

**Vocabulary**

dilation, p. 747  
 center of dilation,  
 p. 747  
 scale factor, p. 747

# Dilations

**BEFORE**

You translated, reflected, and rotated figures.

**Now**

You'll dilate figures in a coordinate plane.

**WHY?**

So you can create an illusion of a moving object, as in Ex. 11.

A **dilation** is a transformation in which a figure stretches or shrinks with respect to a fixed point, called the **center of dilation**. In this book, the origin of a coordinate plane is the center of dilation. In a dilation, a figure and its image are similar.

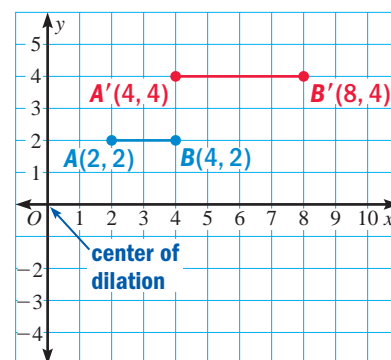
The **scale factor** of a dilation is the ratio of a side length of the image to the corresponding side length of the original figure. In the diagram,  $\overline{A'B'}$  is the image of  $\overline{AB}$  after a dilation.

Because  $\frac{A'B'}{AB} = 2$ , the scale factor is 2.

You can describe a dilation with respect to the origin using the notation

$$(x, y) \rightarrow (kx, ky)$$

where  $k$  is the scale factor.

**Study Strategy**

Notice in Example 1 that when  $k > 1$ , the new figure is an enlargement of the original figure. As you will see in Example 2, when  $k < 1$ , the new figure is a reduction of the original figure.

**Example 1****Dilating a Quadrilateral**

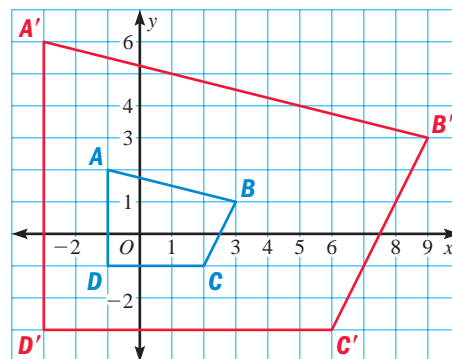
Draw quadrilateral  $ABCD$  with vertices  $A(-1, 2)$ ,  $B(3, 1)$ ,  $C(2, -1)$ , and  $D(-1, -1)$ . Then find the coordinates of the vertices of the image after a dilation having a scale factor of 3, and draw the image.

**Solution**

First draw quadrilateral  $ABCD$ . Then, to dilate  $ABCD$ , multiply the  $x$ - and  $y$ -coordinates of each vertex by 3.

Original	Image
$(x, y)$	$\rightarrow (3x, 3y)$
$A(-1, 2)$	$\rightarrow A'(-3, 6)$
$B(3, 1)$	$\rightarrow B'(9, 3)$
$C(2, -1)$	$\rightarrow C'(6, -3)$
$D(-1, -1)$	$\rightarrow D'(-3, -3)$

Finally, draw quadrilateral  $A'B'C'D'$ , as shown.



**Example 2****Using a Scale Factor Less than 1**

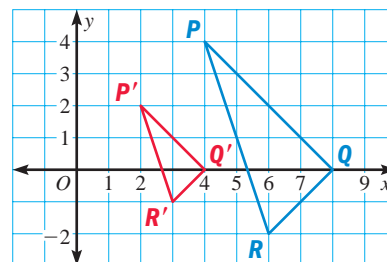
Draw  $\triangle PQR$  with vertices  $P(4, 4)$ ,  $Q(8, 0)$ , and  $R(6, -2)$ . Then find the coordinates of the image after a dilation having a scale factor of 0.5, and draw the image.

**Solution**

Draw  $\triangle PQR$ . Then, to dilate  $\triangle PQR$ , multiply the  $x$ - and the  $y$ -coordinates of each vertex by 0.5.

Original	Image
$(x, y)$	$\rightarrow (0.5x, 0.5y)$
$P(4, 4)$	$\rightarrow P'(2, 2)$
$Q(8, 0)$	$\rightarrow Q'(4, 0)$
$R(6, -2)$	$\rightarrow R'(3, -1)$

Finally, draw  $\triangle P'Q'R'$ , as shown.

**Checkpoint**

Draw  $\triangle ABC$  with vertices  $A(4, 0)$ ,  $B(4, 4)$ , and  $C(-4, 0)$ . Then find the coordinates of the vertices of the image after a dilation having the given scale factor, and draw the image.

- $k = 4$
- $k = \frac{1}{4}$

**In the Real World****Computer Graphics**

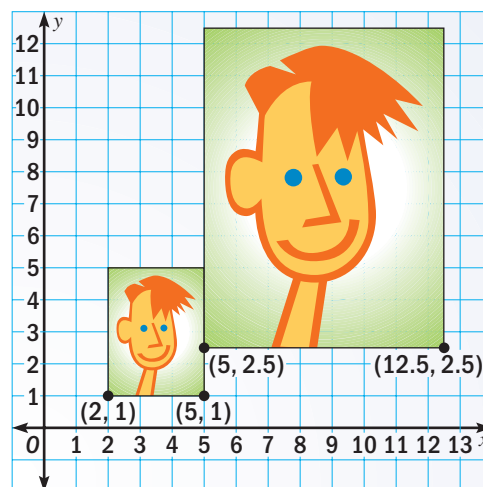
Computer graphics designers may create pictures called bit graphics. A 4 bit graphic can have  $2^4 = 16$  colors, and an 8 bit graphic can have  $2^8 = 256$  colors. How many colors can a 16 bit graphic have?

**Example 3****Finding a Scale Factor**

**Computer Graphics** An artist uses a computer program to enlarge a design, as shown. What is the scale factor of the dilation?

**Solution**

The width of the original design is  $5 - 2 = 3$  units. The width of the image is  $12.5 - 5 = 7.5$  units. So, the scale factor is  $\frac{7.5 \text{ units}}{3 \text{ units}}$ , or 2.5.

**Checkpoint**

- Given  $\overline{AB}$  with endpoints  $A(0.5, 1)$  and  $B(1.5, 1)$ , let  $\overline{A'B'}$  with endpoints  $A'(3, 6)$  and  $B'(9, 6)$  be the image of  $\overline{AB}$  after a dilation. Find the scale factor.

### Note Worthy

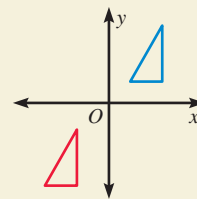
For each transformation that you studied in this chapter, you should include an example in your notebook along with a summary of the characteristics of the transformation.

## SUMMARY Transformations in a Coordinate Plane

### Translations

In a translation, each point of a figure is moved the same distance in the same direction.

$$(x, y) \rightarrow (x + a, y + b)$$

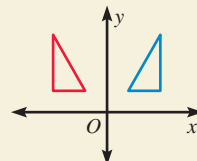


### Reflections

In a reflection, a figure is flipped over a line.

Reflection in  $x$ -axis:  $(x, y) \rightarrow (x, -y)$

Reflection in  $y$ -axis (shown):  $(x, y) \rightarrow (-x, y)$



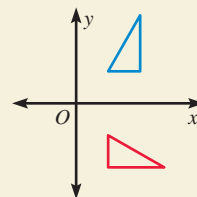
### Rotations

In the rotations below, a figure is turned about the origin through a given angle and direction.

$90^\circ$  clockwise rotation (shown):  $(x, y) \rightarrow (y, -x)$

$90^\circ$  counterclockwise rotation:  $(x, y) \rightarrow (-y, x)$

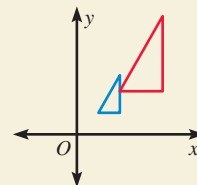
$180^\circ$  rotation:  $(x, y) \rightarrow (-x, -y)$



### Dilations

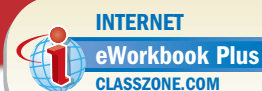
In the dilation below, a figure stretches or shrinks with respect to the origin.

$$(x, y) \rightarrow (kx, ky), \text{ where } k \text{ is the scale factor}$$



## 13.7 Exercises

More Practice, p. 815



### Guided Practice

#### Vocabulary Check

1. Copy and complete: In a translation, a figure and its image are congruent. In a dilation, a figure and its image are ?.
2. Let  $P(2, 3)$  be a point on a figure. The figure is dilated by a scale factor of 4. What are the coordinates of  $P'$ ?

#### Skill Check

3. Draw  $\triangle ABC$  with vertices  $A(-2, 0)$ ,  $B(1, 1)$ , and  $C(2, -1)$ . Then find the coordinates of the vertices of the image after a dilation having a scale factor of 3, and draw the image.
4. Given  $\overline{AB}$  with endpoints  $A(-2, 3)$  and  $B(-2, -4)$ , let  $\overline{A'B'}$  with endpoints  $A'(-5, 7.5)$  and  $B'(-5, -10)$  be the image of  $\overline{AB}$  after a dilation. What is the scale factor of the dilation?

## Practice and Problem Solving

### Homework Help

Example	Exercises
1	5-6, 11-12
2	7-8, 12
3	9-10



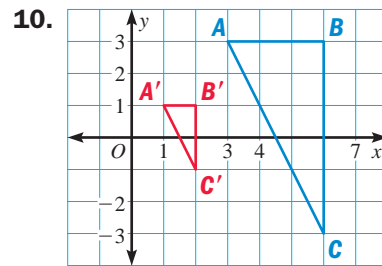
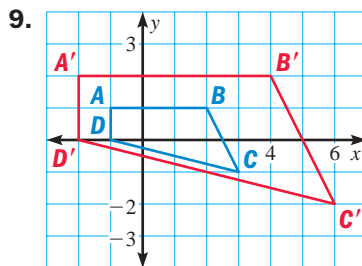
Online Resources  
CLASSZONE.COM

- More Examples
- eTutorial Plus

The vertices of a polygon are given. Draw the polygon. Then find the coordinates of the vertices of the image after a dilation having the given scale factor, and draw the image.

- $A(-1, 2)$ ,  $B(3, 1)$ ,  $C(1, -4)$ ;  $k = 2$
- $X(-1, 2)$ ,  $Y(2, 1)$ ,  $Z(-1, -3)$ ;  $k = 3$
- $P(-6, 2)$ ,  $Q(2, 2)$ ,  $R(2, 0)$ ,  $S(-6, 0)$ ;  $k = 0.5$
- $E(-8, 4)$ ,  $F(4, 4)$ ,  $G(0, -4)$ ,  $H(-4, -4)$ ;  $k = \frac{1}{4}$

Find the scale factor of the dilation.



11. **Illusions** You can use dilations to create the illusion of an object moving toward you.

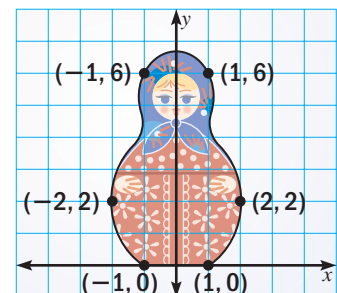
- Draw rectangle  $ABCD$  with vertices  $A(-2, -1)$ ,  $B(-1, -1)$ ,  $C(-1, -1.5)$ , and  $D(-2, -1.5)$ .
- On the same coordinate plane, draw the images of rectangle  $ABCD$  using the following scale factors: 2, 4, 8.

12. **Writing** Is an image *smaller than*, *larger than*, or *congruent to* the original figure when the scale factor is 3? 0.5? 1? Explain.

13. Draw  $\triangle ABC$  with vertices  $A(-2, 4)$ ,  $B(4, 0)$ , and  $C(2, -4)$ .

- You dilate  $\triangle ABC$  using a scale factor of 0.25. You then dilate its image using a scale factor of 2. Find the coordinates of the final image, and draw the image.
- Use the scale factors given in part (a) to find the scale factor you could use to dilate  $\triangle ABC$  to the final image in one step.
- Critical Thinking** Do you get the same final image if you switch the order of the dilations in part (a)? Explain your reasoning.

14. **Nesting Dolls** The figure is the front view of one of the dolls in a set of nesting dolls. Draw the outline of the figure. Then, on the same coordinate plane, draw the images of the outline after dilations having the following scale factors:  $\frac{1}{2}$ ,  $1\frac{1}{2}$ , 2.

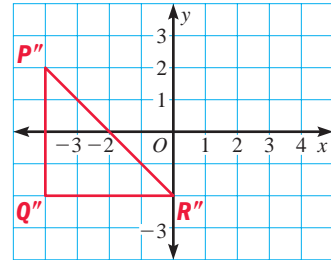


In Exercises 15 and 16,  $\triangle DEF$  has vertices  $D(-2, -4)$ ,  $E(6, 2)$ , and  $F(0, 4)$ . Draw  $\triangle DEF$ . Then find the coordinates of the vertices of the final image after the specified transformations, and draw the final image.

15. Dilate  $\triangle DEF$  using a scale factor of 2, then translate its image using  $(x, y) \rightarrow (x - 2, y + 3)$ .

16. Dilate  $\triangle DEF$  using a scale factor of 0.5, then rotate its image  $180^\circ$ .

17. **Challenge** A triangle is dilated using a scale factor of 2, then its image is reflected in the  $y$ -axis. The figure shown is the final image. Find the coordinates of the vertices of the original triangle, and draw the original triangle.



18. **Extended Problem Solving** Draw  $\triangle ABC$  with vertices  $A(0, -3)$ ,  $B(3, 1)$ , and  $C(3, -3)$ .

- a. **Calculate** Find the perimeter and the area of  $\triangle ABC$ .
- b. Find the coordinates of the vertices of the image of  $\triangle ABC$  after a dilation having a scale factor of 3, and draw the image. Then find the perimeter and the area of the image.

- c. **Compare** How is the scale factor related to the ratios

$$\frac{\text{Perimeter of image of } \triangle ABC}{\text{Perimeter of } \triangle ABC} \text{ and } \frac{\text{Area of image of } \triangle ABC}{\text{Area of } \triangle ABC}?$$

## Mixed Review

Find the number of permutations or combinations. (Lessons 11.6, 11.7)

19.  ${}_4P_2$       20.  ${}_8P_5$       21.  ${}_9C_9$       22.  ${}_{25}C_3$

Write the expression as a polynomial in standard form. (Lesson 12.1)

23.  $4t + 1 - 6t + t^4 - 4$       24.  $2(b - 6b^2) - 9b$

25. Draw  $\triangle DEF$  with vertices  $D(4, 3)$ ,  $E(6, 2)$ , and  $F(5, 1)$ . Then find the coordinates of the vertices of the image after a  $90^\circ$  counterclockwise rotation about the origin, and draw the image. (Lesson 13.6)

## Standardized Test Practice

26. **Multiple Choice** Let  $P(2, 4)$  be a point on a figure, and let  $P'$  be the corresponding point on the image. The figure is dilated by a scale factor of 4. What are the coordinates of  $P'$ ?

- A.  $(-2, 0)$       B.  $\left(\frac{1}{2}, 1\right)$       C.  $(6, 8)$       D.  $(8, 16)$

27. **Multiple Choice** In the diagram, quadrilateral  $A'B'C'D'$  is the image of quadrilateral  $ABCD$  after a dilation. What is the scale factor?

- F.  $\frac{1}{4}$       G.  $\frac{1}{2}$   
H. 2      I. 3

