

Analytical Methods for Materials

Lesson 13 Crystallography and Crystal Structures – continued

Suggested Reading

- Chapter 6 in Waseda
- Excerpt from ASM Metals Handbook.

Notation for crystal structures

- 1. Hermann-Mauguin (HM)... aka the 'International' system
- 2. Pearson
- 3. Strukturbericht
- 4. Schoenflies

The International (i.e., <u>Hermann-Maugin</u>) system is preferred for describing crystal structures in metals and ceramics.

The other systems have their uses too.

We use all of them to describe space groups!

Space group data allows us to define allowed and missing reflections in a diffraction pattern.

Tells us everything we need to know about symmetry and motif placement!

In this class, the first place you'll see this is on X-ray (i.e., ICDD, JCPDS, or PDF) cards.

ICDD (i.e., JCPDF)card



ICDD (i.e., JCPDF)card



ICDD (i.e., JCPDF)card for Al

00-004-0787

Oct 18, 2011 10:36 AM (CAF User)

Status Primary QM: Star (S) Pressure/Temperature: Ambient Chemical Formula: Al Empirical Formula: Al Weight %: Al100.00 Atomic %: Al100.00 Compound Name: Aluminum Mineral Name: Aluminum, syn

Radiation: CuKα1 λ: 1.5406Å Intensity: Diffractometer I/Ic: 3.62

 SYS:
 Cubic
 SPGR:
 Fm-3m (225)

 Author's Cell
 [AuthCell a: 4.0494Å
 AuthCell Vol:
 66.40Å³
 AuthCell Z: 4.00
 AuthCell MolVol:
 16.60
]

 Dcalc:
 2.699g/cm³
 SS/FOM:
 F(9) = 91.8(0.0109, 9)
)



 Space Group:
 Fm-3m (225)
 Molecular Weight:
 26.98

 Crystal Data [
 XtlCell a:
 4.049Å
 XtlCell b:
 4.049Å
 XtlCell c:
 4.049Å
 XtlCell a:
 90.00°
 XtlCell β:
 90.00°

 XtlCell y:
 90.00°
 XtlCell Vol:
 66.40Å³
 XtlCell Z:
 4.00
 Crystal Data Axial Ratio [
 a/b:
 1.000
 c/b:
 1.000
 j
 1.0

Crystal (Symmetry Allowed): Centrosymmetric

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      CAS:
      7429-90-5
      Pearson:
      cF4.00
      Prototype Structure:
      Cu
      Prototype Structure (Alpha Order):
      Cu

      LPF Prototype Structure:
      Cu,cF4,225
      LPF Prototype Structure (Alpha Order):
      Cu

      Mineral Classification:
      Gold (Supergroup), 1C-disordered (Group)
      Cu

      Subfile(c):
      Primary Pattern, Inorganic, Forensic, Mineral Related (Mineral , Synthetic), Pigment/Dye, Common Phase, Metals &
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Subfile(s): Primary Pattern, Inorganic, Forensic, Mineral Related (Mineral, Synthetic), Pigment/Dye, Common Phase, Metals & Alloys, Educational Pattern, Explosive, NBS Pattern

Last Modification Date: 01/15/2010

	01-085-1327 04-002-3717 04-003-4137	(Alternate), (Alternate), (Alternate)	04-001-0564	(Alternate), (Alternate),	04-001-1007 04-002-8919	(Alternate), ∣ (Alternate), ∣	04-001-7364 (04-003-1376 (Alternate), Alternate),
Cross-Ref PDF #'s:	04-003-4137 04-004-2353 04-005-9305	(Alternate), (Alternate), (Alternate),	04-003-4850 04-004-8465 04-006-2586	(Alternate), (Alternate), (Alternate),	04-003-6600 04-004-8742 04-006-6522	(Alternate), (Alternate), (Alternate),	04-003-7059 (04-004-8743 (04-007-0410 (Alternate), Alternate), Alternate),
	04-007-5139 04-012-3402	(Alternate), (Alternate),	04-007-9709	(Alternate), (Alternate),	04-007-9967 04-012-7848	(Alternate), ⊟ (Primary)	04-010-6160 (Alternate),

Experimental Data Reference: Swanson, Tatge. Natl. Bur. Stand. (U.S.), Circ. 539 I, 11 (1953). Physical Data Reference(s): Ibid.

Additional Patterns: See PDF 01-085-1327. Analysis: The chemical analysis (%): Si 0.011, Cu 0.006, Fe 0.007, Ti 0.0001, Zr 0.003, Ga 0.004, Mo 0.00002, S 0.0001, Al 99.9+ (by difference). Color: Light gray metallic. General Comments: Mineral species of doubtful validity, Am. Mineral., 65 205 (1980). Sample Preparation: The material used for the NBS sample was a melting point standard sample of aluminum prepared at NBS, Gaithersburg, Maryland, USA. Temperature of Data Collection: Pattern taken at 298 K. Unit Cell Data Source: Powder Diffraction.

00-004-0787 (Fixed Slit Intensity) - Cu Ka1 1.54056Å

<u>20</u>	d(Å)	I	h	k	I	*	20	d(Å)	I	h	k	1	*	20	d(Å)	I	h	k	L	*
38.4721	2.338000	100	1	1	1		78.2272	1.221000	24	3	1	1		112.0413	0.928900	8	3	3	1	_
44.7384	2.024000	47	2	0	0		82.4354	1.169000	7	2	2	2		116.5688	0.905500	8	4	2	0	
65.1334	1.431000	22	2	2	0		99.0776	1.012400	2	4	0	0		137.4550	0.826600	8	4	2	2	

Space group data allows us to define allowed and missing reflections in a diffraction pattern.

Tells us everything we need to know about symmetry and motif placement!



Another place you'll see this is in the International Tables for Crystallography.

International Tables for Crystallography

From the data tabulated in the International Tables, we can get the following:

- Herman-Mauguin (HM) Symbol (Long, Short)
- Point Group (HM, Schoenflies)
- Location and identification of symmetry elements
- Wyckoff site multiplicity and symmetry
- Location of general and special positions
- Extinction conditions for X-rays
- Possible subgroups and supergroups



You don't need to worry about some of these things... "yet."

RECALL

Herman - Mauguin (International) Symbol

The HM symbol is derived from the type of Bravais lattice and the symmetry elements present in the crystal.

(Letter) \cdot (primary) \cdot (secondary) \cdot (tertiary)

Each <u>letter describes centering</u> present in the lattice:

- P Primitive I Body centered
- C C-centered F Face centered
- B B-centered R Rhombohedral
- A A-centered H Hexagonal

See the next viewgraph.

Letter		Number of lattice	"Centering"
symbol	Lattice type	points per unit cell	Coordinates of lattice points
P	Р	1	0,0,0
A	A-face centered	2	$0, 0, 0; 0, \frac{1}{2}, \frac{1}{2}$
В	B-face centered	2	$0, 0, 0; \frac{1}{2}, 0, \frac{1}{2}$
С	C-face centered	2	$0,0,0;\frac{1}{2},\frac{1}{2},0$
Ι	Body centered	2	$0, 0, 0; \frac{1}{2}, \frac{1}{2}, \frac{1}{2}$
F	Face centered (all)	4	$0, 0, 0; \frac{1}{2}, \frac{1}{2}, 0; \frac{1}{2}, 0, \frac{1}{2}; 0, \frac{1}{2}, \frac{1}{2}$
R	Primitive (Rhombohedral axes)	1	0,0,0
	Centered (Hexagonal axes)	3	$0, 0, 0; \frac{2}{3}, \frac{1}{3}, \frac{1}{3}; \frac{1}{3}, \frac{2}{3}, \frac{2}{3}$ (obverse setting) $0, 0, 0; \frac{1}{3}, \frac{2}{3}, \frac{1}{3}; \frac{2}{3}, \frac{1}{3}, \frac{2}{3}$ (reverse setting)
Н	Centered Hexagonal	3	$0,0,0;\frac{2}{3},\frac{1}{3},0;\frac{1}{3},\frac{2}{3},0$

We use specific symbols to denote crystal systems

The next three symbols denote symmetry elements present in certain directions:

	Symmetry Direction						
Crystal System	Primary	Secondary	Tertiary				
Triclinic	None						
Monoclinic*	[010]						
Orthorhombic	[100]	[010]	[001]				
Tetragonal	[001]	[100]/[010]	[110]				
Hexagonal/ Trigonal	[001]	[100]/[010]	[120]/[1(-1)0]				
Cubic	[100]/[010]/ [001]	[111]	[110]				

* For this structure list primary in secondary slot. Place "1" in primary and tertiary slots for Long notation.

The bracketed numbers on the previous page represent directions originating from the crystal origin:

- [100] Axis parallel or plane perpendicular to the *a*-axis.
- [010] Axis parallel or plane perpendicular to the *b*-axis.
- [001] Axis parallel or plane perpendicular to the *c*-axis.
- [110] Axis parallel or plane perpendicular to the line running at 45° to the *a*-axis and *b*-axis.
- $[1\overline{1}0]$ Axis parallel or plane perpendicular to the long face diagonal of the *ab* face of a hexagonal cell.
- [111] Axis parallel or plane perpendicular to the body diagonal.

Symbol Comments Lattice type reflection Mirror plane т Axial glide plane \perp [010],[001] Glide vector a/2а Glide vector b/2b Axial glide plane \perp [001],[100] Axial glide plane \perp [100],[010] Glide vector c/2С \perp [1 $\overline{1}$ 0],[110] Glide vector c/2Glide vector c/2, hexagonal axes \perp [100], [010], [$\overline{1}\overline{1}0$] Glide vector c/2, hexagonal axes \perp [1 $\overline{10}$],[120],[$\overline{2}\overline{10}$],[$\overline{1}\overline{10}$] Diagonal glide plane \perp [001]; [100]; [010] Glide vector $\frac{1}{2}(a+b)$; $\frac{1}{2}(b+c)$; $\frac{1}{2}(a+c)$ п Diagonal glide plane \perp [1 $\overline{10}$]; [01 $\overline{1}$]; [$\overline{101}$] Glide vector $\frac{1}{2}(a+b+c)$; $\frac{1}{2}(a+c)$ Glide vector $\frac{1}{2}(-a+b+c)$; $\frac{1}{2}(a-b+c)$; $\frac{1}{2}(a+b-c)$ Diagonal glide plane \perp [110]; [011]; [101] Glide vector $\frac{1}{4}(a \pm b)$; $\frac{1}{4}(b \pm c)$; $\frac{1}{4}(a \pm c)$ d Diamond glide plane \perp [001]; [100]; [010] Diamond glide plane \perp [1 $\overline{10}$]; [01 $\overline{1}$]; [$\overline{101}$] Glide vector $\frac{1}{4}(a+b\pm c); \frac{1}{4}(\pm a+b+c); \frac{1}{4}(a\pm b+c)$ Glide vector $\frac{1}{4}(-a+b\pm c)$; $\frac{1}{4}(\pm a-b+c)$; $\frac{1}{4}(a\pm b-c)$ Diamond glide plane \perp [110]; [011]; [101] 1 None _ _ _ 2, 3, 4, 6 n – fold rotation axis A counter clockwise rotation of $360^{\circ}/n$ $\overline{1}$ Center of symmetry _ _ _ $\overline{2} = m, \overline{3}, \overline{4}, \overline{6}$ \overline{n} – fold rotoinversion axis A counter clockwise rotation of $360^{\circ}/n$ followed by inversion $2_1, 3_1, 3_2, 4_1,$ A counter clockwise right-handed screw rotation of $360^{\circ}/n$ followed n- fold screw axis, n_n $4_2, 4_3, 6_1, 6_2,$ by translation by (p/n)T $6_3, 6_4, 6_5$

Symmetry elements in space group symbols

We use specific symbols to denote internal symmetry.

The <u>full</u> Herman-Mauguin symbol shows:

- Both rotation/screw axes running parallel to the specified direction.
- Any mirror/glide plane perpendicular to the same direction.

or

• The two symbols separated by a forward slash (/)

	С	1	2/m	1	
	Р	21	21	2	
Some examples:	Р	2/m	2/n	2 ₁ /a	
	Ι	4 ₁ /a	2/m	2/d	
	Р	6 ₃ /m	2/m	2/c	
	F	4/m	3	2/m	



The short Herman-Mauguin symbol

- Short HM symbols contain only the mirror/glide plane components for primary, secondary and tertiary directions.
- If no mirror/glide plane, contains the rotation/screw axis.
- In certain cases (monoclinic, tetragonal and hexagonal), both the rotation/screw axis and the mirror/glide plane are retained for the primary direction only.

Long

- C 1 2/m 1
- P 2₁ 2₁ 2
- P 2/m 2/n 2₁/a
- I 4₁/a 2/m 2/d
- P 6₃/m 2/m 2/c
- F 4/m 3 2/m



- I4₁/amd
- P6₃/mmc
- Fm3m



Fig. (a) Cubic unit cell. (b) Space group P $4/m \ \overline{3} \ 2/m$. In diagrams (c), (d) and (e), only the symetry elements corresponding to the symmetry directions <100>, <111>, <110> are shown. P $4/m \ \overline{3} \ 2/m$.

 $\langle 100 \rangle \langle 111 \rangle \langle 110 \rangle$

Adapted from W. Borchardt-Ott, <u>*Crystallography, 2nd Edition,*</u> Springer, Berlin, Germany, 1995, p. 99



Fig. 6.18 The location of 4-, 3-, and 2-fold symmetry axes with respect to a cubic outline for 432. Note that the axes connect symbols on the opposite sides of the crystal and run through the center. Adapted from C. Klein and B. Dutrow, <u>Manual of Mineral Science</u>, 23rd Edition (John Wiley & Sons, 2007)

Galena (PbS)



http://crystal-cure.com/pics/galena.jpg



http://en.wikipedia.org/wiki/File:Galena-unit-cell-3Dionic.png $4/m \overline{3} 2/m$

Fm3m



http://www.minservice.com/cartshop3/catalog/FERR0240_galena_p.jpg

Crystal Forms





In a, only those symmetry elements which relate to the a-axis and equivalent directions (i.e. the b- and c-axes) have been drawn in $(4/m \rightarrow \langle a \rangle)$; in **b**, only those relating to the [111] and equivalent directions $(\bar{3} \rightarrow \langle 111 \rangle)$; in **c**, only those relating to the [110] and equivalent directions $(2/m \rightarrow \langle 110 \rangle)$. The stereogram of the symmetry elements is given in **d**

Adapted from W. Borchardt-Ott, <u>Crystallography, 2nd</u> <u>Edition</u>, Springer, Berlin, Germany, 1995, p. 124 **Fig. 1.9.** (a) the three planes of symmetry parallel to the faces of a cube. (b) The six diagonal planes of symmetry of a cube. (c) One of the tetrad axes of a cube. (d) one of the triad axes of a cube. (e) one of the diad axes of a cube. (f) The thirteen axes of symmetry shown by a cube. From C. Kittel, <u>Introduction to Solid State Physics, 2nd Edition</u>, (John Wiley & Sons, New York, 1959) p. 23.



How do we identify the crystal system from a HM symbol?

- Cubic Secondary symmetry symbol 3 or $\overline{3}$ (i.e. Ia3, Pm3m, Fd3m)
- Tetragonal Primary symmetry symbol 4, $\overline{4}$, 4_1 , 4_2 or 4_3 (i.e. P4₁2₁2, I4/*m*, P4/*mcc*)
- Hexagonal Primary symmetry symbol 6, $\overline{6}$, 6_1 , 6_2 , 6_3 or 6_5 (i.e. P6mm, P $6_3 / m3m$)
- **Trigonal** Primary symmetry symbol 3, $\overline{3}$, 3_1 , or 3_2 (i.e. P31*m*, R3, R3*c*, P312)
- Orthorhombic All three symbols following the lattice descriptor are mirror planes, glide planes, 2-fold rotation or screw axes (i.e. Pnma, Cmc2₁, Pnc2)
- Monoclinic Lattice descriptor followed by a single mirror plane, glide plane, 2-fold rotation or screw axis or an axis/plane symbol (i.e. Cc, P2, $P2_1/n$)
- Triclinic Lattice descriptor followed by either a 1 or $\overline{1}$

Pearson Symbols

- W.B. Pearson, *The Crystal Chemistry and Physics of Metals and Alloys,* Wiley-Interscience, New York, 1972.
- Classification of structures based on:
 - Crystal system
 - Bravais lattice
 - Number of atoms per unit cell



Notation for Crystal Structures – cont'd

Symbols for the 14 Bravais Lattices

Symbol	System	Lattice Symbol
aP	Triclinic	Р
mP	Simple monoclinic	Р
mC	Base-centered monoclinic	C
оР	Simple orthorhombic	Р
oC	Base-centered orthorhombic	C
oF	Face-centered orthorhombic	F
oI	Body-centered orthorhombic	Ι
tP	Simple tetragonal	Р
tI	Body-centered tetragonal	Ι
hP	Hexagonal	Р
hR	Rhombohedral	R
cP	Simple cubic	Р
cF	Face-centered cubic	F
cI	Body-centered cubic	Ι

Crystal structure	Pearson Symbol
Simple cubic	cP1
Body-centered cubic	cI2
Face-centered cubic	cF4
Hexagonal close-paced	hP2
CsCl	cP2
NaCl	cF8
Diamond cubic	cF8
Zinc blende	cF8
Fluorite	cF12

Pearson Symbols for Some Typical Crystal Structures



What's the Pearson symbol for this one?





Strukturbericht

- Symbols designate the type of crystal structure.
 - A elements;
 - B AB type compounds (i.e., composition near 50-50 at.%);
 - $C AB_2$ compounds;
 - D A_mB_n compounds;
 - E, F, G, H \cdots K More complex compounds ;
 - L Alloys;
 - O Organic compounds ;
 - S-Silicates.
- Many visual examples can be viewed at:
 - http://cst-www.nrl.navy.mil/lattice/struk/index.html
- This system is inconsistent. For example, TiAl is denoted as L1₀ even though it has AB stoichiometry (see the next viewgraph).

What's the Strukturbericht symbol for this one? What is the international symbol for this one?







Fig. 1. Atom positions, structure symbols, prototypes, space-group notations, Pearson symbols, and lattice parameters for some of the simple metallic crystals (continued on next two pages)

Excerpt from C.S. Barrett, "Crystal Structure of Metals," in <u>ASM Handbook, 8th Edition</u>, vol. 8 (1973), ASM International, Metals Park, OH, pp. 233-250.

A1 Copper type. Face-centered cubic: Fm3m; cF4. Four atoms per cell, at 0,0,0; $\frac{1}{2},0,\frac{1}{2}$; $0,\frac{1}{2},\frac{1}{2}$ and $\frac{1}{2},\frac{1}{2},0$. For Cu, a =3.61 A. See Fig. 1. **Examples:** Ag, Al, Au, α -Ca, α -Ce, β -Co, Cu, γ -Fe, Ir, Ni, Pb, Pd, Pt, Rh, α -Sr, α -Th.

A2 Tungsten type. Body-centered cubic: Im3m; cI2. Two atoms per cell, at 0,0,0 and $\frac{1}{2},\frac{1}{2}$. For W, a = 3.16 A. See Fig. 1. Examples: Ba, Cb, Cr, Cs, β Cu-Zn (HT), α -Fe, δ -Fe, K, β -Li, Mo, β -Na, Rb, Ta, V, W.

A4 Carbon (diamond) type. Face-centered cubic: Fd3m; cF8. Eight atoms per cell, at 0,0,0; 0,½,½; ½,0,½; ½,12,½,0; ¼,14,¼; ¼,¾,¾; ¾,¼,¾ and ¾,¾,¼. For C (diamond), a = 3.57 A. See Fig. 1. Examples: C (diamond), Ge, Si, α -Sn.

 $C11_b \quad \begin{array}{l} \text{MoSi}_2 \text{ type. Tetragonal: } I4/mmm; \\ ti6. \text{Two molybdenum atoms at 0,0,0} \\ \text{and } \frac{1/2}{2}, \frac{1/2}{2}, \frac{1/2}{2}; \text{ four silicon atoms at 0,0,z; } 0, 0, \bar{z}; \\ \frac{1/2}{2}, \frac{1/2}{2}, \frac{1/2}{2} + z \text{ and } \frac{1/2}{2}, \frac{1/2}{2} - z \text{ (with } z = 0.333). \text{ For } \\ \text{MoSi}_2, a = 3.20 \text{ A and } c = 7.86 \text{ A. See Fig. 1.} \\ \text{Examples: } \text{AgZr}_2, \text{ AlCr}_2, \text{ Au}_2\text{Be, } \text{Au}_2\text{Mn}, \\ \text{CuTi}_2, \text{ Hg}_2\text{Mg, MoSi}_2, \text{ Ni}_2\text{Ta, Si}_2\text{W}. \end{array}$

L1 AuCu I type. Tetragonal superlattice: P4/mmm; tP4. Two gold atoms at 0,0,0 and $\frac{1}{2},\frac{1}{2},0$; two copper atoms at 0, $\frac{1}{2},\frac{1}{2}$ and $\frac{1}{2},0,\frac{1}{2}$. See Fig. 1. Examples: AgTi, AlTi, AuCu I, θ CdPt, FePd, $\gamma^{"}$ FePt, θ MnNi, NiPt.

L1₂ AuCu₃ I type. Cubic superlattice: Pm3m; cP4. One gold atom at 0,0,0; three copper atoms at $0,\frac{1}{2},\frac{1}{2}$; $\frac{1}{2},0,\frac{1}{2}$ and $\frac{1}{2},\frac{1}{2},0$. See Fig. 1. Examples: α' AlNi₃, AlZr₃, Au₃Cu, AuCu₃ I, CoPt₃, Cr₃Pt, Fe₃Ga, FePd₃, Ni₃Fe, Ni₃Mn, Sn₃U. B2 CsCl or β' Cu-Zn type. Cubic: Pm3m; cP2. One cesium atom at 0,0,0; and one chlorine atom at $\frac{1}{2},\frac{1}{2},\frac{1}{2}$. For CsCl, a = 4.11 A. See Fig. 1. Examples: AgCd, CoTi, CsCl, FeAl, FeCo, FeTi, FeV, β NiAl, β NiGa, δ NiIn, NiTi, β' Cu-Zn.

B3 ZnS (sphalerite, or zinc blende) type. Face-centered cubic: $F\overline{43}m$; cF8. Four zinc atoms at 0,0,0; 0,12,12; 12,0,12 and 12,12,0; four sulfur atoms at 14,14,14; 14,34,34; 34,14,34 and 34,34,14. For ZnS (sphalerite), a = 5.42 A. See Fig. 1. Examples: CdS, CdSe, CdTe, CuFeS₂ (HT), GaP, GaSb, InAs, InP, InSb, β MnS, β SiC, ZnO, ZnS (sphalerite), ZnSe.

Excerpt from C.S. Barrett, "Crystal Structure of Metals," in <u>ASM Handbook, 8th Edition</u>, vol. 8 (1973), ASM International, Metals Park, OH, pp. 233-250.

A very good place to look for crystallography data is the Bilbao Crystallographic Server

http://www.cryst.ehu.es/