Lesson 11.2 - Solving Systems Of Linear Equations By Substitution

In the system of linear equations shown, the value of y is given.
Use this value of y to find the value of x and the solution of the system.

$$\begin{cases} y = 2 \\ x + y = 6 \end{cases}$$

(A) Substitute the value of y in the second equation and solve for x.

$$x + y = 6$$
$$x + \boxed{} = 6$$
$$x = \boxed{}$$

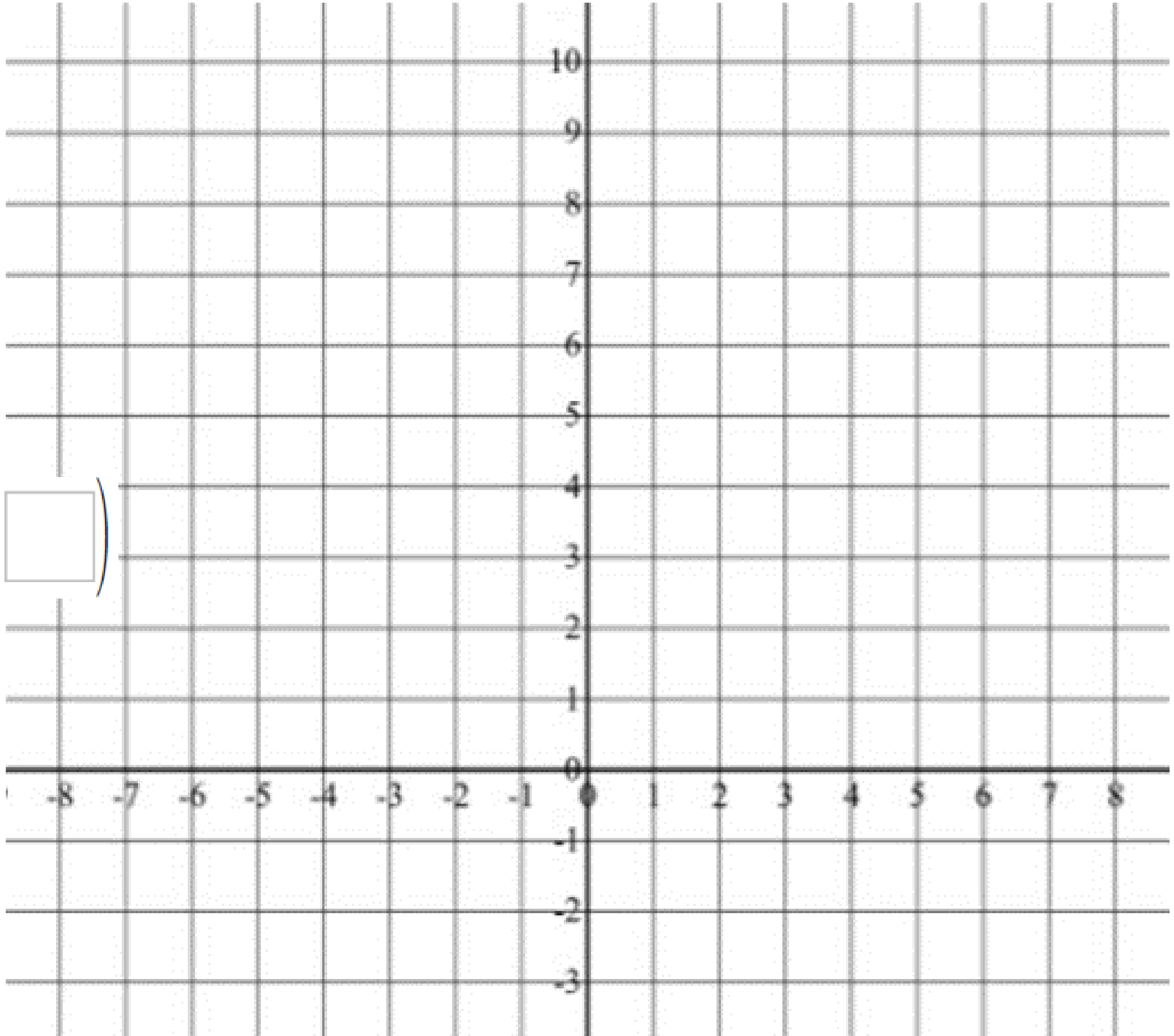
(B) Solution: $\left(\boxed{}, \boxed{}\right)$

(C) Graph the system of linear equations.
How do your solutions compare?

$x + y = 6$

x	y
0	
	0

$y = 2$
$x =$



Use substitution to find the values of *x* and *y* in this system of linear equations.

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$$\begin{cases} y = 4x \\ 5x + 2y = 39 \end{cases}$$

Solution: $(\square, \square)$

How could you check your solution?

Put it in parenthesis!

$$5x + 2(\mathbf{4x}) = 39$$

$$5x + 8x = 39$$

$$\frac{13x}{13} = \frac{39}{13}$$

$$x = 3$$

And Y = ?

$$y = 4x$$

$$y = 4(\mathbf{3})$$

$$y = 12$$

Solving Systems of Equations by Substitution

Step 1	Solve one of the equations for one of its variables, if necessary.
Step 2	Substitute the expression from Step 1 into the other equation, and solve for the other variable.
Step 3	Substitute the value from Step 2 into one of the original equations, and solve for the other variable.
Step 4	Write the values from Steps 2 and 3 in an ordered pair (x, y).
Step 5	Check the solution by substituting into both equations or by graphing.

Example 1 Solve each system of linear equations by substitution.

A $\begin{cases} 3x + y = -3 \\ -2x + y = 7 \end{cases}$

Step 1

Solve an equation for one variable.

$3x + y = -3$ Select one of the equations.

$y = -3x - 3$ Solve for y. Isolate y on one side.

Solving Systems of Equations by Substitution

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Step 1	Solve one of the equations for one of its variables, if necessary.
Step 2	Substitute the expression from Step 1 into the other equation, and solve for the other variable.
Step 3	Substitute the value from Step 2 into one of the original equations, and solve for the other variable.
Step 4	Write the values from Steps 2 and 3 in an ordered pair (x, y).
Step 5	Check the solution by substituting into both equations or by graphing.

From the previous step: $y = -3x - 3$

Step 2

Substitute the expression for y in the other equation and solve.

$$3x + y = -3$$
$$-2x + y = 7$$

Put it in parenthesis!

$-2x + (-3x - 3) = 7$	Substitute the expression for y.
$-5x - 3 = 7$	Combine like terms.
$-5x = 10$	Add 3 to both sides.
$x = -2$	Divide each side by -5.

Solving Systems of Equations by Substitution

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Step 1	Solve one of the equations for one of its variables, if necessary.
Step 2	Substitute the expression from Step 1 into the other equation, and solve for the other variable.
Step 3	Substitute the value from Step 2 into one of the original equations, and solve for the other variable.
Step 4	Write the values from Steps 2 and 3 in an ordered pair (x, y).
Step 5	Check the solution by substituting into both equations or by graphing.

From the previous step: $\mathbf{x = -2}$

Step 3

Substitute the value for x into one of the equations and solve for y.

$3x + y = -3$

$-2x + y = 7$

Put it in parenthesis!

$3(-2) + y = -3$ Substitute the value of x into the first equation.

$-6 + y = -3$ Simplify.

$y = 3$ Add 6 to both sides.

Step 4

So, $(-2, 3)$ is the solution of the system.

Solving Systems of Equations by Substitution	
Step 1	Solve one of the equations for one of its variables, if necessary.
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Step 3	Substitute the value from Step 2 into one of the original equations, and solve for the other variable.
Step 4	Write the values from Steps 2 and 3 in an ordered pair (x, y).
Step 5	Check the solution by substituting into both equations or by graphing.

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From the previous step: (–2,3) is the solution of the system

Step 5

Check the solution by substituting into both equations

$3x + y = -3$	$3(-2) + 3 = -3\,?$	$-6 + 3 = 3$	$-3 = -3$	$\mathit{Correct!}$
$-2x + y = 7$	$-2(-2) + 3 = 7$	$4 + 3 = 7$	$7 = 7$	$\mathit{Correct!}$

The solution is correct.

Solving Systems of Equations by Substitution

Step 1	Solve one of the equations for one of its variables, if necessary.
Step 2	Substitute the expression from Step 1 into the other equation, and solve for the other variable.
Step 3	Substitute the value from Step 2 into one of the original equations, and solve for the other variable.
Step 4	Write the values from Steps 2 and 3 in an ordered pair (x, y).
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B $\begin{cases} x - 3y = 9 \\ x + 4y = 2 \end{cases}$

Step 1

Solve an equation for one variable.

$x - 3y = 9$	Select one of the equations.
$x = \boxed{}$	Solve for x. Isolate x on one side.

Solving Systems of Equations by Substitution

Step 1	Solve one of the equations for one of its variables, if necessary.
Step 2	Substitute the expression from Step 1 into the other equation, and solve for the other variable.
Step 3	Substitute the value from Step 2 into one of the original equations, and solve for the other variable.
Step 4	Write the values from Steps 2 and 3 in an ordered pair (x, y).
Step 5	Check the solution by substituting into both equations or by graphing.

From the previous step: $x = 9 + 3y$

Step 2

Substitute the expression for _____ in the other equation and solve.

$x - 3y = 9$

$x + 4y = 2$

Put it in parenthesis!

$\left(\boxed{}\right) + 4y = 2$ Substitute the expression for _____.

$\boxed{} = 2$ Combine like terms.

$\boxed{} = \boxed{}$ Subtract $\boxed{}$ from both sides.

$y = \boxed{}$ Divide each side by $\boxed{}$.

Solving Systems of Equations by Substitution

Step 1	Solve one of the equations for one of its variables, if necessary.
Step 2	Substitute the expression from Step 1 into the other equation, and solve for the other variable.
Step 3	Substitute the value from Step 2 into one of the original equations, and solve for the other variable.
Step 4	Write the values from Steps 2 and 3 in an ordered pair (x, y).
Step 5	Check the solution by substituting into both equations or by graphing.

From the previous step: $y = -1$

Step 3

Substitute the value for y into one of the equations and solve for x.

$x - 3y = 9$

$x + 4y = 2$

$x - 3(\boxed{}) = 9$ Substitute the value of y into the first equation.

$\boxed{} = 9$ Simplify.

$x = \boxed{}$ Subtract $\boxed{}$ from both sides.

Put it in parenthesis!

Step 4

So, $(\boxed{}, \boxed{})$ is the solution

Solving Systems of Equations by Substitution	
Step 1	Solve one of the equations for one of its variables, if necessary.
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Step 3	Substitute the value from Step 2 into one of the original equations, and solve for the other variable.
Step 4	Write the values from Steps 2 and 3 in an ordered pair (x, y).
Step 5	Check the solution by substituting into both equations or by graphing.

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From the previous step: (6,–1) is the solution to the system

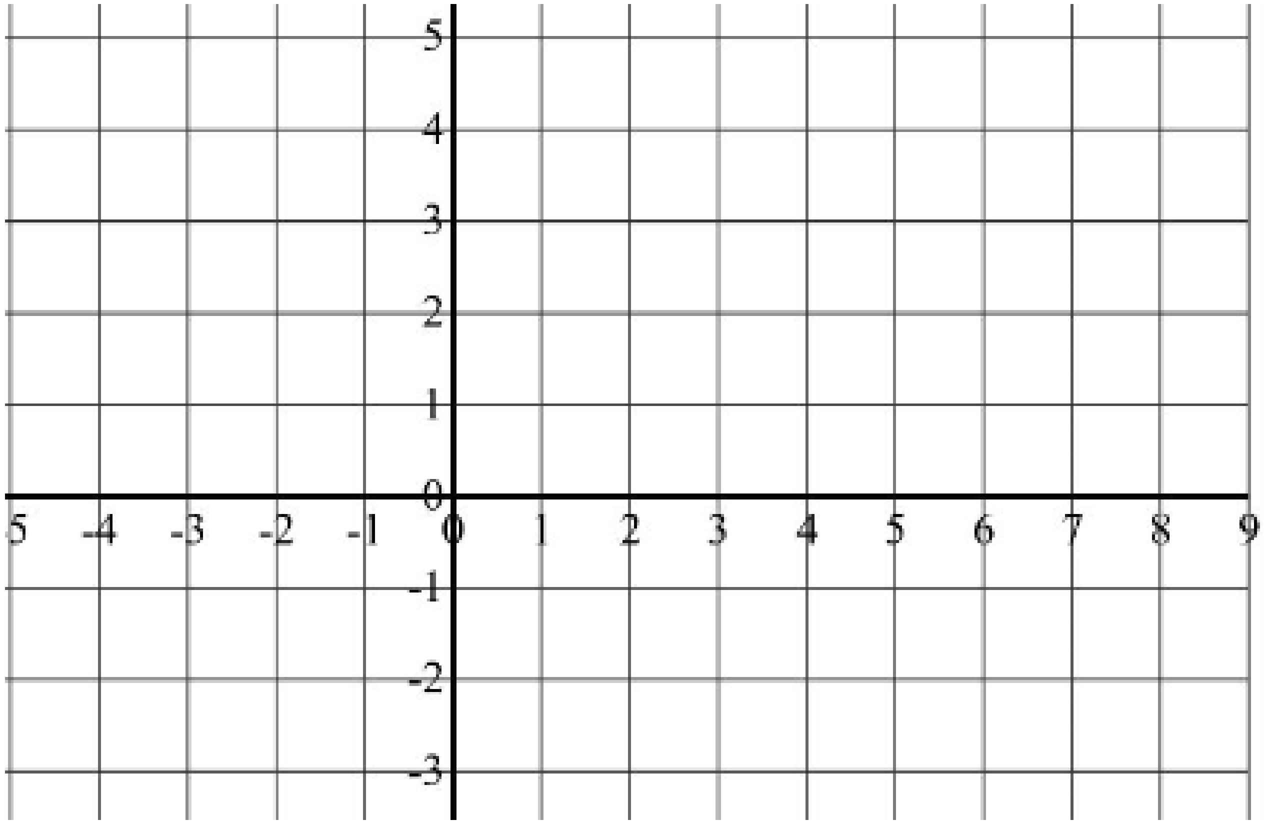
Step 5

Check the solution by graphing.

x	y
0	
	0

$$x - 3y = 9$$
$$x + 4y = 2$$

x	y
0	
	0



How do you choose which equation to solve first, and which variable to solve for?

Look for an equation that can be easily solved for one variable, such as one where the variable's coefficient is 1 or -1.

For example:

	Original Equation	Coefficient	What To Do	What You Get	Which When Swapped Is
1	$3x + \boldsymbol{y} = -3$	1	Subtract $3x$ from both sides	$\boldsymbol{y} = -3 - 3x$	$y = -3x - 3$
2	$-2x + \boldsymbol{y} = 7$	1	Add $2x$ to both sides	$y = -7 + 2x$	$y = 2x - 7$
3	$\boldsymbol{x} - 3y = 9$	1	Add $3y$ to both sides	$x = 9 + 3y$	$x = 3y + 9$
4	$9x - \boldsymbol{y} = 2$	-1	Subtract $9x$ from both sides	$-y = 2 - 9x$	$-y = -9x + 2$ Now multiply it all by -1 (which flips all the signs) $y = 9x - 2$

Solve this system of linear equations by substitution.

$$\begin{aligned} 3x + y &= 14 \\ 2x - 6y &= -24 \end{aligned}$$

Step 1: Solve one of the equations for one of its variables.

Step 2: Substitute the expression from Step 1 into the other equation, and solve for the other variable.

Step 3: Substitute the value from Step 2 into one of the original equations, and solve for the other variable.

Step 4: Write the values from Steps 2 and 3 in an ordered pair (x, y).

Step 5: Check the solution by substituting into both equations or by graphing.

$$\begin{aligned} 3x + y &= 14 \\ 2x - 6y &= -24 \end{aligned}$$

$3, 5$

There are two types of Special Linear Systems.

1) $$\begin{aligned} y &= 2x - 2 \\ -2x + y &= 4 \end{aligned}$$

Substitute the first into the second.

$$-2x + (2x - 2) = 4$$
$$-2x + 2x - 2 = 4$$
$$-2 = 4$$

Since these two numbers can't equal each other, **there is no solution.**

2) $$\begin{aligned} y &= 3x - 3 \\ -3x + y &= -3 \end{aligned}$$

Substitute the first into the second.

$$-3x + (3x - 3) = -3$$
$$-3x + 3x - 3 = -3$$
$$-3 = -3$$

Since these two numbers always equal each other, **there is an infinite number of solutions.**

Solve these: State whether there is **NO** solution, **ONE** solution (specify it), or **INFINITELY MANY** solutions.

$$x - 4y = 2$$
$$4x - 12y = 8$$

$$4x + 3y = 0$$
$$2x - y = 0$$

$$2x - 3y = -13$$
$$y = 2x + 7$$

$$y = 3x + 2$$
$$3x = y - 3$$

$$x = 3y + 7$$
$$x = 2y - 1$$

$$6x = 3y + 21$$
$$y = 2x - 7$$

What if there's no coefficient with a value of 1 or -1?
For example:

$$\mathbf{2x + 6y = 16}$$

What do you do in this case?
You may be able to reduce the entire equation!
Choose the variable you want, and then divide all the terms by its coefficient.
Here, if you want "x", then divide all the terms by **2**:

$$\frac{2x + 6y = 16}{2} = \frac{2x}{2} + \frac{6y}{2} = \frac{16}{2} = x + 3y = 8$$

Do you want "y"? No!
Imagine dividing all the terms by its coefficient of 6!

$$\frac{2x + 6y = 16}{6} = \frac{2x}{6} + \frac{6y}{6} = \frac{16}{6} = \frac{1}{3}x + y = \frac{8}{3}$$

What about this case? $\frac{1}{3}x + 6y = 20$

You can eliminate the fraction (and use the "x") by multiplying all terms by its reciprocal.

This becomes:

$$3\left(\frac{1}{3}x + 6y = 20\right) = \mathbf{x + 18y = 60}$$

(simplified)

What about this case? $\frac{1}{3}x + \frac{1}{4}y = 20$

Multiply all terms by both denominators, one at a time.

$$3\left(\frac{1}{3}x + \frac{1}{4}y = 20\right) = x + \frac{3}{4}y = 60$$

$$4\left(x + \frac{3}{4}y = 60\right) = \mathbf{4x + 3y = 240}$$

(simplified)

Geometry The length of a rectangular room is 5 feet more than its width. The perimeter of the room is 66 feet. Let L represent the length of the room and W represent the width in feet. The system of equations $\begin{cases} L = W + 5 \\ 66 = 2L + 2W \end{cases}$ can be used to represent this situation. What are the room's dimensions?

The sum of two numbers is 2. If one number is subtracted from the other, their difference is 8. Find the numbers.