  2,  3,  4,  5,  6],
##        [ 7,  8,  9, 10, 11, 12, 13],
##        [14,  0,  0,  0,  0,  0,  0]])
```

`np.append(array, value)`: Appends value to the end of array.
`np.insert(array, index, value)`: Inserts values before index.
`np.delete(array, index, axis)`: Deletes row or column on index.

Naming

```
a = np.arange(5)
a = np.append(a, 8)
a = np.insert(a, 3, 77)
print(a)

## [ 0  1  2 77  3  4  8]

a.resize((3, 3))
np.delete(a, 1, axis=0)

## array([[0, 1, 2],
##        [8, 0, 0]])
```

`np.concatenate((arr1, arr2), axis)`: Joins a sequence of arrays along an existing axis.
`np.split(array, n)`: Splits an array into multiple sub-arrays.
`np.hsplit(array, n)`: Splits an array into multiple sub-arrays horizontally.

Naming

```
np.concatenate((a, np.arange(6).reshape(2, 3)), axis=0)

## array([[ 0,  1,  2],
##        [77,  3,  4],
##        [ 8,  0,  0],
##        [ 0,  1,  2],
##        [ 3,  4,  5]])

np.split(np.arange(8), 4)

## [array([0, 1]), array([2, 3]), array([4, 5]), array([6, 7])]
```

`array.T`: Returns the transposed array (as a view).

Transpose

```
arr3

## array([[4, 8, 5],
##        [9, 3, 4],
##        [1, 0, 6]])

arr3.T

## array([[4, 9, 1],
##        [8, 3, 0],
##        [5, 4, 6]])

np.eye(3).T

## array([[1., 0., 0.],
##        [0., 1., 0.],
##        [0., 0., 1.]])
```

`np.dot(arr1, arr2)`: Conducts a matrix multiplication of `arr1` and `arr2`. The `@` operator can be used instead of the `np.dot()` function.

Matrix multiplication

```
res = np.dot(arr3, np.arange(18).reshape((3, 6)))
res

## array([[108, 125, 142, 159, 176, 193],
##        [ 66,  82,  98, 114, 130, 146],
##        [ 72,  79,  86,  93, 100, 107]])

res2 = arr3 @ np.arange(18).reshape((3, 6))
res2

## array([[108, 125, 142, 159, 176, 193],
##        [ 66,  82,  98, 114, 130, 146],
##        [ 72,  79,  86,  93, 100, 107]])

np.allclose(res, res2)

## True
```

Element-wise functions

```
arr3

## array([[4, 8, 5],
##        [9, 3, 4],
##        [1, 0, 6]])

np.sqrt(arr3)

## array([[2.        , 2.82842712, 2.23606798],
##        [3.        , 1.73205081, 2.        ],
##        [1.        , 0.        , 2.44948974]])

np.exp(arr3)

## array([[5.45981500e+01, 2.98095799e+03, 1.48413159e+02],
##        [8.10308393e+03, 2.00855369e+01, 5.45981500e+01],
##        [2.71828183e+00, 1.00000000e+00, 4.03428793e+02]])
```

Function	Description
abs	Absolute value of integer and floating point
sqrt	Sqare root
exp	Exponential function
log, log10, log2	Natural logarithm, log base 10, log base 2
sign	Sign (1 : positiv, 0: zero, -1 : negative)
ceil	Rounding up to integer
floor	Round down to integer
rint	Round to nearest integer
modf	Returns fractional parts
sin, cos, tan, sinh, cosh, tanh, arcsin, ...	

Binary

```
x = np.array([3, -6, 8, 4, 3, 5])
y = np.array([3, 5, 7, 3, 5, 9])
np.maximum(x, y)

## array([3, 5, 8, 4, 5, 9])

np.greater_equal(x, y)

## array([ True, False,  True,  True, False, False])

np.add(x, y)

## array([ 6, -1, 15,  7,  8, 14])

np.mod(x, y)

## array([0, 4, 1, 1, 3, 5])
```

Function	Description
add	Add elements of arrays
subtract	Subtract elements in the second from the first array
multiply	Multiply elements
divide	Divide elements
power	Raise elements in first array to powers in second
maximum	Element-wise maximum
minimum	Element-wise minimum
mod	Element-wise modulus
greater, less, equal gives boolean	

`np.meshgrid(array1, array2)`: Returns coordinate matrices from coordinate arrays.

Evaluate the function $f(x, y) = \sqrt{x^2 + y^2}$ on a 10 x 10 grid

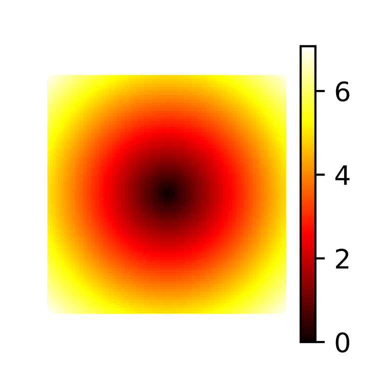
```
p = np.arange(-5, 5, 0.01)
x, y = np.meshgrid(p, p)
x

## array([[-5.  , -4.99, -4.98, ...,  4.97,  4.98,  4.99],
##        [-5.  , -4.99, -4.98, ...,  4.97,  4.98,  4.99],
##        [-5.  , -4.99, -4.98, ...,  4.97,  4.98,  4.99],
##        ...,
##        [-5.  , -4.99, -4.98, ...,  4.97,  4.98,  4.99],
##        [-5.  , -4.99, -4.98, ...,  4.97,  4.98,  4.99],
##        [-5.  , -4.99, -4.98, ...,  4.97,  4.98,  4.99]])
```

Evaluate the function $f(x, y) = \sqrt{x^2 + y^2}$ on a 10 x 10 grid.

```
import matplotlib.pyplot as plt
val = np.sqrt(x**2 + y**2)
plt.figure(figsize=(2, 2))
plt.imshow(val, cmap="hot")
plt.colorbar()

## <matplotlib.colorbar.Colorbar object at 0x16984cb80>
```

Evaluate the function $f(x, y) = \sqrt{x^2 + y^2}$ on a 10 x 10 grid.

```
plt.show()
```



`np.where(condition, a, b)`: If condition is `True`, returns value `a`, otherwise returns `b`.

Conditional logic

```
a = np.array([4, 7, 5, -7, 9, 0])
b = np.array([-1, 9, 8, 3, 3, 3])
cond = np.array([True, True, False, True, False, False])
res = np.where(cond, a, b)
res

## array([ 4,  7,  8, -7,  3,  3])

res = np.where(a <= b, b, a)
res

## array([4, 9, 8, 3, 9, 3])
```

Conditional logic, examples

```
arr3

## array([[4, 8, 5],
##        [9, 3, 4],
##        [1, 0, 6]])

res = np.where(arr3 < 5, 0, arr3)
res

## array([[0, 8, 5],
##        [9, 0, 0],
##        [0, 0, 6]])

even = np.where(arr3 % 2 == 0, arr3, arr3 + 1)
even

## array([[ 4,  8,  6],
##        [10,  4,  4],
##        [ 2,  0,  6]])
```

`array.mean()`: Computes the mean of all array elements.
`array.sum()`: Computes the sum of all array elements.

Statistical methods

```
arr3

## array([[4, 8, 5],
##        [9, 3, 4],
##        [1, 0, 6]])

arr3.mean()

## 4.444444444444445

arr3.sum()

## 40

arr3.argmin()

## 7
```

Overview: Statistical methods

Method	Description
sum	Sum of all array elements
mean	Mean of all array elements
std, var	Standard deviation, variance
min, max	Minimum and Maximum value in array
argmin, argmax	Indices of Minimum and Maximum value

Axes are defined for arrays with more than one dimension. A two-dimensional array has two axes. The first one is running vertically downwards across the rows (`axis=0`), the second one running horizontally across the columns (`axis=1`).

Axis

```
arr3

## array([[4, 8, 5],
##        [9, 3, 4],
##        [1, 0, 6]])

arr3.sum(axis=0)

## array([14, 11, 15])

arr3.sum(axis=1)

## array([17, 16,  7])
```

`array.sort(axis)`: Sorts array by an `axis`.

Sorting one-dimensional arrays

```
arr2

## array([24.3 ,  0.  ,  8.9 ,  4.4 ,  1.65, 45.  ])

arr2.sort()
arr2

## array([ 0.  ,  1.65,  4.4 ,  8.9 , 24.3 , 45.  ])
```

Sorting two-dimensional arrays

```
arr3

## array([[4, 8, 5],
##        [9, 3, 4],
##        [1, 0, 6]])

arr3.sort()
arr3

## array([[4, 5, 8],
##        [3, 4, 9],
##        [0, 1, 6]])

arr3.sort(axis=0)
arr3

## array([[0, 1, 6],
##        [3, 4, 8],
##        [4, 5, 9]])
```

The default axis using `sort()` is `-1`, which means to sort along the last axis (in this case axis 1).

Numerical programming

▶ Linear algebra

Import numpy.linalg

```
import numpy.linalg as nplin
```

`nplin.inv(array)`: Computes the inverse matrix.
`np.allclose(array1, array2)`: Returns `True` if two arrays are element-wise equal within a tolerance.

Inverse

```
inv = nplin.inv(arr3)
inv

## array([[  4., -21.,  16.],
##        [ -5.,  24., -18.],
##        [  1.,  -4.,   3.]])

np.allclose(np.identity(3), np.dot(inv, arr3))

## True
```

`nplin.det(array)`: Computes the determinant.
`np.trace(array)`: Computes the trace.
`np.diag(array)`: Returns the diagonal elements as an array.

Linear algebra functions

```
nplin.det(arr3)

## -1.0

np.trace(arr3)

## 13

np.diag(arr3)

## array([0, 4, 9])
```

`nplin.eig(array)`: Returns the array of eigenvalues and the array of eigenvectors as a list.

Get eigenvalues and eigenvectors

```
A = np.array([[3, -1, 0], [2, 0, 0], [-2, 2, -1]])
eigenval, eigenvec = nplin.eig(A)
eigenval

## array([-1.,  1.,  2.])

eigenvec

## array([[ 0.00000000e+00, -4.08248290e-01, -7.07106781e-01],
##        [ 0.00000000e+00, -8.16496581e-01, -7.07106781e-01],
##        [ 1.00000000e+00, -4.08248290e-01,  1.17027782e-17]])
```

Check eigenvalues and eigenvectors

```
eigenval * eigenvec

## array([[-0.00000000e+00, -4.08248290e-01, -1.41421356e+00],
##        [-0.00000000e+00, -8.16496581e-01, -1.41421356e+00],
##        [-1.00000000e+00, -4.08248290e-01,  2.34055565e-17]])

np.dot(A, eigenvec)

## array([[ 0.00000000e+00, -4.08248290e-01, -1.41421356e+00],
##        [ 0.00000000e+00, -8.16496581e-01, -1.41421356e+00],
##        [-1.00000000e+00, -4.08248290e-01, -1.17027782e-17]])
```

$$\begin{pmatrix} 3 & -1 & 0 \\ 2 & 0 & 0 \\ -2 & 2 & -1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = (-1) \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

`nplin.qr(array)`: Conducts a QR decomposition and returns Q and R as lists.

QR decomposition

```
Q, R = nplin.qr(arr3)
Q

## array([[ 0.        ,  0.98058068,  0.19611614],
##        [-0.6       ,  0.15689291, -0.78446454],
##        [-0.8       , -0.11766968,  0.58834841]])

R

## array([[ -5.        ,  -6.4       , -12.        ],
##        [  0.        ,   1.0198039 ,   6.07960019],
##        [  0.        ,   0.        ,   0.19611614]])

np.allclose(arr3, np.dot(Q, R))

## True
```

`nplin.solve(A, b)`: Returns the solution of the linearsystem $Ax = b$.

Solve linearsystems

```
b = np.array([7, 4, 8])
x = nplin.solve(A, b)
x

## array([  2.,  -1., -14.])

np.allclose(np.dot(A, x), b)

## True
```

$$\begin{aligned} 3x_1 - 1x_2 + 0x_3 &= 7 \\ 2x_1 - 0x_2 + 0x_3 &= 4 \\ -2x_1 + 2x_2 - 1x_3 &= 8 \end{aligned} \rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ -14 \end{pmatrix}$$

Function	Description
np.dot	Matrix multiplication
np.trace	Sum of the diagonal elements
np.diag	Diagonal elements as an array
nplin.det	Matrix determinant
nplin.eig	Eigenvalues and eigenvectors
nplin.inv	Inverse matrix
nplin.qr	QR decomposition
nplin.solve	Solve linearsystem

Data formats and handling

3.1 Pandas package
3.2 Series
3.3 DataFrame
3.4 Import/Export data

Data formats and handling

▶ Pandas package

The package `pandas` is a free software library for Python including the following features:

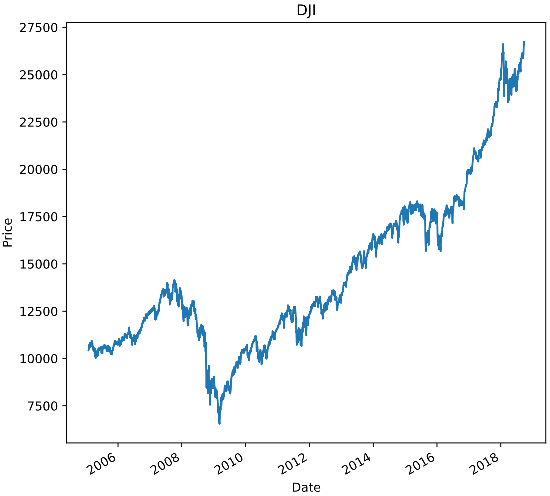
- Data manipulation and analysis,
- DataFrame objects and Series,
- Export and import data from files and web,
- Handling of missing data.

→ Provides high-performance data structures and data analysis tools.

With pandas you can import and visualize financial data in only a few lines of code.

Motivation

```python
import pandas as pd
import matplotlib.pyplot as plt

fig = plt.figure()
ax = fig.add_subplot(1, 1, 1)
dow = pd.read_csv("data/dji.csv", index_col=0, parse_dates=True)
close = dow["Close"]
close.plot(ax=ax)
ax.set_xlabel("Date")
ax.set_ylabel("Price")
ax.set_title("DJI")
fig.savefig("out/dji.pdf", format="pdf")
```


DJI
Price
27500
25000
22500
20000
17500
15000
12500
10000
7500
2006
2008
2010
2012
2014
2016
2018
Date

Data formats and handling

▶ Series

Series are a data structure in pandas.

- One-dimensional array-like object,
- Containing a sequence of values and a corresponding array of labels, called the index,
- The string representation of a Series displays the index on the left and the values on the right,
- The default index consists of the integers 0 through N-1.

String representation of a Series

```
## 0     3
## 1     7
## 2    -8
## 3     4
## 4    26
## dtype: int64
```

`pd.Series()`: Creates one-dimensional array-like object including values and an index.

Importing Pandas and creating a Series

```
import numpy as np
import pandas as pd

obj = pd.Series([2, -5, 9, 4])
obj

## 0    2
## 1   -5
## 2    9
## 3    4
## dtype: int64
```

- Simple Series formed only from a list,
- An index is added automatically.

Series indexing vs. Numpy indexing

```
obj2 = pd.Series([2, -5, 9, 4], index=["a", "b", "c", "d"])
npobj = np.array([2, -5, 9, 4])
obj2

## a    2
## b   -5
## c    9
## d    4
## dtype: int64

obj2["b"]

## -5

npobj[1]

## -5
```

- NumPy arrays can only be indexed by integers while Series can be indexed by the manually set index.

Pandas Series can be created from:

- Lists,
- NumPy arrays,
- Dicts.

Series creation from Numpy arrays

```
npobj = np.array([2, -5, 9, 4])
obj2 = pd.Series(npobj, index=["a", "b", "c", "d"])
obj2

## a    2
## b   -5
## c    9
## d    4
## dtype: int64
```

Series from dicts

```
dictdata = {"Göttingen": 117665, "Northeim": 28920,
            "Hannover": 532163, "Berlin": 3574830}
obj3 = pd.Series(dictdata)
obj3

## Göttingen     117665
## Northeim       28920
## Hannover      532163
## Berlin       3574830
## dtype: int64
```

- The index of the Series can be set manually,
- Compared to NumPy array you can use the set index to select single values,
- Data contained in a dict can be passed to a Series. The index of the resulting Series consists of the dict's keys.

Dict to Series with manual index

```
cities = ["Hamburg", "Göttingen", "Berlin", "Hannover"]
obj4 = pd.Series(dictdata, index=cities)
obj4

## Hamburg              NaN
## Göttingen       117665.0
## Berlin         3574830.0
## Hannover        532163.0
## dtype: float64
```

- Passing a dict to a Series, the index can be set manually,
- `NaN` (not a number) marks missing values where the index and the dict do not match.

`Series.values`: Returns the values of a Series.
`Series.index`: Returns the index of a Series.

Series properties

```
obj.values

## array([ 2, -5,  9,  4])

obj.index

## RangeIndex(start=0, stop=4, step=1)

obj2.index

## Index(['a', 'b', 'c', 'd'], dtype='object')
```

- The values and the index of a Series can be printed separately.
- The default index, if none was explicitly specified, is a `RangeIndex`.
- `RangeIndex` inherits from `Index` class.

Series manipulation

```
obj2[["c", "d", "a"]]

## c    9
## d    4
## a    2
## dtype: int64

obj2[obj2 < 0]

## b   -5
## dtype: int64
```

NumPy-like functions can be applied on Series

- For filtering data,
- To do scalar multiplications or applying math functions,
- The index-value link will be preserved.

Series functions

```
obj2 * 2

## a     4
## b   -10
## c    18
## d     8
## dtype: int64

np.exp(obj2)["a":"c"]

## a       7.389056
## b       0.006738
## c    8103.083928
## dtype: float64

"c" in obj2

## True
```

- Mathematical functions applied to a Series will only be applied on its *values* – not on its index.

Series manipulation

```
obj4["Hamburg"] = 1900000
obj4

## Hamburg      1900000.0
## Göttingen     117665.0
## Berlin       3574830.0
## Hannover      532163.0
## dtype: float64

obj4[["Berlin", "Hannover"]] = [3600000, 1100000]
obj4

## Hamburg      1900000.0
## Göttingen     117665.0
## Berlin       3600000.0
## Hannover     1100000.0
## dtype: float64
```

- Values can be manipulated by using the labels in the index,
- Sets of values can be set in one line.

`pd.isnull()`: `True` if data is missing.
`pd.notnull()`: `False` if data is missing.

NaN

```
pd.isnull(obj4)

## Hamburg      False
## Göttingen    False
## Berlin       False
## Hannover     False
## dtype: bool

pd.notnull(obj4)

## Hamburg      True
## Göttingen    True
## Berlin       True
## Hannover     True
## dtype: bool
```

Align differently indexed data

There are not two values to align for `Hamburg` and `Northeim` – so they are marked with `NaN` (not a number).

Data 1

```
obj3

## Göttingen      117665
## Northeim        28920
## Hannover       532163
## Berlin        3574830
## dtype: int64
```

Data 2

```
obj4

## Hamburg       1900000.0
## Göttingen      117665.0
## Berlin        3600000.0
## Hannover      1100000.0
## dtype: float64
```

Align data

```
obj3 + obj4

## Berlin       7174830.0
## Göttingen     235330.0
## Hamburg            NaN
## Hannover     1632163.0
## Northeim           NaN
## dtype: float64
```

`Series.name`: Returns name of the Series.
`Series.index.name`: Returns name of the Series' index.

Naming

```
obj4.name = "population"
obj4.index.name = "city"
obj4

## city
## Hamburg      1900000.0
## Göttingen     117665.0
## Berlin       3600000.0
## Hannover     1100000.0
## Name: population, dtype: float64
```

- The attribute `name` will change the name of the existing Series,
- There is no default name of the Series or the index.

Series vs. NumPy arrays

- NumPy arrays are accessed by their integer positions,
- Series can be accessed by a user defined index, including letters and numbers,
- Different Series can be aligned efficiently by the index,
- Series can work with missing values, so operations do not automatically fail.

Data formats and handling

▶ DataFrame

- *DataFrames* are the primary structure of pandas,
- It represents a table of data with an ordered collection of columns,
- Each column can have a different data type,
- A DataFrame can be thought of as a dict of Series sharing the same index,
- Physically a DataFrame is two-dimensional but by using hierarchical indexing it can respresent higher dimensional data.

String representation of a DataFrame

```
##    company   price   volume
## 0  Daimler   69.20  4456290
## 1     E.ON    8.11  3667975
## 2  Siemens  110.92  3669487
## 3     BASF   87.28  1778058
## 4      BMW   87.81  1824582
```

`pd.DataFrame()`: Creates a DataFrame which is a two-dimensional tabular-like structure with labeled axis (rows and columns).

Creating a DataFrame

```
data = {"company": ["Daimler", "E.ON", "Siemens", "BASF", "BMW"],
        "price": [69.2, 8.11, 110.92, 87.28, 87.81],
        "volume": [4456290, 3667975, 3669487, 1778058, 1824582]}
frame = pd.DataFrame(data)
frame

##    company   price   volume
## 0  Daimler   69.20  4456290
## 1     E.ON    8.11  3667975
## 2  Siemens  110.92  3669487
## 3     BASF   87.28  1778058
## 4      BMW   87.81  1824582
```

- In this example the construction of the DataFrame `frame` is done by passing a dict of equal-length lists,
- Instead of passing a dict of lists, it is also possible to pass a dict of NumPy arrays.

Print DataFrame

```
frame2 = pd.DataFrame(data, columns=["company", "volume",
                                     "price", "change"])
frame2

##     company   volume   price change
## 0   Daimler  4456290   69.20    NaN
## 1      E.ON  3667975    8.11    NaN
## 2   Siemens  3669487  110.92    NaN
## 3      BASF  1778058   87.28    NaN
## 4       BMW  1824582   87.81    NaN
```

- Passing a column that is not contained in the dict, it will be marked with `NaN`,
- The default index will be assigned automatically as with Series.

Inputs to DataFrame constructor

Type	Description
2D NumPy arrays	A matrix of data
dict of arrays, lists, or tuples	Each sequence becomes a column
dict of Series	Each value becomes a column
dict of dicts	Each inner dict becomes a column
List of dicts or Series	Each item becomes a row
List of lists or tuples	Treated as the 2D NumPy arrays
Another DataFrame	Same indexes

Add data to DataFrame

```
frame2["change"] = [1.2, -3.2, 0.4, -0.12, 2.4]
frame2["change"]

## 0    1.20
## 1   -3.20
## 2    0.40
## 3   -0.12
## 4    2.40
## Name: change, dtype: float64
```

- Selecting the column of DataFrame, a Series is returned,
- A attribute-like access, e.g., `frame2.change`, is also possible,
- The returned Series has the same index as the initial DataFrame.

Indexing DataFrames

```
frame2[["company", "change"]]

##    company  change
## 0  Daimler    1.20
## 1     E.ON   -3.20
## 2  Siemens    0.40
## 3     BASF   -0.12
## 4      BMW    2.40
```

- Using a list of multiple columns while indexing, the result is a DataFrame,
- The returned DataFrame has the same index as the initial one.

`del DataFrame[column]`: Deletes column from DataFrame.

DataFrame delete column

```
del frame2["volume"]
frame2

##    company   price  change
## 0  Daimler   69.20    1.20
## 1     E.ON    8.11   -3.20
## 2  Siemens  110.92    0.40
## 3     BASF   87.28   -0.12
## 4      BMW   87.81    2.40

frame2.columns

## Index(['company', 'price', 'change'], dtype='object')
```

Naming properties

```
frame2.index.name = "number:"
frame2.columns.name = "feature:"
frame2

## feature:   company   price  change
## number:
## 0          Daimler   69.20    1.20
## 1             E.ON    8.11   -3.20
## 2          Siemens  110.92    0.40
## 3             BASF   87.28   -0.12
## 4              BMW   87.81    2.40
```

- In DataFrames there is no default name for the index or the columns.

`DataFrame.reindex()`: Creates new DataFrame with data conformed to a new index, while the initial DataFrame will not be changed.

Reindexing

```
frame3 = frame.reindex([0, 2, 3, 4])
frame3

##    company   price   volume
## 0  Daimler   69.20  4456290
## 2  Siemens  110.92  3669487
## 3     BASF   87.28  1778058
## 4      BMW   87.81  1824582
```

- Index values that are not already present will be filled with `NaN` by default,
- There are many options for filling missing values.

Filling missing values

```
frame4 = frame.reindex(index=[0, 2, 3, 4, 5], fill_value=0,
                       columns=["company", "price", "market cap"])
frame4

##     company   price  market cap
## 0  Daimler   69.20           0
## 2  Siemens  110.92           0
## 3     BASF   87.28           0
## 4      BMW   87.81           0
## 5        0    0.00           0

frame4 = frame.reindex(index=[0, 2, 3, 4], fill_value=np.nan,
                       columns=["company", "price", "market cap"])
frame4

##     company   price  market cap
## 0  Daimler   69.20         NaN
## 2  Siemens  110.92         NaN
## 3     BASF   87.28         NaN
## 4      BMW   87.81         NaN
```

`DataFrame.fillna(value)`: Fills `NaN`s with `value`.

Filling NaN

```
frame4[:3]

##    company   price  market cap
## 0  Daimler   69.20         NaN
## 2  Siemens  110.92         NaN
## 3     BASF   87.28         NaN

frame4.fillna(1000000, inplace=True)
frame4[:3]

##    company   price  market cap
## 0  Daimler   69.20   1000000.0
## 2  Siemens  110.92   1000000.0
## 3     BASF   87.28   1000000.0
```

- The option `inplace=True` fills the current DafaFrame (here `frame4`). Without using `inplace` a new DataFrame will be created, filled with `NaN` values.

`DataFrame.drop(index, axis)`: Returns a new object with labels in requested axis removed.

Dropping index

```
frame5 = frame
frame5

##    company   price   volume
## 0  Daimler   69.20  4456290
## 1     E.ON    8.11  3667975
## 2  Siemens  110.92  3669487
## 3     BASF   87.28  1778058
## 4      BMW   87.81  1824582

frame5.drop([1, 2])

##    company  price   volume
## 0  Daimler  69.20  4456290
## 3     BASF  87.28  1778058
## 4      BMW  87.81  1824582
```

Dropping column

```
frame5[:2]

##    company  price   volume
## 0  Daimler  69.20  4456290
## 1     E.ON   8.11  3667975

frame5.drop("price", axis=1)[:3]

##    company   volume
## 0  Daimler  4456290
## 1     E.ON  3667975
## 2  Siemens  3669487

frame5.drop(2, axis=0)

##    company  price   volume
## 0  Daimler  69.20  4456290
## 1     E.ON   8.11  3667975
## 3     BASF  87.28  1778058
## 4      BMW  87.81  1824582
```

Indexing of DataFrames works like indexing an `numpy` array, you can use the default index values and a manually set index.

Indexing

```
frame

##    company   price   volume
## 0  Daimler   69.20  4456290
## 1     E.ON    8.11  3667975
## 2  Siemens  110.92  3669487
## 3     BASF   87.28  1778058
## 4      BMW   87.81  1824582

frame[2:]

##    company   price   volume
## 2  Siemens  110.92  3669487
## 3     BASF   87.28  1778058
## 4      BMW   87.81  1824582
```

Indexing

```
frame6 = pd.DataFrame(data, index=["a", "b", "c", "d", "e"])
frame6

##     company   price   volume
## a   Daimler   69.20  4456290
## b      E.ON    8.11  3667975
## c   Siemens  110.92  3669487
## d      BASF   87.28  1778058
## e       BMW   87.81  1824582

frame6["b":"d"]

##     company   price   volume
## b      E.ON    8.11  3667975
## c   Siemens  110.92  3669487
## d      BASF   87.28  1778058
```

- When *slicing with labels* the end element is inclusive.

`DataFrame.loc()`: Selects a subset of rows and columns from a DataFrame using axis labels.
`DataFrame.iloc()`: Selects a subset of rows and columns from a DataFrame using integers.

Selection with loc and iloc

```
frame6.loc["c", ["company", "price"]]

## company    Siemens
## price       110.92
## Name: c, dtype: object

frame6.iloc[2, [0, 1]]

## company    Siemens
## price       110.92
## Name: c, dtype: object
```

Selection with loc and iloc

```
frame6.loc[["c", "d", "e"], ["volume", "price", "company"]]

##      volume   price  company
## c   3669487  110.92  Siemens
## d   1778058   87.28     BASF
## e   1824582   87.81      BMW

frame6.iloc[2:, ::-1]

##      volume   price  company
## c   3669487  110.92  Siemens
## d   1778058   87.28     BASF
## e   1824582   87.81      BMW
```

- Both of the indexing functions work with slices or lists of labels,
- Many ways to select and rearrange pandas objects.

DataFrame indexing options

Type	Description
df[val]	Select single column or set of columns
df.loc[val]	Select single row or set of rows
df.loc[:, val]	Select single column or set of columns
df.loc[val1, val2]	Select row and column by label
df.iloc[where]	Select row or set of rows by integer position
df.iloc[:, where]	Select column or set of columns by integer pos.
df.iloc[w1, w2]	Select row and column by integer position

Hierarchical indexing enables you to have multiple index levels.

Multiindex

```
ind = [["a", "a", "a", "b", "b"], [1, 2, 3, 1, 2]]
frame6 = pd.DataFrame(np.arange(15).reshape((5, 3)), index=ind,
                      columns=["first", "second", "third"])
frame6

##      first  second  third
## a 1      0       1      2
##   2      3       4      5
##   3      6       7      8
## b 1      9      10     11
##   2     12      13     14

frame6.index.names = ["index1", "index2"]
frame6.index

## MultiIndex([('a', 1),
##             ('a', 2),
##             ('a', 3),
##             ('b', 1),
##             ('b', 2)],
##            names=['index1', 'index2'])
```

Selecting of a multiindex

```
frame6.loc["a"]

##         first  second  third
## index2
## 1           0       1      2
## 2           3       4      5
## 3           6       7      8

frame6.loc["b", 1]

## first      9
## second    10
## third     11
## Name: (b, 1), dtype: int64
```

Series and DataFrames

```
frame7 = frame[["price", "volume"]]
frame7.index = ["Daimler", "E.ON", "Siemens", "BASF", "BMW"]
series = frame7.iloc[2]
frame7

##            price   volume
## Daimler    69.20  4456290
## E.ON        8.11  3667975
## Siemens   110.92  3669487
## BASF       87.28  1778058
## BMW        87.81  1824582

series

## price          110.92
## volume     3669487.00
## Name: Siemens, dtype: float64
```

- Here the Series was generated from the first row of the DataFrame.

Operations between Series and DataFrames down the rows

```
frame7 + series

##             price     volume
## Daimler   180.12  8125777.0
## E.ON      119.03  7337462.0
## Siemens   221.84  7338974.0
## BASF      198.20  5447545.0
## BMW       198.73  5494069.0
```

- By default arithmetic operations between DataFrames and Series match the index of the Series on the DataFrame's columns,
- The operations will be broadcasted along the rows.

Operations between Series and DataFrames down the columns

```
series2 = frame7["price"]
frame7.add(series2, axis=0)

##           price      volume
## Daimler  138.40  4456359.20
## E.ON      16.22  3667983.11
## Siemens  221.84  3669597.92
## BASF     174.56  1778145.28
## BMW      175.62  1824669.81
```

- Here, the Series was generated from the `price` column,
- The arithmetic operation will be broadcasted along a column matching the DataFrame's row index (`axis=0`).

Pandas vs Numpy

```
nparr = np.arange(12.).reshape((3, 4))
row = nparr[0]
nparr - row

## array([[0., 0., 0., 0.],
##        [4., 4., 4., 4.],
##        [8., 8., 8., 8.]])
```

- Operations between DataFrames are similar to operations between one- and two-dimensional Numpy arrays,
- As in DataFrames and Series the arithmetic operations will be broadcasted along the rows.

`DataFrame.apply(np.function, axis)`: Applies a NumPy function on the DataFrame axis. See also statistical and mathematical NumPy functions.

Numpy functions on DataFrames

```
frame7[:2]

##           price   volume
## Daimler  69.20  4456290
## E.ON      8.11  3667975

frame7.apply(np.mean)

## price          72.664
## volume    3079278.400
## dtype: float64

frame7.apply(np.sqrt)[:2]

##              price       volume
## Daimler  8.318654  2110.992657
## E.ON     2.847806  1915.195812
```

`DataFrame.groupby(col1, col2)`: Groups DataFrame by columns (grouping by one or more than two columns is also possible). See also how to import data from CSV files.

Groupby

```
vote = pd.read_csv("data/vote.csv")[["Party", "Member", "Vote"]]
vote.head()

##       Party     Member    Vote
## 0   CDU/CSU   Abercron     yes
## 1   CDU/CSU     Albani     yes
## 2   CDU/CSU  Altenkamp     yes
## 3   CDU/CSU   Altmaier  absent
## 4   CDU/CSU     Amthor     yes
```

Adding the functions `count()` or `mean()` to `groupby()` returns the sum or the mean of the grouped columns.

Groupby

```
res = vote.groupby(["Party", "Vote"]).count()
res

##                          Member
## Party        Vote
## AfD          absent           6
##              no              86
## BÜ90/GR      absent           9
##              no              58
## CDU/CSU      absent           7
##              yes            239
## DIE LINKE.   absent           7
##              no              62
## FDP          absent           5
##              no              75
## Fraktionslos absent           1
##              no               1
## SPD          absent           6
##              yes            147
```

Data formats and handling

▶ Import/Export data

ex1.csv

```
a, b, c, d, hello
1, 2, 3, 4, world
5, 6, 7, 8, python
2, 3, 5, 7, pandas
```

`pd.read_csv("file")`: Reads CSV into DataFrame.

Read comma-separated values

```
df = pd.read_csv("data/ex1.csv")
df

##    a  b  c  d   hello
## 0  1  2  3  4   world
## 1  5  6  7  8  python
## 2  2  3  5  7  pandas
```

tab.txt

```
a| b| c| d| hello
1| 2| 3| 4| world
5| 6| 7| 8| python
2| 3| 5| 7| pandas
```

`pd.read_table("file", sep)`: Reads table with any seperators into DataFrame.

Read table values

```
df = pd.read_table("data/tab.txt", sep="|")
df

##    a   b   c   d    hello
## 0  1   2   3   4    world
## 1  5   6   7   8   python
## 2  2   3   5   7   pandas
```

ex2.csv

```
1, 2, 3, 4, world
5, 6, 7, 8, python
2, 3, 5, 7, pandas
```

CSV file without header row:

Read CSV and header settings

```
df = pd.read_csv("data/ex2.csv", header=None)
df

##     0  1  2  3       4
## 0   1  2  3  4   world
## 1   5  6  7  8  python
## 2   2  3  5  7  pandas
```

ex2.csv

```
1, 2, 3, 4, world
5, 6, 7, 8, python
2, 3, 5, 7, pandas
```

Specify header:

Read CSV and header names

```
df = pd.read_csv("data/ex2.csv",
                 names=["a", "b", "c", "d", "hello"])
df

##    a  b  c  d   hello
## 0  1  2  3  4   world
## 1  5  6  7  8  python
## 2  2  3  5  7  pandas
```

ex2.csv

```
1, 2, 3, 4, world
5, 6, 7, 8, python
2, 3, 5, 7, pandas
```

Use `hello`-column as the index:

Read CSV and specify index

```
df = pd.read_csv("data/ex2.csv",
                 names=["a", "b", "c", "d", "hello"],
                 index_col="hello")
df

##         a  b  c  d
## hello
##  world   1  2  3  4
##  python  5  6  7  8
##  pandas  2  3  5  7
```

ex3.csv

```
1, 2, 3, 4, world
#+#-.,.-'*'-.,
5, 6, 7, 8, python
87646756754456978
2, 3, 5, 7, pandas
```

Skip rows while reading:

Read CSV and choose rows

```
df = pd.read_csv("data/ex3.csv", skiprows=[1, 3])
df

##    1   2   3   4    world
## 0  5   6   7   8   python
## 1  2   3   5   7   pandas
```

`DataFrame.to_csv("filename")`: Writes DataFrame to CSV.

Write to CSV

```
df = pd.read_csv("data/ex3.csv", skiprows=[1, 3])
df.to_csv("out/out1.csv")
```

out1.csv

```
,1, 2, 3, 4, world
0,5,6,7,8, python
1,2,3,5,7, pandas
```

In the .csv file, the index and header is included (reason why `,1`).

Write to CSV and settings

```
df = pd.read_csv("data/ex3.csv", skiprows=[1, 3])
df.to_csv("out/out2.csv", index=False, header=False)
```

out2.csv

```
5,6,7,8, python
2,3,5,7, pandas
```

Write to CSV and specify header

```
df = pd.read_csv("data/ex3.csv", skiprows=[1, 3, 4])
df.to_csv("out/out3.csv", index=False,
          header=["a", "b", "c", "d", "e"])
```

out3.csv

```
a,b,c,d,e
5,6,7,8, python
```

`pd.read_excel("file.xls")`: Reads .xls files.

	A	B	C	D	E	F	G
1	Date	Open	High	Low	Close	Adj Close	Volume
2	2018-01-31	1170.569946	1173	1159.130005	1169.939941	1169.939941	1538700
3	2018-02-01	1162.609985	1174	1157.52002	1167.699951	1167.699951	2412100
4	2018-02-02	1122	1123.069946	1107.277954	1111.900024	1111.900024	4857900
5	2018-02-05	1090.599976	1110	1052.030029	1055.800049	1055.800049	3798300
6	2018-02-06	1027.180054	1081.709961	1023.137024	1080.599976	1080.599976	3448000
7	2018-02-07	1081.540039	1081.780029	1048.26001	1048.579956	1048.579956	2341700

Figure: goog.xls

Reading Excel

```
xls_frame = pd.read_excel("data/goog.xls")
```

Excel as a DataFrame

```
xls_frame[["Adj Close", "Volume", "High"]]

##      Adj Close   Volume         High
## 0  1169.939941  1538700  1173.000000
## 1  1167.699951  2412100  1174.000000
## 2  1111.900024  4857900  1123.069946
## 3  1055.800049  3798300  1110.000000
## 4  1080.599976  3448000  1081.709961
## 5  1048.579956  2341700  1081.780029
```

Extract financial data from Internet sources into a DataFrame. There are different sources offering different kind of data. Some sources are:

- Robinhood
- IEX
- Yahoo Finance
- World Bank
- OECD
- Eurostat

A complete list of the sources and the usage can be found here: pandas-datareader

Import pandas-datareader

```
from pandas_datareader import data
```

`data.DataReader("symbol", "source", "start", "end")`: Returns financial data of a stock in a certain time period.

Get data of Ford

```
ford = data.DataReader("F", "yahoo", "2020-01-01", "2020-01-31")
ford.head()[["Close", "Volume"]]

##             Close      Volume
## Date
## 2020-01-02   9.42  43425700.0
## 2020-01-03   9.21  45040800.0
## 2020-01-06   9.16  43372300.0
## 2020-01-07   9.25  44984100.0
## 2020-01-08   9.25  45994900.0
```

▸ Stock code list

Explore Ford dataset

```
ford.index
```

```
## DatetimeIndex(['2020-01-02', '2020-01-03',...
## ...dtype='datetime64[ns]', name='Date',...
```

```
ford.loc["2020-01-28"]
```

```
## High          9.000000e+00
## Low           8.860000e+00
## Open          8.940000e+00
## Close         8.970000e+00
## Volume        8.516340e+07
## Adj Close     8.730923e+00
## Name: 2020-01-28 00:00:00, dtype: float64
```

DataFrame index

Index of the DataFrame is different at different sources. Always check `DataFrame.index`!

Download and explore SAP data

```
sap = data.DataReader("SAP", "yahoo", "2020-01-01", "2020-06-30")
sap[25:27]

##                   High         Low  ...    Volume   Adj Close
## Date                                ...
## 2020-02-07  136.020004  134.639999  ...  511700.0  130.987106
## 2020-02-10  135.369995  134.679993  ...  381200.0  131.151978
##
## [2 rows x 6 columns]

sap.loc["2020-03-09"]

## High          1.161900e+02
## Low           1.105500e+02
## Open          1.136100e+02
## Close         1.115000e+02
## Volume        1.571800e+06
## Adj Close     1.081376e+02
## Name: 2020-03-09 00:00:00, dtype: float64
```

Eurostat

```
population = data.DataReader("tps00001", "eurostat", "2010-01-01",
                             "2020-01-01")
```

```
population.columns
```

```
## MultiIndex(levels=[[Population on 1 January - total], [Albania,
## Andorra, Armenia, Austria, Azerbaijan, Belarus, Belgium, ...
```

```
population["Population on 1 January - total", "France"][-5:]
```

```
## FREQ                 Annual
## TIME_PERIOD
## 2016-01-01    66638391.0
## 2017-01-01    66809816.0
## 2018-01-01    66918941.0
## 2019-01-01    67012883.0
## 2020-01-01    67098824.0
```

Eurostat Database

Website used for the example: Econometrics

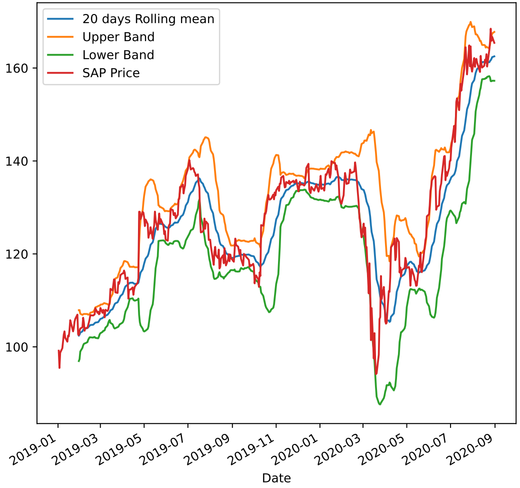
Beautiful Soup

```
from bs4 import BeautifulSoup
import requests
url = "www.uni-goettingen.de/de/applied-econometrics/412565.html"
r = requests.get("https://" + url)
d = r.text
soup = BeautifulSoup(d, "lxml")

soup.title

## <title>Applied Econometrics - Georg-August-... ...</title>
```

Reading data from HTML in detail exceeds the content of this course. If you are interested in this kind of importing data, you can find detailed information on *Beautiful Soup* here.

Bollinger

```
sap = data.DataReader("SAP", "yahoo", "2019-01-01", "2020-08-31")
sap.index = pd.to_datetime(sap.index)
boll = sap["Close"].rolling(window=20, center=False).mean()
std = sap["Close"].rolling(window=20, center=False).std()
upp = boll + std * 2
low = boll - std * 2
fig = plt.figure()
ax = fig.add_subplot(1, 1, 1)
boll.plot(ax=ax, label="20 days Rolling mean")
upp.plot(ax=ax, label="Upper Band")
low.plot(ax=ax, label="Lower Band")
sap["Close"].plot(ax=ax, label="SAP Price")
ax.legend(loc="best")
fig.savefig("out/boll.pdf")
```


20 days Rolling mean
Upper Band
Lower Band
SAP Price
160
140
120
100
2019-01
2019-03
2019-05
2019-07
2019-09
2019-11
2020-01
2020-03
2020-05
2020-07
2020-09
Date

Visual illustrations

Visual illustrations

▶ Matplotlib package

The package `matplotlib` is a free software library for python including the following functions:

- Image plots, Contour plots, Scatter plots, Polar plots, Line plots, 3D plots,
- Variety of hardcopy formats,
- Works in Python scripts, the Python and IPython shell and the Jupyter notebook,
- Interactive environments.

Usage of matplotlib

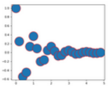
matplotlib has a vast number of functions and options, which is hard to remember. But for almost every task there is an example you can take code from. A great source of information is the examples gallery on the `matplotlib` homepage. Also note the best practice quick start guide.

Gallery

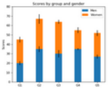
This gallery contains examples of the many things you can do with Matplotlib. Click on any image to see the full image and source code.

For longer tutorials, see our tutorials page. You can also find external resources and a FAQ in our user guide.

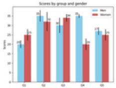
Lines, bars and markers



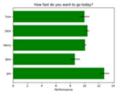
Arctest



Stacked Bar Graph



Barchart

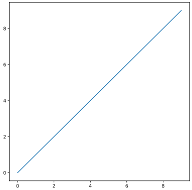


Horizontal bar chart

`plt.plot(array)`: Plots the values of a list, the X-axis has by default the range `[0, 1, ..., n-1]`.

Import matplotlib and simple example

```
import matplotlib.pyplot as plt
import numpy as np
plt.plot(np.arange(10))
plt.savefig("out/list.pdf")
```



Visual illustrations

▶ Figures and subplots

Figures

Plots in `matplotlib` reside in a *Figure object*:
`plt.figure(...)`: Creates new Figure object allowing for multiple parameters.
`plt.gcf()`: Returns the reference of the active figure.

Create Figures

```
fig = plt.figure(figsize=(16, 8))
print(plt.gcf())

## Figure(1600x800)
```

- A Figure object can be considered as an empty window,
- The Figure object has a number of options, such as the size or the aspect ratio,
- You cannot draw a plot in a blank figure. There has to be a subplot in the Figure object.

Saving plots to file

`plt.savefig("filename")`: Saves active figure to file.
Available file formats are among others:

Filename extension	Description
.png	Portable Network Graphics
.pdf	Portable Document Format
.svg	Scalable Vector Graphics
.jpeg	JPEG File Interchange Format
.jpg	JPEG File Interchange Format
.ps	PostScript
.raw	Raw Image Format

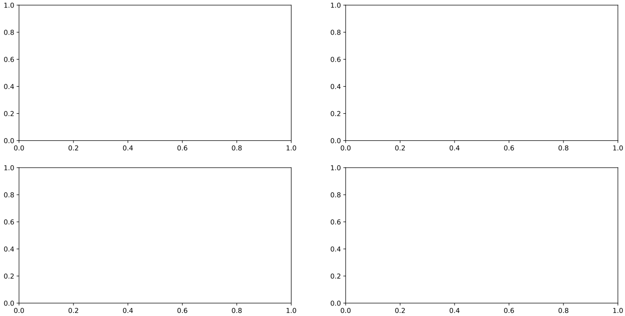
`fig.add_subplot()`: Adds subplot to the Figure `fig`.
Example: `fig.add_subplot(2, 2, 1)` creates four subplots and selects the first.

Adding subplots

```
ax1 = fig.add_subplot(2, 2, 1)
ax2 = fig.add_subplot(2, 2, 2)
ax3 = fig.add_subplot(2, 2, 3)
ax4 = fig.add_subplot(2, 2, 4)
fig.savefig("out/subplots.pdf")
```

- The Figure object is filled with subplots in which the plots reside,
- Using the `plt.plot()` command without creating a subplot in advance, matplotlib will create a Figure object and a subplot automatically,
- The Figure object and its subplots can be created in one line.

Subplots

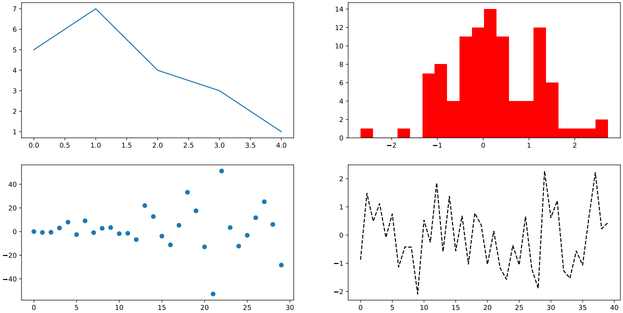


Filling subplots with content

```
from numpy.random import randn
ax1.plot([5, 7, 4, 3, 1])
ax2.hist(randn(100), bins=20, color="r")
ax3.scatter(np.arange(30), np.arange(30) * randn(30))
ax4.plot(randn(40), "k--")
fig.savefig("out/content.pdf")
```

- The subplots in one Figure object can be filled with different plot types,
- Using only `plt.plot()` matplotlib draws the plot in the last Figure object and last subplot selected.

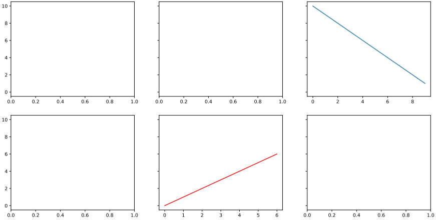
Subplots


7
6
5
4
3
2
1
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0
14
12
10
8
6
4
2
0
−2 −1 0 1 2
40
20
0
−20
−40
0 5 10 15 20 25 30
2
1
0
−1
−2
0 5 10 15 20 25 30 35 40

`plt.subplots(nrows, ncols, sharex, sharey)`: Creates figure and subplots in one line. If `sharex` or `sharey` are `True`, all subplots share the same X- or Y-ticks.

Standard creation

```
fig, axes = plt.subplots(2, 3, figsize=(16, 8), sharey=True)
axes[1, 1].plot(np.arange(7), color="r")
axes[0, 2].plot(np.arange(10, 0, -1))
fig.savefig("out/standard.pdf")
```



Visual illustrations

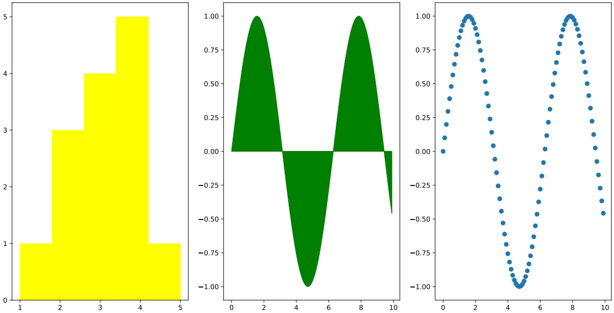
- Plot types and styles

`ax.scatter(x, y)`: Creates a scatter plot of `x` vs `y`.
`ax.hist(x, bins)`: Creates a histogram.
`ax.fill_between(x, y, a)`: Creates a plot of `x` vs `y` and fills plot between `a` and `y`.

Types

```
fig, ax = plt.subplots(1, 3, figsize=(16, 8))
ax[0].hist([1, 2, 3, 4, 5, 4, 3, 2, 3, 4, 2, 3, 4, 4],
           bins=5, color="yellow")
x = np.arange(0, 10, 0.1)
y = np.sin(x)
ax[1].fill_between(x, y, 0, color="green")
ax[2].scatter(x, y)
fig.savefig("out/types.pdf")
```

A vast number of plot types can be found in the examples gallery.

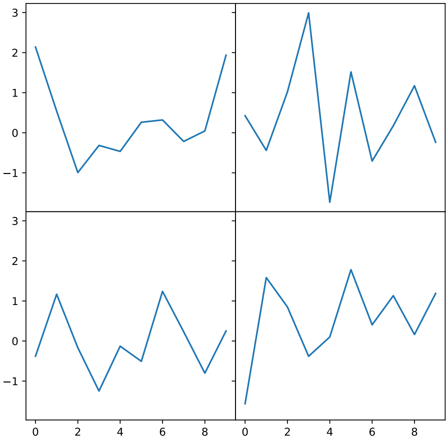

0
1
2
3
4
5
1
2
3
4
5
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
0
2
4
6
8
10
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
0
2
4
6
8
10

`plt.subplots_adjust(left, bottom, ..., hspace)`: Sets the space between the subplots. `wspace` and `hspace` control the percentage of the figure width and figure height, respectively, to use as spacing between subplots.

Adjust spacing

```
fig, axes = plt.subplots(2, 2, sharex=True, sharey=True)
for i in range(2):
    for j in range(2):
        axes[i][j].plot(randn(10))
plt.subplots_adjust(wspace=0, hspace=0)
fig.savefig("out/spacing.pdf")
```

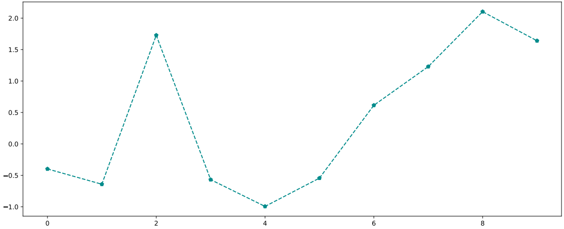
Adjusting the spacing around subplots



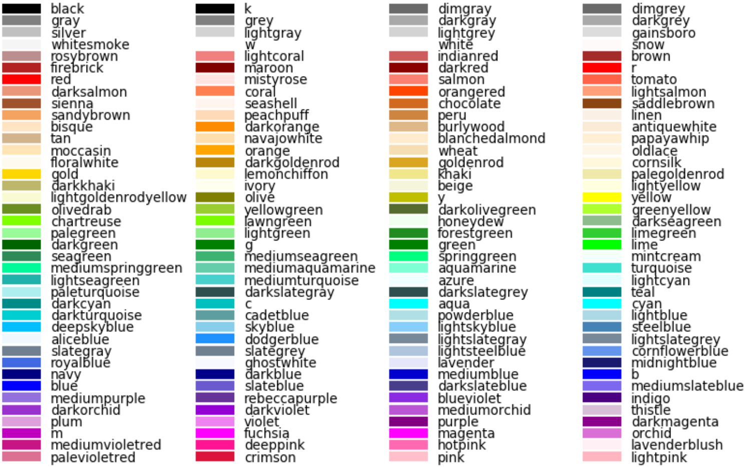
`ax.plot(data, linestyle, color, marker)`: Sets data and styles of subplot `ax`.

Styles

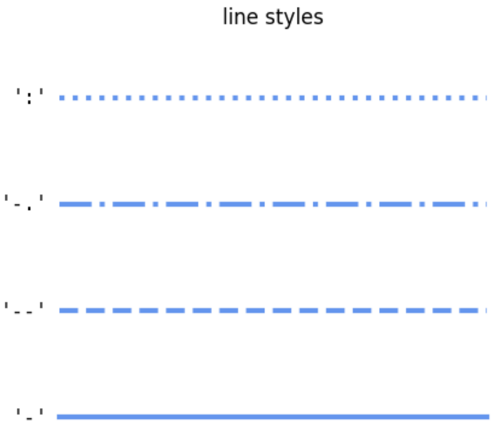
```
fig, ax = plt.subplots(1, figsize=(15, 6))
ax.plot(randn(10), linestyle="--", color="darkcyan", marker="p")
fig.savefig("out/style.pdf")
```



Plot colors


black
gray
silver
whitesmoke
rosybrown
firebrick
red
darksalmon
sienna
sandybrown
bisque
tan
moccasin
floralwhite
gold
darkkhaki
lightgoldenrodyellow
olivedrab
chartreuse
palegreen
darkgreen
seagreen
mediumspringgreen
lightseagreen
paleturquoise
darkcyan
darkturquoise
deepskyblue
aliceblue
slategray
royalblue
navy
blue
mediumpurple
darkorchid
plum
m
mediumvioletred
palevioletred
k
grey
lightgray
w
lightcoral
maroon
mistyrose
coral
seashell
peachpuff
darkorange
navajowhite
orange
darkgoldenrod
lemonchiffon
ivory
olive
yellowgreen
lawngreen
lightgreen
g
mediumseagreen
mediumaquamarine
mediumturquoise
darkslategray
c
cadetblue
skyblue
dodgerblue
slategrey
ghostwhite
darkblue
slateblue
rebeccapurple
darkviolet
violet
fuchsia
deeppink
crimson
dimgray
darkgray
lightgrey
white
indianred
darkred
salmon
orangered
chocolate
peru
burlywood
blanchedalmond
wheat
goldenrod
khaki
beige
y
darkolivegreen
honeydew
forestgreen
green
springgreen
aquamarine
azure
darkslategrey
aqua
powderblue
lightskyblue
lightslategray
lightsteelblue
lavender
mediumblue
darkslateblue
blueviolet
mediumorchid
purple
magenta
hotpink
pink
dimgrey
darkgrey
gainsboro
snow
brown
r
tomato
lightsalmon
saddlebrown
linen
antiquewhite
papayawhip
oldlace
cornsilk
palegoldenrod
lightyellow
yellow
greenyellow
darkseagreen
limegreen
lime
mintcream
turquoise
lightcyan
teal
cyan
lightblue
steelblue
lightslategrey
cornflowerblue
midnightblue
b
mediumslateblue
indigo
thistle
darkmagenta
orchid
lavenderblush
lightpink

Plot line styles



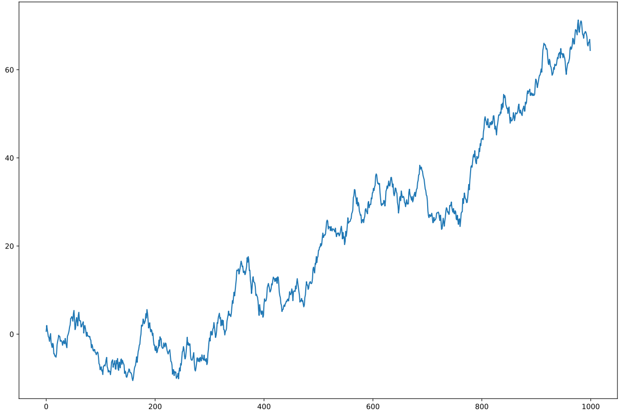
Marker	Description
"."	point
","	pixel
"o"	circle
"v"	triangle_down
"8"	octagon
"s"	square
"p"	pentagon
"P"	plus (filled)
"*"	star
"h"	hexagon1
"H"	hexagon2
"+"	plus
"x"	x
"X"	x (filled)
"D"	diamond

`ax.set_xticks()`: Sets list of X-ticks, analogously for Y-axis.
`ax.set_xlabel()`: Sets the X-label.
`ax.set_title()`: Sets the subplot title.

Ticks and labels - default

```
fig, ax = plt.subplots(1, figsize=(15, 10))
ax.plot(randn(1000).cumsum())
fig.savefig("out/withoutlabels.pdf")
```

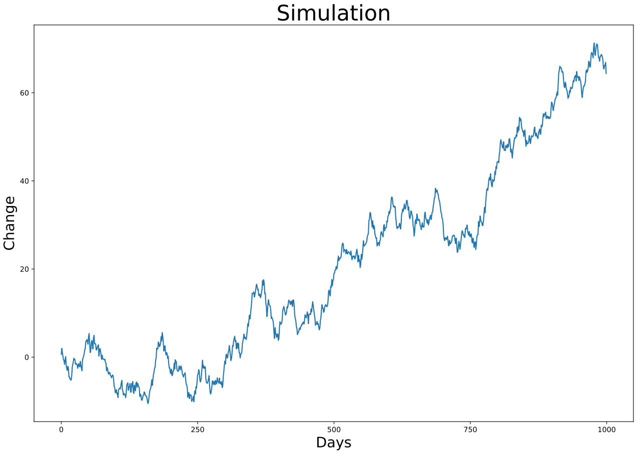
- Here, we create a Figure object as well as a subplot and fill it with a line plot of a *random walk*,
- By default matplotlib places the ticks evenly distributed along the data range. Individual ticks can be set as follows,
- By default there is no axis label or title.


60
40
20
0
0
200
400
600
800
1000

Set ticks and labels

```
ax.set_xticks([0, 250, 500, 750, 1000])
ax.set_xlabel("Days", fontsize=20)
ax.set_ylabel("Change", fontsize=20)
ax.set_title("Simulation", fontsize=30)
fig.savefig("out/labels.pdf")
```

- The individual ticks are given as a list to `ax.set_xticks()`,
- The label and title can be set to an individual size using the argument `fontsize`.

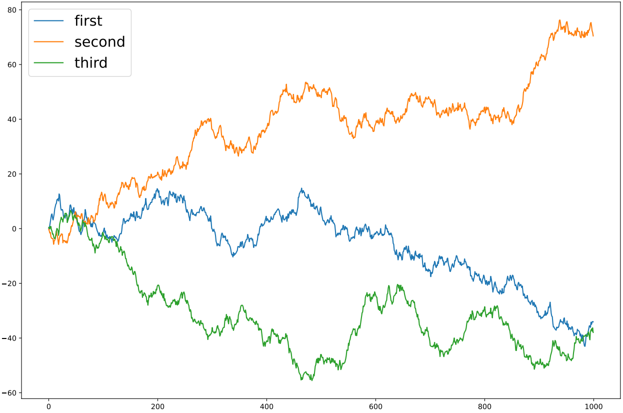

Simulation
Change
Days
0
20
40
60
0
250
500
750
1000

Using multiple plots in one subplot one needs a legend.
`ax.legend(loc)`: Shows the legend at location `loc`.
Some options: `"best"`, `"upper right"`, `"center left"`, ...

Set legend

```
fig = plt.figure(figsize=(15, 10))
ax = fig.add_subplot(1, 1, 1)
ax.plot(randn(1000).cumsum(), label="first")
ax.plot(randn(1000).cumsum(), label="second")
ax.plot(randn(1000).cumsum(), label="third")
ax.legend(loc="best", fontsize=20)
fig.savefig("out/legend.pdf")
```

- The legend displays the label and the color of the associated plot,
- Using the option `"best"` the legend will placed in a corner where is does not interfere the plots.

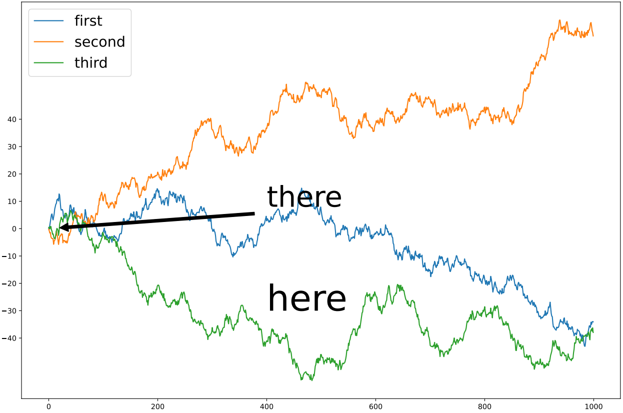

first
second
third
80
60
40
20
0
−20
−40
−60
0
200
400
600
800
1000

`ax.text(x, y, "text", fontsize)`: Inserts a text into a subplot.
`ax.annotate("text", xy, xytext, arrwoprops)`: Inserts an arrow with annotations.

Annotations

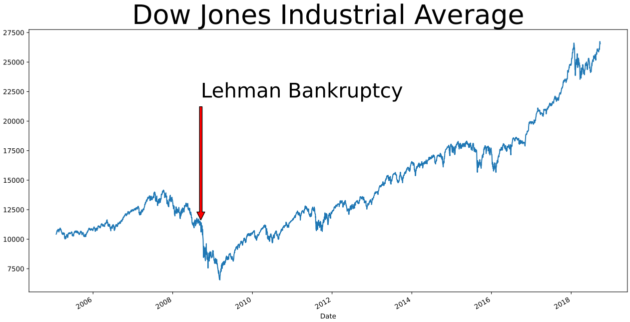
```
ax.text(400, -30, "here", fontsize=50)
ax.annotate("there",
            fontsize=40,
            xy=(0, 0),
            xytext=(400, 8),
            arrowprops=dict(facecolor="black",
                            shrink=0.05))
ax.set_yticks([-40, -30, -20, -10, 0, 10, 20, 30, 40])
fig.savefig("out/arrow.pdf")
```

- Using `ax.annotate()` the arrow head points at `xy` and the bottom left corner of the text will be placed at `xytext`.


first
second
third
there
here
40
30
20
10
0
−10
−20
−30
−40
0
200
400
600
800
1000

Annotation Lehman

```
import pandas as pd
from datetime import datetime

date = datetime(2008, 9, 15)
fig = plt.figure(figsize=(16, 8))
ax = fig.add_subplot(1, 1, 1)
dow = pd.read_csv("data/dji.csv", index_col=0, parse_dates=True)
close = dow["Close"]
close.plot(ax=ax)
ax.annotate("Lehman Bankruptcy",
            fontsize=30,
            xy=(date, close.loc[date] + 400),
            xytext=(date, 22000),
            arrowprops=dict(facecolor="red",
                            shrink=0.03))
ax.set_title("Dow Jones Industrial Average", size=40)
fig.savefig("out/lehman.pdf")
```

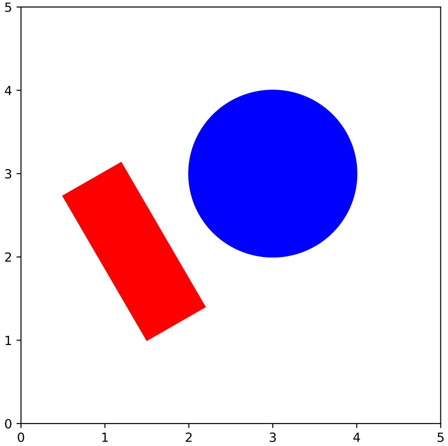

Dow Jones Industrial Average
Lehman Bankruptcy
27500
25000
22500
20000
17500
15000
12500
10000
7500
2006
2008
2010
2012
2014
2016
2018
Date

`plt.Rectangle((x, y), width, height, angle)`: Creates a rectangle
`plt.Circle((x,y), radius)`: Creates a circle.

Drawing

```
fig = plt.figure(figsize=(6, 6))
ax = fig.add_subplot(1, 1, 1)
ax.set_xticks([0, 1, 2, 3, 4, 5])
ax.set_yticks([0, 1, 2, 3, 4, 5])
rectangle = plt.Rectangle((1.5, 1),
                          width=0.8, height=2,
                          color="red", angle=30)
circ = plt.Circle((3, 3),
                  radius=1, color="blue")
ax.add_patch(rectangle)
ax.add_patch(circ)
fig.savefig("out/draw.pdf")
```

A list of all available patches can be found here: ▸ matplotlib-patches



0
1
2
3
4
5
0
1
2
3
4
5

Step 1
Create a Figure object and subplots

Best practice Step 1

```
fig, ax = plt.subplots(1, 1, figsize=(16, 8))
```

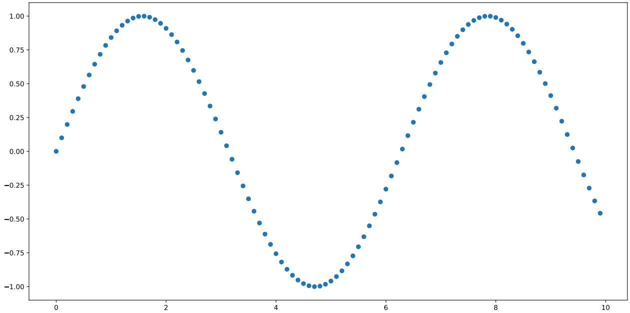
Step 2
Plot data using different plot types

An overview of plot types can be found in the examples gallery.

Best practice Step 2

```
x = np.arange(0, 10, 0.1)
y = np.sin(x)
ax.scatter(x, y)
```

Best practice: Visual illustrations



Step 3
Set colors, markers and line styles

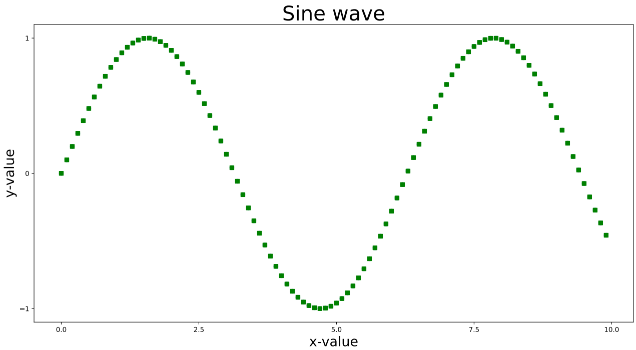
Best practice Step 3

```
ax.scatter(x, y, color="green", marker="s")
```

Step 4
Set title, axis labels and ticks

Best practice Step 4

```
ax.set_title("Sine wave", fontsize=30)
ax.set_xticks([0, 2.5, 5, 7.5, 10])
ax.set_yticks([-1, 0, 1])
ax.set_ylabel("y-value", fontsize=20)
ax.set_xlabel("x-value", fontsize=20)
```


Sine wave
1
0
−1
y-value
0.0
2.5
5.0
7.5
10.0
x-value

Step 5
Set labels

Best practice Step 5

```
ax.scatter(x, y, color="green", marker="s", label="Sine")
```

Step 6
Set legend (if you add another plot to an existing figure)

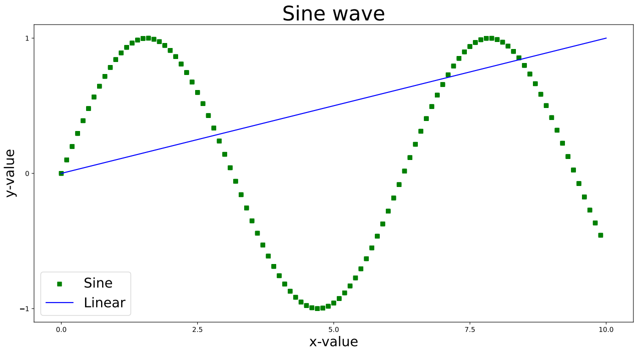
Best practice Step 6

```
ax.plot(np.arange(11) / 10, color="blue", linestyle="-",
        label="Linear")
ax.legend(fontsize=20)
```

Step 7
Save plot to file

Best practice Step 7

```
fig.savefig("out/sinewave.pdf")
```



Sine wave
1
0
−1
y-value
0.0
2.5
5.0
7.5
10.0
x-value
Sine
Linear

Visual illustrations

▶ Pandas layers

Plotting with layers

Plotting with `matplotlib` is often tedious and requires some research: You need to recall parameter details to create a professional charts. For recurring, everyday tasks, you might prefer another level of abstraction: Layer frameworks, which operate on top of matplotlib, produce pretty looking results with short methods and less code. The most popular packages are:

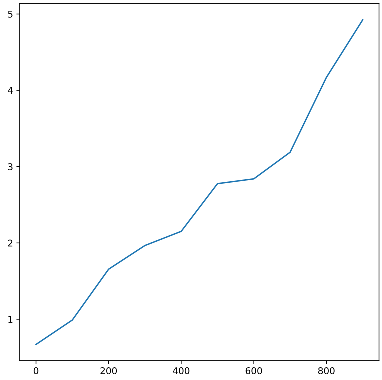
- `pandas` provides a convenient layer with frequently demanded plotting methods for its objects, such as Series and DataFrames.
- `Seaborn` is a powerful graphics framework that allows you to easily create beautiful, complex graphics using a simple interface.

→ In this section, we will have a look at pandas' integrated layer methods. However, Seaborn also works very well with pandas objects.

`DataFrame/Series.plot()`: Plots a DataFrame or a Series.

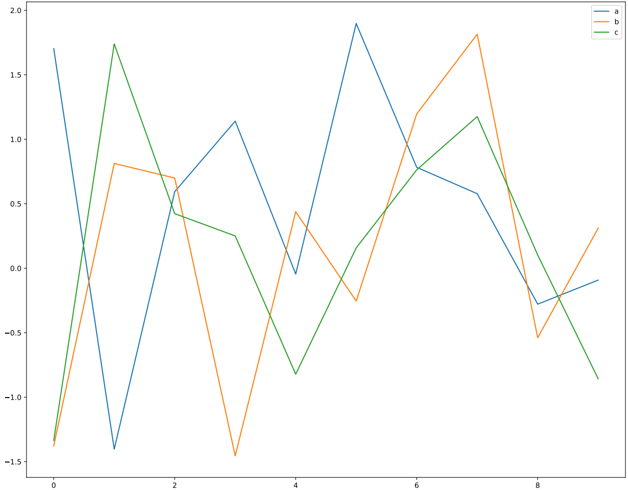
Simple line plot

```
plt.close("all")
p = pd.Series(np.random.rand(10).cumsum(),
              index=np.arange(0, 1000, 100))
p

## 0      0.669761
## 100    0.989702
## 200    1.655715
## 300    1.966073
## 400    2.151883
## 500    2.776987
## 600    2.839751
## 700    3.188431
## 800    4.169061
## 900    4.923286
## dtype: float64

p.plot()
plt.savefig("out/line.pdf")
```


5
4
3
2
1
0
200
400
600
800

Line plots

```
df = pd.DataFrame(np.random.randn(10, 3), index=np.arange(10),
                  columns=["a", "b", "c"])
df

##           a         b         c
## 0  1.703615 -1.376905 -1.336154
## 1 -1.402924  0.812501  1.739143
## 2  0.593504  0.699582  0.423217
## 3  1.140647 -1.454363  0.250578
## 4 -0.044809  0.438279 -0.821514
## 5  1.897959 -0.254581  0.157704
## 6  0.782639  1.196116  0.763081
## 7  0.577947  1.815039  1.175842
## 8 -0.278585 -0.538956  0.102930
## 9 -0.091891  0.310788 -0.857167

df.plot(figsize=(15, 12))
plt.savefig("out/line2.pdf")
```


a
b
c
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0
2
4
6
8

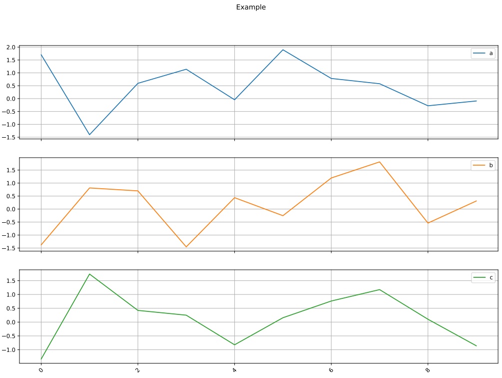
The plot method applied to a DataFrame plots each column as a different line and shows the legend automatically. Plotting DataFrames, there are serveral arguments to change the style of the plot:

Argument	Description
kind	"line", "bar", etc
logy	logarithmic scale on Y-axis
use_index	If True, use index for tick labels
rot	Rotation of tick labels
xticks	Values for x ticks
yticks	Values for y ticks
grid	Set grid True or False
xlim	X-axis limits
ylim	Y-axis limits
subplots	Plot each DataFrame column in a new subplot

Table: Pandas plot arguments

Separated line plots

```
df.plot(grid=True, rot=45, subplots=True, title="Example",
        figsize=(15, 10))
plt.savefig("out/pandas.pdf")
```



`dataframe.plot(ax=subplot)`: Plots a dataframe into `subplot`.

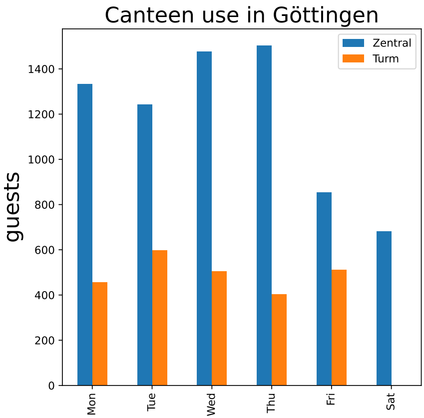
Standard creation

```
fig = plt.figure(figsize=(6, 6))
ax = fig.add_subplot(1, 1, 1)
guests = np.array([[1334, 456], [1243, 597], [1477, 505],
                   [1502, 404], [854, 512], [682, 0]])
canteen = pd.DataFrame(guests,
                       index=["Mon", "Tue", "Wed",
                              "Thu", "Fri", "Sat"],
                       columns=["Zentral", "Turm"])
canteen

##      Zentral  Turm
## Mon     1334   456
## Tue     1243   597
## Wed     1477   505
## Thu     1502   404
## Fri      854   512
## Sat      682     0
```

Bar plot

```
canteen.plot(ax=ax, kind="bar")
ax.set_ylabel("guests", fontsize=20)
ax.set_title("Canteen use in Göttingen", fontsize=20)
fig.savefig("out/canteen.pdf")
```

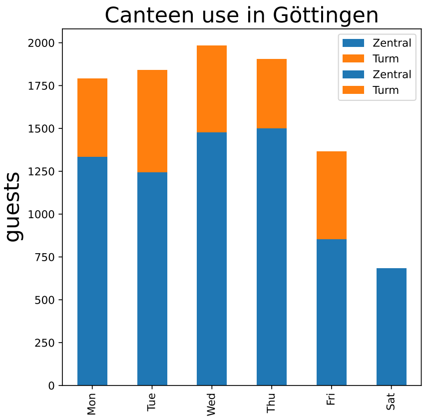
- The bar plot resides in the subplot `ax`,
- The label and title are set as shown before without using pandas.


Canteen use in Göttingen
Zentral
Turm
guests
0
200
400
600
800
1000
1200
1400
Mon
Tue
Wed
Thu
Fri
Sat

Bar plot - stacked

```
canteen.plot(ax=ax, kind="bar", stacked=True)
ax.set_ylabel("guests", fontsize=20)
ax.set_title("Canteen use in Göttingen", fontsize=20)
fig.savefig("out/canteenstacked.pdf")
```

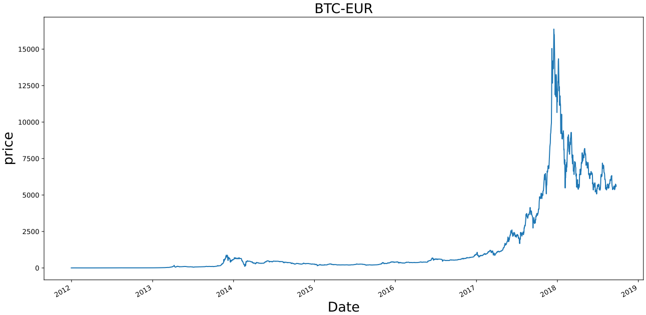
Bar plot


Canteen use in Göttingen
guests
2000
1750
1500
1250
1000
750
500
250
0
Mon
Tue
Wed
Thu
Fri
Sat
Zentral
Turm
Zentral
Turm

BTC chart

```
fig = plt.figure(figsize=(16, 8))
ax = fig.add_subplot(1, 1, 1)
ax.set_ylabel("price", fontsize=20)
ax.set_xlabel("Date", fontsize=20)
BTC = pd.read_csv("data/btc-eur.csv", index_col=0, parse_dates=True)
BTCclose = BTC["Close"]
BTCclose.plot(ax=ax)
ax.set_title("BTC-EUR", fontsize=20)
fig.savefig("out/btc.pdf")
```

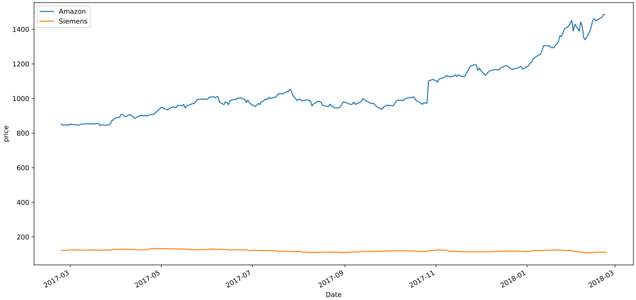
Plot financial data



Compare - bad illustration

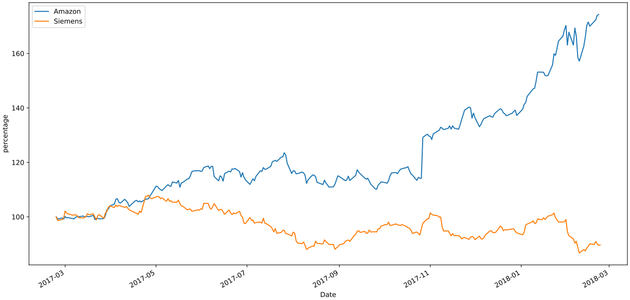
```
amazon = pd.read_csv("data/amzn.csv", index_col=0,
                     parse_dates=True)["Close"]
siemens = pd.read_csv("data/sie.de.csv", index_col=0,
                      parse_dates=True)["Close"]
fig = plt.figure(figsize=(16, 8))
ax = fig.add_subplot(1, 1, 1)
ax.set_ylabel("price")
amazon.plot(ax=ax, label="Amazon")
siemens.plot(ax=ax, label="Siemens")
ax.legend(loc="best")
fig.savefig("out/compare.pdf")
```

- In this illustration you can hardly compare the trend of the two stocks,
- Using pandas you can standardize both dataframes in one line.


Amazon
Siemens
price
Date
1400
1200
1000
800
600
400
200
2017-03
2017-05
2017-07
2017-09
2017-11
2018-01
2018-03

Compare - good illustration

```
amazon = amazon / amazon[0] * 100
siemens = siemens / siemens[0] * 100
fig = plt.figure(figsize=(16, 8))
ax = fig.add_subplot(1, 1, 1)
ax.set_ylabel("percentage")
amazon.plot(ax=ax, label="Amazon")
siemens.plot(ax=ax, label="Siemens")
ax.legend(loc="best")
fig.savefig("out/comparenew.pdf")
```


Amazon
Siemens
160
140
120
100
percentage
2017-03
2017-05
2017-07
2017-09
2017-11
2018-01
2018-03
Date

Applications

5.1 Time series
5.2 Moving window
5.3 Financial applications
5.4 Optimization

Applications

▶ Time series

Data types for date and time are included in the Python standard library.

Datetime creation

```
from datetime import datetime
now = datetime.now()
now

## datetime.datetime(2022, 2, 14, 0, 36, 9, 153276)

now.day

## 14

now.hour

## 0
```

From datetime you can get the attributes `year`, `month`, `day`, `hour`, `minute`, `second`, `microsecond`.

`datetime(year, month, day, ..., microsecond)`: Sets date and time.

Datetime representation

```
holiday = datetime(2020, 12, 24, 8, 30)
holiday

## datetime.datetime(2020, 12, 24, 8, 30)

exam = datetime(2020, 12, 9, 10)
print("The exam will be on the " + "{:%Y-%m-%d}".format(exam))

## The exam will be on the 2020-12-09
```

`timedelta(days, seconds, microseconds)`: Represents difference between two datetime objects.

Datetime difference

```
from datetime import timedelta
delta = exam - now
delta

## datetime.timedelta(days=-432, seconds=33830, microseconds=846724)

print("The exam will take place in " + str(delta.days) + " days.")

## The exam will take place in -432 days.

now

## datetime.datetime(2022, 2, 14, 0, 36, 9, 153276)

now + timedelta(10, 120)

## datetime.datetime(2022, 2, 24, 0, 38, 9, 153276)
```

`datetime.strftime("format")`: Converts datetime object into string.
`datetime.strptime(datestring, "format")`: Converts date as a string into a datetime object.

Convert Datetime

```
stamp = datetime(2020, 4, 12)
stamp

## datetime.datetime(2020, 4, 12, 0, 0)

print("German date format: " + stamp.strftime("%d.%m.%Y"))

## German date format: 12.04.2020

val = "2020-5-5"
d = datetime.strptime(val, "%Y-%m-%d")
d

## datetime.datetime(2020, 5, 5, 0, 0)
```

Converting examples

```
val = "31.01.2012"
d = datetime.strptime(val, "%d.%m.%Y")
d

## datetime.datetime(2012, 1, 31, 0, 0)

now.strftime("Today is %A and we are in week %W of the year %Y.")

## 'Today is Monday and we are in week 07 of the year 2022.'

now.strftime("%c")

## 'Mon Feb 14 00:36:09 2022'
```

Type	Description
%Y	4-digit year
%m	2-digit month [01, 12]
%d	2-digit day [01, 31]
%H	Hour (24-hour clock) [00, 23]
%I	Hour (12-hour clock) [01, 12]
%M	2-digit minute [00, 59]
%S	Second [00, 61]
%W	Week number of the year [00, 53]
%F	Shortcut for %Y-%m-%d

Type	Description
%a	Abbreviated weekday name
%A	Full weekday name
%b	Abbreviated month name
%B	Full month name
%c	Full date and time
%x	Locale-appropriate formatted date

`pd.date_range(start, end, freq)`: Generates a date range.

Date ranges

```
import pandas as pd
index = pd.date_range("2020-01-01", now)
index[0:2]
index[15:16]
index = pd.date_range("2020-01-01", now, freq="M")
index[0:2]

## DatetimeIndex(['2020-01-01', '2...ype='datetime64[ns]', freq='D')
## DatetimeIndex(['2020-01-16'], dtype='datetime64[ns]', freq='D')
## DatetimeIndex(['2020-01-31', '2...ype='datetime64[ns]', freq='M')
```

Alias	Offset type
D	Day
B	Business day
H	Hour
T	Minute
S	Second
M	Month end
BM	Business month end
Q-JAN, Q-FEB, ...	Quarter end
A-JAN, A-FEB, ...	Year end
AS-JAN, AS-FEB, ...	Year begin
BA-JAN, BA-FEB, ...	Business year end
BAS-JAN, BAS-FEB, ...	Business year begin

`DataFrame.resample("frequency")`: Resamples time series by a specified frequency.

Resample date ranges

```
import numpy as np
start = datetime(2016, 1, 1)
ind = pd.date_range(start, now)
numbers = np.arange((now - start).days + 1)
df = pd.DataFrame(numbers, index=ind)
```

```
df.head()

##             0
## 2016-01-01  0
## 2016-01-02  1
## 2016-01-03  2
## 2016-01-04  3
## 2016-01-05  4
```

```
df.resample("3BM").sum().head()

##                 0
## 2016-01-29    406
## 2016-04-29   6734
## 2016-07-29  15015
## 2016-10-31  24205
## 2017-01-31  32246
```

Applications

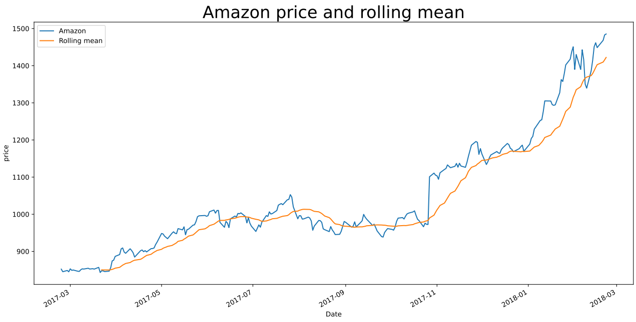
▶ Moving window

`DataFrame.rolling(window)`: Conducts rolling window computations.

Rolling mean

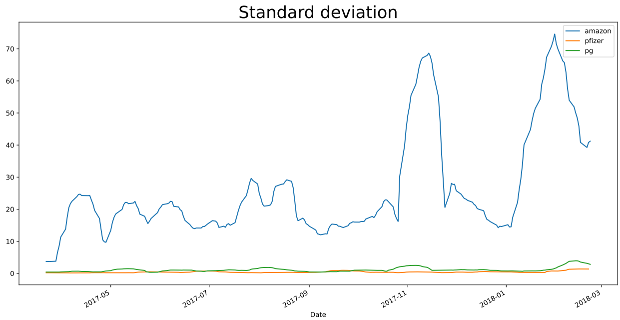
```
import matplotlib.pyplot as plt
amazon = pd.read_csv("data/amzn.csv", index_col=0,
                     parse_dates=True)["Adj Close"]
fig = plt.figure(figsize=(16, 8))
ax = fig.add_subplot(1, 1, 1)
ax.set_ylabel("price")
amazon.plot(ax=ax, label="Amazon")
amazon.rolling(window=20).mean().plot(ax=ax, label="Rolling mean")
ax.legend(loc="best")
ax.set_title("Amazon price and rolling mean", fontsize=25)
fig.savefig("out/amzn.pdf")
```

Frequently used rolling functions: `mean()`, `median()`, `sum()`, `var()`, `std()`, `min()`, `max()`.


Amazon price and rolling mean
Amazon
Rolling mean
price
1500
1400
1300
1200
1100
1000
900
Date
2017-03
2017-05
2017-07
2017-09
2017-11
2018-01
2018-03

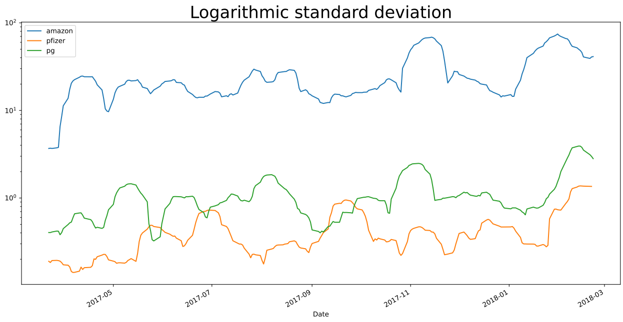
Standard deviation

```
fig = plt.figure(figsize=(16, 8))
ax = fig.add_subplot(1, 1, 1)
pfizer = pd.read_csv("data/pfe.csv", index_col=0,
                     parse_dates=True)["Adj Close"]
pg = pd.read_csv("data/pg.csv", index_col=0,
                 parse_dates=True)["Adj Close"]
prices = pd.DataFrame(index=amazon.index)
prices["amazon"] = pd.DataFrame(amazon)
prices["pfizer"] = pd.DataFrame(pfizer)
prices["pg"] = pd.DataFrame(pg)
prices_std = prices.rolling(window=20).std()
prices_std.plot(ax=ax)
ax.set_title("Standard deviation", fontsize=25)
fig.savefig("out/std.pdf")
```


Standard deviation
amazon
pfizer
pg
70
60
50
40
30
20
10
0
2017-05
2017-07
2017-09
2017-11
2018-01
2018-03
Date

Logarithmic standard deviation

```
fig = plt.figure(figsize=(16, 8))
ax = fig.add_subplot(1, 1, 1)
prices_std.plot(ax=ax, logy=True)
ax.set_title("Logarithmic standard deviation", fontsize=25)
fig.savefig("out/std_log.pdf")
```

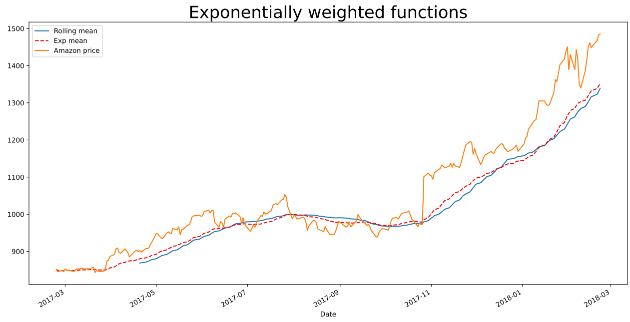


Logarithmic standard deviation
amazon
pfizer
pg
10^2
10^1
10^0
2017-05
2017-07
2017-09
2017-11
2018-01
2018-03
Date

`DataFrame.ewm(span)`: Computes exponentially weighted rolling window functions.

Exponentially weighted functions

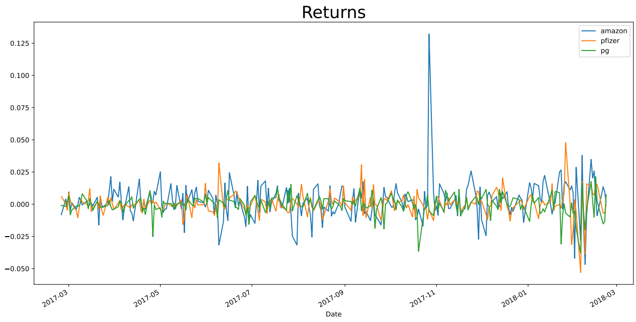
```
fig = plt.figure(figsize=(16, 8))
ax = fig.add_subplot(1, 1, 1)
amazon.rolling(window=40).mean().plot(ax=ax, label="Rolling mean")
amazon.ewm(span=40).mean().plot(ax=ax, label="Exp mean",
                                linestyle="--", color="red")
amazon.plot(ax=ax, label="Amazon price")
ax.legend(loc="best")
ax.set_title("Exponentially weighted functions", fontsize=25)
fig.savefig("out/mean.pdf")
```


Exponentially weighted functions
Rolling mean
Exp mean
Amazon price
1500
1400
1300
1200
1100
1000
900
2017-03
2017-05
2017-07
2017-09
2017-11
2018-01
2018-03
Date

DataFrame.pct_change(): Computes the percentage changes per period.

Percentage change

```
fig = plt.figure(figsize=(16, 8))
ax = fig.add_subplot(1, 1, 1)
returns = prices.pct_change()
returns.head()

##                amazon    pfizer        pg
## Date
## 2017-02-23        NaN       NaN       NaN
## 2017-02-24 -0.008155  0.005872 -0.000878
## 2017-02-27  0.004023  0.000584 -0.001757
## 2017-02-28 -0.004242 -0.004668  0.001980
## 2017-03-01  0.009514  0.008792  0.006479

returns.plot(ax=ax)
ax.set_title("Returns", fontsize=25)
fig.savefig("out/returns.pdf")
```

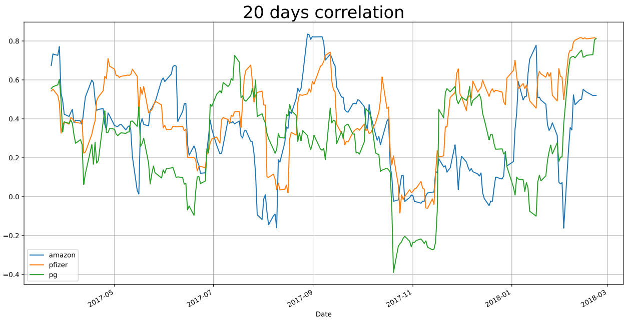

Returns
amazon
pfizer
pg
0.125
0.100
0.075
0.050
0.025
0.000
−0.025
−0.050
2017-03
2017-05
2017-07
2017-09
2017-11
2018-01
2018-03
Date

`DataFrame.rolling().corr(benchmark)`: Computes correlation between two time series.

Correlation

```
fig = plt.figure(figsize=(16, 8))
ax = fig.add_subplot(1, 1, 1)
DJI = pd.read_csv("data/dji.csv", index_col=0,
                  parse_dates=True)["Adj Close"]
DJI_ret = DJI.pct_change()
corr = returns.rolling(window=20).corr(DJI_ret)
corr.plot(ax=ax)
ax.grid()
ax.set_title("20 days correlation", fontsize=25)
fig.savefig("out/corr.pdf")
```

Binary moving window functions

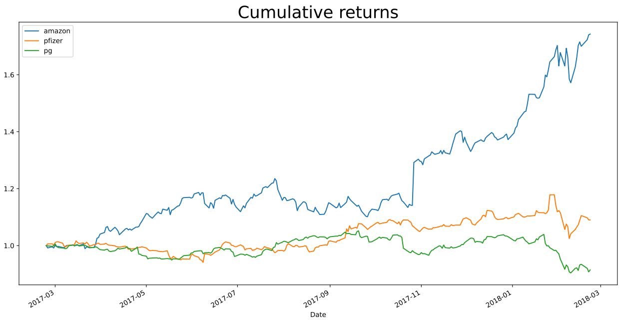


Applications

▶ Financial applications

Returns

```
fig = plt.figure(figsize=(16, 8))
ax = fig.add_subplot(1, 1, 1)
ret_index = (1 + returns).cumprod()
stocks = ["amazon", "pfizer", "pg"]
for i in stocks:
    ret_index[i][0] = 1
ret_index.tail()

##                amazon    pfizer        pg
## Date
## 2018-02-15  1.715298  1.088693  0.932322
## 2018-02-16  1.699961  1.105461  0.934471
## 2018-02-20  1.723031  1.097840  0.920217
## 2018-02-21  1.740128  1.090218  0.907772
## 2018-02-22  1.742968  1.090218  0.914560

ret_index.plot(ax=ax)
ax.set_title("Cumulative returns", fontsize=25)
fig.savefig("out/cumret.pdf")
```

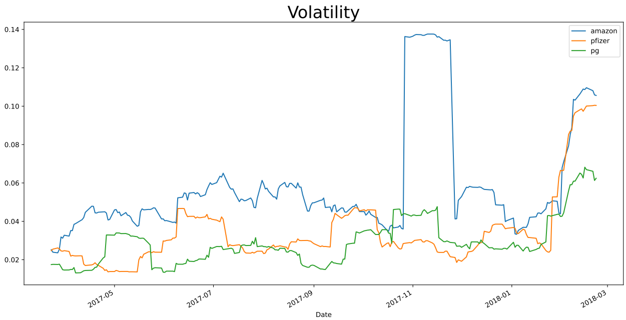

Cumulative returns
amazon
pfizer
pg
1.6
1.4
1.2
1.0
2017-03
2017-05
2017-07
2017-09
2017-11
2018-01
2018-03
Date

Monthly returns

```
returns_m = ret_index.resample("BM").last().pct_change()
returns_m.head()

##               amazon    pfizer        pg
## Date
## 2017-02-28       NaN       NaN       NaN
## 2017-03-31  0.049110  0.002638 -0.013396
## 2017-04-28  0.043371 -0.008477 -0.020604
## 2017-05-31  0.075276 -0.028124  0.008703
## 2017-06-30 -0.026764  0.028790 -0.010671
```

Volatility

```
fig = plt.figure(figsize=(16, 8))
ax = fig.add_subplot(1, 1, 1)
vola = returns.rolling(window=20).std() * np.sqrt(20)
vola.plot(ax=ax)
ax.set_title("Volatility", fontsize=25)
fig.savefig("out/vola.pdf")
```



Volatility
amazon
pfizer
pg
0.14
0.12
0.10
0.08
0.06
0.04
0.02
2017-05
2017-07
2017-09
2017-11
2018-01
2018-03
Date

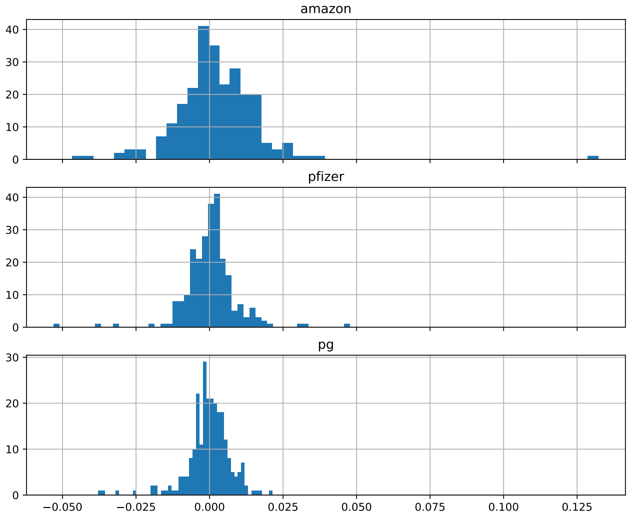
`DataFrame.describe()`: Shows a statistical summary.

Describe

```
prices.describe()

##             amazon      pfizer          pg
## count   252.000000  251.000000  252.000000
## mean   1044.521903   33.892665   87.934304
## std     158.041844    1.694680    2.728659
## min     843.200012   30.872143   79.919998
## 25%     953.567474   32.593733   86.241475
## 50%     988.680023   33.147469   87.863598
## 75%    1136.952484   35.331834   90.363035
## max    1485.339966   38.661823   92.988976
```

Histogram

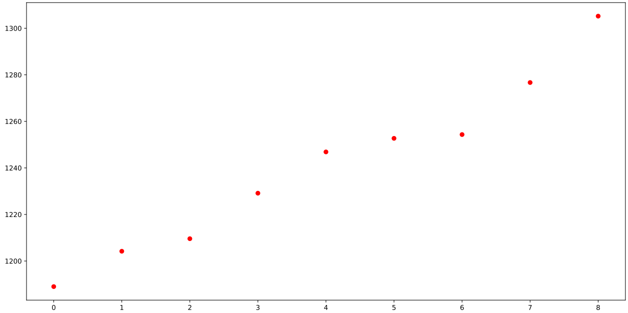
```
fig, ax = plt.subplots(3, 1, figsize=(10, 8), sharex=True)
for i in range(3):
    ax[i].set_title(stocks[i])
    returns[stocks[i]].hist(ax=ax[i], bins=50)
fig.savefig("out/return_hist.pdf")
```


amazon
pfizer
pg
0
10
20
30
40
−0.050
−0.025
0.000
0.025
0.050
0.075
0.100
0.125

Using the statsmodels module to determine regressions:
`Series.tolist()`: Returns a list containing the DataFrame values.
`sm.OLS(Y, X).fit()`: Computes OLS fit of data (`X`, `Y`).

Regression data

```
import statsmodels.api as sm

fig = plt.figure(figsize=(16, 8))
ax = fig.add_subplot(1, 1, 1)
Y = np.array(amazon.loc["2018-1-1":"2018-1-15"].tolist())
X = np.arange(len(Y))
ax.scatter(x=X, y=Y, marker="o", color="red")
fig.savefig("out/reg_data.pdf")
```

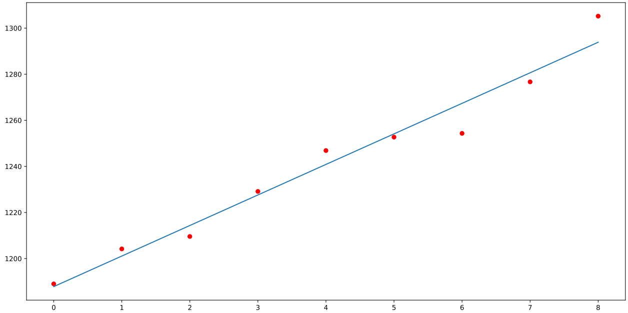

1300
1280
1260
1240
1220
1200
0
1
2
3
4
5
6
7
8

Regression

```
X_reg = sm.add_constant(X)
res = sm.OLS(Y, X_reg).fit()
b, a = res.params
ax.plot(X, a * X + b)
fig.savefig("out/ols.pdf")
```

Summary of OLS regression. To print in python use `res.summary()`.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.965
Model:                            OLS   Adj. R-squared:                  0.959
Method:                 Least Squares   F-statistic:                     190.2
Date:                Mo, 19 Mär 2018   Prob (F-statistic):           2.49e-06
Time:                        15:21:30   Log-Likelihood:                -29.706
No. Observations:                   9   AIC:                             63.41
Df Residuals:                       7   BIC:                             63.81
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const       1187.8418      4.575    259.617      0.000    1177.023    1198.661
x1            13.2540      0.961     13.792      0.000      10.982      15.526
==============================================================================
Omnibus:                        0.788   Durbin-Watson:                   1.627
Prob(Omnibus):                  0.674   Jarque-Bera (JB):                0.117
Skew:                          -0.268   Prob(JB):                        0.943
Kurtosis:                       2.841   Cond. No.                         9.06
==============================================================================
```


1300
1280
1260
1240
1220
1200
0
1
2
3
4
5
6
7
8

Applications

- Optimization

The Newton-Raphson method is an algorithm for finding successively better approximations to the roots of real-valued functions.

Let $F : \mathbb{R}^k \to \mathbb{R}^k$ be a continuously differentiable function and $J_F(x_n)$ the Jacobian matrix of F. The recursive Newton-Raphson method to find the root of F is given by:

$$\boldsymbol{x}_{n+1} := \boldsymbol{x}_n - \left(J(\boldsymbol{x}_n)^{-1} F(\boldsymbol{x}_n)\right)$$

with an initial guess x_0.

For $f : \mathbb{R} \to \mathbb{R}$ the process is repeated as

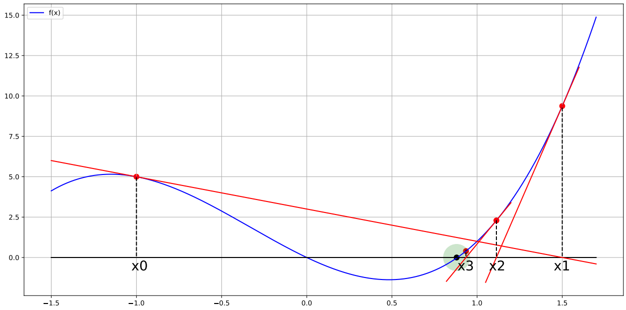
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

Accordingly, we can determine the *optimum* of the function f by applying the method instead to $f' = df/dx$.

As an illustrative application, we consider the function

$$f(x) = 3x^3 + 3x^2 - 5x, \quad x \in \mathbb{R},$$

which is represented by the blue line in the following diagram. The figure depicts the iterative solution path applying the Newton-Raphson method to find the root, e.g., x solving $f(x) = 0$, by tangent points and tangents starting from the initial guess $x_0 = -1$.



The first step involves the definition of the function $f(x)$ and its derivation $f'(x)$ in Python:

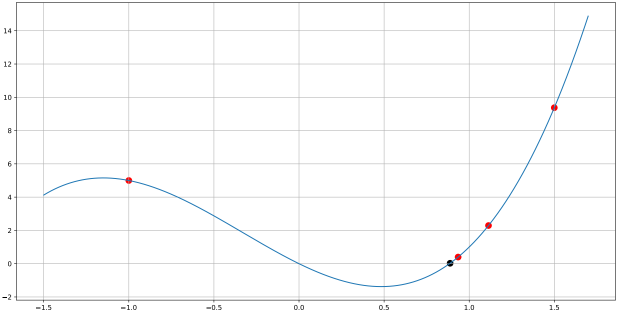
Newton-Raphson requirements

```
def f(x):
    return 3 * x**3 + 3 * x**2 - 5 * x


def df(x):
    return 9 * x**2 + 6 * x - 5
```

Finally, we implement the Newton-Raphson algorithm as outlined above. We allow for a (small) absolute deviation between the target function and its target value, i.e., 0. In addition, for a better understanding, we plot the solution path using the tangent points for $x_0, x_1, \dots, x_N$. The solution point is colored black. Hence, the lines starting with `ax.scatter()` are not part of the algorithm – they take global variables and are included just for the visual illustration.

Newton-Raphson

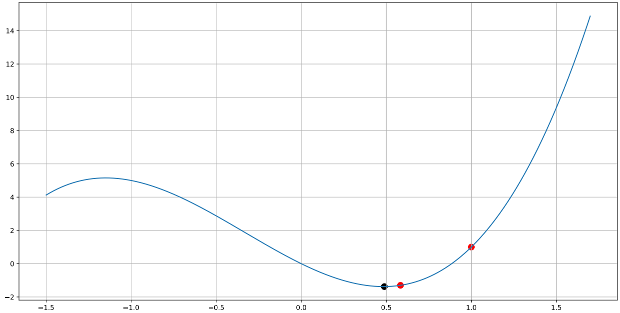
```
def newton_raphson(fun, dfun, x0, e):
    delta = abs(fun(x0))
    while delta > e:
        ax.scatter(x0, f(x0), color="red", s=80)
        x0 = x0 - fun(x0) / dfun(x0)
        delta = abs(fun(x0))
    ax.scatter(x0, f(x0), color="black", s=80)
    return(x0)

fig = plt.figure(figsize=(16, 8))
ax = fig.add_subplot(1, 1, 1)
x = np.arange(-1.5, 1.7, 0.001)
ax.plot(x, f(x))
ax.grid()
x_root = newton_raphson(f, df, -1, 0.1)
fig.savefig("out/newton_raphson_root.pdf")
print(f"Root at: {x_root:.4f}")

## Root at: 0.8878
```


14
12
10
8
6
4
2
0
−2
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

With the definition of the second derivative f'', i.e., the derivative of the derivative, we can employ the Newton-Raphson method to obtain an optimum of the target function $f(x)$ numerically. Hence, the previous example needs only minimal modifications:

Newton-Raphson

```
def ddf(x):
    return 18 * x + 6

fig = plt.figure(figsize=(16, 8))
ax = fig.add_subplot(1, 1, 1)
x = np.arange(-1.5, 1.7, 0.001)
ax.plot(x, f(x))
ax.grid()
x_opt = newton_raphson(df, ddf, 1, 0.1)
fig.savefig("out/newton_raphson_optimum.pdf")
print(f"Minimum at: {x_opt:.4f}")

## Minimum at: 0.4886
```


14
12
10
8
6
4
2
0
−2
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

The `scipy.optimize` package provides several optimization algorithms. A detailed list of the functions is available here.

The module contains:

- Unconstrained and constrained minimization of multivariate scalar functions using a variety of algorithms.
- Global optimization routines.
- Least-squares minimization and curve fitting algorithms.
- Scalar univariate functions minimizers and root finders.
- Multivariate equation system solvers.

`scipy.optimize.minimize()`: Provides minimization algorithms for multivariate scalar functions.

Import minimize

```
from scipy.optimize import minimize
```

The `minimize()` functions has several parameters, of which the most important are:

- `fun`: The objective function to be minimized.
- `x0`: The initial guess.
- `method`: The solver algorithm, such as: BFGS, Nelder-Mead (Simplex), Newton Conjugate Gradient, and many more.
- `constraints`: The constraints definition.

A detailed list of all parameters can be found here.

Let's get started by finding the minimum of the simple scalar function $f(x) = (x-4)^2 + 3$ using `minimize()`:

1D optimization using minimize

```
def f(x):
    return (x - 4)**2 + 3

x0 = [1]  # the initial guess
result = minimize(f, x0)
result

##       fun: 3.0000000000000036
##  hess_inv: array([[0.49999999]])
##       jac: array([-8.94069672e-08])
##   message: 'Optimization terminated successfully.'
##      nfev: 6
##       nit: 2
##      njev: 3
##    status: 0
##   success: True
##         x: array([3.99999994])
```

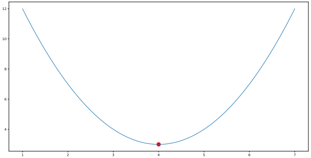
1D optimization

Essential concepts
Getting started
Procedural programming
Object-orientation
Numerical programming
NumPy package
Array basics
Linear algebra
Data formats and handling
Pandas package
Series
DataFrame
Import/Export data
Visual illustrations
Matplotlib package
Figures and subplots
Plot types and styles
Pandas layers
Applications
Time series
Moving window
Financial applications
Optimization

Let's check if the minimum is correct by plotting the function and marking the minima:

1D optimization using minimize

```
min_y = result.fun  # get minimum of the function f
min_x = result.x  # get the x value of the minimum

fig = plt.figure(figsize=(16, 8))
ax = fig.add_subplot(1, 1, 1)
x = np.arange(1, 7, 0.001)
ax.plot(x, f(x))
ax.scatter(min_x, min_y, color="red", s=120)
fig.savefig("out/minimize_1D.pdf")
```



Now, we consider a simple scalar function of two variables $f(x_1, x_2) = (x_1 - 1)^2 + (x_2 - 2.5)^2$:

2D optimization using minimize

```
def f(x):
    return (x[0] - 1)**2 + (x[1] - 2.5)**2

x0 = [0, 0]  # the initial guess
result = minimize(f, x0)
result

##       fun: 1.968344227868139e-15
##  hess_inv: array([[ 0.93103448, -0.1724138 ],
##        [-0.1724138 ,  0.56896552]])
##       jac: array([-6.95567350e-08,  4.21085256e-08])
##   message: 'Optimization terminated successfully.'
##      nfev: 9
##       nit: 2
##      njev: 3
##    status: 0
##   success: True
##         x: array([0.99999996, 2.50000001])
```

A famous performance test problem for optimization algorithms is the Rosenbrock function, which is defined by

$$f(x, y) = (a - x)^2 + b(y - x^2)^2.$$

It has a global minimum at $(x, y) = (a, a^2)$ and the parameters are usually set to $a = 1$ and $b = 100$:

Comparison

```
def rosen(x):
    return (1 - x[0])**2 + 100 * (x[1] - x[0]**2)**2

x0 = [1.3, 0.4]  # random initial guess

res_1 = minimize(rosen, x0, method="Nelder-Mead")
res_2 = minimize(rosen, x0, method="Powell")
res_3 = minimize(rosen, x0, method="CG")
res_4 = minimize(rosen, x0, method="BFGS")
```

Comparison results

```
# The perfect solution would be (1, 1)

res_1.x

## array([1.00000287, 1.00000496])

res_2.x

## array([1., 1.])

res_3.x

## array([0.99999552, 0.99999104])

res_4.x

## array([0.99999554, 0.99999108])
```

There is no "best" solver algorithm, it always depends on the problem.

One key feature of the `minimize()` function is minimization with constraints.

Imagine being a producer of tin cans. Your goal is to create a tin can which has a capacity of 500 ml while consuming as little material as possible. There are two variables which describe the volume v and the surface s of a tin can: the radius r and the height h.

The volume is given by

$$v(r,h) = \pi \cdot r^2 \cdot h,$$

and the surface can be computed with

$$s(r,h) = 2 \cdot \pi \cdot r \cdot (r+h).$$

Your goal is to minimize the function $s(r,h)$ with the constraint $v(r,h) = 500$.

We can easily implement the target and constraining function:

Tin can optimization

```
def s(x):
    r = x[0]
    h = x[1]
    return 2 * np.pi * r * (r + h)


def v(x):
    r = x[0]
    h = x[1]
    return np.pi * r**2 * h - 500  # as it is compared to zero
```

The constraint is defined in a special dictionary. The type is either inequal (`"ineq"`) or equal (`"eq"`) to zero:

Constraints

```
con = {"type": "eq", "fun": v}
```

The last step is to set the initial guess and to choose a solver algorithm. Only a few solver algorithms work with constraints, one of which is abbreviated by `"SLSQP"`:

Tin can optimization

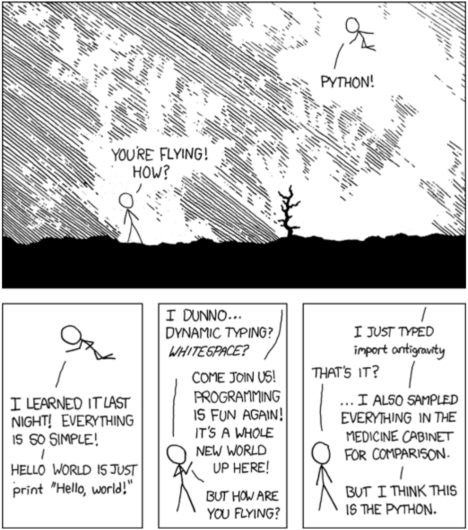
```
x0 = [1, 1]
result = minimize(s, x0, method="SLSQP", constraints=con)
result

##      fun: 348.7342054449393
##      jac: array([108.10270309,  27.02567673])
##  message: 'Optimization terminated successfully'
##     nfev: 29
##      nit: 9
##     njev: 9
##   status: 0
##  success: True
##        x: array([4.3012702 , 8.60253961])

x = result.x
```

We have found a combination of radius and height which gives us a minimal surface of the tin can:

Tin can optimization result

```
r, h = x
r

## 4.301270202292404

h

## 8.60253960537927

np.pi * r**2 * h

## 499.99999998290457

s(x)

## 348.7342054449393
```

A tin can with $r = 4.3$ cm and $h = 8.6$ cm has a volume of 500 ml and a minimal material consumption of 348.7 qcm.


YOU'RE FLYING! HOW?
PYTHON!
I LEARNED IT LAST NIGHT! EVERYTHING IS SO SIMPLE!
HELLO WORLD IS JUST print "Hello, world!"
I DUNNO... DYNAMIC TYPING? WHITESPACE?
COME JOIN US! PROGRAMMING IS FUN AGAIN! IT'S A WHOLE NEW WORLD UP HERE!
BUT HOW ARE YOU FLYING?
I JUST TYPED import antigravity
THAT'S IT?
... I ALSO SAMPLED EVERYTHING IN THE MEDICINE CABINET FOR COMPARISON.
BUT I THINK THIS IS THE PYTHON.