Lesson 12: Linear Equations in Two Variables

Classwork

Opening Exercise

Students complete the Opening Exercise independently in preparation for the discussion about standard form and solutions that follows.

Opening Exercise

Emily tells you that she scored 32 points in a basketball game with only two- and three-point baskets (no free throws). How many of each type of basket did she score? Use the table below to organize your work.

Number of Two-Pointers	Number of Three-Pointers
16	**0**
13	**2**
10	**4**
7	**6**
4	**8**
1	**10**

Let x be the number of two-pointers and y be the number of three-pointers that Emily scored. Write an equation to represent the situation.

$$2x + 3y = 32$$

Discussion

- An equation in the form of $ax + by = c$ is called a *linear equation in two variables*, where a, b, and c are constants, and at least one of a and b are not zero. In this lesson, neither a nor b will be equal to zero. In the Opening Exercise, what equation did you write to represent Emily's score at the basketball game?
  - $2x + 3y = 32$
- The equation $2x + 3y = 32$ is an example of a linear equation in two variables.
- An equation of this form, $ax + by = c$, is also referred to as an equation in *standard form*. Is the equation you wrote in the opening exercise in standard form?
  - *Yes, it is in the same form as* $ax + by = c$.
- In the equation $ax + by = c$, the symbols a, b, and c are constants. What, then, are x and y?
  - *The symbols x and y are numbers. Since they are not constants, it means they are unknown numbers, typically called variables, in the equation* $ax + by = c$.
- For example, $-50x + y = 15$ is a linear equation in x and y. As you can easily see, not just *any* pair of numbers x and y will make the equation true. Consider $x = 1$ and $y = 2$. Does it make the equation true?
  - *No, because* $-50(1) + 2 = -50 + 2 = -48 \neq 15$.

- What pairs of numbers did you find that worked for Emily's basketball score? Did just any pair of numbers work? Explain.
  - *Students should identify the pairs of numbers in the table of the Opening Exercise. No, not just any pair of numbers worked. For example, I couldn't say that Emily scored* 15 *two-pointers and* 1 *three-pointer because that would mean she scored* 33 *points in the game, and she only scored* 32 *points.*
- A *solution* to the linear equation in two variables is an ordered pair of numbers (x, y) so that x and y makes the equation a true statement. The pairs of numbers that you wrote in the table for Emily are solutions to the equation $2x + 3y = 32$ because they are pairs of numbers that make the equation true. The question becomes, how do we find an unlimited number of solutions to a given linear equation?
  - *Guess numbers until you find a pair that makes the equation true.*
- A strategy that will help us find solutions to a linear equation in two variables is as follows: We fix a number for x. That means we pick any number we want and call it x. Since we know how to solve a linear equation in one variable, then we solve for y. The number we picked for x and the number we get when we solve for y is the ordered pair (x, y), which is a solution to the two variable linear equation.
- For example, let $x = 5$. Then, in the equation $-50x + y = 15$ we have

$$\begin{aligned} -50(5) + y &= 15 \\ -250 + y &= 15 \\ -250 + 250 + y &= 15 + 250 \\ y &= 265. \end{aligned}$$

  Therefore, $(5, 265)$ is a solution to the equation $-50x + y = 15$.
- Similarly, we can fix a number for y and solve for x. Let $y = 10$, then

$$\begin{aligned} -50x + 10 &= 15 \\ -50x + 10 - 10 &= 15 - 10 \\ -50x &= 5 \\ \frac{-50}{-50}x &= \frac{5}{-50} \\ x &= -\frac{1}{10}. \end{aligned}$$

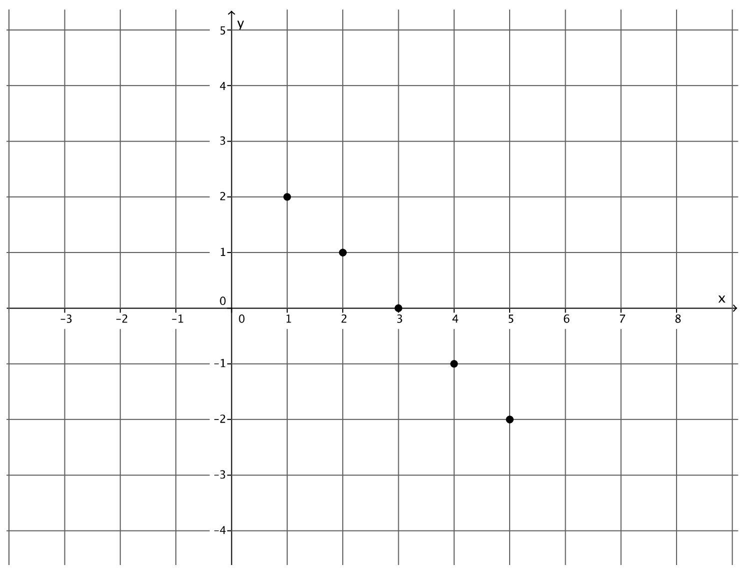
  Therefore, $(-\frac{1}{10}, 10)$ is a solution to the equation $-50x + y = 15$.

Exploratory Challenge/Exercises 1–5

Exploratory Challenge/Exercises

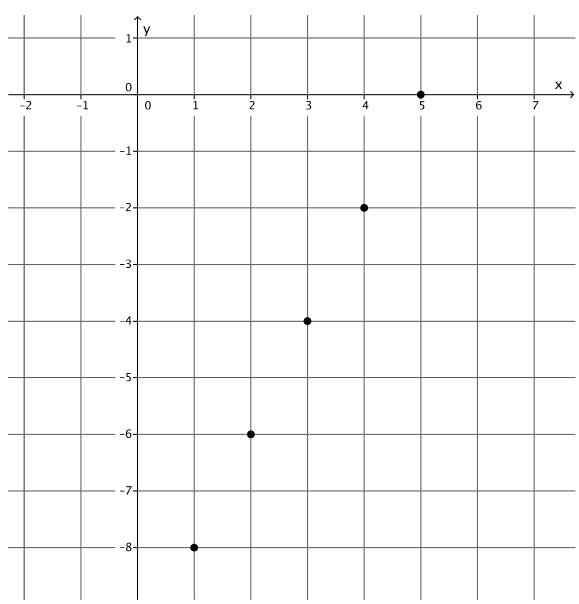
1. Find five solutions for the linear equation $x + y = 3$, and plot the solutions as points on a coordinate plane.

x	Linear equation: $x + y = 3$	y
1	$1 + y = 3$	2
2	$2 + y = 3$	1
3	$3 + y = 3$	0
4	$4 + y = 3$	-1
5	$5 + y = 3$	-2



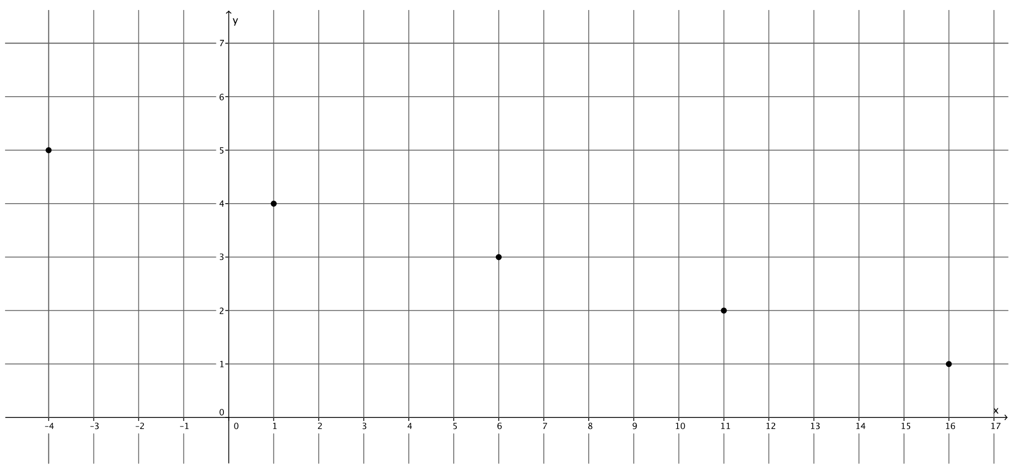
2. **Find five solutions for the linear equation $2x - y = 10$, and plot the solutions as points on a coordinate plane.**

x	Linear equation: $2x - y = 10$	y
1	$2(1) - y = 10$ $2 - y = 10$ $2 - 2 - y = 10 - 2$ $-y = 8$ $y = -8$	-8
2	$2(2) - y = 10$ $4 - y = 10$ $4 - 4 - y = 10 - 4$ $-y = 6$ $y = -6$	-6
3	$2(3) - y = 10$ $6 - y = 10$ $6 - 6 - y = 10 - 6$ $-y = 4$ $y = -4$	-4
4	$2(4) - y = 10$ $8 - y = 10$ $8 - 8 - y = 10 - 8$ $-y = 2$ $y = -2$	-2
5	$2(5) - y = 10$ $10 - y = 10$ $y = 0$	0



3. Find five solutions for the linear equation $x + 5y = 21$, and plot the solutions as points on a coordinate plane.

x	Linear equation: $x + 5y = 21$	y
16	$x + 5(1) = 21$ $x + 5 = 21$ $x = 16$	1
11	$x + 5(2) = 21$ $x + 10 = 21$ $x = 11$	2
6	$x + 5(3) = 21$ $x + 15 = 21$ $x = 6$	3
1	$x + 5(4) = 21$ $x + 20 = 21$ $x = 1$	4
-4	$x + 5(5) = 21$ $x + 25 = 21$ $x = -4$	5



4. Consider the linear equation $\frac{2}{5}x + y = 11$.

a. Will you choose to fix values for x or y? Explain.

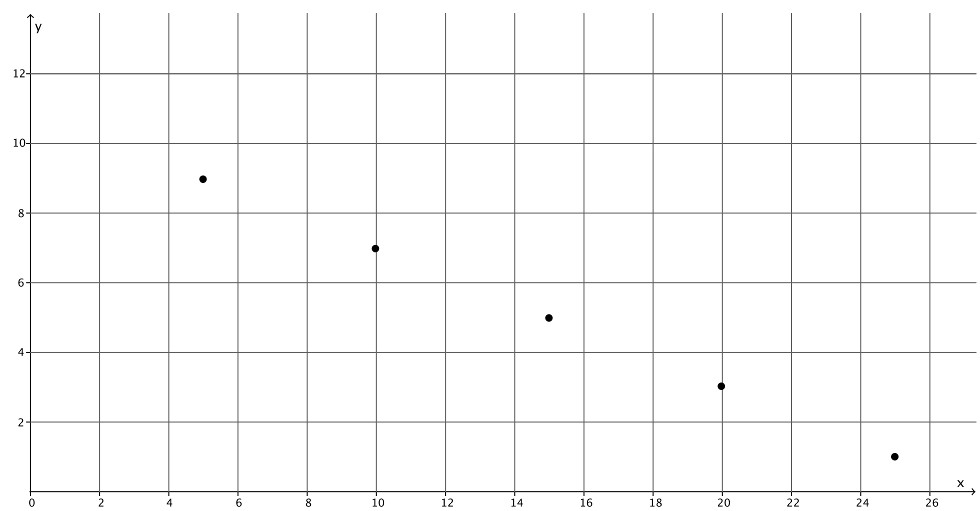
If I fix values for x, it will make the computations easier. Solving for y can be done in one step.

b. Are there specific numbers that would make your computational work easier? Explain.

Values for x that are multiples of 5 will make the computations easier. When I multiply $\frac{2}{5}$ by a multiple of 5, I will get an integer.

c. Find five solutions to the linear equation $\frac{2}{5}x + y = 11$, and plot the solutions as points on a coordinate plane.

x	Linear equation: $\frac{2}{5}x + y = 11$	y
5	$\frac{2}{5}(5) + y = 11$ $2 + y = 11$ $y = 9$	9
10	$\frac{2}{5}(10) + y = 11$ $4 + y = 11$ $y = 7$	7
15	$\frac{2}{5}(15) + y = 11$ $6 + y = 11$ $y = 5$	5
20	$\frac{2}{5}(20) + y = 11$ $8 + y = 11$ $y = 3$	3
25	$\frac{2}{5}(25) + y = 11$ $10 + y = 11$ $y = 1$	1



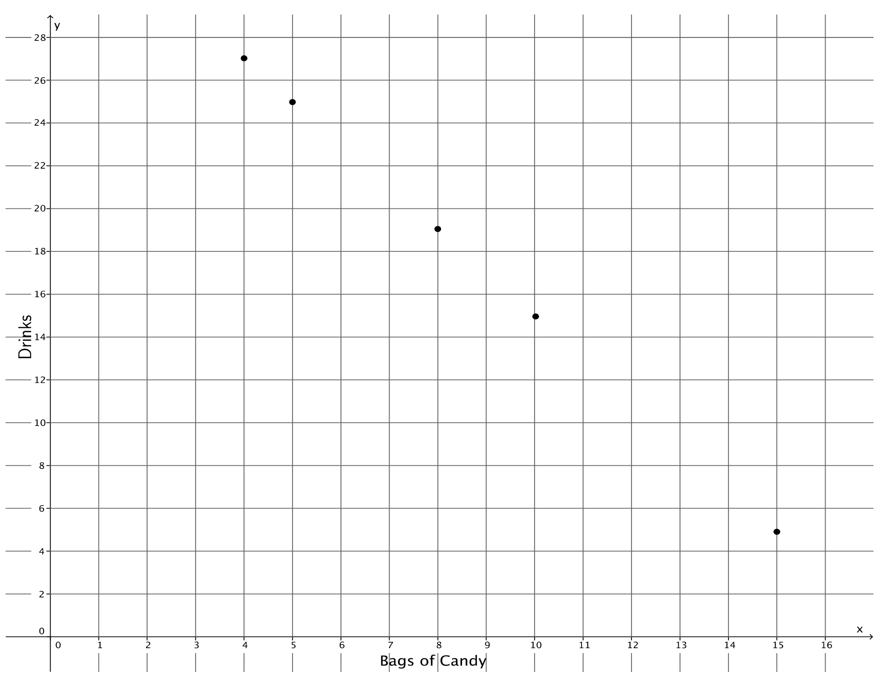
5. At the store, you see that you can buy a bag of candy for $\$2$ and a drink for $\$1$. Assume you have a total of $\$35$ to spend. You are feeling generous and want to buy some snacks for you and your friends.

a. Write an equation in standard form to represent the number of bags of candy, x, and the number of drinks, y, that you can buy with $\$35$.

$$2x + y = 35$$

b. Find five solutions to the linear equation from part (a), and plot the solutions as points on a coordinate plane.

x	Linear equation: $2x + y = 35$	y
4	$2(4) + y = 35$ $8 + y = 35$ $y = 27$	27
5	$2(5) + y = 35$ $10 + y = 35$ $y = 25$	25
8	$2(8) + y = 35$ $16 + y = 35$ $y = 19$	19
10	$2(10) + y = 35$ $20 + y = 35$ $y = 15$	15
15	$2(15) + y = 35$ $30 + y = 35$ $y = 5$	5



Closing

Summarize, or ask students to summarize, the main points from the lesson:

- A two-variable equation in the form of $ax + by = c$ is known as a linear equation in standard form.
- A solution to a linear equation in two variables is an ordered pair (x, y) that makes the given equation true.
- We can find solutions by fixing a number for x or y, then solving for the other variable. Our work can be made easier by thinking about the computations we will need to make before fixing a number for x or y. For example, if x has a coefficient of $\frac{1}{3}$, we should select values for x that are multiples of 3.

Lesson Summary

A two-variable linear equation in the form $ax + by = c$ is said to be in *standard form*.

A solution to a linear equation in two variables is the ordered pair (x, y) that makes the given equation true. Solutions can be found by fixing a number for x and solving for y or fixing a number for y and solving for x.

Problem Set Sample Solutions

Students practice finding and graphing solutions for linear equations that are in standard form.

1. Consider the linear equation $x - \frac{3}{2}y = -2$.

a. Will you choose to fix values for x or y? Explain.

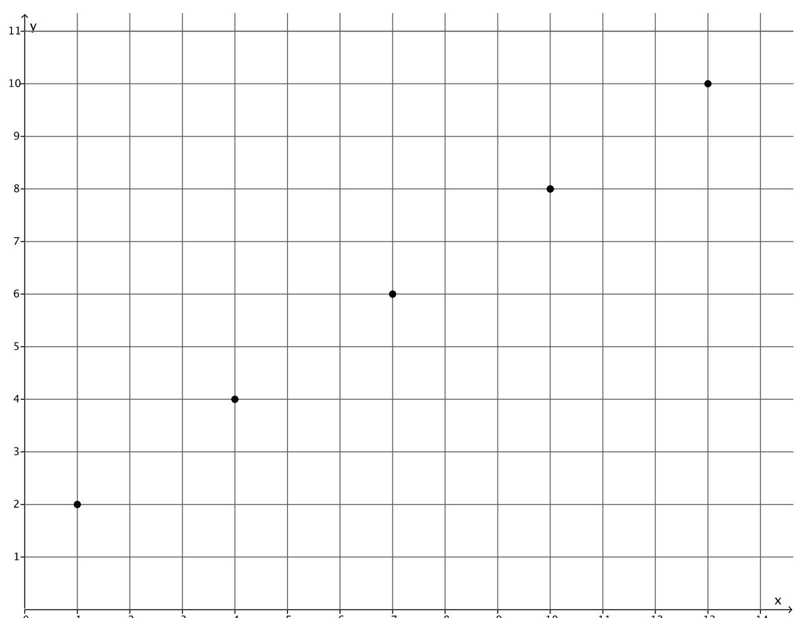
If I fix values for y, it will make the computations easier. Solving for x can be done in one step.

b. Are there specific numbers that would make your computational work easier? Explain.

Values for y that are multiples of 2 will make the computations easier. When I multiply $\frac{3}{2}$ by a multiple of 2, I will get a whole number.

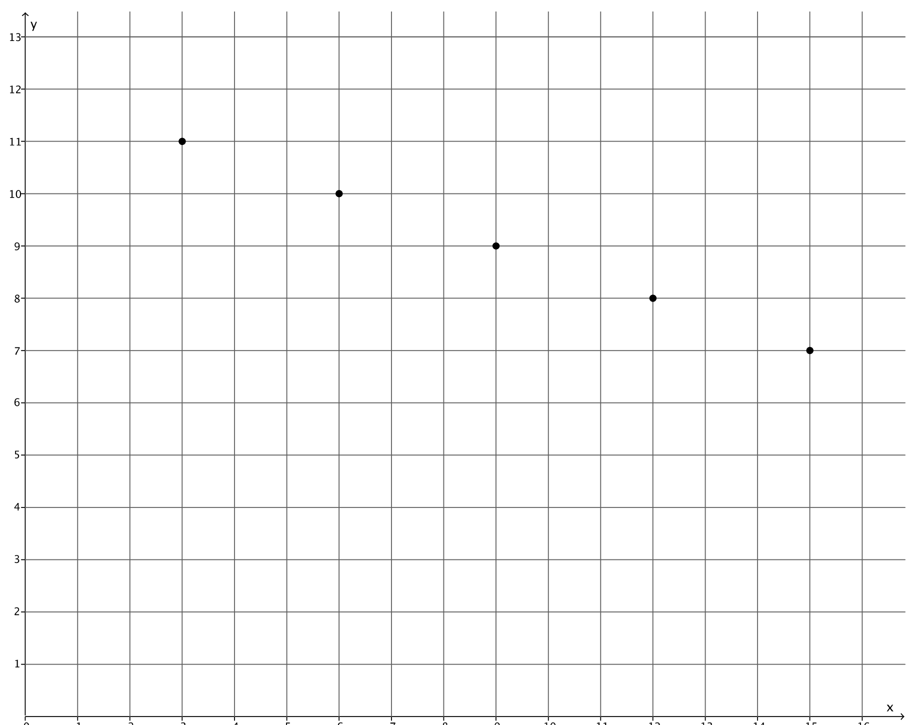
c. Find five solutions to the linear equation $x - \frac{3}{2}y = -2$, and plot the solutions as points on a coordinate plane.

x	Linear equation: $x - \frac{3}{2}y = -2$	y
1	$x - \frac{3}{2}(2) = -2$ $x - 3 = -2$ $x - 3 + 3 = -2 + 3$ $x = 1$	2
4	$x - \frac{3}{2}(4) = -2$ $x - 6 = -2$ $x - 6 + 6 = -2 + 6$ $x = 4$	4
7	$x - \frac{3}{2}(6) = -2$ $x - 9 = -2$ $x - 9 + 9 = -2 + 9$ $x = 7$	6
10	$x - \frac{3}{2}(8) = -2$ $x - 12 = -2$ $x - 12 + 12 = -2 + 12$ $x = 10$	8
13	$x - \frac{3}{2}(10) = -2$ $x - 15 = -2$ $x - 15 + 15 = -2 + 15$ $x = 13$	10



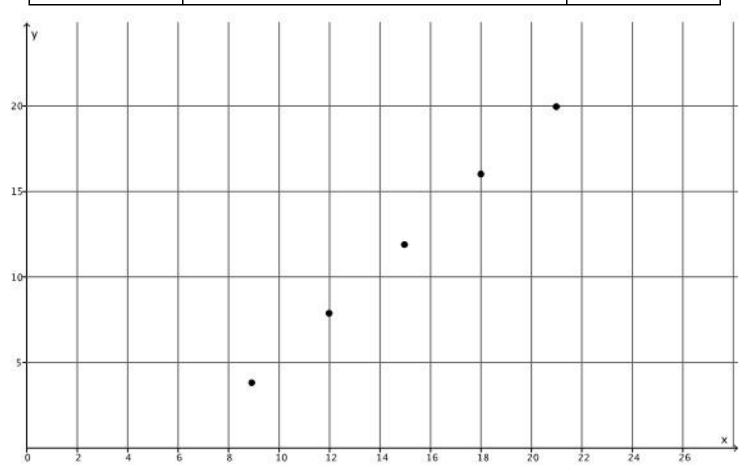
2. Find five solutions for the linear equation $\frac{1}{3}x + y = 12$, and plot the solutions as points on a coordinate plane.

x	Linear equation: $\frac{1}{3}x + y = 12$	y
3	$\frac{1}{3}(3) + y = 12$ $1 + y = 12$ $y = 11$	11
6	$\frac{1}{3}(6) + y = 12$ $2 + y = 12$ $y = 10$	10
9	$\frac{1}{3}(9) + y = 12$ $3 + y = 12$ $y = 9$	9
12	$\frac{1}{3}(12) + y = 12$ $4 + y = 12$ $y = 8$	8
15	$\frac{1}{3}(15) + y = 12$ $5 + y = 12$ $y = 7$	7



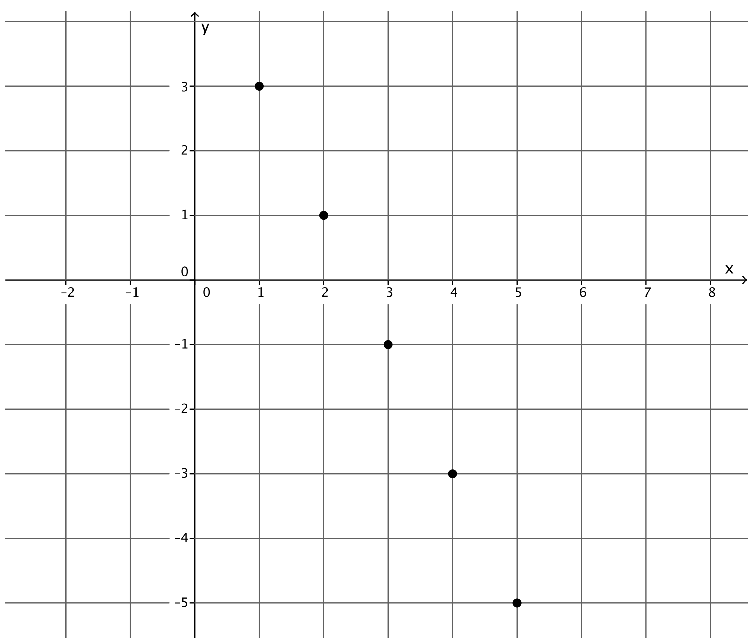
3. Find five solutions for the linear equation $-x+\frac{3}{4}y=-6$, and plot the solutions as points on a coordinate plane.

x	Linear equation: $-x+\frac{3}{4}y=-6$	y
9	$-x+\frac{3}{4}(4)=-6$ $-x+3=-6$ $-x+x+3=-6+x$ $3=-6+x$ $3+6=-6+6+x$ $9=x$	4
12	$-x+\frac{3}{4}(8)=-6$ $-x+6=-6$ $-x+x+6=-6+x$ $6=-6+x$ $6+6=-6+6+x$ $12=x$	8
15	$-x+\frac{3}{4}(12)=-6$ $-x+9=-6$ $-x+x+9=-6+x$ $9=-6+x$ $9+6=-6+6+x$ $15=x$	12
18	$-x+\frac{3}{4}(16)=-6$ $-x+12=-6$ $-x+x+12=-6+x$ $12=-6+x$ $12+6=-6+6+x$ $18=x$	16
21	$-x+\frac{3}{4}(20)=-6$ $-x+15=-6$ $-x+x+15=-6+x$ $15=-6+x$ $15+6=-6+6+x$ $21=x$	20



4. **Find five solutions for the linear equation $2x + y = 5$, and plot the solutions as points on a coordinate plane.**

x	Linear equation: $2x + y = 5$	y
1	$2(1) + y = 5$ $2 + y = 5$ $y = 3$	3
2	$2(2) + y = 5$ $4 + y = 5$ $y = 1$	1
3	$2(3) + y = 5$ $6 + y = 5$ $y = -1$	-1
4	$2(4) + y = 5$ $8 + y = 5$ $y = -3$	-3
5	$2(5) + y = 5$ $10 + y = 5$ $y = -5$	-5



5. Find five solutions for the linear equation $3x - 5y = 15$, and plot the solutions as points on a coordinate plane.

x	Linear equation: $3x - 5y = 15$	y
$\frac{20}{3}$	$3x - 5(1) = 15$ $3x - 5 = 15$ $3x - 5 + 5 = 15 + 5$ $3x = 20$ $\frac{3}{3}x = \frac{20}{3}$ $x = \frac{20}{3}$	1
$\frac{25}{3}$	$3x - 5(2) = 15$ $3x - 10 = 15$ $3x - 10 + 10 = 15 + 10$ $3x = 25$ $\frac{3}{3}x = \frac{25}{3}$ $x = \frac{25}{3}$	2
10	$3x - 5(3) = 15$ $3x - 15 = 15$ $3x - 15 + 15 = 15 + 15$ $3x = 30$ $x = 10$	3
$\frac{35}{3}$	$3x - 5(4) = 15$ $3x - 20 = 15$ $3x - 20 + 20 = 15 + 20$ $3x = 35$ $\frac{3}{3}x = \frac{35}{3}$ $x = \frac{35}{3}$	4
$\frac{40}{3}$	$3x - 5(5) = 15$ $3x - 25 = 15$ $3x - 25 + 25 = 15 + 25$ $3x = 40$ $\frac{3}{3}x = \frac{40}{3}$ $x = \frac{40}{3}$	5