Lesson 13

Linear Patterns

Lesson Topics: Representations of Linear Functions.

Lesson Length: Three 50-minute periods

Student Objectives:

Students will:

- Recognize a linear relationship from a table or graph.
- Find the slope and y-intercept of a linear function from a table or graph.
- Find an equation for a linear function.
- Use a table, graph or equation to answer questions about a linear relationship.

Grouping for Instruction:

- Whole group for launch and closure.
- Small groups for the investigation.

Overview of Lesson:

- Students will use K'NEX materials to create a train of right triangles and find the linear relationship between the number of rods and the number of triangles in the train.
- They will then find the linear relationship between the number of connectors and the number of triangles in the train. These linear functions will be represented as tables, graphs, and equations. The representations will be used to extend the patterns.
- The investigation is repeated with a train of squares.

Materials and Equipment:

- *K'NEX Middle School Math set and Instructions Booklets*
- *Copies of the Lesson #13 Student Inquiry Sheets*
- *Graph paper*
- *Rulers*

A – Motivation and Introduction:

"A book club charges $20 to join the club. You then receive four books each month for $15. A plumber charges $50 for a service call plus $65 per hour. These are just two of many examples of linear relationships. In this lesson we will explore linear patterns and learn different ways of representing them."

B – Development:

1. Place the students in (heterogeneous) cooperative groups of about four students each. Assign a task to each person in a group.
2. Instruct the groups to complete the activities outlined in the Student Inquiry Sheets.
3. Circulate among the groups, guiding them as they complete the project.
4. Ask each group to report their discoveries and their findings to the rest of the class.

C – Summary and Closure:

Ask students to represent one of the problems from the introduction as a table, graph, and equation. Ask them to justify why each representation of a train of triangles or squares is an accurate model of a linear pattern and to explain similarities between the models.

Assessment:

Observe the students during the group work. Use a checklist to record whether students can recognize a linear function from a table. Each group should receive a group grade on the project. Ask the students to explain in their Math Journal what they learned during the lesson and any concepts that are still unclear.

Extension:

Provide students with K'NEX models or charts, graphs or tables of data that may or may not represent linear patterns. Instruct students to demonstrate whether the information represents a linear pattern or not. If the pattern is linear, can the students determine the slope?

Linear Patterns: Representing Linear Functions

Train 1:

Create a right triangle using 2 blue rods, 1 yellow rod and 3 yellow or white connectors. (See figure 1.) We are going to create a train of these right triangles. Create the second figure which is formed using 4 blue rods and 1 yellow rod and connectors. Finally, add another triangle to the train to create figure 3.

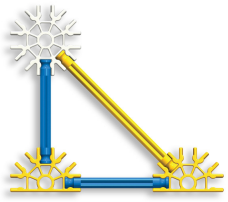

Figure 1

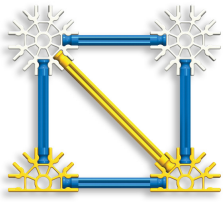

Figure 2

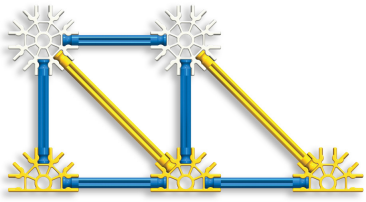

Figure 3

1. What patterns do you see in this train? Describe as many patterns as you can find.

1. Use these patterns to create the next two figures in the pattern. Sketch them on a sheet of paper using stick figures representations.

2. Complete the table below for these 5 figures.

Figure (Input Values)	Number of Rods (Output Values)	Number of Connectors (Output Values)
1		
2		
3		
4		
5		

Look at the table above. A table represents a linear function if the input values are an arithmetic sequence and the output values are also an arithmetic sequence. That is, the table represents a linear function if the differences between successive input values are constant and the differences in successive output values are constant. In this table the inputs are the figure numbers. Since each figure number after the first is 1 more than the previous one, the inputs have a common difference of 1 and this is an arithmetic sequence. Let the "number of rods" column represent the output values.

3. Is the number of rods an arithmetic sequence? If it is, what is the common difference?

4. Is the number of rods a linear function of the figure number? Why?

A second way of determining whether or not a relationship between two variables is a linear relationship is to graph ordered pairs (figure #, number of rods) that satisfy the relationship. The ordered pair (1, 3) tells us that in figure 1 there are 3 rods.

5. What are two other ordered pairs in the relation between the figure number and the number of rods?

6. Plot the points corresponding to the ordered pairs (figure #, rods) in the table above on a sheet of graph paper. The horizontal axis (*x*) will represent the figure number. The vertical axis (*y*) will represent the number of rods.

7. Do the points on your graph lie in a straight line?

8. Draw a line through these points (use a ruler if you wish). Label the graph "Student Inquiry Sheet - Lesson #13 – Graph 1 – Triangle Rods".

A third representation of a linear function uses symbols. All linear functions can be represented as an equation in the form $y = mx + b$, where x is the input variable (here, the figure number) and y is the output variable (here, the number of rods). The number m is the ratio of the change in the y-values (the outputs) to the change in the x-values (the inputs). That is, it is the ratio of the common difference in the outputs (sometimes called the **rise**) over the common difference in the inputs (sometimes called the **run**). We call m the **slope** of the line, because it tells us **how steep the line is** and whether it is increasing or decreasing.

9. What is the slope of the line showing the linear relationship between the number of rods and the figure number?

$m =$

The number b is the output value that corresponds to an input of 0. Graphically, b is the point where the line crosses the vertical or y-axis. That is, b is the **y-intercept** of the graph of the linear function. If the value of the output when the input is 0 is given, it is easy to find b. If you cannot find b this way, you can put the value of m and the x-value and y-value of a point on the graph into the form $y = mx + b$ to obtain an equation with only one letter – b. Since the point (1, 3) is a point on our graph, we can replace x by 1 and y by 3.

10. Create an equation by doing these substitutions and substituting your value for m into $y = mx + b$. Then solve this equation for b. $y = mx + b$

$b =$

11. Use the values of m and b just found to represent the linear function showing how the number of rods needed is related to the figure number. You will write the formula for the slope of this linear function and substitute actual numbers for the '*m*' and the '*b*' in the formula.

$y =$

12. Use this equation to predict the number of rods needed for figure 10 of this pattern. (Remember, y is the number of rods needed and x is the figure number.)

13. Use this equation to find the figure number, if 25 rods are needed. Show all work.

14. Refer to the table on the first page. Is the number of connectors needed a linear function of the figure number. How do you know?

15. Plot the points corresponding to the ordered pairs (figure #, connectors) on the back of your other graph. The horizontal axis (*x*) will represent the figure number. The vertical axis (*y*) will represent the number of connectors.

16. Do the points lie in a straight line?

17. Draw a line through these points (a ruler will help). Label the graph "Student Inquiry Sheet - Lesson #13 - Graph 2 - Triangle Connectors".

18. What is the slope of the line showing the linear relationship between the number of connectors and the figure number?

$m =$

19. What is the *y*-intercept *b* for this linear function?

$b =$

20. Write an equation that represents the linear relationship between the number of connectors needed and the figure number for this train.

$y =$

21. Use this equation to predict the number of connectors needed for figure 12.

22. If 20 connectors are needed for a figure in this train, what is the figure number? Show how you found the answer.

Train 2:

This train is formed by connecting squares together. Create the first three figures shown below using yellow rods and yellow and white connectors.

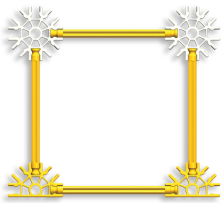

Figure 1

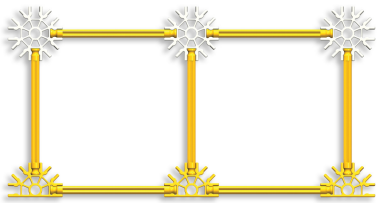

Figure 2

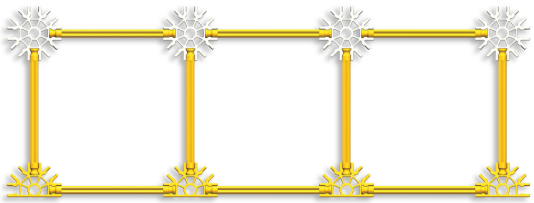

Figure 3

1. What patterns do you see in this train? Describe as many patterns as you can find.

2. Use these patterns to create the next two figures in the pattern. Sketch them on a sheet of paper using stick figure representations.

3. Complete the table below for these 5 figures.

Figure (Input Values)	Number of Rods (Output Values)	Number of Connectors (Output Values)
1		
2		
3		
4		
5		

4. Is the number of rods a linear function of the figure number? How do you know?

5. Use the table to determine how many rods will be needed for figure 6 if you were to build that figure from K'NEX.

6. Plot the points corresponding to the ordered pairs (figure #, rods) found in the table on a sheet of graph paper. The horizontal axis (x) will represent the figure number. The vertical axis (y) will represent the number of rods. Draw a line through these points (a ruler will help). Label the graph "Student Inquiry Sheet - Lesson #13 - Graph 3 - Square Rods".

7. Use the graph to predict how many rods will be needed in figure 8. How did you find the answer?

8. Find the slope m and the y-intercept b of this linear function.

$m =$

$b =$

9. Write the equation that represents this linear function.

$y =$

10. Use this equation to predict how many rods will be needed for the figure 10 train. Show all your work.

11. If 37 rods are needed to create a train in this sequence, what is the figure number? How did you find the answer?

12. Refer to the table for train 2. Is the number of connectors needed a linear function of the figure number? Explain how you know.

13. Use the table to predict how many connectors will be needed for figure 6.

14. Use the table to predict how many connectors will be needed for figure 7.

15. Refer to the table for train 2. Plot the points corresponding to the ordered pairs (figure #, connectors) found in previous table on the back of your other graph. The horizontal axis (x) will represent the figure number. The vertical axis (y) will represent the number of connectors. Draw a line through these points (a ruler will help). Label the graph "Student Inquiry Sheet - Lesson #15 - Graph 4 - Square Connectors".

16. Use the line on your graph to predict how many connectors will be needed in figure 8.

17. Find the slope m and the y-intercept b of this linear function.

$m =$ $b =$

18. Write the equation that represents this linear function.

$y =$

19. Use this equation to predict how many connectors will be needed for figure 10 of the train. Show all your work.

20. If 26 connectors are needed to create a train in this sequence, what is the figure number? How did you find the answer?