Name ____ Date____________________

Lesson 1: Integer Sequences—Should You Believe in Patterns?

Exit Ticket

1. Consider the sequence given by a "plus 8" pattern: $2, 10, 18, 26, \ldots$.

   Shae says that the formula for the sequence is $f(n) = 8n + 2$. Marcus tells Shae that she is wrong because the formula for the sequence is $f(n) = 8n - 6$.

   a. Which formula generates the sequence by starting at $n = 1$? At $n = 0$?

   b. Find the 100^{th} term in the sequence.

2. Write a formula for the sequence of cube numbers: $1, 8, 27, 64, \ldots$.

Name ____ Date____________________

Lesson 2: Recursive Formulas for Sequences

Exit Ticket

1. Consider the sequence following a "minus 8" pattern: $9, 1, -7, -15, \ldots$.
   a. Write an explicit formula for the sequence.

   b. Write a recursive formula for the sequence.

   c. Find the 38^{th} term of the sequence.

2. Consider the sequence given by the formula $a(n+1) = 5a(n)$ and $a(1) = 2$ for $n \geq 1$.
   a. Explain what the formula means.

   b. List the first five terms of the sequence.

Name ____ Date____________________

Lesson 3: Arithmetic and Geometric Sequences

Exit Ticket

1. Write the first three terms in the following sequences. Identify them as arithmetic or geometric.
   a. $A(n+1) = A(n) - 5$ for $n \geq 1$ and $A(1) = 9$.

   b. $A(n+1) = \frac{1}{2}A(n)$ for $n \geq 1$ and $A(1) = 4$.

   c. $A(n+1) = A(n) \div 10$ for $n \geq 1$ and $A(1) = 10$.

2. Identify each sequence as arithmetic or geometric. Explain your answer, and write an explicit formula for the sequence.
   a. $14, 11, 8, 5, \ldots$

   b. $2, 10, 50, 250, \ldots$

   c. $-\frac{1}{2}, -\frac{3}{2}, -\frac{5}{2}, -\frac{7}{2}, \ldots$

Name ____ Date____________________

Lesson 4: Why Do Banks Pay YOU to Provide Their Services?

Exit Ticket

A youth group has a yard sale to raise money for a charity. The group earns $\$800$ but decides to put its money in the bank for a while. Calculate the amount of money the group will have if,

a. Cool Bank pays simple interest at a rate of 4%, and the youth group leaves the money in for 3 years.

b. Hot Bank pays an interest rate of 3% compounded annually, and the youth group leaves the money in for 5 years.

c. If the youth group needs the money quickly, which is the better choice? Why?

Name ____ Date____________________

Lesson 5: The Power of Exponential Growth

Exit Ticket

Chain emails are emails with a message suggesting you will have good luck if you forward the email on to others. Suppose a student started a chain email by sending the message to 3 friends and asking those friends to each send the same email to 3 more friends exactly 1 day after they received it.

a. Write an explicit formula for the sequence that models the number of people who will receive the email on the n^{th} day. (Let the first day be the day the original email was sent.) Assume everyone who receives the email follows the directions.

b. Which day will be the first day that the number of people receiving the email exceeds 100?

Name ____ Date____________________

Lesson 6: Exponential Growth—U.S. Population and World Population

Exit Ticket

Do the examples below require a linear or exponential growth model? State whether each example is linear or exponential, and write an explicit formula for the sequence that models the growth for each case. Include a description of the variables you use.

1. A savings account accumulates no interest but receives a deposit of $\$825$ per month.

2. The value of a house increases by 1.5% per year.

3. Every year, the alligator population is $\frac{9}{7}$ of the previous year's population.

4. The temperature increases by $2°$ every 30 minutes from $8{:}00$ a.m. to $3{:}30$ p.m. each day for the month of July.

5. Every 240 minutes, $\frac{1}{3}$ of the rodent population dies.

Name ____ Date____________________

Lesson 7: Exponential Decay

Exit Ticket

A huge ping-pong tournament is held in Beijing, with $65{,}536$ participants at the start of the tournament. Each round of the tournament eliminates half the participants.

a. If $p(r)$ represents the number of participants remaining after r rounds of play, write a formula to model the number of participants remaining.

b. Use your model to determine how many participants remain after 10 rounds of play.

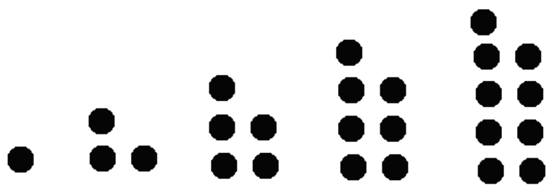
c. How many rounds of play will it take to determine the champion ping-pong player?

Name ____ Date____________________

Lesson 8: Why Stay with Whole Numbers?

Exit Ticket

Recall that an odd number is a number that is one more than or one less than twice an integer. Consider the sequence formed by the odd numbers $\{1, 3, 5, 7, \ldots\}$.



1. Find a formula for $O(n)$, the n^{th} odd number starting with $n = 1$.

2. Write a convincing argument that 121 is an odd number.

3. What is the meaning of $O(17)$?

Name ____ Date____________________

Lesson 9: Representing, Naming, and Evaluating Functions

Exit Ticket

1. Given f as described below.
   $f: \{whole\ numbers\} \rightarrow \{whole\ numbers\}$
   Assign each whole number to its largest place value digit.

   For example, $f(4) = 4$, $f(14) = 4$, and $f(194) = 9$.

   a. What is the domain and range of f?

   b. What is $f(257)$?

   c. What is $f(0)$?

   d. What is $f(999)$?

   e. Find a value of x that makes the equation $f(x) = 7$ a true statement.

2. Is the correspondence described below a function? Explain your reasoning.
   $M: \{women\} \rightarrow \{people\}$
   Assign each woman their child.

Name ____ Date____________________

Lesson 10: Representing, Naming, and Evaluating Functions

Exit Ticket

1. Let $f(x) = 4(3)^x$. Complete the table shown below.

x	-1	0	1	2	3
$f(x)$					

2. Jenna knits scarves and then sells them on Etsy, an online marketplace. Let $C(x) = 4x + 20$ represent the cost C in dollars to produce 1 to 6 scarves.
   a. Create a table to show the relationship between the number of scarves x and the cost C.

   b. What are the domain and range of C?

   c. What is the meaning of $C(3)$?

   d. What is the meaning of the solution to the equation $C(x) = 40$?

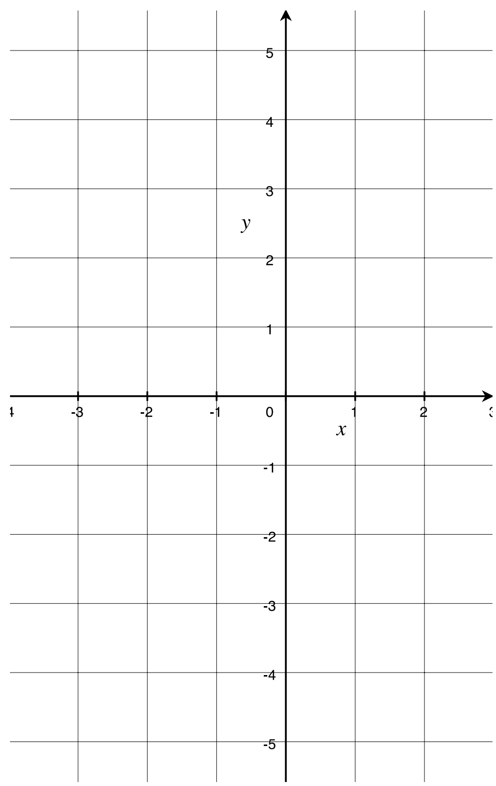
Name ____ Date____________________

Lesson 11: The Graph of a Function

Exit Ticket

1. Perform the instructions for the following programming code as if you were a computer and your paper was the computer screen.

```
Declare x integer
Let f(x) = 2x + 1
Initialize G as {}
For all x from −3 to 2
        Append (x, f(x)) to G
Next x
Plot G
```



2. Write three or four sentences describing in words how the thought code works.

Name ____ Date____________________

Lesson 12: The Graph of the Equation $y = f(x)$

Exit Ticket

1. Perform the instructions in the following programming code as if you were a computer and your paper was the computer screen:

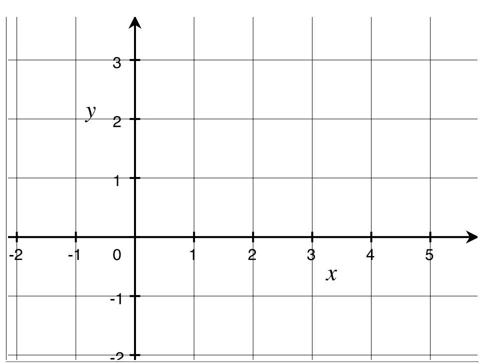
```
Declare x integer
For all x from 2 to 7
    If x + 2 = 7 then
        Print True
     else
        Print False
    End if
Next x
```

2. Let $f(x) = -\frac{1}{2}x + 2$ for x in the domain $0 \le x \le 4$.

   a. Write out in words the meaning of the set notation:

$$\{(x, y) \mid 0 \le x \le 4 \text{ and } y = f(x)\}.$$

   b. Sketch the graph of $y = f(x)$.



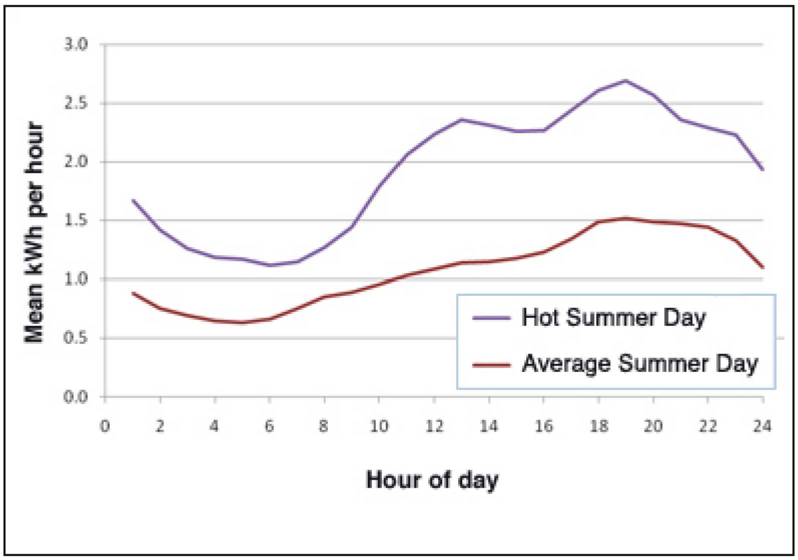
Name ____ Date____________________

Lesson 13: Interpreting the Graph of a Function

Exit Ticket

1. Estimate the time intervals when mean energy use is decreasing on an average summer day. Why would power usage be decreasing during those time intervals?

Power Usage on a Typical Summer Day in Ontario, Canada



Source: National Resource Council Canada, 2011

2. The hot summer day energy use changes from decreasing to increasing and from increasing to decreasing more frequently than it does on an average summer day. Why do you think this occurs?

Name ____ Date____________________

Lesson 14: Linear and Exponential Models—Comparing Growth Rates

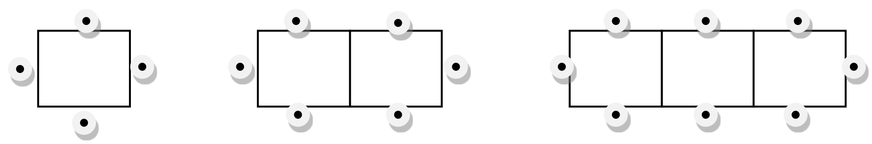
Exit Ticket

A big company settles its new headquarters in a small city. The city council plans road construction based on traffic increasing at a linear rate, but based on the company's massive expansion, traffic is really increasing exponentially.

What will be the repercussions of the city council's current plans? Include what you know about linear and exponential growth in your discussion.

Name ____ Date ______________________

1. The diagram below shows how tables and chairs are arranged in the school cafeteria. One table can seat 4 people, and tables can be pushed together. When two tables are pushed together, 6 people can sit around the table.



1 Table 2 Tables 3 Tables

a. Complete this table to show the relationship between the number of tables, n, and the number of students, S, that can be seated around the table.

n (tables)						
S (students)						

b. If we make a sequence where the first term of the sequence is the number of students that can fit at one table, the second term of the sequence is the number of students that can fit at two tables, and so on, will the sequence be arithmetic, geometric, or neither? Explain your reasoning.

c. Create an explicit formula for a sequence that models this situation. Use $n = 1$ as the first term representing how many students can sit at one table. How do the constants in your formula relate to the situation?

d. Using this seating arrangement, how many students could fit around 15 tables pushed together in a row?

The cafeteria needs to provide seating for 189 students. They can fit up to 15 rows of tables in the cafeteria. Each row can contain at most 9 tables but could contain less than that. The tables on each row must be pushed together. Students will still be seated around the tables as described earlier.

e. If they use exactly 9 tables pushed together to make each row, how many rows will they need to seat 189 students? What will be the total number of tables used to seat all of the students?

f. Is it possible to seat the 189 students with fewer total tables? If so, what is the fewest number of tables needed? How many tables would be used in each row? (Remember that the tables on each row must be pushed together.) Explain your thinking.

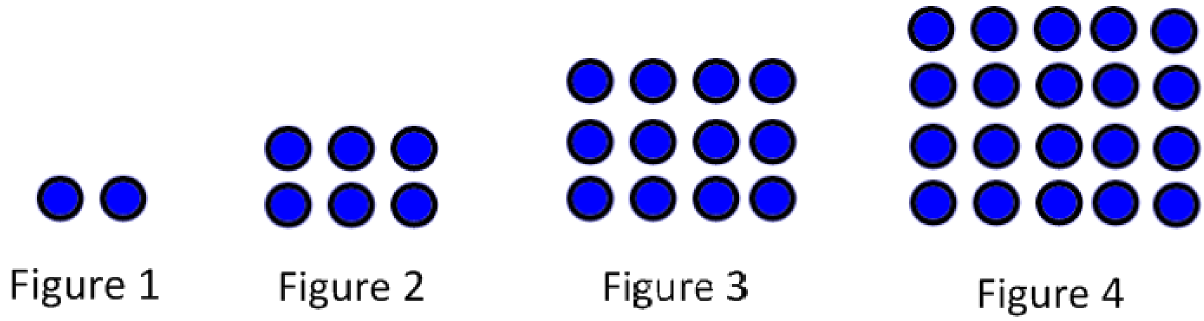
2. Sydney was studying the following functions:

$$f(x) = 2x + 4 \text{ and } g(x) = 2(2)^x + 4$$

She said that linear functions and exponential functions are basically the same. She made her statement based on plotting points at $x = 0$ and $x = 1$ and graphing the functions.

Help Sydney understand the difference between linear functions and exponential functions by comparing and contrasting f and g. Support your answer with a written explanation that includes use of the average rate of change and supporting tables and/or graphs of these functions.

3. Dots can be arranged in rectangular shapes like the one shown below.



a. Assuming the trend continues, draw the next three shapes in this particular sequence of rectangles. How many dots are in each of the shapes you drew?

The numbers that represent the number of dots in this sequence of rectangular shapes are called rectangular numbers. For example, 2 is the first rectangular number and 6 is the second rectangular number.

b. What is the fiftieth rectangular number? Explain how you arrived at your answer.

c. Write a recursive formula for the rectangular numbers.

d. Write an explicit formula for the rectangular numbers.

e. Could an explicit formula for the n^{th} rectangular number be considered a function? Explain why or why not. If yes, what would be the domain and range of the function?

4. Stephen is assigning parts for the school musical.

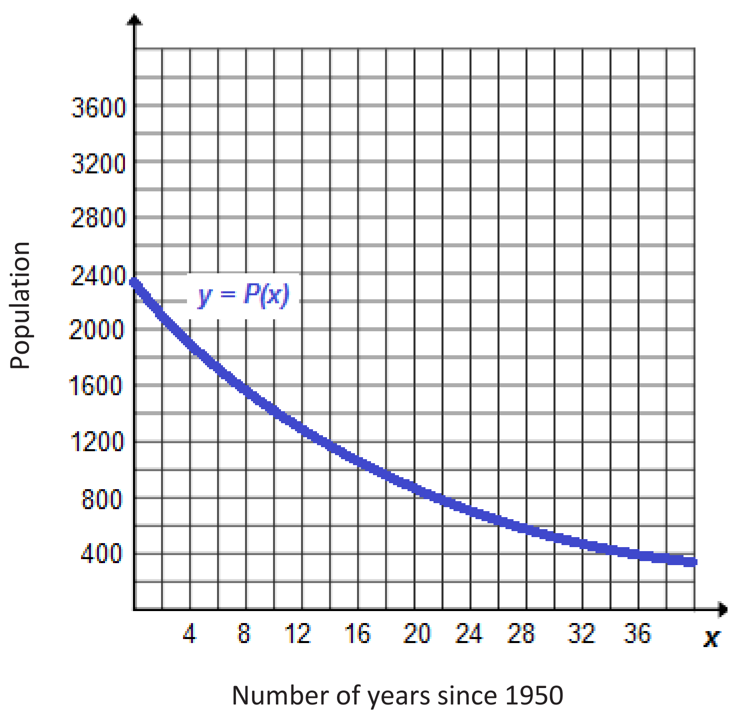
a. Suppose there are 20 students participating, and he has 20 roles available. If each of the 20 students will be assigned to exactly one role in the play, and each role will be played by only one student, is the assignment of the roles to the students in this way certain to be an example of a function? Explain why or why not. If yes, state the domain and range of the function.

The school musical also has a pit orchestra.

b. Suppose there are 10 instrumental parts but only 7 musicians in the orchestra. The conductor assigns an instrumental part to each musician. Some musicians will have to cover two instrumental parts, but no two musicians will have the same instrumental part. If the instrumental parts are the domain and the musicians are the range, is the assignment of instrumental parts to musicians as described sure to be an example of a function? Explain why or why not. If so, what would be the meaning of $A(Piano) = Scott$?

c. Suppose there are 10 instrumental parts but 13 musicians in the orchestra. The conductor assigns an instrumental part to each musician. Some instrumental parts will have two musicians assigned so that all the musicians have instrumental parts. When two musicians are assigned to one part, they alternate who plays at each performance of the play. If the instrumental parts are the domain, and the musicians are the range, is the assignment of instrumental parts to musicians as described sure to be an example of a function? Explain why or why not. If so, what would be the meaning of $A(Piano) = Scott$?

5. The population of a remote island has been experiencing a decline since the year 1950. Scientists used census data from 1950 and 1970 to model the declining population. In 1950 the population was 2,350. In 1962 the population was 1,270. They chose an exponential decay model and arrived at the function: $P(x) = 2350(0.95)^x, x \geq 0$, where x is the number of years since 1950. The graph of this function is given below.



a. What is the y-intercept of the graph? Interpret its meaning in the context of the problem.

b. Over what intervals is the function increasing? What does your answer mean within the context of the problem?

c. Over what intervals is the function decreasing? What does your answer mean within the context of the problem?

Another group of scientists argues that the decline in population would be better modeled by a linear function. They use the same two data points to arrive at a linear function.

d. Write the linear function that this second group of scientists uses.

e. What is an appropriate domain for the function? Explain your choice within the context of the problem.

f. Graph the function on the coordinate plane.

g. What is the x-intercept of the function? Interpret its meaning in the context of the problem.

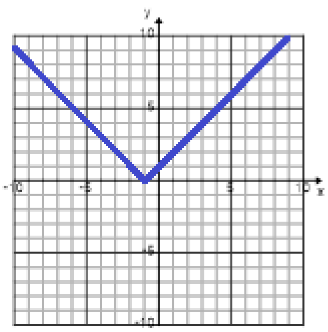
Name ____ Date____________________

Lesson 15: Piecewise Functions

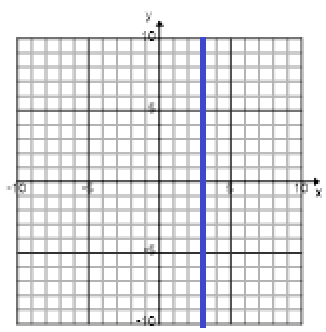
Exit Ticket

Each graph shown below represents the solution set to a two-variable equation.

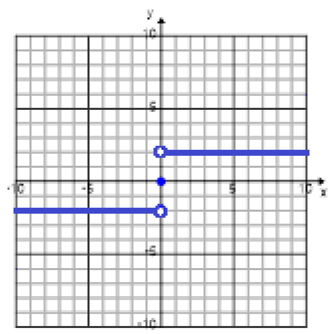
Graph A



Graph B



Graph C



1. Which of these graphs could be represented by a function? Explain your reasoning.

2. For each one that can be represented by a function, define a piecewise function whose graph would be identical to the solution set shown.

Name ____ Date____________________

Lesson 16: Graphs Can Solve Equations Too

Exit Ticket

1. How do intersection points of the graphs of two functions f and g relate to the solution of an equation in the form $f(x) = g(x)$?

2. What are some benefits of solving equations graphically? What are some limitations?

Name ____ Date____________________

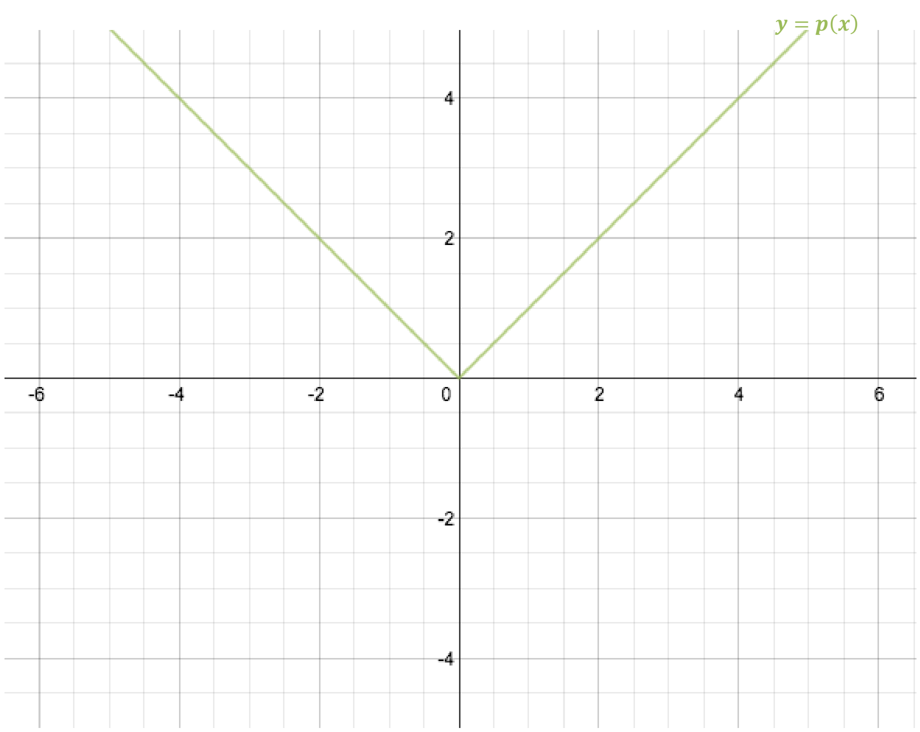
Lesson 17: Four Interesting Transformations of Functions

Exit Ticket

Let $p(x) = |x|$ for every real number x. The graph of $y = p(x)$ is shown below.

1. Let $q(x) = -\frac{1}{2}|x|$ for every real number x. Describe how to obtain the graph of $y = q(x)$ from the graph of $y = p(x)$. Sketch the graph of $y = q(x)$ on the same set of axes as the graph of $y = p(x)$.

2. Let $r(x) = |x| - 1$ for every real number x. Describe how to obtain the graph of $y = r(x)$ from the graph of $y = p(x)$. Sketch the graph of $y = r(x)$ on the same set of axes as the graphs of $y = p(x)$ and $y = q(x)$.



Name ____ Date____________________

Lesson 18: Four Interesting Transformations of Functions

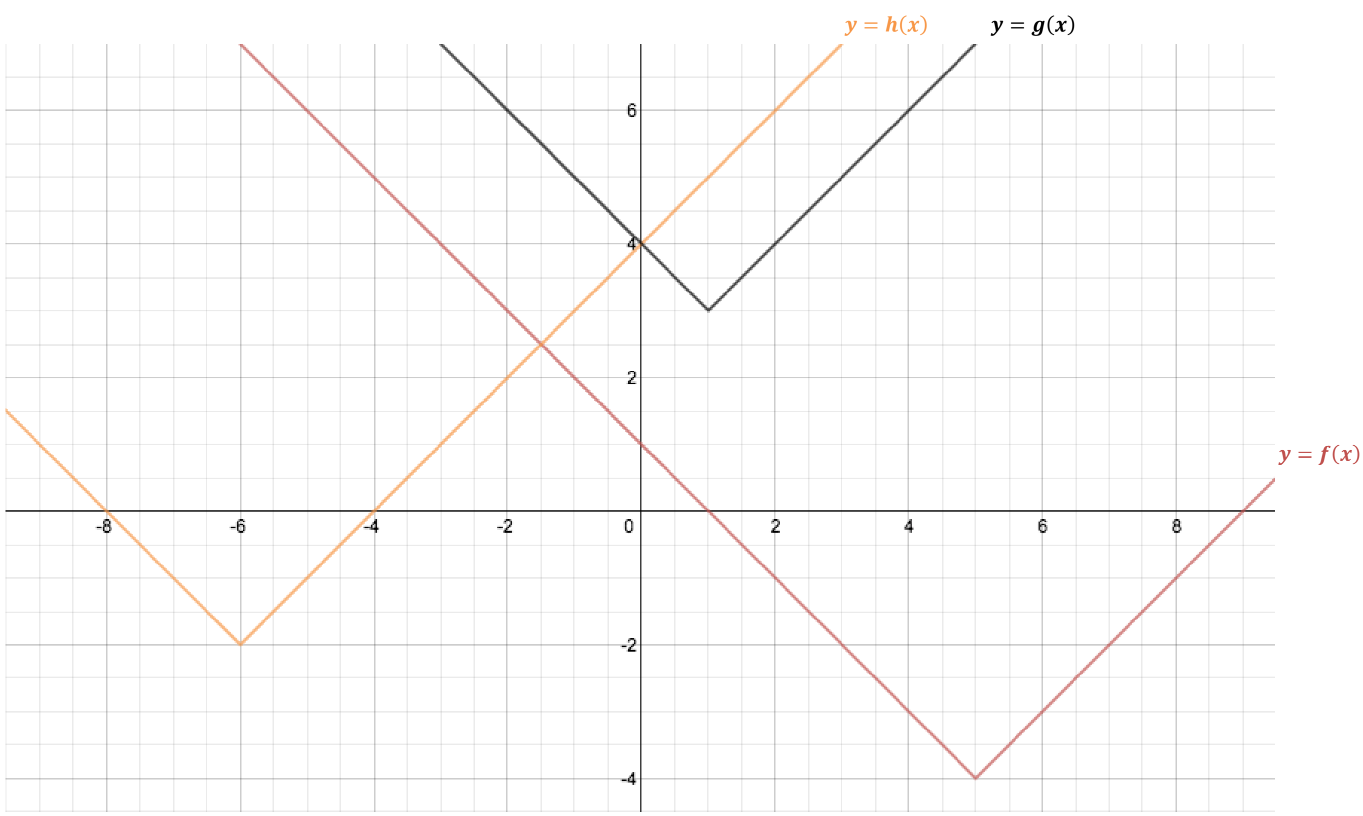
Exit Ticket

Write the formula for the functions depicted by the graphs below:

a. $f(x) =$ ______________________________

b. $g(x) =$ ______________________________

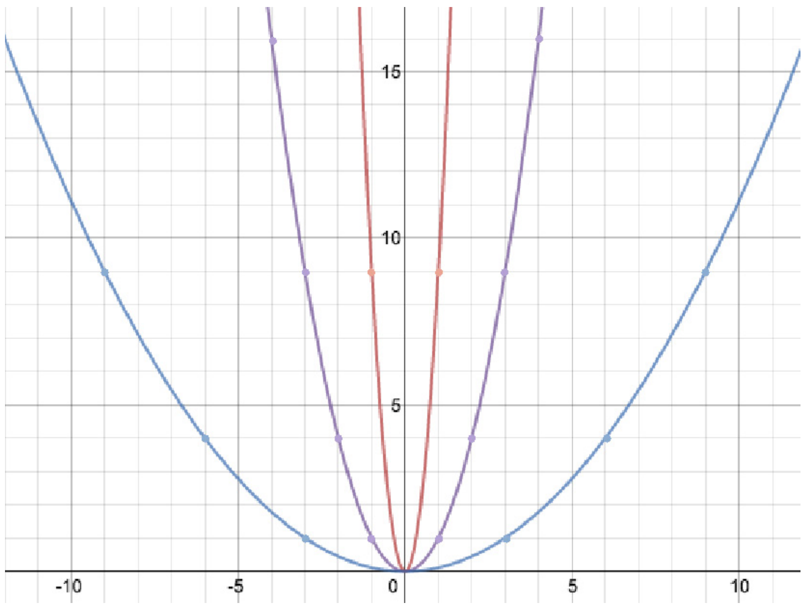
c. $h(x) =$ ______________________________



Name ____ Date____________________

Lesson 19: Four Interesting Transformations of Functions

Exit Ticket



Let $f(x) = x^2$, $g(x) = (3x)^2$, and $h(x) = \left(\frac{1}{3}x\right)^2$, where x can be any real number. The graphs above are of $y = f(x)$, $y = g(x)$, and $y = h(x)$.

1. Label each graph with the appropriate equation.

2. Describe the transformation that takes the graph of $y = f(x)$ to the graph of $y = g(x)$. Use coordinates of each to illustrate an example of the correspondence.

3. Describe the transformation that takes the graph of $y = f(x)$ to the graph of $y = h(x)$. Use coordinates to illustrate an example of the correspondence.

Name ____ Date____________________

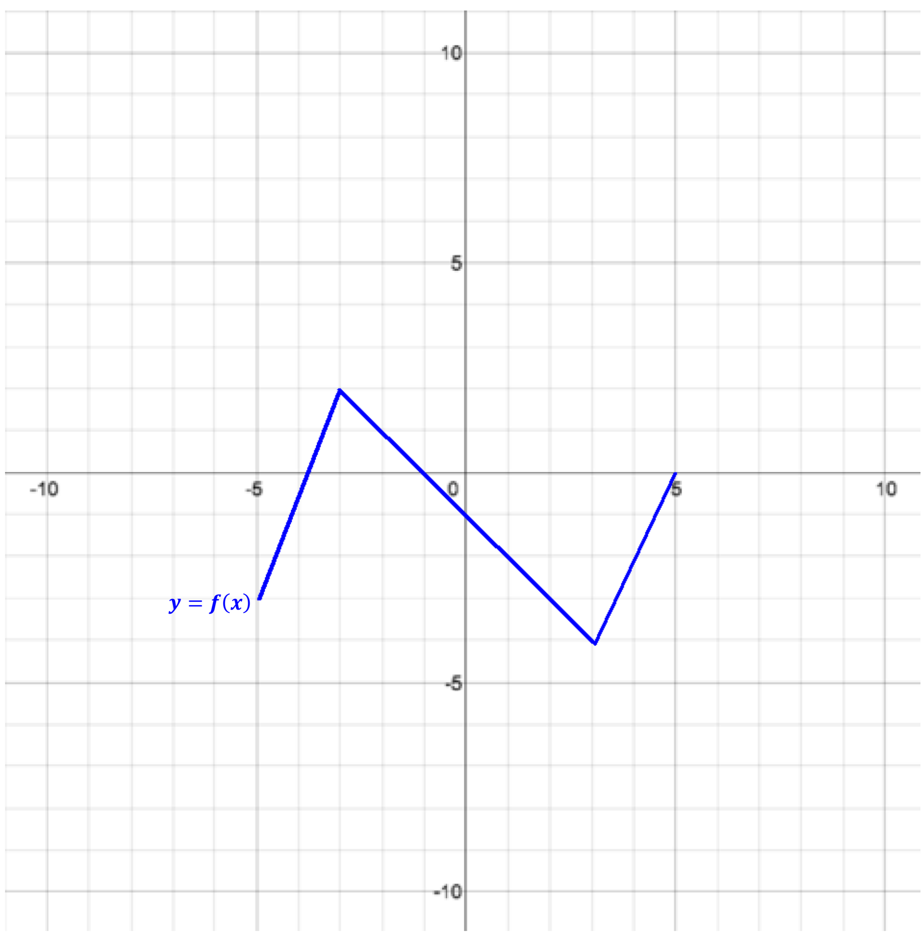
Lesson 20: Four Interesting Transformations of Functions

Exit Ticket

The graph of a piecewise function f is shown below.

Let $p(x) = f(x-2)$, $q(x) = \frac{1}{2}f(x-2)$, and $r(x) = \frac{1}{2}f(x-2) + 3$.

Graph $y = p(x)$, $y = q(x)$, and $y = r(x)$ on the same set of axes as the graph of $y = f(x)$.



Name ____ Date____________________

Lesson 21: Comparing Linear and Exponential Functions Again

Exit Ticket

Here is a classic riddle: Mr. Smith has an apple orchard. He hires his daughter, Lucy, to pick apples and offers her two payment options.

Option A: $\$1.50$ per bushel of apples picked.

Option B: 1 cent for picking one bushel, 3 cents for picking two bushels, 9 cents for picking three bushels, and so on, with the amount of money tripling for each additional bushel picked.

a. Write a function to model each option.

b. If Lucy picks six bushels of apples, which option should she choose?

c. If Lucy picks 12 bushels of apples, which option should she choose?

d. How many bushels of apples does Lucy need to pick to make option B better for her than option A?

Name ____ Date____________________

Lesson 22: Modeling an Invasive Species Population

Exit Ticket

1. For the equation found in Exercise 8, explain the parameters of the equation within the context of the problem.

2. Given each of the following, describe what features in the data or graph make it apparent that an exponential model would be more suitable than a linear model.
   a. The table of data.

   b. The scatterplot.

   c. The average rates of change found in Exercise 6.

3. Use your equation from Exercise 8 to predict the number of lionfish sightings by year 2020. Is this prediction accurate? Explain.

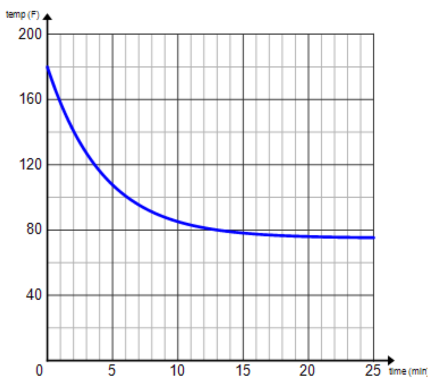
Name ____ Date____________________

Lesson 23: Newton's Law of Cooling

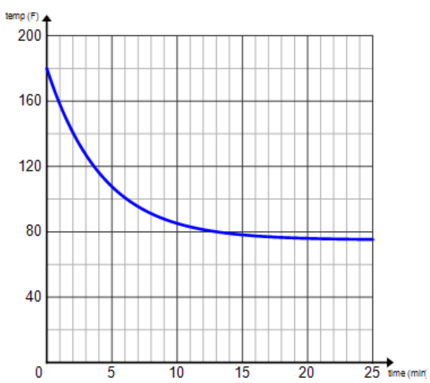
Exit Ticket

Shown below is the graph of cup 1 from the exercise completed in class. For each scenario, sketch and label a graph of cup 2 on the same coordinate plane.

1. Cup 2 is poured 10 minutes after cup 1 (the pot of coffee is maintained at $180°\text{F}$ over the 10 minutes).



2. Cup 2 is immediately taken outside where the temperature is $90°\text{F}$.

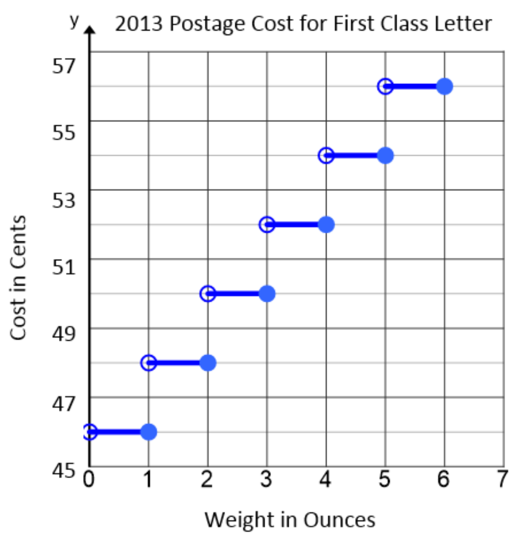


Name ____ Date____________________

Lesson 24: Piecewise and Step Functions in Context

Exit Ticket

1. Use the graph to complete the table.



Weight in ounces, x	2	2.2	3	3.5	4
Cost of postage, $C(x)$					

2. Write a formula involving step functions that represents the cost of postage based on the graph shown above.

3. If it cost Trina $\$0.54$ to mail her letter, how many ounces did it weigh?

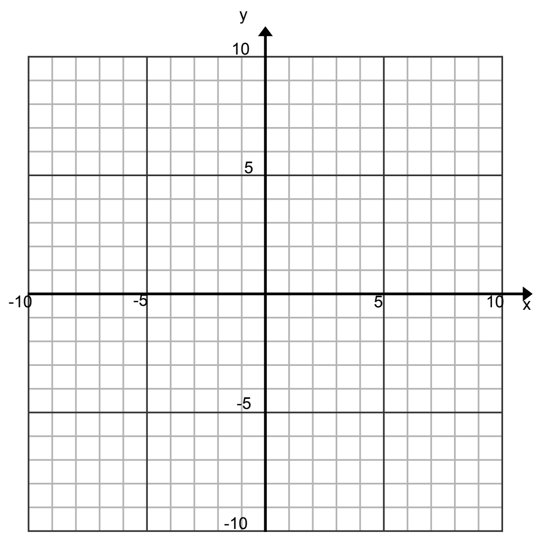
Name ____ Date ____________________

1. Given $h(x) = |x + 2| - 3$ and $g(x) = -|x| + 4$.

   a. Describe how to obtain the graph of g from the graph of $a(x) = |x|$ using transformations.

   b. Describe how to obtain the graph of h from the graph of $a(x) = |x|$ using transformations.

   c. Sketch the graphs of $h(x)$ and $g(x)$ on the same coordinate plane.



d. Use your graphs to estimate the solutions to the equation:

$$|x+2|-3=-|x|+4$$

Explain how you got your answer.

e. Were your estimations in part (d) correct? If yes, explain how you know. If not explain why not.

2. Let f and g be the functions given by $f(x)=x^2$ and $g(x)=x|x|$.

a. Find $f\left(\frac{1}{3}\right)$, $g(4)$, and $g\left(-\sqrt{3}\right)$.

b. What is the domain of f?

c. What is the range of g?

d. Evaluate $f(-67) + g(-67)$.

e. Compare and contrast f and g. How are they alike? How are they different?

f. Is there a value of x, such that $f(x) + g(x) = -100$? If so, find x. If not, explain why no such value exists.

g. Is there a value of x such that $(x) + g(x) = 50$? If so, find x. If not, explain why no such value exists.

3. A boy bought six guppies at the beginning of the month. One month later the number of guppies in his tank had doubled. His guppy population continued to grow in this same manner. His sister bought some tetras at the same time. The table below shows the number of tetras, t, after n months have passed since they bought the fish.

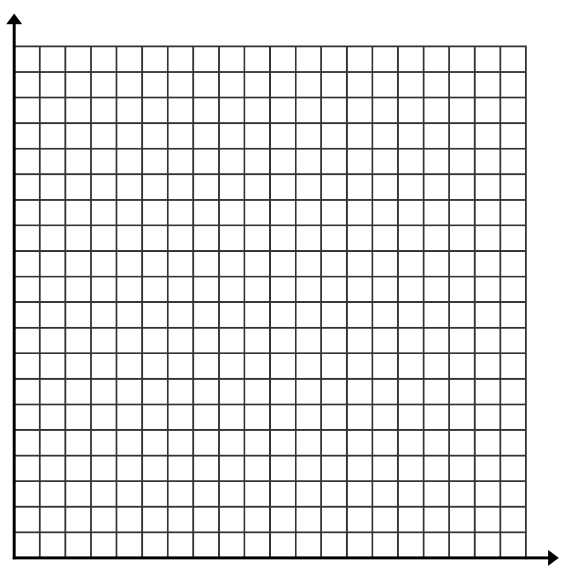
n, months	0	1	2	3
t, tetras	8	16	24	32

a. Create a function g to model the growth of the boy's guppy population, where $g(n)$ is the number of guppies at the beginning of each month and n is the number of months that have passed since he bought the six guppies. What is a reasonable domain for g in this situation?

b. How many guppies will there be one year after he bought the six guppies?

c. Create an equation that could be solved to determine how many months it will take for there to be 100 guppies.

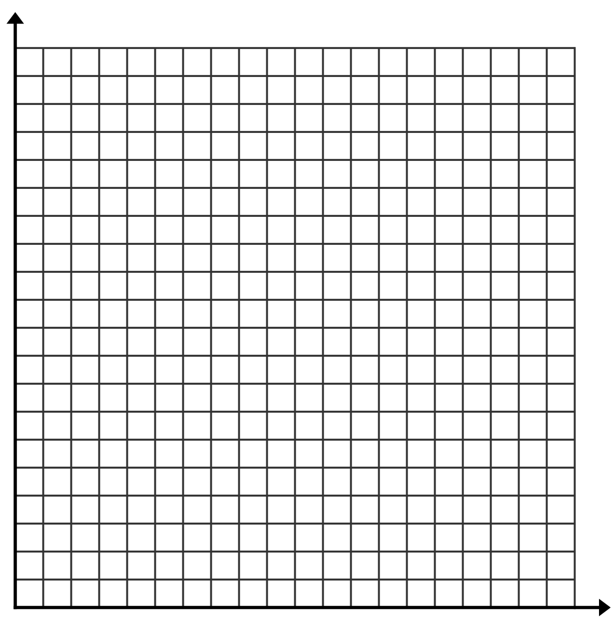
d. Use graphs or tables to approximate a solution to the equation from part (c). Explain how you arrived at your estimate.



e. Create a function, t, to model the growth of the sister's tetra population, where $t(n)$ is the number of tetras after n months have passed since she bought the tetras.

f. Compare the growth of the sister's tetra population to the growth of the guppy population. Include a comparison of the average rate of change for the functions that model each population's growth over time.

g. Use graphs to estimate the number of months that will have passed when the population of guppies and tetras will be the same.



h. Use graphs or tables to explain why the guppy population will eventually exceed the tetra population even though there were more tetras to start with.

i. Write the function $g(n)$ in such a way that the percent increase in the number of guppies per month can be identified. Circle or underline the expression representing percent increase in number of guppies per month.

4. Regard the solid dark equilateral triangle as Figure 0. Then, the first figure in this sequence is the one composed of three dark triangles, the second figure is the one composed of nine dark triangles, and so on.

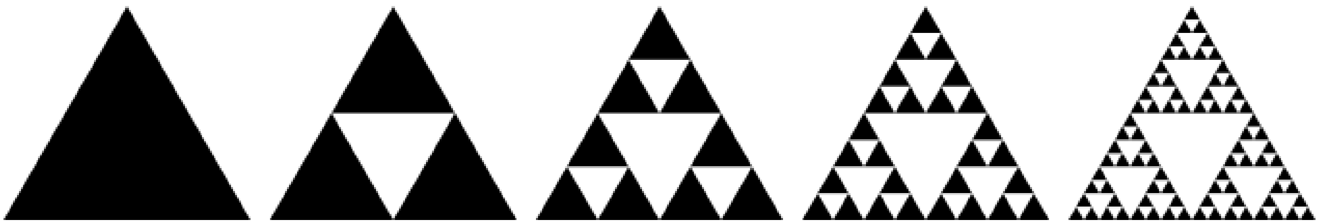


Figure 0 Figure 1 Figure 2 Figure 3 Figure 4

a. How many dark triangles are in each figure? Make a table to show this data.

n (Figure Number)					
T (# of dark triangles)					

b. Given the number of dark triangles in a figure, describe in words how to determine the number of dark triangles in the next figure.

c. Create a function that models this sequence. What is the domain of this function?

d. Suppose the area of the solid dark triangle in Figure 0 is 1 square meter. The areas of one dark triangle in each figure form a sequence. Create an explicit formula that gives the area of just one of the dark triangles in the n^{th} figure in the sequence.

e. The sum of the areas of all the dark triangles in Figure 0 is $1\ \text{m}^2$; there is only one triangle in this case. The sum of the areas of all the dark triangles in Figure 1 is $\frac{3}{4}\ \text{m}^2$. What is the sum of the areas of all the dark triangles in the n^{th} figure in the sequence? Is this total area increasing or decreasing as n increases?

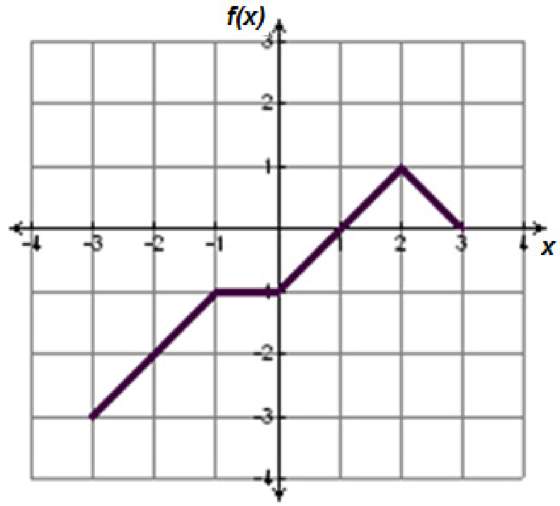
f. Let $P(n)$ be the sum of the perimeters of the all dark triangles in the n^{th} figure in the sequence of figures. There is a real number k so that,

$$P(n+1) = kP(n)$$

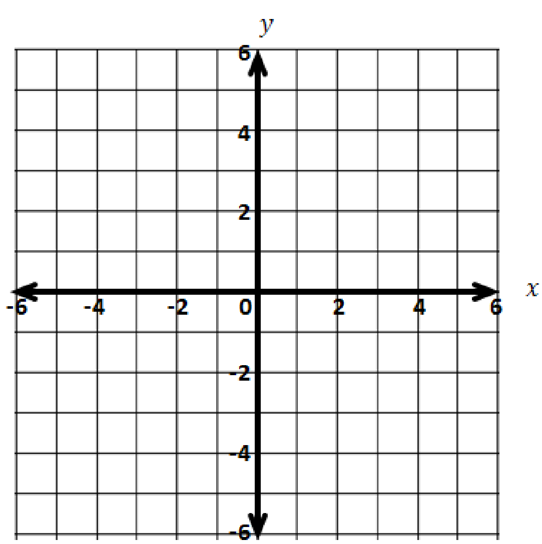
is true for each positive whole number n. What is the value of k?

5. The graph of a piecewise function f is shown to the right. The domain of f is $-3 \leq x \leq 3$.

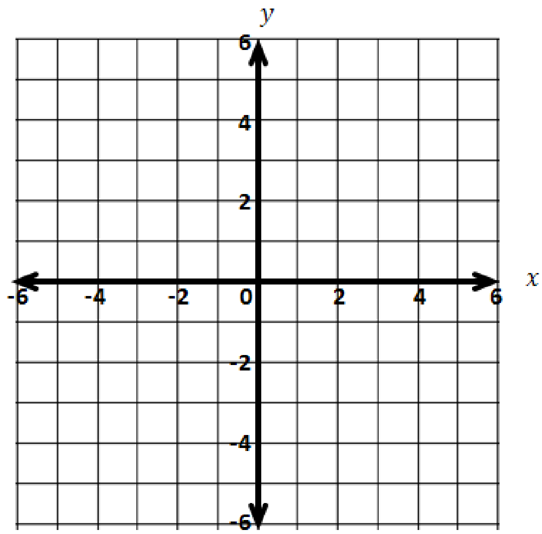
   a. Create an algebraic representation for f. Assume that the graph of f is composed of straight line segments.



   b. Sketch the graph of $y = 2f(x)$, and state the domain and range.



c. Sketch the graph of $y = f(2x)$ and state the domain and range.



d. How does the range of $y = f(x)$ compare to the range of $y = kf(x)$, where $k > 1$?

e. How does the domain of $y = f(x)$ compare to the domain of $y = f(kx)$, where $k > 1$?