LESSON 10 SOLVING TRIGONOMETRIC EQUATIONS

Equations which are solved in this lesson (Click on the number):

1. $\cos\theta = -\dfrac{\sqrt{3}}{2}$
2. $2\sin\alpha - \sqrt{2} = 0$
3. $\sqrt{3}\cot\beta + 1 = 0$
4. $3\sec x - 4 = 2$
5. $14\sin\left(\dfrac{\theta}{3}\right) + 12 = 5$
6. $\tan\left(5\alpha + \dfrac{\pi}{3}\right) = 1$
7. $\csc(4\beta - 280°) = -\dfrac{2}{\sqrt{3}}$
8. $\cos 6\alpha = 0$
9. $\sec^2\theta - 2 = 0$
10. $4\sin^2\beta + 9 = 12$
11. $\cos\alpha(\csc\alpha - 2) = 0$
12. $\sqrt{3}\sin x\tan x = \sin x$
13. $(\cos\beta + 1)(\sin\beta - 1) = 0$
14. $2\cos^2\theta - \cos\theta - 1 = 0$
15. $6\sin^2\alpha + 7\sin\alpha - 5 = 0$
16. $\cos^2\beta + 24 = 11\cos\beta$

Additional examples worked in this lesson:

Example Find all the approximate solutions (in degrees) between $-360°$ and $540°$ for the equation $\tan\theta = -\dfrac{5}{4}$.

Example Find all the approximate solutions (in degrees) between $-270°$ and $630°$ for the equation $3\sec 2\alpha - 7 = 0$.

Example Find all the approximate solutions (in degrees) between $-360°$ and $360°$ for the equation $9\sin(3\beta + 150°) + 4 = 0$.

Example Find all the exact and approximate solutions to the equation $\tan^2\theta - 6\tan\theta - 11 = 0$.

Examples Find all the exact solutions for the following equations.

1. $\cos\theta = -\dfrac{\sqrt{3}}{2}$

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First, determine where the solutions θ will occur. Since $-\dfrac{\sqrt{3}}{2}$ is not the minimum negative number for the cosine function, then the solutions do not occur at the coordinate axes. Thus, the solutions occur in two of the four quadrants. Since cosine is negative in the II and III quadrants, the solutions for this equation occur in those quadrants.

Find the reference angle θ' for the solutions θ:

$$\cos\theta = -\frac{\sqrt{3}}{2} \;\Rightarrow\; \cos\theta' = \frac{\sqrt{3}}{2} \;\Rightarrow\; \theta' = \cos^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{6}$$

The solutions in the II quadrant: The one solution in the II quadrant, that is between 0 and 2π, is $\theta = \pi - \dfrac{\pi}{6} = \dfrac{5\pi}{6}$. Now, all the other solutions in the II quadrant are coterminal to this one solution. Thus, all the solutions in the II quadrant are given by $\theta = \dfrac{5\pi}{6} + 2n\pi$, where *n* is an integer.

Thus, some of the solutions in the II quadrant that are obtained by using $\theta = \dfrac{5\pi}{6} + 2n\pi$, where *n* is an integer, are:

$$n = 0: \quad \theta = \frac{5\pi}{6} + 0 = \frac{5\pi}{6}$$

$$n = 1: \quad \theta = \frac{5\pi}{6} + 2\pi = \frac{5\pi}{6} + \frac{12\pi}{6} = \frac{17\pi}{6}$$

$$n = 2: \quad \theta = \frac{5\pi}{6} + 4\pi = \frac{5\pi}{6} + \frac{24\pi}{6} = \frac{29\pi}{6}$$

$$n = 3: \quad \theta = \frac{5\pi}{6} + 6\pi = \frac{5\pi}{6} + \frac{36\pi}{6} = \frac{41\pi}{6}$$

$$n = 4: \quad \theta = \frac{5\pi}{6} + 8\pi = \frac{5\pi}{6} + \frac{48\pi}{6} = \frac{53\pi}{6}$$

$$\vdots$$

$$n = -1: \quad \theta = \frac{5\pi}{6} - 2\pi = \frac{5\pi}{6} - \frac{12\pi}{6} = -\frac{7\pi}{6}$$

$$n = -2: \quad \theta = \frac{5\pi}{6} - 4\pi = \frac{5\pi}{6} - \frac{24\pi}{6} = -\frac{19\pi}{6}$$

$$n = -3: \quad \theta = \frac{5\pi}{6} - 6\pi = \frac{5\pi}{6} - \frac{36\pi}{6} = -\frac{31\pi}{6}$$

$$n = -4: \quad \theta = \frac{5\pi}{6} - 8\pi = \frac{5\pi}{6} - \frac{48\pi}{6} = -\frac{43\pi}{6}$$

$$\vdots$$

The solutions in the III quadrant: The one solution in the III quadrant, that is between 0 and 2π, is $\theta = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$. Now, all the other solutions in the III quadrant are coterminal to this one solution. Thus, all the solutions in the III quadrant are given by $\theta = \frac{7\pi}{6} + 2n\pi$, where *n* is an integer.

Thus, some of the solutions in the III quadrant that are obtained by using $\theta = \frac{7\pi}{6} + 2n\pi$, where *n* is an integer, are:

$$n = 0: \quad \theta = \frac{7\pi}{6} + 0 = \frac{7\pi}{6}$$

$$n = 1: \quad \theta = \frac{7\pi}{6} + 2\pi = \frac{7\pi}{6} + \frac{12\pi}{6} = \frac{19\pi}{6}$$

$$n = 2: \quad \theta = \frac{7\pi}{6} + 4\pi = \frac{7\pi}{6} + \frac{24\pi}{6} = \frac{31\pi}{6}$$

$$n = 3: \quad \theta = \frac{7\pi}{6} + 6\pi = \frac{7\pi}{6} + \frac{36\pi}{6} = \frac{43\pi}{6}$$

$$n = 4: \quad \theta = \frac{7\pi}{6} + 8\pi = \frac{7\pi}{6} + \frac{48\pi}{6} = \frac{55\pi}{6}$$

$$\vdots$$

$$n = -1: \quad \theta = \frac{7\pi}{6} - 2\pi = \frac{7\pi}{6} - \frac{12\pi}{6} = -\frac{5\pi}{6}$$

$$n = -2: \quad \theta = \frac{7\pi}{6} - 4\pi = \frac{7\pi}{6} - \frac{24\pi}{6} = -\frac{17\pi}{6}$$

$$n = -3: \quad \theta = \frac{7\pi}{6} - 6\pi = \frac{7\pi}{6} - \frac{36\pi}{6} = -\frac{29\pi}{6}$$

$$n = -4: \quad \theta = \frac{7\pi}{6} - 8\pi = \frac{7\pi}{6} - \frac{48\pi}{6} = -\frac{41\pi}{6}$$

$$\vdots$$

Answers: $\theta = \frac{5\pi}{6} + 2n\pi$; $\theta = \frac{7\pi}{6} + 2n\pi$, where *n* is an integer

2. $2\sin\alpha - \sqrt{2} = 0$

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First, do the necessary algebra to isolate $\sin\alpha$ on one side of the equation:

$$2\sin\alpha - \sqrt{2} = 0 \Rightarrow 2\sin\alpha = \sqrt{2} \Rightarrow \sin\alpha = \frac{\sqrt{2}}{2}$$

Now, determine where the solutions α will occur. Since $\frac{\sqrt{2}}{2}$ is not the maximum positive number for the sine function, then the solutions do not occur at the coordinate axes. Thus, the solutions occur in two of the four quadrants. Since sine is positive in the I and II quadrants, the solutions for this equation occur in those quadrants.

Find the reference angle α' for the solutions α:

$$\sin\alpha = \frac{\sqrt{2}}{2} \Rightarrow \sin\alpha' = \frac{\sqrt{2}}{2} \Rightarrow \alpha' = \sin^{-1}\frac{\sqrt{2}}{2} = \frac{\pi}{4}$$

The solutions in the I quadrant: The one solution in the I quadrant, that is between 0 and 2π, is $\alpha = \frac{\pi}{4}$. Now, all the other solutions in the I quadrant are coterminal to this one solution. Thus, all the solutions in the I quadrant are given by $\alpha = \frac{\pi}{4} + 2n\pi$, where *n* is an integer.

Thus, some of the solutions in the I quadrant that are obtained by using $\alpha = \frac{\pi}{4} + 2n\pi$, where *n* is an integer, are:

$$n = 0: \quad \alpha = \frac{\pi}{4} + 0 = \frac{\pi}{4}$$

$$n = 1: \quad \alpha = \frac{\pi}{4} + 2\pi = \frac{\pi}{4} + \frac{8\pi}{4} = \frac{9\pi}{4}$$

$$n = 2: \quad \alpha = \frac{\pi}{4} + 4\pi = \frac{\pi}{4} + \frac{16\pi}{4} = \frac{17\pi}{4}$$

$$n = 3: \quad \alpha = \frac{\pi}{4} + 6\pi = \frac{\pi}{4} + \frac{24\pi}{4} = \frac{25\pi}{4}$$

$$n = 4: \quad \alpha = \frac{\pi}{4} + 8\pi = \frac{\pi}{4} + \frac{32\pi}{4} = \frac{33\pi}{4}$$

$$\vdots$$

$$n = -1: \quad \alpha = \frac{\pi}{4} - 2\pi = \frac{\pi}{4} - \frac{8\pi}{4} = -\frac{7\pi}{4}$$

$$n = -2: \quad \alpha = \frac{\pi}{4} - 4\pi = \frac{\pi}{4} - \frac{16\pi}{4} = -\frac{15\pi}{4}$$

$$n = -3: \quad \alpha = \frac{\pi}{4} - 6\pi = \frac{\pi}{4} - \frac{24\pi}{4} = -\frac{23\pi}{4}$$

$$n = -4: \quad \alpha = \frac{\pi}{4} - 8\pi = \frac{\pi}{4} - \frac{32\pi}{4} = -\frac{31\pi}{4}$$

$$\vdots$$

The solutions in the II quadrant: The one solution in the II quadrant, that is between 0 and 2π, is $\alpha = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$. Now, all the other solutions in the II quadrant are coterminal to this one solution. Thus, all the solutions in the II quadrant are given by $\alpha = \frac{3\pi}{4} + 2n\pi$, where n is an integer.

Thus, some of the solutions in the II quadrant that are obtained by using $\alpha = \frac{3\pi}{4} + 2n\pi$, where n is an integer, are:

$$n = 0: \qquad \alpha = \frac{3\pi}{4} + 0 = \frac{3\pi}{4}$$

$$n = 1: \qquad \alpha = \frac{3\pi}{4} + 2\pi = \frac{3\pi}{4} + \frac{8\pi}{4} = \frac{11\pi}{4}$$

$$n = 2: \qquad \alpha = \frac{3\pi}{4} + 4\pi = \frac{3\pi}{4} + \frac{16\pi}{4} = \frac{19\pi}{4}$$

$$n = 3: \qquad \alpha = \frac{3\pi}{4} + 6\pi = \frac{3\pi}{4} + \frac{24\pi}{4} = \frac{27\pi}{4}$$

$$n = 4: \qquad \alpha = \frac{3\pi}{4} + 8\pi = \frac{3\pi}{4} + \frac{32\pi}{4} = \frac{35\pi}{4}$$

$$\vdots$$

$$n = -1: \qquad \alpha = \frac{3\pi}{4} - 2\pi = \frac{3\pi}{4} - \frac{8\pi}{4} = -\frac{5\pi}{4}$$

$$n = -2: \qquad \alpha = \frac{3\pi}{4} - 4\pi = \frac{3\pi}{4} - \frac{16\pi}{4} = -\frac{13\pi}{4}$$

$$n = -3: \quad \alpha = \frac{3\pi}{4} - 6\pi = \frac{3\pi}{4} - \frac{24\pi}{4} = -\frac{21\pi}{4}$$

$$n = -4: \quad \alpha = \frac{3\pi}{4} - 8\pi = \frac{3\pi}{4} - \frac{32\pi}{4} = -\frac{29\pi}{4}$$

$$\vdots$$

Answers: $\alpha = \dfrac{\pi}{4} + 2n\pi$; $\alpha = \dfrac{3\pi}{4} + 2n\pi$, where *n* is an integer

3. $\sqrt{3}\cot\beta + 1 = 0$ Back to Equations List

First, do the necessary algebra to isolate $\cot\beta$ on one side of the equation:

$$\sqrt{3}\cot\beta + 1 = 0 \;\Rightarrow\; \sqrt{3}\cot\beta = -1 \;\Rightarrow\; \cot\beta = -\frac{1}{\sqrt{3}}$$

Now, take the reciprocal of both sides of this equation to obtain the equation $\tan\beta = -\sqrt{3}$.

Now, determine where the solutions β will occur. Since tangent is negative in the II and IV quadrants, the solutions for this equation occur in those quadrants.

Find the reference angle β' for the solutions β:

$$\tan\beta = -\sqrt{3} \;\Rightarrow\; \tan\beta' = \sqrt{3} \;\Rightarrow\; \beta' = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$$

The solutions in the II quadrant: The one solution in the II quadrant, that is between 0 and 2π, is $\beta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$. Now, all the other solutions in the II quadrant are coterminal to this one solution. Thus, all the solutions in the II quadrant are given by $\beta = \frac{2\pi}{3} + 2n\pi$, where *n* is an integer.

Thus, some of the solutions in the II quadrant that are obtained by using $\beta = \frac{2\pi}{3} + 2n\pi$, where *n* is an integer, are:

$$n = 0: \qquad \beta = \frac{2\pi}{3} + 0 = \frac{2\pi}{3}$$

$$n = 1: \qquad \beta = \frac{2\pi}{3} + 2\pi = \frac{2\pi}{3} + \frac{6\pi}{3} = \frac{8\pi}{3}$$

$$n = 2: \qquad \beta = \frac{2\pi}{3} + 4\pi = \frac{2\pi}{3} + \frac{12\pi}{3} = \frac{14\pi}{3}$$

$$n = 3: \qquad \beta = \frac{2\pi}{3} + 6\pi = \frac{2\pi}{3} + \frac{18\pi}{3} = \frac{20\pi}{3}$$

$$n = 4: \qquad \beta = \frac{2\pi}{3} + 8\pi = \frac{2\pi}{3} + \frac{24\pi}{3} = \frac{26\pi}{3}$$

$$\vdots$$

$$n = -1: \qquad \beta = \frac{2\pi}{3} - 2\pi = \frac{2\pi}{3} - \frac{6\pi}{3} = -\frac{4\pi}{3}$$

$$n = -2: \qquad \beta = \frac{2\pi}{3} - 4\pi = \frac{2\pi}{3} - \frac{12\pi}{3} = -\frac{10\pi}{3}$$

$$n = -3: \quad \beta = \frac{2\pi}{3} - 6\pi = \frac{2\pi}{3} - \frac{18\pi}{3} = -\frac{16\pi}{3}$$

$$n = -4: \quad \beta = \frac{2\pi}{3} - 8\pi = \frac{2\pi}{3} - \frac{24\pi}{3} = -\frac{22\pi}{3}$$

$$\vdots$$

The solutions in the IV quadrant: The one solution in the IV quadrant, that is between 0 and 2π, is $\beta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$. Now, all the other solutions in the IV quadrant are coterminal to this one solution. Thus, all the solutions in the IV quadrant are given by $\beta = \frac{5\pi}{3} + 2n\pi$, where *n* is an integer.

Thus, some of the solutions in the IV quadrant that are obtained by using $\beta = \frac{5\pi}{3} + 2n\pi$, where *n* is an integer, are:

$$n = 0: \quad \beta = \frac{5\pi}{3} + 0 = \frac{5\pi}{3}$$

$$n = 1: \quad \beta = \frac{5\pi}{3} + 2\pi = \frac{5\pi}{3} + \frac{6\pi}{3} = \frac{11\pi}{3}$$

$$n = 2: \quad \beta = \frac{5\pi}{3} + 4\pi = \frac{5\pi}{3} + \frac{12\pi}{3} = \frac{17\pi}{3}$$

$$n = 3: \quad \beta = \frac{5\pi}{3} + 6\pi = \frac{5\pi}{3} + \frac{18\pi}{3} = \frac{23\pi}{3}$$

$$n = 4: \quad \beta = \frac{5\pi}{3} + 8\pi = \frac{5\pi}{3} + \frac{24\pi}{3} = \frac{29\pi}{3}$$

$$\vdots$$

$$n = -1: \quad \beta = \frac{5\pi}{3} - 2\pi = \frac{5\pi}{3} - \frac{6\pi}{3} = -\frac{\pi}{3}$$

$$n = -2: \quad \beta = \frac{5\pi}{3} - 4\pi = \frac{5\pi}{3} - \frac{12\pi}{3} = -\frac{7\pi}{3}$$

$$n = -3: \quad \beta = \frac{5\pi}{3} - 6\pi = \frac{5\pi}{3} - \frac{18\pi}{3} = -\frac{13\pi}{3}$$

$$n = -4: \quad \beta = \frac{5\pi}{3} - 8\pi = \frac{5\pi}{3} - \frac{24\pi}{3} = -\frac{19\pi}{3}$$

$$\vdots$$

Answers: $\beta = \frac{2\pi}{3} + 2n\pi \; ; \quad \beta = \frac{5\pi}{3} + 2n\pi$, where n is an integer

NOTE: Since the period of the tangent function is π, then these two answers may also be written as the one answer $\beta = \frac{2\pi}{3} + n\pi$, where n is an integer.

4. $3\sec x - 4 = 2$ Back to Equations List

First, do the necessary algebra to isolate $\sec x$ on one side of the equation:

$$3\sec x - 4 = 2 \Rightarrow 3\sec x = 6 \Rightarrow \sec x = 2$$

Now, take the reciprocal of both sides of this equation to obtain the equation $\cos x = \frac{1}{2}$.

Now, determine where the solutions x will occur. Since $\frac{1}{2}$ is not the maximum positive number for the cosine function, then the solutions do not occur at the coordinate axes. Thus, the solutions occur in two of the four quadrants. Since cosine is positive in the I and IV quadrants, the solutions for this equation occur in those quadrants.

Find the reference angle x' for the solutions x:

$$\cos x = \frac{1}{2} \quad \Rightarrow \quad \cos x' = \frac{1}{2} \quad \Rightarrow \quad x' = \cos^{-1} \frac{1}{2} = \frac{\pi}{3}$$

The solutions in the I quadrant: The one solution in the I quadrant, that is between 0 and 2π, is $x = \frac{\pi}{3}$. Now, all the other solutions in the I quadrant are coterminal to this one solution. Thus, all the solutions in the I quadrant are given by $x = \frac{\pi}{3} + 2n\pi$, where *n* is an integer.

The solutions in the IV quadrant: The one solution in the IV quadrant, that is between 0 and 2π, is $x = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$. Now, all the other solutions in the IV quadrant are coterminal to this one solution. Thus, all the solutions in the IV quadrant are given by $x = \frac{5\pi}{3} + 2n\pi$, where *n* is an integer.

Answers: $x = \frac{\pi}{3} + 2n\pi$; $x = \frac{5\pi}{3} + 2n\pi$, where *n* is an integer

5. $14\sin\left(\frac{\theta}{3}\right) + 12 = 5$

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First, we must get the angle argument for the trigonometric function to be a single variable. So, let $u = \frac{\theta}{3}$. Thus, we obtain the equation $14\sin u + 12 = 5$. Now, do the necessary algebra to isolate $\sin u$ on one side of the equation:

$$14\sin u + 12 = 5 \Rightarrow 14\sin u = -7 \Rightarrow \sin u = -\frac{1}{2}$$

Now, determine where the solutions u will occur. Since $-\frac{1}{2}$ is not the minimum negative number for the sine function, then the solutions do not occur at the coordinate axes. Thus, the solutions occur in two of the four quadrants. Since sine is negative in the III and IV quadrants, the solutions for this equation occur in those quadrants.

Find the reference angle u' for the solutions u:

$$\sin u = -\frac{1}{2} \Rightarrow \sin u' = \frac{1}{2} \Rightarrow u' = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$$

The solutions in the III quadrant: The one solution u in the III quadrant, that is between 0 and 2π, is $u = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$. Now, all the other solutions u in the III quadrant are coterminal to this one solution. Thus, all the solutions u in the III quadrant are given by $u = \frac{7\pi}{6} + 2n\pi$, where n is an integer. These are the solutions for u. Now, find the solutions for θ. Since $u = \frac{\theta}{3}$ and $u = \frac{7\pi}{6} + 2n\pi$, then $\frac{\theta}{3} = \frac{7\pi}{6} + 2n\pi$. Now, multiply both sides of this equation by 3 in order to solve for θ. Thus, $\theta = \frac{7\pi}{2} + 6n\pi$, where n is an integer.

Thus, some of the solutions in the III quadrant that are obtained by using $\theta = \frac{7\pi}{2} + 6n\pi$, where n is an integer, are:

$n = 0$: $\theta = \frac{7\pi}{2} + 0 = \frac{7\pi}{2}$

NOTE: $\frac{\theta}{3} = \frac{7\pi}{6}$ is in the III quadrant.

$n = 1$: $\theta = \frac{7\pi}{2} + 6\pi = \frac{7\pi}{2} + \frac{12\pi}{2} = \frac{19\pi}{2}$

NOTE: $\frac{\theta}{3} = \frac{19\pi}{6} = 2\pi + \frac{7\pi}{6}$ is in the III quadrant.

$n = 2$: $\theta = \frac{7\pi}{2} + 12\pi = \frac{7\pi}{2} + \frac{24\pi}{2} = \frac{31\pi}{2}$

NOTE: $\frac{\theta}{3} = \frac{31\pi}{6} = 4\pi + \frac{7\pi}{6}$ is in the III quadrant.

$n = 3$: $\theta = \frac{7\pi}{2} + 18\pi = \frac{7\pi}{2} + \frac{36\pi}{2} = \frac{43\pi}{2}$

NOTE: $\frac{\theta}{3} = \frac{43\pi}{6} = 6\pi + \frac{7\pi}{6}$ is in the III quadrant.

$n = 4$: $\theta = \frac{7\pi}{2} + 24\pi = \frac{7\pi}{2} + \frac{48\pi}{2} = \frac{55\pi}{2}$

NOTE: $\frac{\theta}{3} = \frac{55\pi}{6} = 8\pi + \frac{7\pi}{6}$ is in the III quadrant.

$$\vdots$$

$n = -1$: $\theta = \frac{7\pi}{2} - 6\pi = \frac{7\pi}{2} - \frac{12\pi}{2} = -\frac{5\pi}{2}$

NOTE: $\dfrac{\theta}{3} = -\dfrac{5\pi}{6}$ is in the III quadrant.

$n = -2$: $\theta = \dfrac{7\pi}{2} - 12\pi = \dfrac{7\pi}{2} - \dfrac{24\pi}{2} = -\dfrac{17\pi}{2}$

NOTE: $\dfrac{\theta}{3} = -\dfrac{17\pi}{6} = -2\pi - \dfrac{5\pi}{6}$ is in the III quadrant.

$n = -3$: $\theta = \dfrac{7\pi}{2} - 18\pi = \dfrac{7\pi}{2} - \dfrac{36\pi}{2} = -\dfrac{29\pi}{2}$

NOTE: $\dfrac{\theta}{3} = -\dfrac{29\pi}{6} = -4\pi - \dfrac{5\pi}{6}$ is in the III quadrant.

$n = -4$: $\theta = \dfrac{7\pi}{2} - 24\pi = \dfrac{7\pi}{2} - \dfrac{48\pi}{2} = -\dfrac{41\pi}{2}$

NOTE: $\dfrac{\theta}{3} = -\dfrac{41\pi}{6} = -6\pi - \dfrac{5\pi}{6}$ is in the III quadrant.

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The solutions in the IV quadrant: The one solution *u* in the IV quadrant, that is between 0 and 2π, is $u = 2\pi - \dfrac{\pi}{6} = \dfrac{11\pi}{6}$. Now, all the other solutions *u* in the IV quadrant are coterminal to this one solution. Thus, all the solutions *u* in the IV quadrant are given by $u = \dfrac{11\pi}{6} + 2n\pi$, where *n* is an integer. These are the solutions for *u*. Now, find the solutions for θ. Since $u = \dfrac{\theta}{3}$ and $u = \dfrac{11\pi}{6} + 2n\pi$, then $\dfrac{\theta}{3} = \dfrac{11\pi}{6} + 2n\pi$. Now, multiply both sides of this equation by 3 in order to solve for θ. Thus, $\theta = \dfrac{11\pi}{2} + 6n\pi$, where *n* is an integer.

Thus, some of the solutions in the IV quadrant that are obtained by using $\theta = \frac{11\pi}{2} + 6n\pi$, where n is an integer, are:

$n = 0$: $\theta = \frac{11\pi}{2} + 0 = \frac{11\pi}{2}$

NOTE: $\frac{\theta}{3} = \frac{11\pi}{6}$ is in the IV quadrant.

$n = 1$: $\theta = \frac{11\pi}{2} + 6\pi = \frac{11\pi}{2} + \frac{12\pi}{2} = \frac{23\pi}{2}$

NOTE: $\frac{\theta}{3} = \frac{23\pi}{6} = 2\pi + \frac{11\pi}{6}$ is in the IV quadrant.

$n = 2$: $\theta = \frac{11\pi}{2} + 12\pi = \frac{11\pi}{2} + \frac{24\pi}{2} = \frac{35\pi}{2}$

NOTE: $\frac{\theta}{3} = \frac{35\pi}{6} = 4\pi + \frac{11\pi}{6}$ is in the IV quadrant.

$n = 3$: $\theta = \frac{11\pi}{2} + 18\pi = \frac{11\pi}{2} + \frac{36\pi}{2} = \frac{47\pi}{2}$

NOTE: $\frac{\theta}{3} = \frac{47\pi}{6} = 6\pi + \frac{11\pi}{6}$ is in the IV quadrant.

$n = 4$: $\theta = \frac{11\pi}{2} + 24\pi = \frac{11\pi}{2} + \frac{48\pi}{2} = \frac{59\pi}{2}$

NOTE: $\frac{\theta}{3} = \frac{59\pi}{6} = 8\pi + \frac{11\pi}{6}$ is in the IV quadrant.

$\vdots$

$n = -1$: $\theta = \frac{11\pi}{2} - 6\pi = \frac{11\pi}{2} - \frac{12\pi}{2} = -\frac{\pi}{2}$

NOTE: $\frac{\theta}{3} = -\frac{\pi}{6}$ is in the IV quadrant.

$n = -2$: $\theta = \frac{11\pi}{2} - 12\pi = \frac{11\pi}{2} - \frac{24\pi}{2} = -\frac{13\pi}{2}$

NOTE: $\frac{\theta}{3} = -\frac{13\pi}{6} = -2\pi - \frac{\pi}{6}$ is in the IV quadrant.

$n = -3$: $\theta = \frac{11\pi}{2} - 18\pi = \frac{11\pi}{2} - \frac{36\pi}{2} = -\frac{25\pi}{2}$

NOTE: $\frac{\theta}{3} = -\frac{25\pi}{6} = -4\pi - \frac{\pi}{6}$ is in the IV quadrant.

$n = -4$: $\theta = \frac{11\pi}{2} - 24\pi = \frac{11\pi}{2} - \frac{48\pi}{2} = -\frac{37\pi}{2}$

NOTE: $\frac{\theta}{3} = -\frac{37\pi}{6} = -6\pi - \frac{\pi}{6}$ is in the IV quadrant.

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Answers: $\theta = \frac{7\pi}{2} + 6n\pi$; $\theta = \frac{11\pi}{2} + 6n\pi$, where n is an integer

6. $\tan\left(5\alpha + \frac{\pi}{3}\right) = 1$ Back to Equations List

First, we must get the angle argument for the trigonometric function to be a single variable. So, let $u = 5\alpha + \frac{\pi}{3}$. Thus, we obtain the equation $\tan u = 1$.

Now, determine where the solutions u will occur. Since tangent is positive in the I and III quadrants, the solutions for this equation occur in those quadrants.

Find the reference angle u' for the solutions u:

$$\tan u = 1 \Rightarrow \quad \tan u' = 1 \Rightarrow \quad u' = \tan^{-1} 1 = \frac{\pi}{4}$$

The solutions in the I quadrant: The one solution *u* in the I quadrant, that is between 0 and 2π, is $u = \frac{\pi}{4}$. Now, all the other solutions *u* in the I quadrant are coterminal to this one solution. Thus, all the solutions *u* in the I quadrant are given by $u = \frac{\pi}{4} + 2n\pi$, where *n* is an integer. These are the solutions for *u*. Now, find the solutions for α. Since $u = 5\alpha + \frac{\pi}{3}$ and $u = \frac{\pi}{4} + 2n\pi$, then $5\alpha + \frac{\pi}{3} = \frac{\pi}{4} + 2n\pi$. First, subtract $\frac{\pi}{3}$ from both sides of the equation. Thus, $5\alpha = \frac{\pi}{4} + 2n\pi - \frac{\pi}{3} = \frac{3\pi}{12} + 2n\pi - \frac{4\pi}{12} = -\frac{\pi}{12} + 2n\pi$. Now, divide both sides of this equation by 5 in order to solve for α. Thus, $\alpha = -\frac{\pi}{60} + \frac{2n\pi}{5} = -\frac{\pi}{60} + \frac{24n\pi}{60} = \frac{-\pi + 24n\pi}{60} = \frac{(24n - 1)\pi}{60}$, where *n* is an integer.

Thus, some of the solutions in the I quadrant that are obtained by using $\alpha = \frac{(24n - 1)\pi}{60}$, where *n* is an integer, are:

$n = 0$: $\quad \alpha = \frac{(0 - 1)\pi}{60} = \frac{-\pi}{60} = -\frac{\pi}{60}$

NOTE: $5\alpha + \frac{\pi}{3} = -\frac{\pi}{12} + \frac{4\pi}{12} = \frac{3\pi}{12} = \frac{\pi}{4}$ is in the I quadrant.

$n = 1$: $\alpha = \dfrac{(24 - 1)\pi}{60} = \dfrac{23\pi}{60}$

NOTE: $5\alpha + \dfrac{\pi}{3} = \dfrac{23\pi}{12} + \dfrac{4\pi}{12} = \dfrac{27\pi}{12} = \dfrac{9\pi}{4} = 2\pi + \dfrac{\pi}{4}$
is in the I quadrant.

$n = 2$: $\alpha = \dfrac{(48 - 1)\pi}{60} = \dfrac{47\pi}{60}$

NOTE: $5\alpha + \dfrac{\pi}{3} = \dfrac{47\pi}{12} + \dfrac{4\pi}{12} = \dfrac{51\pi}{12} = \dfrac{17\pi}{4} = 4\pi + \dfrac{\pi}{4}$
is in the I quadrant.

$n = 3$: $\alpha = \dfrac{(72 - 1)\pi}{60} = \dfrac{71\pi}{60}$

NOTE: $5\alpha + \dfrac{\pi}{3} = \dfrac{71\pi}{12} + \dfrac{4\pi}{12} = \dfrac{75\pi}{12} = \dfrac{25\pi}{4} = 6\pi + \dfrac{\pi}{4}$
is in the I quadrant.

$n = 4$: $\alpha = \dfrac{(96 - 1)\pi}{60} = \dfrac{95\pi}{60} = \dfrac{19\pi}{12}$

NOTE: $5\alpha + \dfrac{\pi}{3} = \dfrac{95\pi}{12} + \dfrac{4\pi}{12} = \dfrac{99\pi}{12} = \dfrac{33\pi}{4} = 8\pi + \dfrac{\pi}{4}$
is in the I quadrant.

$\vdots$

$n = -1$: $\alpha = \dfrac{(-24 - 1)\pi}{60} = -\dfrac{25\pi}{60} = -\dfrac{5\pi}{12}$

NOTE: $5\alpha + \dfrac{\pi}{3} = -\dfrac{25\pi}{12} + \dfrac{4\pi}{12} = -\dfrac{21\pi}{12} = -\dfrac{7\pi}{4}$
is in the I quadrant.

$n = -2$: $\alpha = \dfrac{(-48 - 1)\pi}{60} = -\dfrac{49\pi}{60}$

NOTE: $5\alpha + \dfrac{\pi}{3} = -\dfrac{49\pi}{12} + \dfrac{4\pi}{12} = -\dfrac{45\pi}{12} = -\dfrac{15\pi}{4} = -2\pi - \dfrac{7\pi}{4}$ is in the I quadrant.

$n = -3$: $\alpha = \dfrac{(-72 - 1)\pi}{60} = -\dfrac{73\pi}{60}$

NOTE: $5\alpha + \dfrac{\pi}{3} = -\dfrac{73\pi}{12} + \dfrac{4\pi}{12} = -\dfrac{69\pi}{12} = -\dfrac{23\pi}{4} = -4\pi - \dfrac{7\pi}{4}$ is in the I quadrant.

$n = -4$: $\alpha = \dfrac{(-96 - 1)\pi}{60} = -\dfrac{97\pi}{60}$

NOTE: $5\alpha + \dfrac{\pi}{3} = -\dfrac{97\pi}{12} + \dfrac{4\pi}{12} = -\dfrac{93\pi}{12} = -\dfrac{31\pi}{4} = -6\pi - \dfrac{7\pi}{4}$ is in the I quadrant.

$\vdots$

The solutions in the III quadrant: The one solution u in the III quadrant, that is between 0 and 2π, is $u = \pi + \dfrac{\pi}{4} = \dfrac{5\pi}{4}$. Now, all the other solutions u in the III quadrant are coterminal to this one solution. Thus, all the solutions u in the III quadrant are given by $u = \dfrac{5\pi}{4} + 2n\pi$, where n is an integer. These are the solutions for u. Now, find the solutions for α. Since $u = 5\alpha + \dfrac{\pi}{3}$ and $u = \dfrac{5\pi}{4} + 2n\pi$, then $5\alpha + \dfrac{\pi}{3} = \dfrac{5\pi}{4} + 2n\pi$. First, subtract $\dfrac{\pi}{3}$ from both sides of the equation. Thus, $5\alpha = \dfrac{5\pi}{4} + 2n\pi - \dfrac{\pi}{3} = \dfrac{15\pi}{12} + 2n\pi - \dfrac{4\pi}{12} = \dfrac{11\pi}{12} + 2n\pi$. Now, divide both sides of this equation by 5 in order to solve for α. Thus, $\alpha = \dfrac{11\pi}{60} + \dfrac{2n\pi}{5} = \dfrac{11\pi}{60} + \dfrac{24n\pi}{60} = \dfrac{11\pi + 24n\pi}{60} = \dfrac{(24n + 11)\pi}{60}$, where n is an integer.

Thus, some of the solutions in the III quadrant that are obtained by using $\alpha = \dfrac{(24n + 11)\pi}{60}$, where n is an integer, are:

$n = 0$: $\alpha = \dfrac{(0 + 11)\pi}{60} = \dfrac{11\pi}{60}$

NOTE: $5\alpha + \dfrac{\pi}{3} = \dfrac{11\pi}{12} + \dfrac{4\pi}{12} = \dfrac{15\pi}{12} = \dfrac{5\pi}{4}$
is in the III quadrant.

$n = 1$: $\alpha = \dfrac{(24 + 11)\pi}{60} = \dfrac{35\pi}{60} = \dfrac{7\pi}{12}$

NOTE: $5\alpha + \dfrac{\pi}{3} = \dfrac{35\pi}{12} + \dfrac{4\pi}{12} = \dfrac{39\pi}{12} = \dfrac{13\pi}{4} = 2\pi + \dfrac{5\pi}{4}$
is in the III quadrant.

$n = 2$: $\alpha = \dfrac{(48 + 11)\pi}{60} = \dfrac{59\pi}{60}$

NOTE: $5\alpha + \dfrac{\pi}{3} = \dfrac{59\pi}{12} + \dfrac{4\pi}{12} = \dfrac{63\pi}{12} = \dfrac{21\pi}{4} = 4\pi + \dfrac{5\pi}{4}$
is in the III quadrant.

$n = 3$: $\alpha = \dfrac{(72 + 11)\pi}{60} = \dfrac{83\pi}{60}$

NOTE: $5\alpha + \dfrac{\pi}{3} = \dfrac{83\pi}{12} + \dfrac{4\pi}{12} = \dfrac{87\pi}{12} = \dfrac{29\pi}{4} = 6\pi + \dfrac{5\pi}{4}$
is in the III quadrant.

$n = 4$: $\alpha = \dfrac{(96 + 11)\pi}{60} = \dfrac{107\pi}{60}$

NOTE: $5\alpha + \frac{\pi}{3} = \frac{107\pi}{12} + \frac{4\pi}{12} = \frac{111\pi}{12} = \frac{37\pi}{4} = 8\pi + \frac{5\pi}{4}$ is in the III quadrant.

⋮

$n = -1$: $\alpha = \frac{(-24 + 11)\pi}{60} = -\frac{13\pi}{60}$

NOTE: $5\alpha + \frac{\pi}{3} = -\frac{13\pi}{12} + \frac{4\pi}{12} = -\frac{9\pi}{12} = -\frac{3\pi}{4}$ is in the III quadrant.

$n = -2$: $\alpha = \frac{(-48 + 11)\pi}{60} = -\frac{37\pi}{60}$

NOTE: $5\alpha + \frac{\pi}{3} = -\frac{37\pi}{12} + \frac{4\pi}{12} = -\frac{33\pi}{12} = -\frac{11\pi}{4} = -2\pi - \frac{3\pi}{4}$ is in the III quadrant.

$n = -3$: $\alpha = \frac{(-72 + 11)\pi}{60} = -\frac{61\pi}{60}$

NOTE: $5\alpha + \frac{\pi}{3} = -\frac{61\pi}{12} + \frac{4\pi}{12} = -\frac{57\pi}{12} = -\frac{19\pi}{4} = -4\pi - \frac{3\pi}{4}$ is in the III quadrant.

$n = -4$: $\alpha = \frac{(-96 + 11)\pi}{60} = -\frac{85\pi}{60} = -\frac{17\pi}{12}$

NOTE: $5\alpha + \frac{\pi}{3} = -\frac{85\pi}{12} + \frac{4\pi}{12} = -\frac{81\pi}{12} = -\frac{27\pi}{4} = -6\pi - \frac{3\pi}{4}$ is in the III quadrant.

⋮

Answers: $\alpha = \dfrac{(24n - 1)\pi}{60}$; $\alpha = \dfrac{(24n + 11)\pi}{60}$, where n is an integer

NOTE: Since the period of the tangent function is π, then these two answers may also be written as the one answer $\alpha = \dfrac{(12n - 1)\pi}{60}$, where n is an integer. This expression is obtained in the following manner. The solutions in the I quadrant in terms of u were given by $u = \dfrac{\pi}{4} + 2n\pi$, where n is an integer. The solutions in the III quadrant in terms of u were given by $u = \dfrac{3\pi}{4} + 2n\pi$, where n is an integer. These two solutions can be written as $u = \dfrac{\pi}{4} + n\pi$, where n is an integer. Now, find the solutions for α. Since $u = 5\alpha + \dfrac{\pi}{3}$ and $u = \dfrac{\pi}{4} + n\pi$, then $5\alpha + \dfrac{\pi}{3} = \dfrac{\pi}{4} + n\pi$. First, subtract $\dfrac{\pi}{3}$ from both sides of the equation. Thus, $5\alpha = \dfrac{\pi}{4} + n\pi - \dfrac{\pi}{3} = \dfrac{3\pi}{12} + n\pi - \dfrac{4\pi}{12} = -\dfrac{\pi}{12} + n\pi$. Now, divide both sides of this equation by 5 in order to solve for α. Thus, $\alpha = -\dfrac{\pi}{60} + \dfrac{n\pi}{5} = -\dfrac{\pi}{60} + \dfrac{12n\pi}{60} = \dfrac{-\pi + 12n\pi}{60} = \dfrac{(12n - 1)\pi}{60}$, where n is an integer.

7. $\csc(4\beta - 280°) = -\dfrac{2}{\sqrt{3}}$ Back to Equations List

First, we must get the angle argument for the trigonometric function to be a single variable. So, let $u = 4\beta - 280°$. Thus, we obtain the equation $\csc u = -\dfrac{2}{\sqrt{3}}$.

Now, take the reciprocal of both sides of this equation to obtain the equation $\sin u = -\dfrac{\sqrt{3}}{2}$.

Now, determine where the solutions u will occur. Since $-\dfrac{\sqrt{3}}{2}$ is not the minimum negative number for the sine function, then the solutions do not occur at the coordinate axes. Thus, the solutions occur in two of the four quadrants. Since sine is negative in the III and IV quadrants, the solutions for this equation occur in those quadrants.

Find the reference angle u' for the solutions u:

$$\sin u = -\frac{\sqrt{3}}{2} \quad \Rightarrow \quad \sin u' = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad u' = \sin^{-1}\frac{\sqrt{3}}{2} = 60°$$

The solutions in the III quadrant: The one solution u in the III quadrant, that is between $0°$ and $360°$, is $u = 180° + 60° = 240°$. Now, all the other solutions u in the III quadrant are coterminal to this one solution. Thus, all the solutions u in the III quadrant are given by $u = 240° + n \cdot 360°$, where n is an integer. These are the solutions for u. Now, find the solutions for β. Since $u = 4\beta - 280°$ and $u = 240° + n \cdot 360°$, then $4\beta - 280° = 240° + n \cdot 360°$. First, add $280°$ to both sides of the equation. Thus, $4\beta = 240° + n \cdot 360° + 280° = 520° + n \cdot 360°$. Now, divide both sides of this equation by 4 in order to solve for β. Thus, $\beta = 130° + n \cdot 90°$, where n is an integer.

Thus, some of the solutions in the III quadrant that are obtained by using $\beta = 130° + n \cdot 90°$, where n is an integer, are:

$n = 0$: $\beta = 130°$
NOTE: $4\beta - 280° = 520° - 280° = 240°$
is in the III quadrant.

$n = 1$: $\beta = 130° + 90° = 220°$
NOTE: $4\beta - 280° = 880° - 280° = 600° = 360° + 240°$

is in the III quadrant.

$n = 2$: $\beta = 130° + 180° = 310°$
NOTE: $4\beta - 280° = 1240° - 280° = 960° = 720° + 240°$
is in the III quadrant.

$n = 3$: $\beta = 130° + 270° = 400°$
NOTE: $4\beta - 280° = 1600° - 280° = 1320° = 3(360°) + 240°$
is in the III quadrant.

$n = 4$: $\beta = 130° + 360° = 490°$
NOTE: $4\beta - 280° = 1960° - 280° = 1680° = 4(360°) + 240°$
is in the III quadrant.

$\vdots$

$n = -1$: $\beta = 130° - 90° = 40°$
NOTE: $4\beta - 280° = 160° - 280° = -120°$
is in the III quadrant.

$n = -2$: $\beta = 130° - 180° = -50°$
NOTE: $4\beta - 280° = -200° - 280° = -480° = -360° - 120°$
is in the III quadrant.

$n = -3$: $\beta = 130° - 270° = -140°$
NOTE: $4\beta - 280° = -560° - 280° = -840° = -720° - 120°$
is in the III quadrant.

$n = -4$: $\beta = 130° - 360° = -230°$
NOTE: $4\beta - 280° = -920° - 280° = -1200° = 3(-360°) - 120°$
is in the III quadrant.

$\vdots$

⋮

The solutions in the IV quadrant: The one solution u in the IV quadrant, that is between $0°$ and $360°$, is $u = 360° - 60° = 300°$. Now, all the other solutions u in the IV quadrant are coterminal to this one solution. Thus, all the solutions u in the IV quadrant are given by $u = 300° + n \cdot 360°$, where n is an integer. These are the solutions for u. Now, find the solutions for β. Since $u = 4\beta - 280°$ and $u = 300° + n \cdot 360°$, then $4\beta - 280° = 300° + n \cdot 360°$. First, add $280°$ to both sides of the equation. Thus, $4\beta = 300° + n \cdot 360° + 280° = 580° + n \cdot 360°$. Now, divide both sides of this equation by 4 in order to solve for β. Thus, $\beta = 145° + n \cdot 90°$, where n is an integer.

Thus, some of the solutions in the IV quadrant that are obtained by using $\beta = 145° + n \cdot 90°$, where n is an integer, are:

$n = 0$: $\beta = 145°$
NOTE: $4\beta - 280° = 580° - 280° = 300°$
is in the IV quadrant.

$n = 1$: $\beta = 145° + 90° = 235°$
NOTE: $4\beta - 280° = 940° - 280° = 660° = 360° + 300°$
is in the IV quadrant.

$n = 2$: $\beta = 145° + 180° = 325°$
NOTE: $4\beta - 280° = 1300° - 280° = 1020° = 720° + 300°$
is in the IV quadrant.

$n = 3$: $\beta = 145° + 270° = 415°$
NOTE: $4\beta - 280° = 1660° - 280° = 1380° = 3(360°) + 300°$
is in the IV quadrant.

$n = 4$: $\beta = 145° + 360° = 505°$
NOTE: $4\beta - 280° = 2020° - 280° = 1740° = 4(360°) + 300°$

is in the IV quadrant.

⋮

$n = -1$: $\beta = 145° - 90° = 55°$
NOTE: $4\beta - 280° = 220° - 280° = -60°$
is in the IV quadrant.

$n = -2$: $\beta = 145° - 180° = -35°$
NOTE: $4\beta - 280° = -140° - 280° = -420° = -360° - 60°$
is in the IV quadrant.

$n = -3$: $\beta = 145° - 270° = -125°$
NOTE: $4\beta - 280° = -500° - 280° = -780° = -720° - 60°$
is in the IV quadrant.

$n = -4$: $\beta = 145° - 360° = -215°$
NOTE: $4\beta - 280° = -860° - 280° = -1140° = 3(-360°) - 60°$
is in the IV quadrant.

⋮

Answers: $\beta = 130° + n \cdot 90°$; $\beta = 145° + n \cdot 90°$, where n is an integer

8. $\cos 6\alpha = 0$ Back to Equations List

First, we must get the angle argument for the trigonometric function to be a single variable. So, let $u = 6\alpha$. Thus, we obtain the equation $\cos u = 0$.

Since 0 is **not** positive, then the solutions u are neither in the I quadrant nor the IV quadrant. Since 0 is **not** negative, then the solutions u are neither in the II quadrant nor the III quadrant. Thus, the solutions u occur at the coordinate axes. The solutions u occur on the positive y-axis and on the negative y-axis.

The solutions on the positive y-axis: The one solution u on the positive y-axis, that is in the interval $[0, 2\pi)$, is $u = \frac{\pi}{2}$. Now, all the other solutions u on the positive y-axis are coterminal to this one solution. Thus, all the solutions u on the positive y-axis are given by $u = \frac{\pi}{2} + 2n\pi$, where n is an integer. These are the solutions for u. Now, find the solutions for α. Since $u = 6\alpha$ and $u = \frac{\pi}{2} + 2n\pi$, then $6\alpha = \frac{\pi}{2} + 2n\pi$. Divide both sides of this equation by 6 in order to solve for α. Thus, $\alpha = \frac{\pi}{12} + \frac{2n\pi}{6} = \frac{\pi}{12} + \frac{n\pi}{3} = \frac{\pi}{12} + \frac{4n\pi}{12} = \frac{\pi + 4n\pi}{12} = \frac{(4n+1)\pi}{12}$, where n is an integer.

The solutions on the negative y-axis: The one solution u on the negative y-axis, that is in the interval $[0, 2\pi)$, is $u = \frac{3\pi}{2}$. Now, all the other solutions u on the negative y-axis are coterminal to this one solution. Thus, all the solutions u on the negative y-axis are given by $u = \frac{3\pi}{2} + 2n\pi$, where n is an integer. These are the solutions for u. Now, find the solutions for α. Since $u = 6\alpha$ and $u = \frac{3\pi}{2} + 2n\pi$, then $6\alpha = \frac{3\pi}{2} + 2n\pi$. Divide both sides of this equation by 6 in order to solve for α. Thus, $\alpha = \frac{3\pi}{12} + \frac{2n\pi}{6} = \frac{\pi}{4} + \frac{n\pi}{3} = \frac{3\pi}{12} + \frac{4n\pi}{12} = \frac{3\pi + 4n\pi}{12} = \frac{(4n+3)\pi}{12}$, where n is an integer.

Answers: $\alpha = \dfrac{(4n + 1)\pi}{12}$ and $\alpha = \dfrac{(4n + 3)\pi}{12}$, where n is an integer

NOTE: This answer is the same as the answer $\alpha = \dfrac{(2n + 1)\pi}{12}$, where n is an integer.

9. $\sec^2\theta - 2 = 0$ Back to Equations List

First, do the necessary algebra to isolate $\sec^2\theta$ on one side of the equation:

$$\sec^2\theta - 2 = 0 \Rightarrow \sec^2\theta = 2 \Rightarrow \cos^2\theta = \frac{1}{2}$$

Now, use square roots to solve for $\cos\theta$. Recall that $\sqrt{x^2} = |x|$. Thus, if $x^2 = a$, where $a > 0$, then $\sqrt{x^2} = \sqrt{a} \Rightarrow |x| = \sqrt{a} \Rightarrow x = \pm\sqrt{a}$. Thus,

$$\cos^2\theta = \frac{1}{2} \Rightarrow \cos\theta = \pm\sqrt{\frac{1}{2}} \Rightarrow \cos\theta = \pm\frac{1}{\sqrt{2}} \Rightarrow \cos\theta = \pm\frac{\sqrt{2}}{2}$$

Thus, there are **two** equations to be solved. The first equation is $\cos\theta = \dfrac{\sqrt{2}}{2}$ and the second equation is $\cos\theta = -\dfrac{\sqrt{2}}{2}$.

To solve the first equation $\cos\theta = \dfrac{\sqrt{2}}{2}$: Since $\dfrac{\sqrt{2}}{2}$ is not the maximum positive number for the cosine function, then the solutions do not occur at the coordinate axes. Thus, the solutions occur in two of the four quadrants. Since cosine is positive in the I and IV quadrants, the solutions for this first equation occur in those quadrants.

Find the reference angle θ' for the solutions θ:

$$\cos\theta = \frac{\sqrt{2}}{2} \Rightarrow \cos\theta' = \frac{\sqrt{2}}{2} \Rightarrow \theta' = \cos^{-1}\frac{\sqrt{2}}{2} = \frac{\pi}{4}$$

The solutions in the I quadrant: The one solution in the I quadrant, that is between 0 and 2π, is $\theta = \dfrac{\pi}{4}$. Now, all the other solutions in the I quadrant are coterminal to this one solution. Thus, all the solutions in the I quadrant are given by $\theta = \dfrac{\pi}{4} + 2n\pi$, where *n* is an integer.

The solutions in the IV quadrant: The one solution in the IV quadrant, that is between 0 and 2π, is $\theta = 2\pi - \dfrac{\pi}{4} = \dfrac{7\pi}{4}$. Now, all the other solutions in the IV quadrant are coterminal to this one solution. Thus, all the solutions in the IV quadrant are given by $\theta = \dfrac{7\pi}{4} + 2n\pi$, where *n* is an integer.

Thus, the solutions of the first equation $\cos\theta = \dfrac{\sqrt{2}}{2}$ are $\theta = \dfrac{\pi}{4} + 2n\pi$ and $\theta = \dfrac{7\pi}{4} + 2n\pi$, where *n* is an integer.

To solve the second equation $\cos\theta = -\dfrac{\sqrt{2}}{2}$: Since $-\dfrac{\sqrt{2}}{2}$ is not the minimum negative number for the cosine function, then the solutions do not occur at the coordinate axes. Thus, the solutions occur in two of the four quadrants. Since cosine is negative in the II and III quadrants, the solutions for this second equation occur in those quadrants.

Find the reference angle θ' for the solutions θ:

$$\cos\theta = -\frac{\sqrt{2}}{2} \quad\Rightarrow\quad \cos\theta' = \frac{\sqrt{2}}{2} \quad\Rightarrow\quad \theta' = \cos^{-1}\frac{\sqrt{2}}{2} = \frac{\pi}{4}$$

The solutions in the II quadrant: The one solution in the II quadrant, that is between 0 and 2π, is $\theta = \pi - \dfrac{\pi}{4} = \dfrac{3\pi}{4}$. Now, all the other solutions in the II quadrant are coterminal to this one solution. Thus, all the solutions in the

II quadrant are given by $\theta = \frac{3\pi}{4} + 2n\pi$, where *n* is an integer.

The solutions in the III quadrant: The one solution in the III quadrant, that is between 0 and 2π, is $\theta = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$. Now, all the other solutions in the III quadrant are coterminal to this one solution. Thus, all the solutions in the III quadrant are given by $\theta = \frac{5\pi}{4} + 2n\pi$, where *n* is an integer.

Thus, the solutions of the second equation $\cos\theta = -\frac{\sqrt{2}}{2}$ are $\theta = \frac{3\pi}{4} + 2n\pi$ and $\theta = \frac{5\pi}{4} + 2n\pi$, where *n* is an integer.

Answer: $\theta = \frac{\pi}{4} + 2n\pi$; $\theta = \frac{3\pi}{4} + 2n\pi$; $\theta = \frac{5\pi}{4} + 2n\pi$; $\theta = \frac{7\pi}{4} + 2n\pi$, where *n* is an integer

NOTE: This answer may also be written in the following two ways:

1. $\theta = \frac{\pi}{4} + n\pi$; $\theta = \frac{3\pi}{4} + n\pi$, where *n* is an integer

2. $\theta = \frac{\pi}{4} + \frac{n\pi}{2} = \frac{(2n+1)\pi}{4}$, where *n* is an integer

10. $4\sin^2\beta + 9 = 12$ Back to Equations List

First, do the necessary algebra to isolate $\sin^2\beta$ on one side of the equation:

$$4\sin^2\beta + 9 = 12 \Rightarrow 4\sin^2\beta = 3 \Rightarrow \sin^2\beta = \frac{3}{4} \Rightarrow \sin\beta = \pm\frac{\sqrt{3}}{2}$$

Thus, there are **two** equations to be solved. The first equation is $\sin\beta = \frac{\sqrt{3}}{2}$ and the second equation is $\sin\beta = -\frac{\sqrt{3}}{2}$.

To solve the first equation $\sin\beta = \frac{\sqrt{3}}{2}$: Since $\frac{\sqrt{3}}{2}$ is not the maximum positive number for the sine function, then the solutions do not occur at the coordinate axes. Thus, the solutions occur in two of the four quadrants. Since sine is positive in the I and II quadrants, the solutions for this first equation occur in those quadrants.

Find the reference angle β' for the solutions β:

$$\sin\beta = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad \sin\beta' = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad \beta' = \sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}$$

The solutions in the I quadrant: The one solution in the I quadrant, that is between 0 and 2π, is $\beta = \frac{\pi}{3}$. Now, all the other solutions in the I quadrant are coterminal to this one solution. Thus, all the solutions in the I quadrant are given by $\beta = \frac{\pi}{3} + 2n\pi$, where *n* is an integer.

The solutions in the II quadrant: The one solution in the II quadrant, that is between 0 and 2π, is $\beta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$. Now, all the other solutions in the II quadrant are coterminal to this one solution. Thus, all the solutions in the II quadrant are given by $\beta = \frac{2\pi}{3} + 2n\pi$, where *n* is an integer.

Thus, the solutions of the first equation $\sin\beta = \frac{\sqrt{3}}{2}$ are $\beta = \frac{\pi}{3} + 2n\pi$ and $\beta = \frac{2\pi}{3} + 2n\pi$, where *n* is an integer.

To solve the second equation $\sin \beta = -\frac{\sqrt{3}}{2}$: Since $-\frac{\sqrt{3}}{2}$ is not the minimum negative number for the sine function, then the solutions do not occur at the coordinate axes. Thus, the solutions occur in two of the four quadrants. Since sine is negative in the III and IV quadrants, the solutions for this second equation occur in those quadrants.

Find the reference angle β' for the solutions β:

$$\sin \beta = -\frac{\sqrt{3}}{2} \Rightarrow \sin \beta' = \frac{\sqrt{3}}{2} \Rightarrow \beta' = \sin^{-1} \frac{\sqrt{3}}{2} = \frac{\pi}{3}$$

The solutions in the III quadrant: The one solution in the III quadrant, that is between 0 and 2π, is $\beta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$. Now, all the other solutions in the III quadrant are coterminal to this one solution. Thus, all the solutions in the III quadrant are given by $\beta = \frac{4\pi}{3} + 2n\pi$, where *n* is an integer.

The solutions in the IV quadrant: The one solution in the IV quadrant, that is between 0 and 2π, is $\beta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$. Now, all the other solutions in the IV quadrant are coterminal to this one solution. Thus, all the solutions in the IV quadrant are given by $\beta = \frac{5\pi}{3} + 2n\pi$, where *n* is an integer.

Thus, the solutions of the second equation $\sin \beta = -\frac{\sqrt{3}}{2}$ are $\beta = \frac{4\pi}{3} + 2n\pi$ and $\beta = \frac{5\pi}{3} + 2n\pi$, where *n* is an integer.

Answer: $\beta = \frac{\pi}{3} + 2n\pi$; $\beta = \frac{2\pi}{3} + 2n\pi$; $\beta = \frac{4\pi}{3} + 2n\pi$; $\beta = \frac{5\pi}{3} + 2n\pi$, where *n* is an integer

NOTE: This answer is the same as the answer $\beta = \dfrac{\pi}{3} + n\pi$ and $\beta = \dfrac{2\pi}{3} + n\pi$, where n is an integer.

We will need the following theorem in order to solve the rest of these examples.

<u>Theorem</u> If $ab = 0$, then either $a = 0$, or $b = 0$, or both are zero.

11. $\cos\alpha\,(\csc\alpha - 2) = 0$ Back to Equations List

Using the theorem above, we have that $\cos\alpha = 0$ or $\csc\alpha - 2 = 0$. Thus, there are **two** equations to be solved.

To solve the first equation $\cos\alpha = 0$: Since 0 is **not** positive, then the solutions are neither in the I quadrant nor the IV quadrant. Since 0 is **not** negative, then the solutions are neither in the II quadrant nor the III quadrant. Thus, the solutions occur at the coordinate axes. The solutions occur on the positive *y*-axis and on the negative *y*-axis.

The solutions on the positive *y*-axis: The one solution on the positive *y*-axis, that is in the interval $[0, 2\pi)$, is $\alpha = \dfrac{\pi}{2}$. Now, all the other solutions on the positive *y*-axis are coterminal to this one solution. Thus, all the solutions on the positive *y*-axis are given by $\alpha = \dfrac{\pi}{2} + 2n\pi$, where n is an integer.

The solutions on the negative *y*-axis: The one solution on the negative *y*-axis, that is in the interval $[0, 2\pi)$, is $\alpha = \dfrac{3\pi}{2}$. Now, all the other solutions on the negative *y*-axis are coterminal to this one solution. Thus, all the solutions on the negative *y*-axis are given by $\alpha = \dfrac{3\pi}{2} + 2n\pi$, where n is an integer.

Thus, the solutions of the first equation $\cos\alpha = 0$ are $\alpha = \dfrac{\pi}{2} + 2n\pi$ and $\alpha = \dfrac{3\pi}{2} + 2n\pi$, where *n* is an integer. NOTE: This answer is the same as the answer $\alpha = \dfrac{\pi}{2} + n\pi$, where *n* is an integer.

To solve the second equation $\csc\alpha - 2 = 0$:

$$\csc\alpha - 2 = 0 \Rightarrow \quad \csc\alpha = 2 \Rightarrow \quad \sin\alpha = \frac{1}{2}$$

Since $\dfrac{1}{2}$ is not the maximum positive number for the sine function, then the solutions do not occur at the coordinate axes. Thus, the solutions occur in two of the four quadrants. Since sine is positive in the I and II quadrants, the solutions for this second equation occur in those quadrants.

Find the reference angle α' for the solutions α:

$$\sin\alpha = \frac{1}{2} \Rightarrow \quad \sin\alpha' = \frac{1}{2} \Rightarrow \quad \alpha' = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$$

The solutions in the I quadrant: The one solution in the I quadrant, that is between 0 and 2π, is $\alpha = \dfrac{\pi}{6}$. Now, all the other solutions in the I quadrant are coterminal to this one solution. Thus, all the solutions in the I quadrant are given by $\alpha = \dfrac{\pi}{6} + 2n\pi$, where *n* is an integer.

The solutions in the II quadrant: The one solution in the II quadrant, that is between 0 and 2π, is $\alpha = \pi - \dfrac{\pi}{6} = \dfrac{5\pi}{6}$. Now, all the other solutions in the II quadrant are coterminal to this one solution. Thus, all the solutions in the II quadrant are given by $\alpha = \dfrac{5\pi}{6} + 2n\pi$, where *n* is an integer.

Thus, the solutions of the second equation $\csc \alpha - 2 = 0$ are $\alpha = \frac{\pi}{6} + 2n\pi$ and $\alpha = \frac{5\pi}{6} + 2n\pi$, where n is an integer.

Answer: $\alpha = \frac{\pi}{2} + n\pi$; $\alpha = \frac{\pi}{6} + 2n\pi$; $\alpha = \frac{5\pi}{6} + 2n\pi$, where n is an integer.

12. $\sqrt{3} \sin x \tan x = \sin x$ Back to Equations List

NOTE: The first step that most students want to do is to divide both sides of the equation by $\sin x$. If you divide both sides of the equation by $\sin x$, you might (and probably will) be dividing by zero. Of course, division by zero is undefined.

Our first step is to produce a zero on one side of the equation. Subtracting $\sin x$ from both sides of the equation, we have that

$$\sqrt{3} \sin x \tan x = \sin x \Rightarrow \sqrt{3} \sin x \tan x - \sin x = 0$$

Our next step is to factor out the common $\sin x$. Thus,

$$\sqrt{3} \sin x \tan x - \sin x = 0 \Rightarrow \sin x (\sqrt{3} \tan x - 1) = 0$$

Now, using the theorem above, we have that $\sin x = 0$ or $\sqrt{3} \tan x - 1 = 0$. Thus, there are **two** equations to be solved.

To solve the first equation $\sin x = 0$: Since 0 is **not** positive, then the solutions are neither in the I quadrant nor the II quadrant. Since 0 is **not** negative, then the solutions are neither in the III quadrant nor the IV quadrant. Thus, the solutions occur at the coordinate axes. The solutions occur on the positive *x*-axis and on the negative *x*-axis.

The solutions on the positive *x*-axis: The one solution on the positive *x*-axis, that is in the interval $[0, 2\pi)$, is $x = 0$. Now, all the other solutions on the positive *x*-axis are coterminal to this one solution. Thus, all the solutions on the positive *x*-axis are given by $x = 0 + 2n\pi = 2n\pi$, where *n* is an integer.

The solutions on the negative *x*-axis: The one solution on the negative *x*-axis, that is in the interval $[0, 2\pi)$, is $x = \pi$. Now, all the other solutions on the negative *x*-axis are coterminal to this one solution. Thus, all the solutions on the negative *x*-axis are given by $x = \pi + 2n\pi = (2n + 1)\pi$, where *n* is an integer.

Thus, the solutions of the first equation $\sin x = 0$ are $x = 2n\pi$ and $x = (2n + 1)\pi$, where *n* is an integer. NOTE: This answer is the same as the answer $x = n\pi$, where *n* is an integer.

To solve the second equation $\sqrt{3}\tan x - 1 = 0$:

$$\sqrt{3}\tan x - 1 = 0 \Rightarrow \sqrt{3}\tan x = 1 \Rightarrow \tan x = \frac{1}{\sqrt{3}}$$

Since tangent is positive in the I and III quadrants, the solutions for this second equation occur in those quadrants.

Find the reference angle x' for the solutions x:

$$\tan x = \frac{1}{\sqrt{3}} \Rightarrow \tan x' = \frac{1}{\sqrt{3}} \Rightarrow x' = \tan^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{6}$$

The solutions in the I quadrant: The one solution in the I quadrant, that is between 0 and 2π, is $x = \frac{\pi}{6}$. Now, all the other solutions in the I quadrant are coterminal to this one solution. Thus, all the solutions in the I quadrant are given by $x = \frac{\pi}{6} + 2n\pi$, where *n* is an integer.

The solutions in the III quadrant: The one solution in the III quadrant, that is between 0 and 2π, is $x = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$. Now, all the other solutions in the III quadrant are coterminal to this one solution. Thus, all the solutions in the III quadrant are given by $x = \frac{7\pi}{6} + 2n\pi$, where n is an integer.

Thus, the solutions of the second equation $\sqrt{3}\tan x - 1 = 0$ are $x = \frac{\pi}{6} + 2n\pi$ and $x = \frac{7\pi}{6} + 2n\pi$, where n is an integer. NOTE: This answer is the same as the answer $x = \frac{\pi}{6} + n\pi$, where n is an integer.

Answer: $x = n\pi$; $x = \frac{\pi}{6} + n\pi$, where n is an integer

13. $(\cos\beta + 1)(\sin\beta - 1) = 0$ Back to Equations List

Using the theorem above, we have that $\cos\beta + 1 = 0$ or $\sin\beta - 1 = 0$. Thus, there are **two** equations to be solved.

To solve the first equation $\cos\beta + 1 = 0$:

$$\cos\beta + 1 = 0 \Rightarrow \cos\beta = -1$$

Since -1 is the minimum negative number for the cosine function, then the solutions occur at the coordinate axes. There is only **one** solution and it occurs on the negative *x*-axis.

The solutions for $\cos\beta + 1 = 0$: The one solution on the negative *x*-axis, that is in the interval $[0, 2\pi)$, is $\beta = \pi$. Now, all the other solutions on the negative *x*-axis are coterminal to this one solution. Thus, all the solutions on the negative *x*-axis are given by $\beta = \pi + 2n\pi = (2n + 1)\pi$, where n is an integer.

Thus, the solutions of the first equation $\cos\beta + 1 = 0$ are $\beta = (2n + 1)\pi$, where *n* is an integer.

To solve the second equation $\sin\beta - 1 = 0$:

$$\sin\beta - 1 = 0 \Rightarrow \quad \sin\beta = 1$$

Since 1 is the maximum positive number for the sine function, then the solutions occur at the coordinate axes. There is only **one** solution and it occurs on the positive *y*-axis.

The solutions for $\sin\beta - 1 = 0$: The one solution on the positive *y*-axis, that is in the interval $[0, 2\pi)$, is $\beta = \dfrac{\pi}{2}$. Now, all the other solutions on the positive *y*-axis are coterminal to this one solution. Thus, all the solutions on the positive *y*-axis are given by $\beta = \dfrac{\pi}{2} + 2n\pi$, where *n* is an integer.

Thus, the solutions of the second equation $\sin\beta - 1 = 0$ are $\beta = \dfrac{\pi}{2} + 2n\pi$, where *n* is an integer.

Answers: $\beta = (2n + 1)\pi$; $\beta = \dfrac{\pi}{2} + 2n\pi$, where *n* is an integer

14. $2\cos^2\theta - \cos\theta - 1 = 0$ Back to Equations List

This equation is a quadratic equation in $\cos\theta$. We must first factor the left side of this equation. The first "term" of $2\cos^2\theta$ has **one** set of factors. This set of factors is $\cos\theta$ and $2\cos\theta$. The last term of 1 also has **one** set of factors. This set of factors is 1 and 1. Since the last term is negative, then the signs separating these factors are different (one is positive and the other is negative.) Thus,

$$2\cos^2\theta - \cos\theta - 1 = (\cos\theta - 1)(2\cos\theta + 1)$$

Thus,

$$2\cos^2\theta - \cos\theta - 1 = 0 \Rightarrow (\cos\theta - 1)(2\cos\theta + 1) = 0$$

Using the theorem above, we have that $\cos\theta - 1 = 0$ or $2\cos\theta + 1 = 0$. Thus, there are **two** equations to be solve.

To solve the first equation $\cos\theta - 1 = 0$:

$$\cos\theta - 1 = 0 \Rightarrow \cos\theta = 1$$

Since 1 is the maximum positive number for the cosine function, then the solutions occur at the coordinate axes. There is only **one** solution and it occurs on the positive *x*-axis.

The solutions for $\cos\theta - 1 = 0$: The one solution on the positive *x*-axis, that is in the interval $[0, 2\pi)$, is $\theta = 0$. Now, all the other solutions on the positive *x*-axis are coterminal to this one solution. Thus, all the solutions on the positive *x*-axis are given by $\theta = 0 + 2n\pi = 2n\pi$, where *n* is an integer.

Thus, the solutions of the first equation $\cos\theta - 1 = 0$ are $\theta = 2n\pi$, where *n* is an integer.

To solve the second equation $2\cos\theta + 1 = 0$:

$$2\cos\theta + 1 = 0 \Rightarrow 2\cos\theta = -1 \Rightarrow \cos\theta = -\frac{1}{2}$$

Determine where the solutions θ will occur. Since $-\frac{1}{2}$ is not the minimum negative number for the cosine function, then the solutions do not occur at the coordinate axes. Thus, the solutions occur in two of the four quadrants. Since cosine is negative in the II and III quadrants, the solutions for this equation occur in those quadrants.

Find the reference angle θ' for the solutions θ:

$$\cos\theta = -\frac{1}{2} \Rightarrow \quad \cos\theta' = \frac{1}{2} \Rightarrow \quad \theta' = \cos^{-1}\frac{1}{2} = \frac{\pi}{3}$$

The solutions in the II quadrant: The one solution in the II quadrant, that is between 0 and 2π, is $\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$. Now, all the other solutions in the II quadrant are coterminal to this one solution. Thus, all the solutions in the II quadrant are given by $\theta = \frac{2\pi}{3} + 2n\pi$, where *n* is an integer.

The solutions in the III quadrant: The one solution in the III quadrant, that is between 0 and 2π, is $\theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$. Now, all the other solutions in the III quadrant are coterminal to this one solution. Thus, all the solutions in the III quadrant are given by $\theta = \frac{4\pi}{3} + 2n\pi$, where *n* is an integer.

Thus, the solutions of the second equation $2\cos\theta + 1 = 0$ are $\theta = \frac{2\pi}{3} + 2n\pi$ and $\theta = \frac{4\pi}{3} + 2n\pi$, where *n* is an integer.

Answers: $\theta = 2n\pi$; $\theta = \frac{2\pi}{3} + 2n\pi$; $\theta = \frac{4\pi}{3} + 2n\pi$, where *n* is an integer

15. $6\sin^2\alpha + 7\sin\alpha - 5 = 0$ Back to Equations List

This equation is a quadratic equation in $\sin\alpha$. We must first factor the left side of this equation. The first "term" of $6\sin^2\alpha$ have **two** sets of factors. The first set of factors is $\sin\alpha$ and $6\sin\alpha$, and the second set is $2\sin\alpha$ and $3\sin\alpha$. The last term of 5 has **one** set of factors. This set of factors is 1 and

5. Since the last term is negative, then the signs separating these factors are different (one is positive and the other is negative.) Thus,

$$6\sin^2\alpha + 7\sin\alpha - 5 = (2\sin\alpha - 1)(3\sin\alpha + 5)$$

Thus,

$$6\sin^2\alpha + 7\sin\alpha - 5 = 0 \Rightarrow (2\sin\alpha - 1)(3\sin\alpha + 5) = 0$$

Using the theorem above, we have that $2\sin\alpha - 1 = 0$ or $3\sin\alpha + 5 = 0$. Thus, there are **two** equations to be solved.

To solve the first equation $2\sin\alpha - 1 = 0$:

$$2\sin\alpha - 1 = 0 \Rightarrow 2\sin\alpha = 1 \Rightarrow \sin\alpha = \frac{1}{2}$$

First, determine where the solutions α will occur. Since $\frac{1}{2}$ is not the maximum positive number for the sine function, then the solutions do not occur at the coordinate axes. Thus, the solutions occur in two of the four quadrants. Since sine is positive in the I and II quadrants, the solutions for this equation occur in those quadrants.

Find the reference angle α' for the solutions α:

$$\sin\alpha = \frac{1}{2} \Rightarrow \sin\alpha' = \frac{1}{2} \Rightarrow \alpha' = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$$

The solutions in the I quadrant: The one solution in the I quadrant, that is between 0 and 2π, is $\alpha = \frac{\pi}{6}$. Now, all the other solutions in the I quadrant are coterminal to this one solution. Thus, all the solutions in the I quadrant are given by $\alpha = \frac{\pi}{6} + 2n\pi$, where n is an integer.

The solutions in the II quadrant: The one solution in the II quadrant, that is between 0 and 2π, is $\alpha = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$. Now, all the other solutions in the II quadrant are coterminal to this one solution. Thus, all the solutions in the II quadrant are given by $\alpha = \frac{5\pi}{6} + 2n\pi$, where *n* is an integer.

Thus, the solutions of the first equation $2\sin\alpha - 1 = 0$ are $\alpha = \frac{\pi}{6} + 2n\pi$ and $\alpha = \frac{5\pi}{6} + 2n\pi$, where *n* is an integer.

To solve the second equation $3\sin\alpha + 5 = 0$:

$$3\sin\alpha + 5 = 0 \Rightarrow \quad 3\sin\alpha = -5 \Rightarrow \quad \sin\alpha = -\frac{5}{3}$$

Since $-\frac{5}{3}$ is less than the minimum negative number for the sine function, then this equation has no solutions.

Thus, there are no solutions for the second equation $3\sin\alpha + 5 = 0$.

Answers: $\alpha = \frac{\pi}{6} + 2n\pi$; $\alpha = \frac{5\pi}{6} + 2n\pi$, where *n* is an integer

16. $\cos^2\beta + 24 = 11\cos\beta$ Back to Equations List

This equation is a quadratic equation in $\cos\beta$. First, we will need to produce a zero on one side of the equation. Subtract $11\cos\beta$ from both sides of the equation. Thus,

$$\cos^2\beta + 24 = 11\cos\beta \Rightarrow \cos^2\beta - 11\cos\beta + 24 = 0$$

Now, we will factor the left side of this equation. The first "term" of $\cos^2 \beta$ have **one** set of factors. This set of factors is $\cos \beta$ and $\cos \beta$. The last term of 24 has **four** sets of factors. The first set of factors is 1 and 24, the second set is 2 and 12, the third set is 3 and 8, and the fourth set is 4 and 6. Since the last term is positive, then the signs separating these factors are the same. This sign is the sign of the middle term, which is negative. Thus,

$$\cos^2 \beta - 11\cos \beta + 24 = (\cos \beta - 3)(\cos \beta - 8)$$

Thus,

$$\cos^2 \beta - 11\cos \beta + 24 = 0 \Rightarrow (\cos \beta - 3)(\cos \beta - 8) = 0$$

Using the theorem above, we have that $\cos \beta - 3 = 0$ or $\cos \beta - 8 = 0$. Thus, there are **two** equations to be solved.

To solve the first equation $\cos \beta - 3 = 0$:

$$\cos \beta - 3 = 0 \Rightarrow \cos \beta = 3$$

Since 3 is greater than the maximum positive number for the cosine function, then this equation has no solutions.

Thus, there are no solutions for the first equation $\cos \beta - 3 = 0$.

To solve the second equation $\cos \beta - 8 = 0$:

$$\cos \beta - 8 = 0 \Rightarrow \cos \beta = 8$$

Since 8 is greater than the maximum positive number for the cosine function, then this equation has no solutions.

Thus, there are no solutions for the second equation $\cos \beta - 8 = 0$.

Answer: No solution

Example Find all the approximate solutions (in degrees) between $-360°$ and $540°$ for the equation $\tan\theta = -\frac{5}{4}$.

Determine where the solutions θ will occur. Since tangent is negative in the II and IV quadrants, the solutions for this equation occur in those quadrants.

Find the reference angle θ' for the solutions θ:

$$\tan\theta = -\frac{5}{4} \Rightarrow \quad \tan\theta' = \frac{5}{4} \Rightarrow \quad \theta' = \tan^{-1}\frac{5}{4} \approx 51.3°$$

The solutions in the II quadrant: The one approximate solution in the II quadrant, that is between $0°$ and $360°$, is $\theta = 180° - \theta' \approx 180° - 51.3° = 128.7°$. Now, all the other approximate solutions in the II quadrant are coterminal to this one solution. Thus, all the approximate solutions in the II quadrant are given by $\theta \approx 128.7° + n \cdot 360°$, where n is an integer.

Now, use $\theta \approx 128.7° + n \cdot 360°$, where n is an integer, to find the approximate solutions in the II quadrant that are between $-360°$ and $540°$:

$$n = 0: \quad \theta \approx 128.7° + 0° = 128.7°$$

$$n = 1: \quad \theta \approx 128.7° + 360° = 488.7°$$

$$n = 2: \quad \theta \approx 128.7° + 720° = 848.7° > 540°$$

$$n = -1: \quad \theta \approx 128.7° - 360° = -231.3°$$

$$n = -2: \quad \theta \approx 128.7° - 720° = -591.3° < -360°$$

Thus, the approximate solutions in the II quadrant that are between $-360°$ and $540°$ are $-231.3°$, $128.7°$, and $488.7°$.

The solutions in the IV quadrant: The one approximate solution in the IV quadrant, that is between $0°$ and $360°$, is $\theta = 360° - \theta' \approx 360° - 51.3° = 308.7°$.

Now, all the other approximate solutions in the IV quadrant are coterminal to this one solution. Thus, all the approximate solutions in the IV quadrant are given by $\theta \approx 308.7^\circ + n \cdot 360^\circ$, where n is an integer.

Now, use $\theta \approx 308.7^\circ + n \cdot 360^\circ$, where n is an integer, to find the approximate solutions in the IV quadrant that are between -360° and 540°:

$$n = 0: \quad \theta \approx 308.7^\circ + 0^\circ = 308.7^\circ$$

$$n = 1: \quad \theta \approx 308.7^\circ + 360^\circ = 668.7^\circ > 540^\circ$$

$$n = -1: \quad \theta \approx 308.7^\circ - 360^\circ = -51.3^\circ$$

$$n = -2: \quad \theta \approx 308.7^\circ - 720^\circ = -411.3^\circ < -360^\circ$$

Thus, the approximate solutions in the IV quadrant that are between -360° and 540° are -51.3° and 308.7°

Answers: -231.3°, -51.3°, 128.7°, 308.7°, 488.7°

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Example Find all the approximate solutions (in degrees) between -270° and 630° for the equation $3\sec 2\alpha - 7 = 0$.

First, we must get the angle argument for the trigonometric function to be a single variable. So, let $u = 2\alpha$. Thus, we obtain the equation $3\sec u - 7 = 0$.

$$3\sec u - 7 = 0 \Rightarrow 3\sec u = 7 \Rightarrow \sec u = \frac{7}{3} \Rightarrow \cos u = \frac{3}{7}$$

Determine where the solutions u will occur. Since $\frac{3}{7}$ is not the maximum positive number for the cosine function, then the solutions do not occur at the coordinate axes. Thus, the solutions occur in two of the four quadrants. Since cosine is positive in the I and IV quadrants, the solutions for this equation occur in those quadrants.

Find the reference angle u' for the solutions u:

$$\cos u = \frac{3}{7} \Rightarrow \cos u' = \frac{3}{7} \Rightarrow u' = \cos^{-1}\frac{3}{7} \approx 64.6°$$

The solutions in the I quadrant: The one approximate solution u in the I quadrant, that is between $0°$ and $360°$, is $u \approx 64.6°$. Now, all the other approximate solutions u in the I quadrant are coterminal to this one solution. Thus, all the approximate solutions in the I quadrant are given by $u \approx 64.6° + n \cdot 360°$, where n is an integer. These are the approximate solutions for u. Now, find the approximate solutions for α. Since $u = 2\alpha$ and $u \approx 64.6° + n \cdot 360°$, then $2\alpha \approx 64.6° + n \cdot 360°$. Divide both sides of this equation by 2 in order to solve for α. Thus, $\alpha \approx 32.3° + n \cdot 180°$.

Now, use $\alpha \approx 32.3° + n \cdot 180°$, where n is an integer, to find the approximate solutions in the I quadrant that are between $-270°$ and $630°$:

$$n = 0: \quad \alpha \approx 32.3° + 0° = 32.3°$$

$$n = 1: \quad \alpha \approx 32.3° + 180° = 212.3°$$

$$n = 2: \quad \alpha \approx 32.3° + 360° = 392.3°$$

$$n = 3: \quad \alpha \approx 32.3° + 540° = 572.3°$$

$$n = 4: \quad \alpha \approx 32.3° + 720° = 752.3° > 630°$$

$$n = -1: \quad \alpha \approx 32.3° - 180° = -147.7°$$

$$n = -2: \quad \alpha \approx 32.3° - 360° = -327.7° < -270°$$

Thus, the approximate solutions in the I quadrant that are between $-270°$ and $630°$ are $-147.7°$, $32.3°$, $212.3°$, $392.3°$ and $572.3°$.

The solutions in the IV quadrant: The one approximate solution u in the IV quadrant, that is between $0°$ and $360°$, is $u = 360° - u' \approx 360° - 64.6° = 295.4°$. Now, all the other approximate solutions u in the IV quadrant are coterminal to this one solution. Thus, all the approximate solutions in the IV quadrant are given by $u \approx 295.4° + n \cdot 360°$, where n is an integer. These are the approximate solutions for u. Now, find the approximate solutions for α. Since $u = 2\alpha$ and $u \approx 295.4° + n \cdot 360°$, then $2\alpha \approx 295.4° + n \cdot 360°$. Divide both sides of this equation by 2 in order to solve for α. Thus, $\alpha \approx 147.7° + n \cdot 180°$.

Now, use $\alpha \approx 147.7° + n \cdot 180°$, where n is an integer, to find the approximate solutions in the IV quadrant that are between $-270°$ and $630°$:

$n = 0$: $\alpha \approx 147.7° + 0° = 147.7°$

$n = 1$: $\alpha \approx 147.7° + 180° = 327.7°$

$n = 2$: $\alpha \approx 147.7° + 360° = 507.7°$

$n = 3$: $\alpha \approx 147.7° + 540° = 687.7° > 630°$

$n = -1$: $\alpha \approx 147.7° - 180° = -32.3°$

$n = -2$: $\alpha \approx 147.7° - 360° = -212.3°$

$n = -3$: $\alpha \approx 147.7° - 540° = -392.3° < -270°$

Thus, the approximate solutions in the IV quadrant that are between $-270°$ and $630°$ are $-212.3°$, $-32.3°$, $147.7°$, $327.7°$, and $507.7°$.

Answers: $-212.3°$, $-147.7°$, $-32.3°$, $32.3°$, $147.7°$, $212.3°$, $327.7°$, $392.3°$, $507.7°$, $572.3°$

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Example Find all the approximate solutions (in degrees) between $-360°$ and $360°$ for the equation $9\sin(3\beta + 150°) + 4 = 0$.

First, we must get the angle argument for the trigonometric function to be a single variable. So, let $u = 3\beta + 150°$. Thus, we obtain the equation $9\sin u + 4 = 0$.

$$9\sin u + 4 = 0 \Rightarrow 9\sin u = -4 \Rightarrow \sin u = -\frac{4}{9}$$

Determine where the solutions u will occur. Since $-\frac{4}{9}$ is not the minimum negative number for the sine function, then the solutions do not occur at the coordinate axes. Thus, the solutions occur in two of the four quadrants. Since sine is negative in the III and IV quadrants, the solutions for this equation occur in those quadrants.

Find the reference angle u' for the solutions u:

$$\sin u = -\frac{4}{9} \Rightarrow \sin u' = \frac{4}{9} \Rightarrow u' = \sin^{-1}\frac{4}{9} \approx 26.4°$$

The solutions in the III quadrant: The one approximate solution u in the III quadrant, that is between $0°$ and $360°$, is $u = 180° + u' \approx 180° + 26.4° = 206.4°$. Now, all the other approximate solutions u in the III quadrant are coterminal to this one solution. Thus, all the approximate solutions in the III quadrant are given by $u \approx 206.4° + n \cdot 360°$, where n is an integer. These are the approximate solutions for u. Now, find the approximate solutions for β. Since $u = 3\beta + 150°$ and $u \approx 206.4° + n \cdot 360°$, then $3\beta + 150° \approx 206.4° + n \cdot 360°$. First, subtract $150°$ from both sides of the equation. Thus, $3\beta \approx 206.4° + n \cdot 360° - 150° = 3\beta \approx 56.4° + n \cdot 360°$. Now, divide both sides of this equation by 3 in order to solve for β. Thus, $\beta \approx 18.8° + n \cdot 120°$.

Now, use $\beta \approx 18.8° + n \cdot 120°$, where n is an integer, to find the approximate solutions in the III quadrant that are between $-360°$ and $360°$:

$$n = 0: \qquad \beta \approx 18.8° + 0° = 18.8°$$

$$n = 1: \quad \beta \approx 18.8° + 120° = 138.8°$$

$$n = 2: \quad \beta \approx 18.8° + 240° = 258.8°$$

$$n = 3: \quad \beta \approx 18.8° + 360° > 360°$$

$$n = -1: \quad \beta \approx 18.8° - 120° = -101.2°$$

$$n = -2: \quad \beta \approx 18.8° - 240° = -221.2°$$

$$n = -3: \quad \beta \approx 18.8° - 360° = -341.2°$$

$$n = -4: \quad \beta \approx 18.8° - 480° < -360°$$

Thus, the approximate solutions in the III quadrant that are between $-360°$ and $360°$ are $-341.2°$, $-221.2°$, $-101.2°$, $18.8°$, $138.8°$, and $258.8°$.

The solutions in the IV quadrant: The one approximate solution u in the IV quadrant, that is between $0°$ and $360°$, is $u = 360° - u' \approx 360° - 26.4° = 333.6°$. Now, all the other approximate solutions u in the IV quadrant are coterminal to this one solution. Thus, all the approximate solutions in the IV quadrant are given by $u \approx 333.6° + n \cdot 360°$, where n is an integer. These are the approximate solutions for u. Now, find the approximate solutions for β. Since $u = 3\beta + 150°$ and $u \approx 333.6° + n \cdot 360°$, then $3\beta + 150° \approx 333.6° + n \cdot 360°$. First, subtract $150°$ from both sides of the equation. Thus, $3\beta \approx 333.6° + n \cdot 360° - 150° = 3\beta \approx 183.6° + n \cdot 360°$. Now, divide both sides of this equation by 3 in order to solve for β. Thus, $\beta \approx 61.2° + n \cdot 120°$.

Now, use $\beta \approx 61.2° + n \cdot 120°$, where n is an integer, to find the approximate solutions in the IV quadrant that are between $-360°$ and $360°$:

$$n = 0: \quad \beta \approx 61.2° + 0° = 61.2°$$

$$n = 1: \quad \beta \approx 61.2° + 120° = 181.2°$$

$$n = 2: \quad \beta \approx 61.2° + 240° = 301.2°$$

$$n = 3: \quad \beta \approx 61.2° + 360° > 360°$$

$$n = -1: \quad \beta \approx 61.2° - 120° = -58.8°$$

$$n = -2: \quad \beta \approx 61.2° - 240° = -178.8°$$

$$n = -3: \quad \beta \approx 61.2° - 360° = -298.8°$$

$$n = -4: \quad \beta \approx 61.2° - 480° < -360°$$

Thus, the approximate solutions in the IV quadrant that are between $-360°$ and $360°$ are $-298.8°$, $-178.8°$, $-58.8°$, $61.2°$, $181.2°$, and $301.2°$.

Answers: $-341.2°$, $-298.8°$, $-221.2°$, $-178.8°$, $-101.2°$, $-58.8°$, $18.8°$, $61.2°$, $138.8°$, $181.2°$, $258.8°$, $301.2°$

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Example Find all the exact and approximate solutions to the equation $\tan^2\theta - 6\tan\theta - 11 = 0$.

This equation is a quadratic equation in $\tan\theta$. However, this equation does not factor. Thus, we will need to use the Quadratic Formula. Recall: If given the quadratic equation $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Thus,

$$\tan\theta = \frac{6 \pm \sqrt{36 - 4(1)(-11)}}{2} = \frac{6 \pm \sqrt{36 + 44}}{2} = \frac{6 \pm \sqrt{80}}{2} =$$

$$\frac{6 \pm 4\sqrt{5}}{2} = \frac{2(3 \pm 2\sqrt{5})}{2} = 3 \pm 2\sqrt{5}$$

Thus, there are **two** equations to be solved. The first equation is $\tan\theta = 3 + 2\sqrt{5}$ and the second equation is $\tan\theta = 3 - 2\sqrt{5}$

To solve the first equation $\tan\theta = 3 + 2\sqrt{5}$:

First, determine where the solutions θ will occur. The number $3 + 2\sqrt{5}$ is positive. Since tangent is positive in the I and III quadrants, the solutions for this first equation occur in those quadrants.

Find the reference angle θ' for the solutions θ:

$$\tan\theta = 3 + 2\sqrt{5} \;\Rightarrow\; \tan\theta' = 3 + 2\sqrt{5} \;\Rightarrow\; \theta' = \tan^{-1}(3 + 2\sqrt{5})$$

The solutions in the I quadrant: The one exact solution in the I quadrant, that is between $0°$ and $360°$, is $\theta = \tan^{-1}(3 + 2\sqrt{5})$. Now, all the other exact solutions in the I quadrant are coterminal to this one solution. Thus, all the exact solutions in the I quadrant are given by $\theta = \tan^{-1}(3 + 2\sqrt{5}) + n \cdot 360°$, where n is an integer. Since $\tan^{-1}(3 + 2\sqrt{5}) \approx 82.4°$, then all the approximate solutions in the I quadrant are given by $\theta \approx 82.4° + n \cdot 360°$, where n is an integer.

The solutions in the III quadrant: The one exact solution in the III quadrant, that is between $0°$ and $360°$, is $\theta = 180° + \theta' = 180° + \tan^{-1}(3 + 2\sqrt{5})$. Now, all the other exact solutions in the III quadrant are coterminal to this one solution. Thus, all the exact solutions in the III quadrant are given by $\theta = 180° + \tan^{-1}(3 + 2\sqrt{5}) + n \cdot 360° = 180° + \tan^{-1}(3 + 2\sqrt{5}) + 2n \cdot 180°$ $= \tan^{-1}(3 + 2\sqrt{5}) + (2n + 1)\,180°$, where n is an integer. Since $\tan^{-1}(3 + 2\sqrt{5}) \approx 82.4°$, then all the approximate solutions in the III quadrant are given by $\theta \approx 82.4° + (2n + 1)180°$, where n is an integer.

The exact solutions of the first equation $\tan\theta = 3 + 2\sqrt{5}$ are $\theta = \tan^{-1}(3 + 2\sqrt{5}) + n \cdot 360°$ and $\theta = \tan^{-1}(3 + 2\sqrt{5}) + (2n + 1)\,180°$, where n is an integer. NOTE: Since the period of the tangent function is π, then this answer may also be written as $\theta = \tan^{-1}(3 + 2\sqrt{5}) + n \cdot 180°$, where n is an integer.

The approximate solutions of the first equation $\tan\theta = 3 + 2\sqrt{5}$ are $\theta \approx 82.4° + n \cdot 360°$ and $\theta \approx 82.4° + (2n + 1)180°$, where n is an integer. NOTE: Since the period of the tangent function is π, then this answer may also be written as $\theta \approx 82.4° + n \cdot 180°$, where n is an integer.

To solve the second equation $\tan\theta = 3 - 2\sqrt{5}$:

First, determine where the solutions θ will occur. The number $3 - 2\sqrt{5}$ is negative. Since tangent is negative in the II and IV quadrants, the solutions for this second equation occur in those quadrants.

Find the reference angle θ' for the solutions θ:

$$\tan\theta = 3 - 2\sqrt{5} \Rightarrow \tan\theta' = -(3 - 2\sqrt{5}) \Rightarrow$$

$$\tan\theta' = 2\sqrt{5} - 3 \Rightarrow \theta' = \tan^{-1}(2\sqrt{5} - 3)$$

The solutions in the II quadrant: The one exact solution in the II quadrant, that is between $0°$ and $360°$, is $\theta = 180° - \theta' = 180° - \tan^{-1}(2\sqrt{5} - 3)$. Now, all the other exact solutions in the II quadrant are coterminal to this one solution. Thus, all the exact solutions in the II quadrant are given by $\theta = 180° - \tan^{-1}(2\sqrt{5} - 3) + n \cdot 360° = 180° - \tan^{-1}(2\sqrt{5} - 3) + 2n \cdot 180°$ $= -\tan^{-1}(2\sqrt{5} - 3) + (2n + 1)180°$, where n is an integer. Since $\tan^{-1}(2\sqrt{5} - 3) \approx 55.8°$, then all the approximate solutions in the II quadrant are given by $\theta \approx -55.8° + (2n + 1)180° = 124.2° + n \cdot 360°$, where n is an integer.

The solutions in the IV quadrant: The one exact solution in the IV quadrant, that is between $0°$ and $360°$, is $\theta = 360° - \theta' = 360° - \tan^{-1}(2\sqrt{5} - 3)$. Now, all the other exact solutions in the IV quadrant are coterminal to this one solution. Thus, all the exact solutions in the IV quadrant are given by $\theta = 360° - \tan^{-1}(2\sqrt{5} - 3) + n \cdot 360° = -\tan^{-1}(2\sqrt{5} - 3) + (n + 1)360°$, where n is an integer. Since $\tan^{-1}(2\sqrt{5} - 3) \approx 55.8°$, then all the approximate solutions in the IV quadrant are given by $\theta \approx -55.8° + (n + 1)360° = 304.2° + n \cdot 360°$, where n is an integer.

Thus, the exact solutions of the second equation $\tan\theta = 3 - 2\sqrt{5}$ are $\theta = -\tan^{-1}(2\sqrt{5} - 3) + (2n + 1)180°$ and $\theta = -\tan^{-1}(2\sqrt{5} - 3) + (n + 1)360°$, where n is an integer. NOTE: Since the period of the tangent function is π, then this answer may also be written as $\theta = -\tan^{-1}(2\sqrt{5} - 3) + (n + 1)180°$, where n is an integer.

Thus, the approximate solutions of the second equation $\tan\theta = 3 - 2\sqrt{5}$ are $\theta \approx 124.2° + n \cdot 360°$ and $\theta \approx 304.2° + n \cdot 360°$, where n is an integer. NOTE: Since the period of the tangent function is π, then this answer may also be written as $\theta \approx 124.2° + n \cdot 180°$, where n is an integer.

Answers: Exact: $\theta = \tan^{-1}(3 + 2\sqrt{5}) + n \cdot 180°$

$\theta = -\tan^{-1}(2\sqrt{5} - 3) + (n + 1)180°$,

where n is an integer

Approximate: $\theta \approx 82.4° + n \cdot 180°$

$\theta \approx 124.2° + n \cdot 180°$,

where n is an integer

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