Lesson 1: Modeling with Polynomials — The Cardboard Box Problem

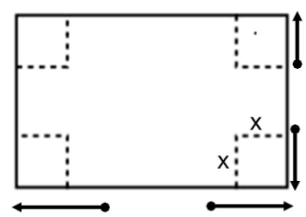
Discussion

1. What is the most important characteristic of a cardboard box?

Mathematical Modeling Exercise

Suppose your company, *EcoBox*, is looking to create an open-topped box with the greatest volume possible. As lead designers, your group is tasked with determining what the dimensions should be. The grid you are starting with is 25 by 13.

The net for the open box, is shown on the right. Each group will be assigned a specific size square to cut out of each corner. Then we'll gather the data from each group to determine the dimensions for the box with the greatest volume.



2. A. Make a prediction about the dimensions that will give the greatest volume. Remember we are limited to a 25 by 13 grid.

My dimensions guess: _______ length

_______ width

_______ height

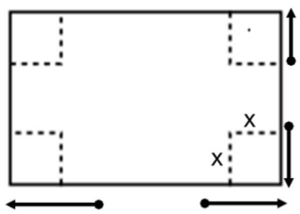
_______ volume

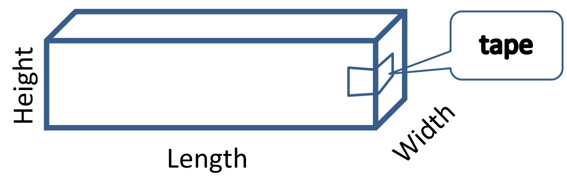
B. Why would a company want the greatest volume of a box?

Your group will need: Grid Sheet Handout, scissors, tape

3. A. With your group, create a box from the Grid Sheet Handout with your assigned size of square to use. Cut out the 25 x 13 grid on the handout, and then cut out congruent squares from each corner. Fold the sides to create an open-topped box as shown in the figure below. Pro Tip: It is easier to determine the dimensions if the grid lines are on the **outside** of the box. Finally, tape the corners so that the box can stand alone.

   B. Determine the volume of your group's box.





4. Record your box's data and the data from all other groups in your class. Don't worry about the last row for now.

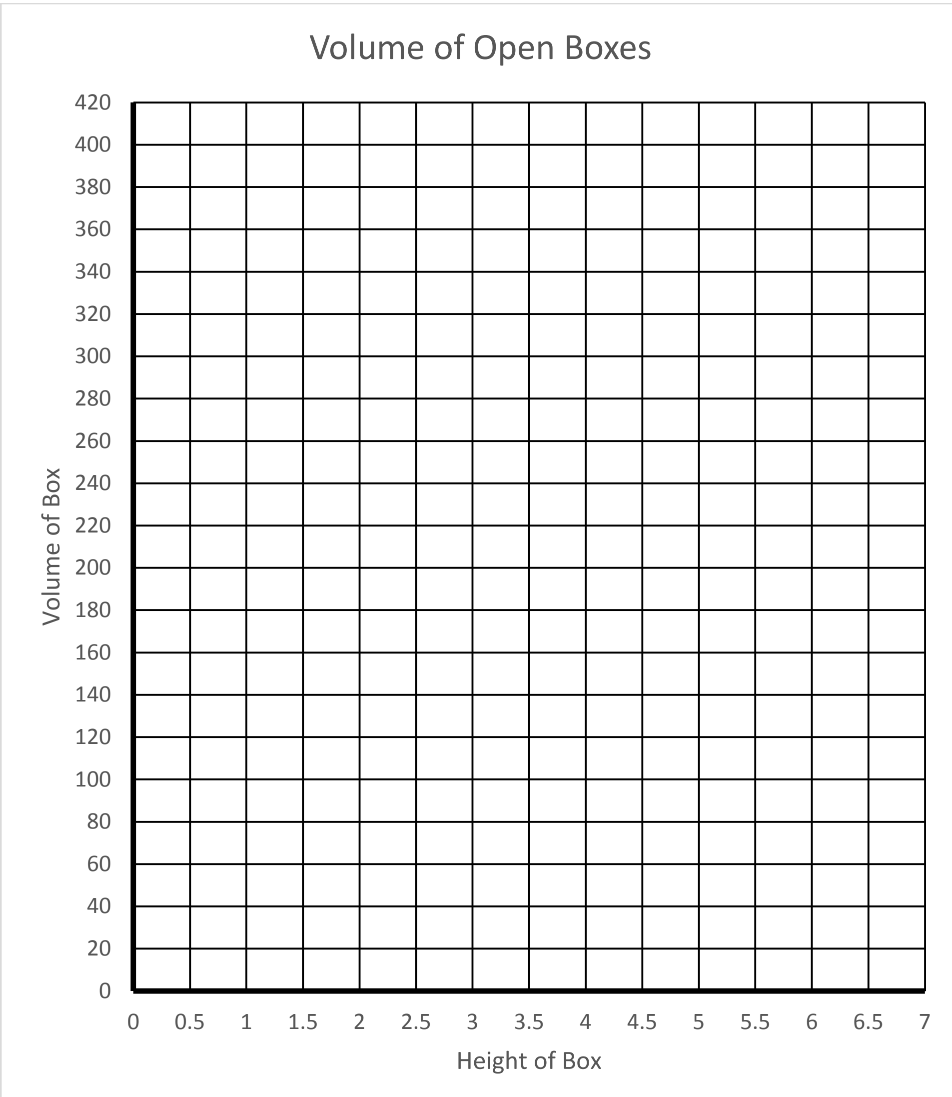
Group	Size of Cut Out Square	Length	Width	Height	Volume
1	1 x 1				
2	1.5 x 1.5				
3	2 x 2				
4	2.5 x 2.5				
5	3 x 3				
6	3.5 x 3.5				
7	4 x 4				
8	4.5 x 4.5				
9	5 x 5				
10	6 x 6				
	n x n				

5. A. Can you tell from the table which dimensions give the greatest volume? Explain your thinking.

   B. How close is your prediction from Exercise 2 to the results from the table?

To better see the patterns in the data, we'll create a scatterplot and see if our prediction of the box with greatest volume is correct.

6. Use the grid below to plot the height of the box versus the volume of the box.



The Invention of the Cardboard Box

The Chinese invented cardboard in the 1600s. The English played off that invention and created the first commercial cardboard box in 1817. Pleated paper, an early form of corrugated board, initially served as lining for men's hats. By the 1870s, corrugated cardboard cushioned delicate glassware during shipment. Stronger, lined corrugated cardboard soon followed. American Robert Gair produced the first really efficient cardboard box in 1879. His die-cut and scored box could be stored flat and then easily folded for use. Refinements followed, enabling cardboard cartons to substitute for labor-intensive, space-consuming, and weighty wooden boxes and crates. Since then, cardboard boxes have been widely appreciated for being strong, light, inexpensive, and recyclable.

[source: http://www.toyhalloffame.org/toys/cardboard-box]

7. Do you believe your prediction from Exercise 5 is correct based on the graph? Explain your thinking.

8. **Discussion** – What type of function is this? Explain your reasoning.

9. A. What would the volume of the open box be if the square cut out was 0.5 x 0.5? Use the graph to make a prediction and then verify your prediction by thinking about the patterns in the table.

B. Mark the height and volume of the 0.5 x 0.5 square on the graph.

10. What would be the volume of an open box if the height was 0? Mark this point on the graph.

11. At what other point would the volume be 0?

12. With your group, determine a rule that could be used to determine the dimensions of any size square that was cut out of the grid. Enter your rule in the last row of the table from Exercise 4. You'll use these rules in the next lesson.

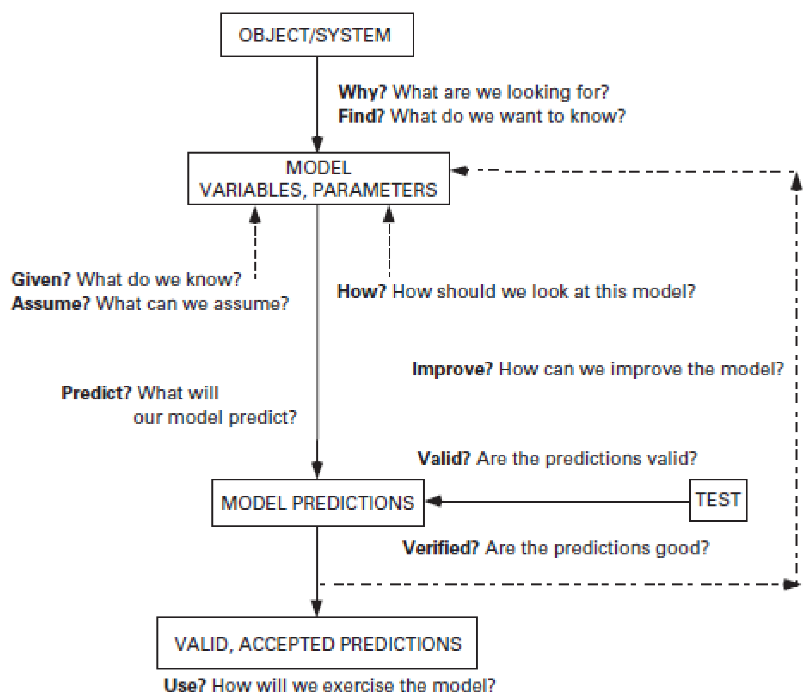
Lesson Summary

Mathematical modeling is when you use ____________________, ________________________, and/or ____________________ to represent real world situations.

Since the modeling of devices and phenomena is essential to both engineering and science, engineers and scientists have very practical reasons for doing mathematical modeling.

[source: http://www.sfu.ca/~vdabbagh/Chap1-modeling.pdf]

MODELING PRINCIPLES

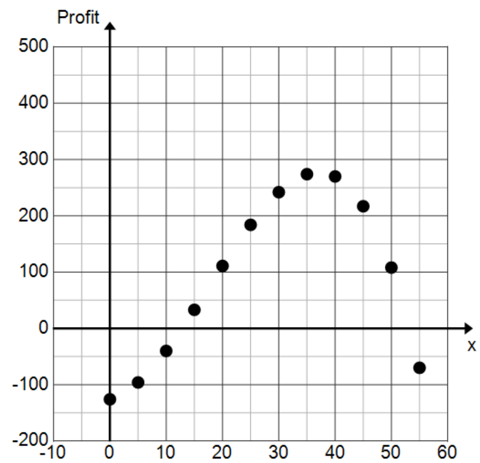


13. Read over the Modeling Principles flowchart above. Then highlight all of steps you completed in this lesson.

Homework Problem Set

1. For a fundraiser, members of the math club decide to make and sell "Pythagoras may have been Fermat's first problem but not his last" t-shirts. They are trying to decide how many t-shirts to make and sell at a fixed price.

They surveyed the level of interest of students around school and made a scatterplot of the number of t-shirts sold (x) versus profit shown at the right.



A. Identify the y-intercept. Interpret its meaning within the context of this problem.

B. If we model this data with a function, what point on the graph represents the number of t-shirts they need to sell in order to break even? Why?

C. What is the smallest number of t-shirts they can sell and still make a profit?

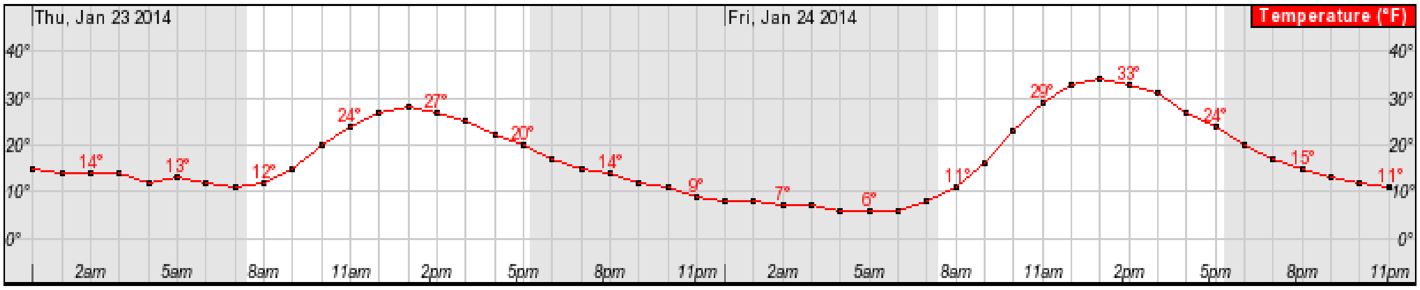
D. How many t-shirts should they sell in order to maximize the profit?

E. What is the maximum profit?

F. What factors would affect the profit?

G. What would cause the profit to start decreasing?

2. The following graph shows the temperature in Aspen, Colorado during a 48-hour period beginning at midnight on Thursday, January 21, 2014. (Source: National Weather Service)



A. Let T be the function that represents the temperature, in degrees Fahrenheit, as a function of time t, in hours. If we let $t = 0$ correspond to midnight on Thursday, interpret the meaning of $T(5)$. What is $T(5)$?

B. What are the relative maximum values? Interpret their meanings.

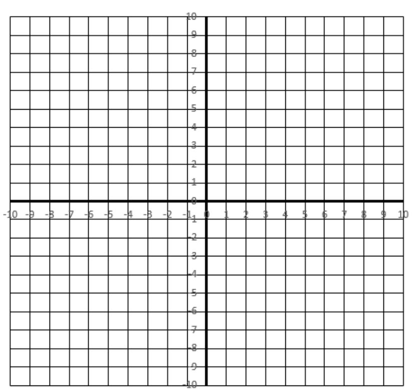
Review

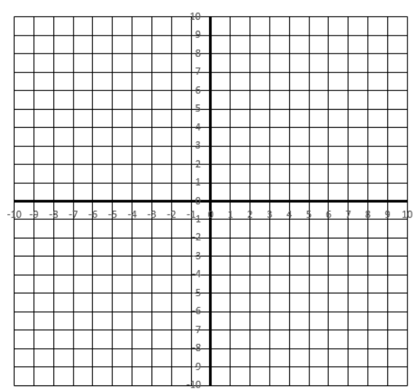
3. Graph each function on the grid provided. You may use what you know about the functions or you can create a table of values to complete the graphs. Then identify the type of graph each one is.

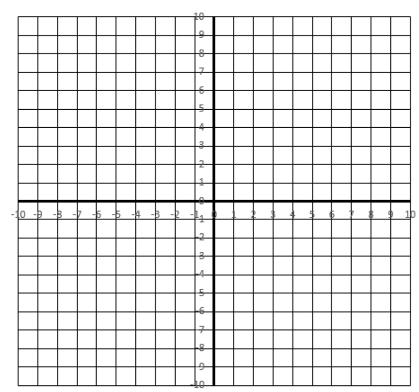
A. $y = (x-3)^2 + 1$

B. $y = -\frac{2}{3}x + 2$

C. $y = \sqrt{x+1}$







	Rule in Symbols	Example
Product Rule	$x^m \bullet x^n = x^{m+n}$	$x^7 \bullet x^3 = x^{10}$
Quotient Rule	$\dfrac{x^m}{x^n} = x^{m-n}$	$\dfrac{x^7}{x^3} = x^4$
Power Rule	$\left(x^m\right)^n = x^{m \bullet n}$	$\left(x^7\right)^3 = x^{21}$
Negative Exponent Rule	$x^{-m} = \dfrac{1}{x^m}$	$x^{-7} = \dfrac{1}{x^7}$
Zero Power Rule	$x^0 = 1$	$(xyz)^0 = 1$

4. Use the rules above to simplify each expression.

A. $x^6 \bullet x^2 \bullet x^3$

B. $y^{-2} \bullet y^2$

C. $\left(m^2 n^3\right)^4$

D. $\left(2g^3 h^{-1}\right)^{-2}$

E. $\dfrac{t^6 u^2}{t^3 u}$

F. $\dfrac{a^2 b^{-3} c d^4}{a^{-1} b^0 c^3}$

5. Write an integer in the empty space to make a true equation.

A. $\left(\dfrac{a^3 \bullet a^2}{a^7}\right)^2 = a^{\_\_} \bullet a^5$

B. $\left(\dfrac{b^{-3} \bullet b^4}{b^0}\right)^{-1} = \dfrac{b^{\_\_} \bullet b^3}{b^2}$

C. $\left(c^{x+y}\right)^2 = c^{2x} \bullet c^{\_\_}$