Overview | Understand Functions

UNDERSTAND: How can you recognize a function?

MATH FOCUS

This Understand lesson extends students' understanding of proportional relationships and linear equations to the concept of functions. Foundational understanding established in this lesson supports students in developing strategies to model real-world behavior with functions.

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

SMP 2, 3, and 7 are integrated into the Understand lesson structure.*

This lesson provides additional support for:

1 Make sense of problems and persevere in solving them.

8 Look for and express regularity in repeated reasoning.

* See page 1o to learn how every lesson includes these SMP.

Objectives

Content Objectives

- Understand that a function is a rule that assigns to each input exactly one output.
- Identify whether a relationship is a function from a verbal description, table of values, graph, or equation.
- Classify a function as linear or nonlinear.
- Interpret the equation $y = mx + b$ as defining a linear function whose graph is a nonvertical straight line.

Language Objectives

- Understand and use lesson vocabulary and the adverbs *exactly* and *only* to define, write, and talk about functions.
- Read descriptions, tables, graphs, and equations to determine if a relationship is a function and write short paragraphs to explain the conclusion.
- Take notes and paraphrase speakers' ideas about how a table can help predict the shape of a graph.

Prior Knowledge

- Know that every nonvertical line can be modeled by an equation of the form $y = mx + b$.
- Understand that the rate of change of a linear relationship is constant.
- Use a linear equation to make a table of values.
- Sketch the graph of the line modeled by a linear equation.

Vocabulary

Math Vocabulary

function a rule in which each input results in exactly one output.

input (of a function) the independent variable of a function.

linear function a function that can be represented by a linear equation. The graph of a linear function is a nonvertical straight line.

nonlinear function a function with a graph that is not a straight line.

output (of a function) the dependent variable of a function.

Review the following key terms.

factors of a number whole numbers that multiply together to get the given number.

prime number a whole number greater than 1 whose only factors are 1 and itself.

rate of change in a linear relationship between x and y, it tells how much y changes when x changes by 1.

supplementary angles two angles whose measures sum to 180°.

Academic Vocabulary

classify to sort or decide that something can be put into a category.

Learning Progression

In Grade 6, students learned how to use variables to represent unknown quantities in expressions and equations.

In Grade 7, students extended their work with algebraic expressions and equations as they explored proportional relationships.

Earlier in Grade 8, students learned to model linear relationships using tables of values, graphs, equations, and verbal descriptions.

In this lesson, students explore what it means to say that an input-output rule is a function. They learn to analyze relationships presented as equations, graphs, tables of values, or verbal rules to determine which represent functions. Students learn to classify functions as linear or nonlinear.

In the next lesson, students will write equations for linear functions and interpret their rates of change and initial values.

Later in Grade 8, students use functions to model and analyze real-world relationships.

In high school, students will continue their work with functions, including quadratic, exponential, and trigonometric functions.

Pacing Guide

Items marked with ✱ are available on the **Teacher Toolbox**.

	MATERIALS	DIFFERENTIATION
SESSION 1 **Explore** Functions (35–50 min)		
• **Start** (5 min) • **Model It** (5 min) • **Discuss It** (5–10 min) • **Model It** (5–10 min) • **Discuss It** (10–15 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 351–352)	Presentation Slides ✱	**PREPARE** Interactive Tutorial ✱ **RETEACH or REINFORCE** Visual Model **Materials** For display: 3 mapping diagrams (pairs of side-by-side elongated ovals)
SESSION 2 **Develop** Understanding of Linear and Nonlinear Functions (45–60 min)		
• **Start** (5 min) • **Model It: Graphs** (5 min) • **Discuss It** (5–10 min) • **Model It: Equations** (5 min) • **Discuss It** (10–15 min) • **Connect It** (10–15 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 355–356)	Presentation Slides ✱	**RETEACH or REINFORCE** Visual Model **Materials** For display: 30 index cards, tape **REINFORCE** Fluency & Skills Practice ✱
SESSION 3 **Refine** Ideas About Functions (45–60 min)		
• **Start** (5 min) • **Apply It** (35–50 min) • **Close: Exit Ticket** (5 min)	**Math Toolkit** graph paper, straightedges Presentation Slides ✱	
Lesson 15 Quiz ✱ or **Digital Comprehension Check**		**RETEACH** Tools for Instruction ✱ **REINFORCE** Math Center Activity ✱ **EXTEND** Enrichment Activity ✱

Overview | Understand Functions

Connect to Culture

➤ **Use these activities to connect with and leverage the diverse backgrounds and experiences of all students. Engage students in sharing what they know about contexts before you add the information given here.**

SESSION 1 ■ □ □

Model It Ask students if they like to eat mangos. Do they prefer eating the raw fruit, drinking mango smoothies, or using mangos in cooking? Invite students to share what they know about this fruit. They may be surprised to learn that the fruit is not the only valuable part of the mango tree. Bark, leaves, flowers, seed pits, and even the skin of the fruit have been used in traditional medicines from ancient times, continuing in this modern age. Many students will know that mangos are grown in the Caribbean. Mangos currently flourish in most tropical regions, but they were first grown in India over 5,000 years ago. India produces more mangos than any other country in the world. Mangos are exchanged as gifts of love and friendship.

SESSION 2 ■ ■ □

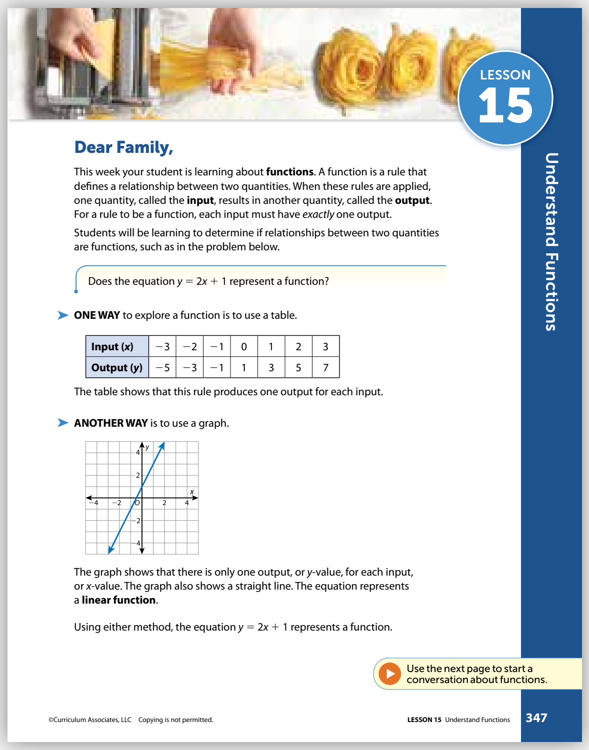
Model It Birdhouses come in many shapes, sizes, and colors. A birdhouse may look like a small house, a tea pot, or even a cat! There is a science behind birdhouses, though. Birdhouses are made for cavity-nesting birds, those that naturally excavate holes in which to build their nests. The entrance hole must be just the right size. It must be big enough to let birds in and small enough to keep predators out. Perches can make it easier for predators to get to the birds, so it is best to leave them off. While the outside may be highly decorated, it is best to leave the interior unpainted. A rough surface makes it easier for new baby birds to move and climb. Invite students to share a time they had a birdhouse on their property or watched birds tending a nest.

SESSION 3 ■ ■ ■

Apply It **Problem 1** Rolling balls at target items, like pins, has been a popular pastime for thousands of years. Evidence in Egypt of an early form of bowling suggests that people have enjoyed this sport since 3200 BCE. The game has changed over time and varies from country to country. In the United States, ten-pin bowling is most popular. European bowlers, however, prefer nine-pin bowling. In the nine-pin version, the center pin is colored red and is called the King Pin. Scoring is more complicated in nine-pin bowling and depends on which specific pins are knocked down and which are left standing. Nine-pin bowling was once wildly popular in the United States. In the 1930s, nine-pin bowling was legally banned across the U.S. because people would skip work to play. Today, nine-pin bowling is still banned in every state except Texas! Ask if any students have seen or played nine-pin bowling and invite students to share their experiences with ten-pin bowling.

Connect to Family and Community

➤ **After the Explore session, have students use the Family Letter to let their families know what they are learning and to encourage family involvement.**


LESSON 15

Understand Functions

Dear Family,

This week your student is learning about **functions**. A function is a rule that defines a relationship between two quantities. When these rules are applied, one quantity, called the **input**, results in another quantity, called the **output**. For a rule to be a function, each input must have *exactly* one output.

Students will be learning to determine if relationships between two quantities are functions, such as in the problem below.

Does the equation $y = 2x + 1$ represent a function?

➤ **ONE WAY** to explore a function is to use a table.

Input (x)	−3	−2	−1	0	1	2	3
Output (y)	−5	−3	−1	1	3	5	7

The table shows that this rule produces one output for each input.

➤ **ANOTHER WAY** is to use a graph.

The graph shows that there is only one output, or y-value, for each input, or x-value. The graph also shows a straight line. The equation represents a **linear function**.

Using either method, the equation $y = 2x + 1$ represents a function.

Use the next page to start a conversation about functions.

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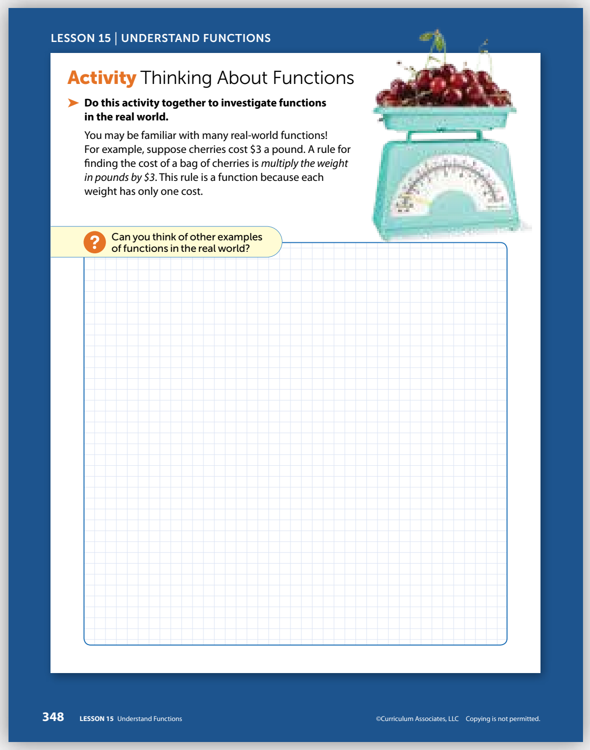
LESSON 15 | UNDERSTAND FUNCTIONS

Activity Thinking About Functions

➤ **Do this activity together to investigate functions in the real world.**

You may be familiar with many real-world functions! For example, suppose cherries cost \$3 a pound. A rule for finding the cost of a bag of cherries is *multiply the weight in pounds by \$3*. This rule is a function because each weight has only one cost.

Can you think of other examples of functions in the real world?

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Connect to Language

➤ **For English language learners, use the Differentiation chart to scaffold the language in each session. Use the Academic Vocabulary routine for academic terms before Session 1.**

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 1** **Discuss It**

Levels 1–3: Speaking/Listening

Prepare students to understand and answer the question in the first Discuss It. Display the lesson terms *input* and *output*. Then read Model It problem 1b. Ask: *How many students can sit at one desk?*

- *____ students can sit at one desk.*

Have students turn to partners. Ask partners to take turns identifying the input and output for each column in the table. Then read the question in Discuss It. Help students identify and explain different inputs that have the same output. Provide sentence frames:

- *For ____ student(s), you need ____ desk(s).*
- *One desk seats ____ students.*
- *For ____ students, you still need ____ .*

Levels 2–4: Speaking/Listening

Prepare students to answer the question in the first Discuss It. Display the lesson terms *input* and *output*. Then read Model It problem 1b with students. Call on volunteers to explain the meaning of *2-student desk*. Then have students turn to partners and take turns explaining the input and output for each column in the table. Next, help students read the question in Discuss It. Have partners talk about different inputs that have the same output. Then help them complete the statement:

- *Inputs ____ and ____ have the same output because ____ .*

Levels 3–5: Speaking/Listening

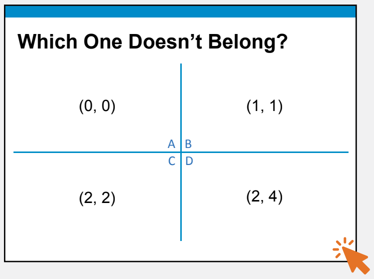
Prepare students to answer the question in the first Discuss It. Display the lesson terms *input* and *output*. Then have students read Model It problem 1b. Have students turn to partners and use the math terms to describe the table. Then ask them to read the question in Discuss It. If students need clarification, call on volunteers to **Say It Another Way**. Encourage students to discuss some examples before they answer the question. Ask: *What are some of the inputs that have the same output? Why is the output the same?* Encourage students to use *for, to, and, both,* and *because* to explain.

Explore Functions

Purpose

- **Explore** the idea that rules that assign outputs to given inputs can be represented in tables and graphs.
- **Understand** that a function is a rule for which each input results in exactly one output.

START CONNECT TO PRIOR KNOWLEDGE



Possible Solutions

A is the only point on an axis.

B is the only point with odd coordinates.

C is the only point whose *y*-coordinate is not equal to the square of its *x*-coordinate.

D is the only point whose coordinates are not equal to each other.

WHY? Encourage students to consider the relationship between the coordinates of a point.

MODEL IT SMP 1, 2, 8

Read the *Understand* question at the top of the page. Remind students that they know how to read a table of values, identify coordinates of points, and plot points on a coordinate grid.

1 – 2 See ***Connect to Culture*** to support student engagement. Tell students that they are going to use what they know about patterns and rules to complete the tables. Read the problems aloud. Call on students to rephrase the definition of a function to confirm understanding. Have students turn and talk to a partner about why each rule is or is not a function.

Common Misconception If students think that every input in a table will have exactly one output, then encourage them to draw a model as they complete problem 2. Remind students that a point can lie to the left or to the right of zero on a number line.

Explore Functions

UNDERSTAND: How can you recognize a function?

Model It

➤ **Complete the problems about functions.**

1 A rule may tell you what to do to a starting value, or **input**, to get a final value, or **output**. Use the rules to complete the tables.

a. Input: number of mangos that cost \$2 each
Output: total cost

Mangos	1	2	3	4
Cost	\$2	\$4	\$6	\$8

b. Input: *x*, number of students
Output: *y*, least number of two-student desks needed

Students (*x*)	1	2	3	4	5	6
Desks (*y*)	1	1	2	2	3	3

c. A **function** is a type of rule in which each input results in exactly one output. Explain why both rules above are functions.
Possible explanation: For each number of mangos, there is only one total cost. For each number of students, there is only one number of desks.

d. In problem 1a, you can say that the total cost is *a function of* the number of mangos. Write a similar statement about the numbers of students and desks.
The number of desks is a function of the number of students.

2 Use the rule to complete the table. Explain why *y* is not a function of *x*.

Input: *x*, a number; Output: *y*, all numbers *x* units from 0 on a number line

Input (*x*)	1	2	3	4	5
Output (*y*)	−1, 1	−2, 2	−3, 3	−4, 4	−5, 5

Possible explanation: For each input *x* there are two outputs, or values of *y*, that are that number of units from 0 on a number line.

DISCUSS IT

Ask: In problem 1b, why do some inputs have the same output?

Share: To find the number of desks needed . . .

Learning Targets SMP 1, SMP 2, SMP 3, SMP 7, SMP 8
- Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
- Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

DISCUSS IT SMP 3, 7

Support Partner Discussion

After students complete problems 1 and 2, have them respond to Discuss It with a partner. Encourage them to draw a picture or act out the rule in problem 1b.

Listen for understanding that:

- in the table for problem 1b, the inputs 1 and 2 both have output 1, the inputs 3 and 4 both have output 2, and so on.
- each desk can seat 1 or 2 students, so 1 desk is needed for 1 or 2 students, 2 desks are needed for 3 or 4 students, and so on.

Facilitate Whole Class Discussion

Prompt students to think about how to tell if a table represents a function.

ASK *Which tables on this page show functions? How do you know?*

LISTEN FOR The tables for problems 1a and 1b show exactly one output for every input value. These tables represent functions. The table in problem 2 has more than one output for a given input value. It does not represent a function.

MODEL IT
SMP 2, 3

3 Tell students that they will now think about how to use a graph to represent a rule. Read the problem out loud and call on students to rephrase to confirm understanding. Encourage students to think about how the graphs of functions are different from the graphs of rules that are not functions.

Common Misconception If students think that the rule on the right is not a function because output 2 has three input values, then ask them to state the definition of a function. Point out that a function may have several inputs for a given output.

DISCUSS IT
SMP 3, 7, 8

Support Partner Discussion
After students complete problem 3, have them respond to Discuss It with a partner. Encourage them to discuss what a graph looks like when it shows a rule that is not a function.

Listen for understanding that:

- if an input-output rule is not a function, then at least one input value will have more than one output value.
- if an input-output rule is not a function, the graph of the relationship will have at least two points that share the same x-coordinate.

DIFFERENTIATION | RETEACH or REINFORCE

Visual Model
Use mapping diagrams to model input-output relationships.

If students are unsure about how to identify a function, then use this activity to show a different way of looking at the relationships between input and output values.

Materials For display: 3 mapping diagrams (pairs of side-by-side elongated ovals)

- Direct students' attention to the rules and tables displayed on the first page of the lesson.
- For each rule, have students help you enter the inputs into one oval and outputs into the other oval of one diagram. For each rule, write each input value and each output value only once. Draw an arrow from each input to its output(s).
- Ask: *How does the diagram show that a rule is a function?* [If there is only one arrow starting from each input, the relationship is a function.]
- Invite students to share which model they prefer to use for analyzing rules to determine whether they are functions.

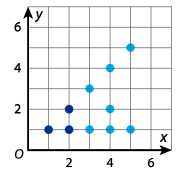
Model It

➤ **Complete the problems about showing a rule with a graph.**

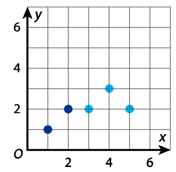
3 You can make a graph of the (input, output) pairs for a rule.

a. Complete both graphs by plotting (input, output) pairs for input values of 3, 4, and 5.

Input: x, a whole number
Output: y, all factors of x



Input: x, a whole number
Output: y, the number of factors of x



b. Look at the graph on the left. Is y a function of x? Explain how you know.
No; Possible answer: There is more than one point above some x-values. This means some inputs have more than one output.

c. Look at the graph on the right. Is y a function of x? Explain how you know.
Yes; Possible answer: For any whole number, there is only one number of factors. On the graph, there is only one output value above each input value.

DISCUSS IT

Ask: If you extend the graph on the right to include more inputs, will you ever find an input with more than one point above it? Why?

Share: I know that a rule is not a function if its graph . . .

4 **Reflect** How can making a table or graph help you decide whether a rule is a function?
Possible answer: Making a table or graph helps me organize the input and output values. If I make a table and get more than one output for an input, then I know the rule is not a function. If I get only one output for each input, then the rule may be a function. If I make a graph of (input, output) pairs and there is more than one point above a given input value, then I know the rule is not a function. If there is only one point above each input value, then the graph may be a function.

Facilitate Whole Class Discussion
Prompt students to **Compare and Connect** tables and graphs of functions and nonfunctions.

ASK *What will you see when a table of values or a graph shows a nonfunction?*

LISTEN FOR A function rule has exactly one output for every input. In a table of a nonfunction, at least one input value will have more than one output value. In the graph of a nonfunction, there will be at least two points that have the same x-coordinate.

CLOSE EXIT TICKET

4 **Reflect** Look for understanding of how to identify a function by inspection from a table or graph.

Error Alert If students think that a function cannot have the same output value for two different inputs, then have them review the definition. Working with a partner, have them use the definition to explain why the table in problem 1b shows a function, while the table in problem 2 does not. Have them repeat this for the graphs in problem 3.

Prepare for Functions

Support Vocabulary Development

Assign **Prepare for Functions** as extra practice in class or as homework.

If you have students complete this in class, then use the guidance below.

Ask students to consider the term *rate of change*. Have students talk about what it would mean to have a constant rate of change.

Have students work individually to complete the graphic organizer. Invite pairs to share their completed organizers and prompt a whole-class comparative discussion of the examples given. Encourage a variety of responses, including graphs, tables of values, equations, and verbal descriptions.

Have students look at the equations in problem 2 and discuss with a partner whether the slope-intercept form of a linear equation may help to identify the rate of change of the relationship.

Problem Notes

1. Students should understand that rate of change describes how one quantity changes with respect to the change in another quantity. Student responses might include connections to the slope of a line. Students may mention that a linear relationship has a constant rate of change.

2. Students may recognize that the rate of change of a linear equation can be identified when the equation is written in slope-intercept form. The rate of change of the first equation is $\frac{2}{3}$. In the second equation, the rate of change is 6. Only the third equation has a rate of change equal to 3.

LESSON 15 | SESSION 1 Name:

Prepare for Functions

1. Think about what you know about rates of change. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.

Possible answers:

What Is It?
the rate at which one quantity changes with respect to change in another quantity

What I Know About It
The rate of change of a line graphed in a coordinate plane is the slope of the line.

The rate of change in the verbal description of a problem is often expressed as a rate such as *miles per gallon* or *dollars per hour*.

rate of change

Examples
the value of m in the slope-intercept form of an equation, $y = mx + b$

Examples
Danny travels at a constant rate of 35 miles per hour. The rate of change is 35.

Stella paints at a constant rate of 2 paintings per week. The rate of change is 2.

2. Circle the equation with a rate of change of 3.

$3y = 2x + 5$ $y = 6x - 3$ $y = 3x + 12$

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REAL-WORLD CONNECTION

When buying a car, people often consider the car's fuel economy, or the average number of miles it can travel on a gallon of gas. A fuel economy rating of 32 mpg means the car can travel an average of 32 miles per gallon. Once you know a car's fuel economy rating, you can write a function that lets you estimate how far you can drive on a given number of gallons of gas. The number of miles is a function of the number of gallons of gas. Ask students to think of other real-world examples when functions might be useful.

3 **a.** Students may find it helpful to describe the rule verbally as *Multiply by 9*.

b. Student responses should clearly show that the situation satisfies the definition of a function. It has exactly one output for every input.

4 **a.** Students may choose to write an equation to help them complete the table: $y = 180 - x$.

b. Students may reason that, with respect to measure, there can be only one angle supplementary to a given angle.

5 **a.** Students may make an organized list or table of (input, output) pairs before graphing.

b. Students may list the coordinates of points with the same input and two different outputs to help justify their response.

➤ Complete problems 3–5.

3 **a.** Use the rule to complete the table.

Input: x, number of tickets that cost \$9 each
Output: y, total cost

Tickets (x)	1	2	3	10
Cost (y)	\$9	\$18	\$27	\$90

b. Explain why the rule is a function.
For any number of tickets, there is only one possible total cost.

4 **a.** Use the rule to complete the table.

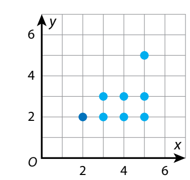
Input: x, an angle
Output: y, the angle supplementary to x

Input (x)	10°	26°	45°	90°	112°	175°
Output (y)	170°	154°	135°	90°	68°	5°

b. Is the rule a function? Explain.
Yes; Possible explanation: Every angle measure has just one supplement.

5 **a.** Complete the graph by plotting (input, output) pairs for input values of 3, 4, and 5.

Input: x, a whole number
Output: y, all prime numbers less than or equal to x



b. Is y a function of x? How can you tell by looking at the graph?
No; Possible answer: There is more than one point above some x-values. This means some inputs have more than one output.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 2** **Model It**

ACADEMIC VOCABULARY

Calculate is to find a value using mathematics.

Determine is to decide something based on evidence or facts.

Represent is to use a sign, symbol, or example to show something.

Levels 1–3: Reading/Speaking

Prepare students to answer Model It problem 3a. Display the Academic Vocabulary and the lesson terms *linear function, nonlinear function,* and *classify*. Read the problem and ask: *What do the equations represent? What do you need to determine?*

- *The equations represent* ____.
- *I need to determine if* ____.

Make a two-column chart with the headings *Linear* and *Nonlinear*. Have students graph and classify the equations. Have them explain their reasoning using:

- ____ *is linear/nonlinear because* ____.

Levels 2–4: Reading/Speaking

Prepare students to answer Model It problem 3a. Display the Academic Vocabulary and the lesson terms *linear function, nonlinear function,* and *classify*. Read the problem with students. Help them use *show* and *decide* to **Say It Another Way**. Ask: *What do the equations show? What do you need to decide?*

Make a two-column chart with the headings *Linear* and *Nonlinear*. Then have students graph and classify the equations. Have partners talk about the charts and paraphrase to check if they understood each other's ideas.

Levels 3–5: Reading/Speaking

Prepare students to answer Model It problem 3a. Display the Academic Vocabulary and the lesson terms *linear function, nonlinear function,* and *classify*. Have students read the problem. Encourage them to use *find, show, decide, sort, linear,* and *nonlinear* to **Say It Another Way**. Then have students make a two-column chart to sort the equations. Have them graph and classify the equations and then turn to partners to discuss. Remind students that they can paraphrase to check if they understood their partner's ideas. Have students take turns describing the charts and paraphrasing ideas.

Develop Understanding of Linear and Nonlinear Functions

Purpose

- **Develop** the idea that functions can be linear or nonlinear.
- **Understand** that a linear function has a constant rate of change, a straight-line pattern of (input, output) pairs, and an equation that can be written in the form $y = mx + b$.

START CONNECT TO PRIOR KNOWLEDGE

Same and Different

A: $y = 3x - 1$	B: $5x + 5y = 3$
C: $y = 1.8x$	D: $x = 4y$

Possible Solutions

All are linear equations.

A and C are in slope-intercept form.

A and B have integer values for slope.

C and D represent proportional relationships.

WHY? Activate previous knowledge of linear equations.

DEVELOP ACADEMIC LANGUAGE

WHY? Build listening skills by asking students to paraphrase and confirm understanding.

HOW? Ask students to share ways they try to understand another person's ideas during a discussion. Introduce the idea that the listener can take notes as the speaker talks and use the notes to paraphrase the explanation. Ask students to practice this approach when answering the question in Discuss It. Have partners confirm the paraphrased explanations.

MODEL IT

SMP 2

1 – 2 See ***Connect to Culture*** to support student engagement. As students complete the problems, have them recognize that the graphs of linear functions are lines.

Common Misconception If students think that the rule in problem 2 is not a function, then review the definition of function with them. Ask whether this rule assigns exactly one output to each input. Remind students that a function may assign the same output to different inputs.

UNDERSTAND: How can you recognize a function?

Develop Understanding of Linear and Nonlinear Functions

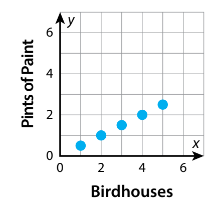
Model It: Graphs

➤ **Try these two problems involving linear and nonlinear functions.**

1 a. Functions can be classified as linear or nonlinear. Look at this **linear function**. Then complete the table and graph.

Input: x, number of birdhouses Aimee paints with 0.5 pint of paint each; Output: y, total pints of paint she needs

Birdhouses (x)	Pints of Paint (y)
1	0.5
2	1
3	1.5
4	2
5	2.5



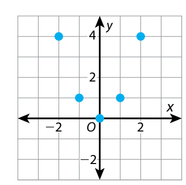
b. Look at the shape of the graph. Why do you think this function is classified as a linear function?

Possible answer: The points fall on a straight line.

2 a. Complete the table and graph for this **nonlinear function**.

Input: x, an integer; Output: y, the square of x

Input (x)	−2	−1	0	1	2
Output (y)	4	1	0	1	4



b. How is the graph different from the graph of the linear function in problem 1?

Possible answer: The points are not all on a straight line.

DISCUSS IT

Ask: How can you tell the shape of the graph from the table?

Share: The rate of change shown in the table . . .

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DISCUSS IT

SMP 1, 3, 7

Support Partner Discussion

After students complete problems 1 and 2, have them respond to Discuss It with a partner. Support as needed with questions such as:

- *How can you calculate the rate of change between two pairs of values in a table?*
- *What do you notice when you calculate the rate of change for the linear function and for the nonlinear function?*

Facilitate Whole Class Discussion

For each problem, have students talk about how to recognize linear and nonlinear functions.

ASK *How are linear and nonlinear functions alike?*

LISTEN FOR Every input has exactly one output.

ASK *How are linear and nonlinear functions different?*

LISTEN FOR A linear function has a constant rate of change and a graph that is a line. The rate of change of a nonlinear function varies, and the graph is not a line.

MODEL IT
SMP 2

3 As students complete the problems, have them think about how to use equations to recognize linear and nonlinear functions.

DISCUSS IT
SMP 2, 3, 7

Support Partner Discussion
After students complete problem 3, have them respond to Discuss It with a partner. Encourage them to think about the difference between linear and nonlinear relationships. Listen for understanding that:

- the points on the graph of an equation of the form $y = x^2$ do not form a line.
- outputs of a linear function change at a constant rate.

Facilitate Whole Class Discussion
Have students talk about how to determine whether a function is linear based on its equation, graph, or a table of values.

ASK *How do you know when a function is linear?*

LISTEN FOR Its graph is a line. The rate of change is the same between any two pairs of values. The equation can be written in the form $y = mx + b$.

DIFFERENTIATION | RETEACH or REINFORCE

Visual Model
Classify functions as linear or nonlinear.

If students are unsure about how to identify a function as linear or nonlinear, then use this activity to provide additional practice with classifying functions.

Materials For display: 30 index cards, tape

- On one side of each index card, write either *linear function* or *nonlinear function*, and write one of these representations: *equation*, *graph*, *table*, and *verbal description.* Make at least one card for each of the 8 possible pairings. Leave the other side blank.
- Give each student a card. On the blank side, have them provide an example of the assigned function type and representation. Have students trade cards with a partner to check each other's work.
- Divide the board into sections labeled *Linear* and *Nonlinear*.
- Collect the cards and have students classify each example as linear or nonlinear. Then, tape the card in the correct section of the board.
- Discuss how to decide whether a function is linear or nonlinear for each type of representation.

LESSON 15 | SESSION 2

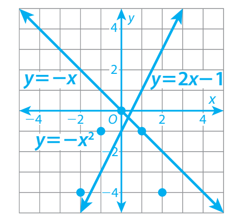
Model It: Equations

➤ **Try this problem about functions.**

3 Many functions can be represented by equations that show how to calculate the output y for the input x.

a. Determine whether each equation represents a linear function. Show your work.

$y = 2x - 1$ $\quad$ $y = -x^2$ $\quad$ $y = -x$



Possible work: See graph.

The graphs of $y = 2x - 1$ and $y = -x$ are lines. They are linear functions.

The graph of $y = -x^2$ is not a line. It is a nonlinear function.

SOLUTION $y = 2x - 1$ and $y = -x$ are linear functions.

b. Explain how you know that equations of the form $y = mx + b$ always represent linear functions.

$y = mx + b$ is an equation for a line with slope m and y-intercept b. Each value of x results in only one value of y.

DISCUSS IT

Ask: How do you know that equations of the form $y = x^2$ do not represent linear functions?

Share: Not all functions are linear because . . .

CONNECT IT

➤ **Complete the problems below.**

4 You want to determine whether a function is linear. How can making a graph or writing an equation help?

Possible answer: I can plot several (input, output) pairs of the function to see if they fall on a straight line. I can rewrite the function equation to see if it can be written in the form $y = mx + b$.

5 Does the rule describe a linear function? Use a model to explain.

Input: x, a number; Output: y, 6 more than 5 times x

Yes; Possible answer: You can represent the rule with the equation $y = 5x + 6$. This is the equation of a line, so the rule is a linear function.

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CONNECT IT
SMP 2, 3, 7

4 Student responses should show understanding that it is possible to classify a function as linear or nonlinear by looking at its graph or an equation that models the function.

CLOSE | EXIT TICKET

5 Look for understanding that the rule describes a linear function. Students may determine this by making a table and observing that the rule has a constant rate of change, by making a graph and noting that the points form a straight line, or by writing an equation of the form $y = mx + b$ to represent the rule.

Common Misconception If students think the function is linear only because each input has one output, explain the difference between *function* and *nonlinear function*.

Practice Linear and Nonlinear Functions

Problem Notes

Assign **Practice Linear and Nonlinear Functions** as extra practice in class or as homework.

1. **a.** Students may use the table of values or the graph of the line to identify the slope and y-intercept. ***Basic***

   b. Student responses may show understanding that any equation of the form $y = mx + b$ models a linear function. ***Medium***

2. The rule directs students to divide 6 by the input value, which gives $y = \frac{6}{x}$, a nonlinear function. ***Medium***

Practice Linear and Nonlinear Functions

➤ **Study how the Example shows how to determine whether a function is linear or nonlinear. Then solve problems 1–4.**

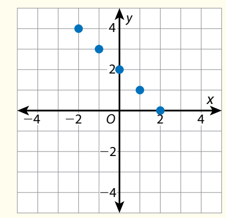
Example

Use a graph to determine whether the function is a linear function.

Input: *x*, a number; Output: *y*, 2 more than −1 times *x*

Make a table of input and output values.

Input (x)	−2	−1	0	1	2
Output (y)	4	3	2	1	0



Graph the (input, output) pairs. The points lie on a straight line. Plotting more points will continue to follow the same straight line. The function is linear.

1. **a.** What is an equation that represents the rule in the Example?

   $y = -x + 2$

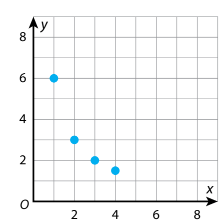
   b. Use the equation to explain why the rule is a linear function.

   The equation can be written in the form $y = mx + b$. All equations in this form represent linear functions.

2. Complete the table and graph for the function. Tell whether the function is linear or nonlinear. Explain your reasoning.

   Input: x, a number; Output: y, 6 divided by x

Input (x)	Output (y)
1	6
2	3
3	2
4	1.5

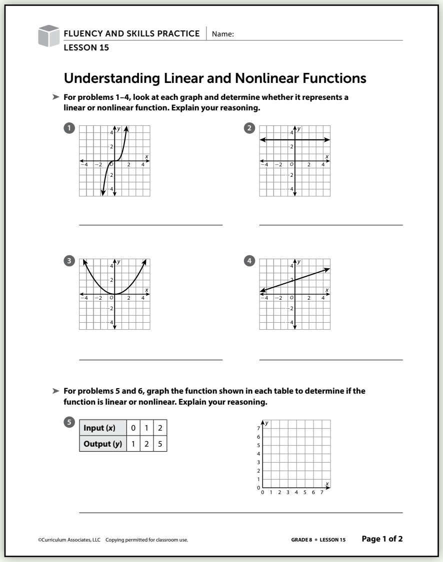


Nonlinear; Possible explanation: The function is nonlinear because the points are not on a straight line.

Fluency & Skills Practice

Understanding Linear and Nonlinear Functions

In this activity, students examine graphs, tables, and equations to determine if they represent a linear function.


FLUENCY AND SKILLS PRACTICE | Name:
LESSON 15

Understanding Linear and Nonlinear Functions

➤ For problems 1–4, look at each graph and determine whether it represents a linear or nonlinear function. Explain your reasoning.

➤ For problems 5 and 6, graph the function shown in each table to determine if the function is linear or nonlinear. Explain your reasoning.

Input (x)	0	1	2
Output (y)	1	2	5

Page 1 of 2

Additional Practice

UNDERSTAND LESSON

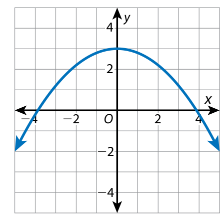
3 Students may recognize that the graph in problem 3d models a linear function, even though the line is horizontal. The horizontal line has a constant rate of change, 0. It can be modeled with the equation $y = 2$, which is $y = mx + b$ with $m = 0$ and $b = 2$. ***Basic***

4 **a.** Students should recognize that 2 points are not sufficient to demonstrate that the function is linear, because any 2 points define a unique line. ***Medium***

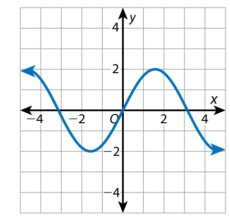
b. Students may apply the distributive property to show that the equation can be written as $y = x^2 + 2x$. It is necessary only to find three sets of ordered pairs that satisfy the equation but do not lie on a line. ***Challenge***

LESSON 15 | SESSION 2

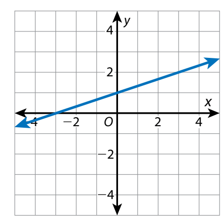
3 Each graph represents a function. Tell whether the function is *linear* or *nonlinear*.



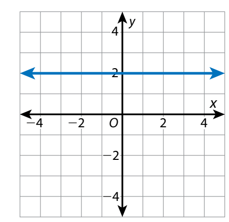
a. nonlinear



b. nonlinear



c. linear

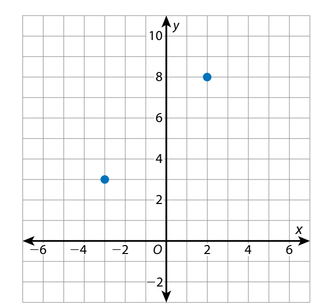


d. linear

4 Felipe wants to figure out if the equation $y = x(x + 2)$ represents a linear function.

He finds two (x, y) pairs and plots them.

x	-3	2
y	3	8



Felipe says he can draw a line through these two points, so the equation represents a linear function.

a. Explain why Felipe's reasoning is incorrect.

Possible answer: It is possible to draw a line through any two points. Felipe needs to plot more points to tell if the function is linear.

b. Does the equation represent a linear function? Explain your reasoning.

No; Possible explanation: Another possible (x, y) pair is $(0, 0)$. If I plot this point on the graph, all points do not fall on a straight line. So, the equation does not represent a linear function.

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DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 3** **Apply It**

Levels 1–3: Reading/Writing

Prepare students to answer Apply It problem 5. Read the problem and review *function*. Make a **Co-Constructed Word Bank** with words students can use to answer the problem, for example, *graph, table, equation, point, value, above, below, exactly,* and *only*. Then use **Act It Out** with objects and words to demonstrate *exactly* and *only*: *I have* exactly *two pens. Do I have more than two? Less?* [No] *I have* only *two pens. Do I have more pens?* [No] Next, reread the problem and help students discuss and write what needs to happen in each case:

- *A graph of a function will have only* _____.
- *In a table, there will be only* _____.
- *In an equation, for any value of the input, there will be only* _____.

Levels 2–4: Reading/Writing

Prepare students to answer Apply It problem 5. Read the problem and review *function*. Make a **Co-Constructed Word Bank** with words students can use to answer the problem, for example, *graph, table, equation, point, value, above, below, exactly,* and *only*. Then use **Act It Out** to help students demonstrate *exactly* and *only*. Say: *Show me* exactly *two pens.* Ask: *Do you have more pens?* Encourage students to use *only* to answer. Then reread the problem and have students turn to partners to discuss and write what needs to happen in each case:

- *A graph of a function will* _____.
- *In a table, there will* _____.
- *In an equation, for* _____*, there will* _____.

Levels 3–5: Reading/Writing

Prepare students to answer Apply It problem 5. Have students read the problem and make a **Co-Constructed Word Bank**. Call on volunteers to explain *exactly* and *only*. Then use **Stronger and Clearer Each Time**. Remind students to refer to the word bank as they draft their answers. Allow think time for students to plan their explanations. Have students consider if they need to add *exactly* and *only*. Then have them turn to partners to explain their drafts. Remind students to be respectful as they provide feedback to their partners. Then have students revise and comment on the feedback:

- *For a graph/table/equation, I wrote that* _____*. My partner said that* _____.
- *I revised/did not revise because* _____.

Refine Ideas About Functions

Purpose

- **Refine** understanding of how to recognize functions and how to distinguish between linear and nonlinear functions using tables, graphs, and equations.

START CONNECT TO PRIOR KNOWLEDGE

Which Would You Rather?

Be given a function represented by

A a table	B a graph
C an equation	D a word description

Possible Solutions

A to calculate rates of change between points

B to tell whether the function is linear or nonlinear

C to find (input, output) pairs

D to understand what the function means in a context

WHY? Encourage students to think of the advantages of each function representation.

APPLY IT

SMP 1, 2, 3, 7

Have students work independently or with a partner for problems 1–3.

1 **Apply** See ***Connect to Culture*** to support student engagement. Look for understanding that each input value for a function is associated with exactly one output. Have students explain why the relationship is a function even though the output $19 is associated with more than one input value.

LESSON 15 | SESSION 3

UNDERSTAND: How can you recognize a function?

Refine Ideas About Functions

Apply It

Math Toolkit graph paper, straightedges

➤ Complete problems 1–5.

1 **Apply** Ana is bowling. She pays $4 per game and $7 to rent shoes. After three games, each additional game is free. Is the total cost to bowl a function of the number of games played? Use a table or graph to help explain your answer.

Yes; Possible explanation: The table shows there is one output for each input.

Games (input)	1	2	3	4	5
Cost (input)	$11	$15	$19	$19	$19

2 **Identify** The tables show inputs and outputs for two functions. One of the functions is linear. Explain how you can tell without a graph which represents a linear function.

Table 1

Input	1	2	3	4	5
Output	6	7	9	12	16

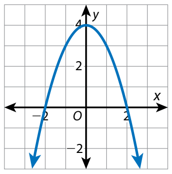
Table 2

Input	2	4	6	8	10
Output	5	9	13	17	21

Table 2; Possible explanation: A linear function has a constant rate of change. In Table 2, no matter which two (input, output) pairs I choose, the unit rate is 2. In Table 1, the rate of change is not constant.

3 **Analyze** Fiona says the graph does not represent a function because inputs −1 and 1 have the same output, 3. Is Fiona correct? Explain.

Fiona is not correct; Possible explanation: A function can have the same output for two different inputs. The definition of a function is that it has only one output for each input. This graph does have only one output for each input, so it is a function.



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2 **Identify** Look for understanding that linear functions have a constant rate of change. Have students explain their methods for evaluating rates of change using values in a table.

Error Alert If students say that Table 1 represents a linear function, then ask them to calculate the rate of change between the first and second input-output pairs and between the second and third input-output pairs. Ask if the rates of change are the same. Remind them that a linear function has a constant rate of change.

3 **Analyze** Look for understanding that a function may have the same output for two different input values. The definition requires that every input value has exactly one output, not that every output corresponds to only one input.

4 Before students begin, read the first part of the problem aloud and engage them in a discussion about what it means to say that a function is nonlinear. Then have students read the directions for Parts A, B, and C and rephrase to confirm that they understand each part of the task.

As students work on their own, walk around to confirm that they have identified a nonlinear function. Challenge students to identify a nonlinear function that has not been discussed in the lesson.

Have students share their function rule and explain their strategy for Part B.

CLOSE **EXIT TICKET**

5 **Math Journal** Look for understanding of how each representation can help you determine whether each input for a rule has exactly one output.

Error Alert If students struggle to explain how to tell whether a rule represents a function, encourage them to model the same function in different ways. Working with more than one model will help students make connections among representations and better understand what it means to say that a relationship is a function.

End of Lesson Checklist

INTERACTIVE GLOSSARY Support students by suggesting they look back through the lesson to identify examples of linear and nonlinear functions and of a nonfunction and then use those examples as models to help them come up with their own examples.

4 **PART A** Give an example of a nonlinear function. You may represent the function by giving the rule in words or by writing an equation.
Possible answer: Input x, a number; output y, the cube of x.

PART B Make a graph or table to represent your function.
Possible answer:

Input (x)	1	2	3	4	5
Output (y)	1	8	27	64	125

PART C Use your graph or table to help you explain why your function is nonlinear.
Possible answer: For the function to be linear, the rate of change needs to be constant.

Rate of change from $x = 1$ to $x = 2$: $\frac{8-1}{2-1} = 7$

Rate of change from $x = 2$ to $x = 3$: $\frac{27-8}{3-2} = 19$

The rate of change is not constant, so the function is not linear.

5 **Math Journal** How can you tell if a rule represents a function? Explain how a graph, table, or equation for a rule can help you determine whether the rule is a function.
Possible explanation: A function is a rule that has exactly one output for each input. In a graph of a function, there will be only one point above or below each input value. In a table, there will be only one output value for each input value. In an equation, for any value I substitute for the input variable, there will be only one possible value of the output variable.

End of Lesson Checklist

☐ **INTERACTIVE GLOSSARY** Find the entry for *function*. Show an example of a linear function, a nonlinear function, and a relationship that is not a function.

Short Response Scoring Rubric (2 points) — **Problem 4**

Points	Expectations
PART A	
2	The rule defines a nonlinear function.
1	The rule defines a relationship that is nonlinear **OR** is a function.
PART B	
2	The graph **OR** table accurately represents the rule from Part A.
1	The graph **OR** table represents the rule from Part A, with minor errors.
PART C	
2	Specific information from the graph or table is used to support the explanation.
1	A general explanation is given without reference to specific evidence from the graph or table.

Overview | Use Functions to Model Linear Relationships

STANDARDS FOR MATHEMATICAL PRACTICE (SMP)

SMP 1, 2, 3, 4, 5, and 6 are integrated into the Try-Discuss-Connect routine.*

This lesson provides additional support for:

2 Reason abstractly and quantitatively.

4 Model with mathematics.

7 Look for and make use of structure.

* See page 1o to learn how every lesson includes these SMP.

Objectives

Content Objectives

- Write an equation for a linear function from a graph, from two points, or from a verbal description.
- Identify and interpret the rate of change and initial value of a linear function from a verbal description, equation, graph, or table.

Language Objectives

- Explain the relationship between graphs, points, verbal descriptions, and equations that model linear functions.
- Use lesson vocabulary and the phrase *is a function of* to talk and write about equations, variables, rates of change, and initial values that represent real-life situations.
- Share solutions with a partner and state reasons to justify them.

Prior Knowledge

- Identify linear functions from tables, graphs, and equations.
- Write equations for lines.
- Find the slope of a line from a graph.
- Find the slope of a line using two points and the slope formula.
- Identify the *y*-intercept from a linear equation or graph.

Vocabulary

Math Vocabulary

initial value in a linear function, the value of the output when the input is 0.

Review the following key terms.

linear function a function that can be represented by a linear equation. The graph of a linear function is a nonvertical straight line.

quadrants the four regions of the coordinate plane that are formed when the *x*-axis and *y*-axis intersect at the origin.

rate of change in a linear relationship between *x* and *y*, it tells how much *y* changes when *x* changes by 1.

slope for any two points on a line, the $\frac{\text{rise}}{\text{run}}$ or $\frac{\text{change in } y}{\text{change in } x}$. It is a measure of the steepness of a line. It is also called the rate of change of a linear function.

slope-intercept form a linear equation in the form $y = mx + b$, where m is the slope and b is the *y*-intercept.

***y*-intercept** the *y*-coordinate of the point where a line, or a graph of a function, intersects the *y*-axis.

Academic Vocabulary

model to represent.

Learning Progression

Earlier in Grade 8, students identified slopes and *y*-intercepts of lines from graphs and equations. They graphed linear equations and rewrote linear equations in slope-intercept form. Students also modeled real-world situations by defining variables and writing linear equations.

In this lesson, students construct equations for linear functions. They interpret rates of change and initial values for these functions in context.

Later in Grade 8, students will compare functions that are represented in different ways. They will analyze graphs of functional relationships qualitatively and sketch a graph of a function from a qualitative description. They will also write equations of functions to model two-variable linear data and make predictions.

Pacing Guide

Items marked with are available on the **Teacher Toolbox**.

	MATERIALS	DIFFERENTIATION
SESSION 1 Explore Using Functions to Model Linear Relationships (35–50 min)		
• **Start** (5 min) • **Try It** (5–10 min) • **Discuss It** (10–15 min) • **Connect It** (10–15 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 363–364)	Presentation Slides	**PREPARE** Interactive Tutorial **RETEACH or REINFORCE** Visual Model **Materials** For display: large first-quadrant coordinate plane
SESSION 2 Develop Interpreting a Linear Function (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 369–370)	Presentation Slides	**RETEACH or REINFORCE** Visual Model **REINFORCE** Fluency & Skills Practice **EXTEND** Deepen Understanding
SESSION 3 Develop Writing an Equation for a Linear Function from Two Points (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 375–376)	**Math Toolkit** graph paper, straightedges Presentation Slides	**RETEACH or REINFORCE** Visual Model **REINFORCE** Fluency & Skills Practice **EXTEND** Deepen Understanding
SESSION 4 Develop Writing an Equation for a Linear Function from a Verbal Description (45–60 min)		
• **Start** (5 min) • **Try It** (10–15 min) • **Discuss It** (10–15 min) • **Connect It** (15–20 min) • **Close: Exit Ticket** (5 min) **Additional Practice** (pages 381–382)	**Math Toolkit** graph paper, straightedges, tracing paper Presentation Slides	**RETEACH or REINFORCE** Visual Model **REINFORCE** Fluency & Skills Practice **EXTEND** Deepen Understanding
SESSION 5 Refine Using Functions to Model Linear Relationships (45–60 min)		
• **Start** (5 min) • **Monitor & Guide** (15–20 min) • **Group & Differentiate** (20–30 min) • **Close: Exit Ticket** (5 min)	**Math Toolkit** Have items from previous sessions available for students. Presentation Slides	**RETEACH** Visual Model **Materials** For display: large four-quadrant coordinate plane **REINFORCE** Problems 4–8 **EXTEND** Challenge **PERSONALIZE** i-Ready
Lesson 16 Quiz or **Digital Comprehension Check**		**RETEACH** Tools for Instruction **REINFORCE** Math Center Activity **EXTEND** Enrichment Activity

Overview | Use Functions to Model Linear Relationships

Connect to Culture

➤ **Use these activities to connect with and leverage the diverse backgrounds and experiences of all students. Engage students in sharing what they know about contexts before you add the information given here.**

SESSION 1 ■ □ □ □ □

Try It Take a survey to find out students' favorite pizza toppings. Record the results on the board. The most common pizza toppings in the United States are pepperoni, mushrooms, onions, sausage, bacon, and extra cheese. For people who cannot agree on which topping to include on their pizza, many restaurants allow certain toppings on half the pizza and other toppings on the other half. Some of the more obscure toppings to find on pizza are pineapple, seafood, or condiments such as ketchup, mustard, or peanut butter.

SESSION 2 ■ ■ □ □ □

Try It Rappelling is a method of sliding down a rope to make a smooth, controlled descent down a rough or steep patch of land, such as a mountain. Rappellers wear a harness on their bodies and attach an anchor to a sturdy area in the ground. Then they slowly push themselves away from the face of whatever they are descending from and control their speed with a hand brake on the rappelling device. Ask students who have ever rappelled or hiked on rough terrain to describe what it was like.

SESSION 3 ■ ■ ■ □ □

Try It Alpaca are members of the camel family and are most similar to llamas. They are raised mainly for their soft wool. Alpaca make great pets if trained because they are very social animals. Alpacas hum under normal conditions and shriek when trying to warn others. They make a sound similar to *wark* when excited. Most alpaca are found in South America, but they are now raised on farms all over the world. Have students raise their hands if they have ever seen an alpaca and allow others to ask those students questions.

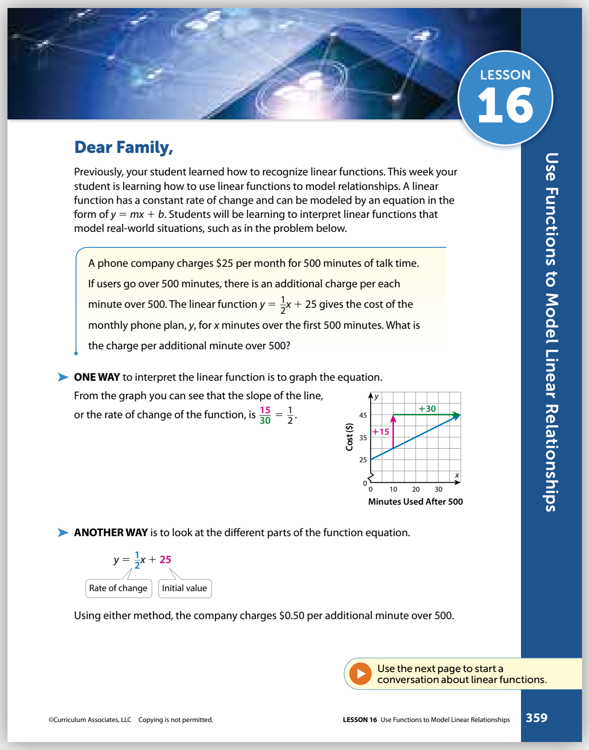
Apply It **Problem 6** Ask any students who have done cross-stitching to share their experiences with the class. Cross stitch is a form of decorative sewing in a pattern that uses X-shaped stitches. It is easy to learn, and many projects are sold as kits. Usually the kits have charts that show the pattern of stitches on a numbered grid representing the background fabric, including specific colors of embroidery floss to use.

SESSION 5 ■ ■ ■ ■ ■

Apply It **Problem 1** Ask if any students have ever lived in a country that uses Celsius degrees. If so, ask them to share what the Celsius temperature would be on a very cold day and what it would be on a very hot day. The Celsius scale, introduced in the mid-1700s by Andres Celsius, is a metric measure of temperature. The Fahrenheit scale, introduced in 1724 by Daniel Gabriel Fahrenheit, is another measure of temperature. In the United States, temperatures are usually reported in degrees Fahrenheit, but most other countries use degrees Celsius. The only value for which the temperatures are the same is $-40°$.

Connect to Family and Community

➤ **After the Explore session, have students use the Family Letter to let their families know what they are learning and to encourage family involvement.**



LESSON 16

Dear Family,

Previously, your student learned how to recognize linear functions. This week your student is learning how to use linear functions to model relationships. A linear function has a constant rate of change and can be modeled by an equation in the form of $y = mx + b$. Students will be learning to interpret linear functions that model real-world situations, such as in the problem below.

A phone company charges $25 per month for 500 minutes of talk time. If users go over 500 minutes, there is an additional charge per each minute over 500. The linear function $y = \frac{1}{2}x + 25$ gives the cost of the monthly phone plan, y, for x minutes over the first 500 minutes. What is the charge per additional minute over 500?

➤ **ONE WAY** to interpret the linear function is to graph the equation. From the graph you can see that the slope of the line, or the rate of change of the function, is $\frac{15}{30} = \frac{1}{2}$.

➤ **ANOTHER WAY** is to look at the different parts of the function equation.

$y = \frac{1}{2}x + 25$

Rate of change | Initial value

Using either method, the company charges $0.50 per additional minute over 500.

Use the next page to start a conversation about linear functions.

Use Functions to Model Linear Relationships

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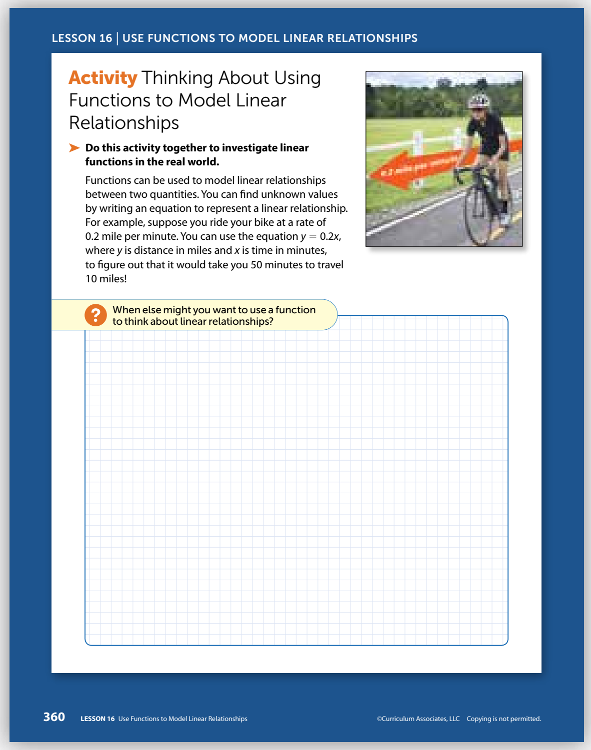

LESSON 16 | USE FUNCTIONS TO MODEL LINEAR RELATIONSHIPS

Activity Thinking About Using Functions to Model Linear Relationships

➤ **Do this activity together to investigate linear functions in the real world.**

Functions can be used to model linear relationships between two quantities. You can find unknown values by writing an equation to represent a linear relationship. For example, suppose you ride your bike at a rate of 0.2 mile per minute. You can use the equation $y = 0.2x$, where y is distance in miles and x is time in minutes, to figure out that it would take you 50 minutes to travel 10 miles!

When else might you want to use a function to think about linear relationships?

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Connect to Language

➤ **For English language learners, use the Differentiation chart to scaffold the language in each session. Use the Academic Vocabulary routine for academic terms before Session 1.**

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 1** **Connect It**

Levels 1–3: Reading/Writing

Prepare students to respond to Connect It problem 2c. Read the problem. Have students brainstorm examples of quantities. Write examples on the board. Then help students define the word:

- *Quantities are ____.*

Review that variables are letters that represent quantities. Have students turn to partners to reread and answer the question:

- *The variable ____ represents ____.*

Help students discuss and write about the situation. *When does the price, y, change?*

- *The price changes when ____ changes.*
- *The price is a function of ____.*

Levels 2–4: Reading/Writing

Prepare students to respond to Connect It problem 2c. Read the problem with students. Call on volunteers to define the word *quantities*. Provide examples if needed. Then repeat for *variables*. Have students turn to partners to reread and answer the question. Allow think time for students to say what the variables represent and to discuss the situation. Then help them understand the meaning of *a function of* [depends on] and describe the relationship between the variables:

- *The price changes when ____.*
- *This means that the price depends on or is a function of ____.*

Levels 3–5: Reading/Writing

Prepare students to respond to Connect It problem 2c. Have students read the problem. Call on volunteers to read the equations that model or represent the situation. Then have other students name the quantities that the variables represent. Have students turn to partners to discuss the situation. Ask: *What is the relationship between variables y and x?* Encourage partners to use *relationship*, *change(s)*, and *a function of* to answer. Then have them write individual responses.

Call on volunteers to tell how they can paraphrase one of their sentences using *depends on*.

Explore Using Functions to Model Linear Relationships

Purpose

- **Explore** the idea that the slope and y-intercept of a linear graph and the values m and b in an equation of the form $y = mx + b$ represent the rate of change and initial value of a linear function.
- **Understand** that identifying the rate of change and initial value in the description of a linear situation can help you construct a linear function to model the situation.

START CONNECT TO PRIOR KNOWLEDGE

Same and Different

A	B
$y = 2x + 1$	$y = 2x + 3$
C	**D**
$y = -5x + 1$	$y = -x + 3$

Possible Solutions

All of the equations are in the form $y = mx + b$.

A and B represent lines with the same slope.

A and C represent lines with the same y-intercept, as do B and D.

C and D represent lines with negative slopes.

WHY? Support students' understanding of equations in the form $y = mx + b$.

TRY IT SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Co-Craft Questions** to help them make sense of the problem. As students write their questions, encourage them to consider the problem statement, graph, and the information in the advertisement. Instruct students to compare their questions to those of a partner and revise them as needed.

DISCUSS IT SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, have them respond to Discuss It with a partner. Listen for understanding that:

- the costs of pizzas without toppings are the y-intercepts of the lines and the values of b in the equations.
- the costs of each topping are the slopes of the lines and the values of m in the equations.

Explore Using Functions to Model Linear Relationships

Previously, you learned about functions. In this lesson, you will learn about writing equations to model linear functions.

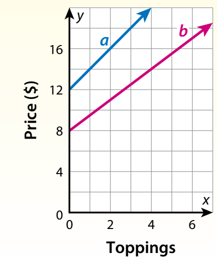
➤ **Use what you know to try to solve the problem below.**

A customer can use the menu above to call in a pizza order. He or she chooses a size and then adds toppings. The graphs and equations model the prices of the two sizes of pizza.

$y = 1.5x + 8$

$y = 2x + 12$

Which equation and which line model the price of a small pizza?
Which equation and which line model the price of a large pizza?



TRY IT Possible work:

SAMPLE A

A small pizza with no toppings is $8.
Line b passes through (0, 8), which is a point on $y = 1.5x + 8$.

A large pizza with no toppings is $12.
Line a passes through (0, 12), which is a point on $y = 2x + 12$.

SAMPLE B

Small pizza: Toppings cost $1.50 each.

Slope of line $b = \frac{14 - 8}{4 - 0} = \frac{6}{4} = 1.5$. So, line b is the graph of $y = 1.5x + 8$.

Large pizza: Toppings cost $2 each.

Slope of line $a = \frac{16 - 12}{2 - 0} = \frac{4}{2} = 2$. So, line a is the graph of $y = 2x + 12$.

DISCUSS IT

Ask: Which numbers in the problem helped you answer the questions?

Share: The numbers I used were . . .

Learning Target SMP 1, SMP 2, SMP 3, SMP 4, SMP 5, SMP 6, SMP 7
Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Error Alert If students specify the incorrect equation, table, or line for each pizza, work with them to use the information on the menu to write an equation of the form $y = mx + b$ for the price, y, of a pizza with x toppings. Then, help them connect the values of m and b in each equation to the slope and y-intercept of a line.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- menu used to find (toppings, price) pairs for each size pizza; then those pairs matched to a particular equation and line
- cost of each pizza connected to y-intercept of a line and value of b in an equation
- cost per topping connected to the slope of a line and value of m in an equation
- menu information used to write equations; then those equations matched to lines

Facilitate Whole Class Discussion

Call on students to share selected strategies. Remind students that they can ask questions about strategies that are not clear.

Guide students to **Compare and Connect** the representations. Use turn and talk to help students think through their responses before sharing with the group.

> **ASK** *How did each strategy use the information on the menu to match each size pizza to an equation and a line?*
>
> **LISTEN FOR** The price of the pizza is the y-intercept of the line and the value of b in the equation. The price of each topping is the slope of the line and the value of m in the equation.

CONNECT IT

SMP 1, 2, 4, 5

1 **Look Back** Look for understanding that the equation $y = 1.5x + 8$ and line b represent the price of a small pizza, and the equation $y = 2x + 12$ and line a represent the price of a large pizza.

DIFFERENTIATION | RETEACH or REINFORCE

Visual Model

Interpret the slope/rate of change and the y-intercept.

If students are unsure about how to interpret the slopes/rates of change and y-intercepts, then use this activity to help students make the connection.

Materials For display: large first-quadrant coordinate plane

- Label the horizontal axis of the coordinate plane Toppings and the vertical axis Price ($).
- Ask: *How much is a small pizza with no toppings? What point represents this?* [$8; (0, 8)] Invite a student to plot (0, 8) on the graph.
- Ask: *How much is a small pizza with 1 topping? What point represents this?* [$9.50; (1, 9.5)] Invite a student to plot (1, 9.5) on the graph.
- Continue until 6 points are plotted and then draw a line through them.
- Ask: *What is the y-intercept for this situation?* [8] *What does it mean in this situation?* [The price of a small pizza with no toppings is $8.]
- Ask: *What is the slope or rate of change for this situation?* [1.5] *What does it mean in this situation?* [The price of each topping is $1.50.]
- Repeat the activity for the costs of a large pizza with different numbers of toppings.

LESSON 16 | SESSION 1

CONNECT IT

1 **Look Back** Which equation and graph represent the price of each size pizza? How do you know?

$y = 1.5x + 8$ and line b represent the price of a small pizza; $y = 2x + 12$ and line a represent the price of a large pizza; Possible explanation: I matched the slopes and y-intercepts of the lines with the numbers in each equation and the numbers on the pizza menu.

2 **Look Ahead** The graphs and equations in the **Try It** problem are linear models because they model, or represent, linear functions.

a. A linear function has a constant rate of change. What do the rates of change represent in this situation?

The cost of each additional topping

b. The **initial value** of each function in the **Try It** is the value of y when $x = 0$. What do the initial values represent in this situation?

The price of a pizza with no toppings

c. What quantities do the variables x and y represent in this situation? Use the phrase *is a function of* to describe the relationship between these quantities.

x represents the number of toppings, y represents the total price of a pizza; The price of a pizza is a function of the number of toppings on the pizza.

3 **Reflect** Look back at the equations and graphs in the **Try It** problem. Which type of model would you rather use to find the price of a pizza? Explain.

Responses will vary. Some students may prefer using the graphs because there are no calculations involved. Others may prefer using the equations to ensure accuracy.

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2 **Look Ahead** Point out that in these linear functions, the independent variable, or input, is the number of toppings and the dependent variable, or output, is the price. Students should recognize that the rate of change in this situation is the price of each topping and the initial value is the cost of the pizza without any toppings. When the number of toppings increases, the price increases.

Ask a volunteer to rephrase the definition of *initial value*. Support student understanding by pointing out that the initial value in this situation is the cost of a pizza with 0 toppings.

CLOSE EXIT TICKET

3 **Reflect** Look for understanding that the equation and graph for each pizza represent the same relationship between number of toppings and price. Either may be used to find the price of a pizza.

Common Misconception If students think that only the equation can be used to find the price of the pizza, remind them that the equation and graph represent the same relationship and either can be used to find the price, although using the graph may require using estimation.

Prepare for Using Functions to Model Linear Relationships

Support Vocabulary Development

Assign **Prepare for Using Functions to Model Linear Relationships** as extra practice in class or as homework.

If you have students complete this in class, then use the guidance below.

Ask students to consider the term *slope-intercept form*. They may define the term in their own words and include examples of equations in slope-intercept form and non-examples, which could be linear equations in forms other than slope-intercept form.

Have students work individually to complete the graphic organizer. Invite students to share their completed organizers, and prompt a whole-class comparative discussion of the definitions, known information, examples, and non-examples.

Have students look at the graph in problem 2 and discuss with a partner what information they need to extract from the graph to be able to write the equation.

Problem Notes

1. Students should understand that a linear equation is in slope-intercept form when it is written in the form $y = mx + b$, where m is the line's slope and b is the y-intercept of the line.
2. Students may observe that the y-intercept is 2 by noting the y-coordinate where the line crosses the y-axis. They may count units for the rise and run to find the slope or may use the slope formula with two points such as (0, 2) and (4, −1). Students should be able to see that the slope is negative because the line is decreasing.

LESSON 16 | SESSION 1 Name:

Prepare for Using Functions to Model Linear Relationships

1. Think about what you know about linear equations. Fill in each box. Use words, numbers, and pictures. Show as many ideas as you can.
Possible answers:

What Is It?
A linear equation is in slope-intercept form when it is written in the form $y = mx + b$.

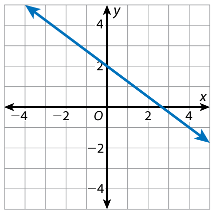
What I Know About It
When an equation of the form $y = mx + b$ is graphed, m is the slope and b is the y-intercept of the line.

slope-intercept form

Examples
$y = 3x - 7$
$y = -9x + 2.5$
$y = x$
$y = -14x$
$y = 37$

Non-Examples
$4x + y = 100$
$x - 11y + 22 = 0$
$x = 12$

2. Write an equation for the graph in slope-intercept form.
$y = -\frac{3}{4}x + 2$



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REAL-WORLD CONNECTION

Mechanical engineers use functions to solve problems involving the motion of objects. A function can be used to relate the final speed of a car after the driver applies constant acceleration over a set amount of time when the initial speed is known. This function is modeled by $v = at + r$, where v is the final speed, a is the constant acceleration, t is time, and r is the initial speed of the car. For example, suppose a car was traveling at a speed of 20 miles per hour and the driver accelerates the car 3 miles per hour per second for 10 seconds. The function would reveal the car is now traveling at a speed of 50 miles per hour. Ask students to think of other real-world examples when using a function to model a linear relationship might be useful.

Additional Practice

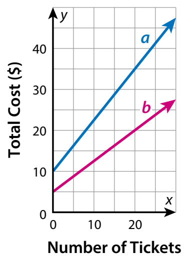
3 Problem 3 provides another look at matching equations and graphs to verbal descriptions of linear functions. This problem is similar to the problem about costs of small and large pizzas based on the number of toppings. In both problems, an advertisement with the needed information is given and students match an equation and line to that information. This problem asks for an equation and line that represents the prices for adults and children attending a carnival based on admission fee and number of ride tickets.

Suggest that students use **Three Reads** to help them understand the situation.

3 A poster announces a carnival is coming to town. People pay an admission fee to enter the carnival. Then they buy tickets to go on rides. The total cost of attending the carnival is a function of the number of tickets bought. The graphs and equations model the total costs for children and for adults.

$y = 1.25x + 10$

$y = 0.75x + 5$



a. Which equation and which line model the total cost for a child? Which equation and line model the total cost for an adult? Show your work.

Possible work:

A child admission fee is $5.

Line *b* passes through (0, 5), which is a point on $y = 0.75x + 5$.

An adult admission fee is $10.

Line *a* passes through (0, 10), which is a point on $y = 1.25x + 10$.

SOLUTION **The total cost for a child is modeled by $y = 0.75x + 5$ and line *b*. The total cost for an adult is modeled by $y = 1.25x + 10$ and line *a*.**

b. Check your answer to problem 3a. Show your work.

Possible work:

Child: Ride tickets cost $0.75 each.

Slope of line *b* = $\frac{20 - 5}{20 - 0} = \frac{15}{20} = 0.75$. So, line *b* is the graph of $y = 0.75x + 5$.

Adult: Ride tickets cost $1.25 each.

Slope of line *a* = $\frac{35 - 10}{20 - 0} = \frac{25}{20} = 1.25$. So, line *a* is the graph of $y = 1.25x + 10$.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 2** **Apply It**

Levels 1–3: Speaking/Writing

Prepare students to write a response to Apply It problem 6. Use **Notice and Wonder** to help students share ideas about the graph. Guide them to comment on the axes, units, and scales. Next, help them ask questions about the line. Then read the first sentence and ask: *What is the relationship between distance and time?* Have partners discuss how the distance changes over time. Then have them write the equation. Next, discuss examples of things that can move up over time, such as an elevator. Help partners use an example to answer the problem:

- *The initial value represents* _____.
- *The rate of change represents* _____.

Levels 2–4: Speaking/Writing

Prepare students to write a response to Apply It problem 6. Use **Notice and Wonder** to help students share ideas and ask questions about the graph. Then read the problem and have partners discuss how the distance changes over time. Have partners write and justify the equation using:

- *The equation is reasonable because* _____.

Next, discuss examples of things that can increase or move up over time. Help students use an example to answer the problem. Invite them to share answers using:

- *The situation is* _____.
- *The equation represents this situation by* _____.

Levels 3–5: Speaking/Writing

Prepare students to write a response to Apply It problem 6. Use **Notice and Wonder** to help students share ideas and ask questions about the graph. Then have students read the problem. Allow think time for students to think about the relationship in the graph. Then have them write the equation. Have students turn to partners to share and justify the equation. Ask: *Why is your equation reasonable?*

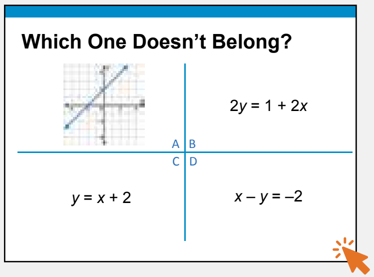
Next, have students brainstorm examples of things that can increase or move up over time. Ask students to think of an example to answer the problem. Then have them use the words *change, function, quantity, model*, and *represent* to explain the situations.

Develop Interpreting a Linear Function

Purpose

- **Develop** strategies for interpreting the slope/rate of change and y-intercept/initial value from a function's graph or equation in terms of the context.
- **Recognize** that the y-intercept is the initial value and the slope is the rate of change for a linear function.

START CONNECT TO PRIOR KNOWLEDGE



Possible Solutions

All represent linear functions. A is the only graph.

C is the only equation in slope-intercept form.

B is the only model that does not represent the function $y = x + 2$.

D is the only equation with x and y on the same side.

WHY? Support students' facility with recognizing graphs and equations that represent the same linear function.

DEVELOP ACADEMIC LANGUAGE

WHY? Support students as they use reasons to justify their strategies.

HOW? When answering the question in Discuss It, have students give reasons to justify how they used the information from the graph. Have students turn and talk to share strategies, using:

- *I know my approach is reasonable because* _____.

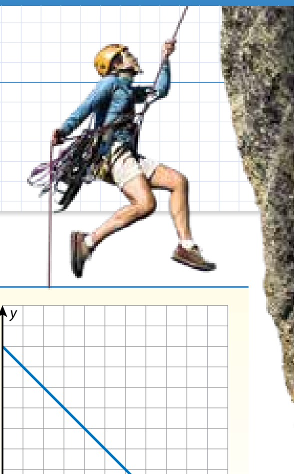
Invite partners to compare and contrast their strategies and reasons.

TRY IT SMP 1, 2, 4, 5, 6, 7

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Co-Craft Questions** to help them make sense of the problem. Draw their attention to the graph to ensure they are making the connections between it and the verbal description.

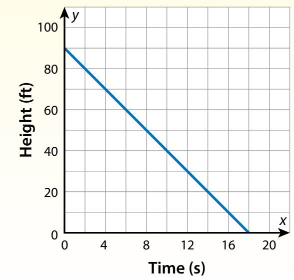
Develop Interpreting a Linear Function



Read and try to solve the problem below.

Aretha is rappelling down the side of a cliff. The graph shows her height above the ground in feet as a function of time in seconds as she descends.

What is Aretha's height above ground when she begins rappelling down? At what rate does she descend?



TRY IT

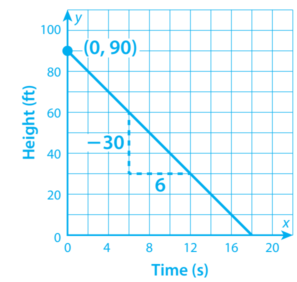
Possible work:

SAMPLE A

The y-intercept is 90. Her height at the beginning is 90 feet.

Slope: $\frac{\text{rise}}{\text{run}} = \frac{-30}{6} = -5$

She is descending at a rate of 5 feet per second.



SAMPLE B

Time (s)	0	2	4	6	8	10	12	14	16	18
Height (ft)	90	80	70	60	50	40	30	20	10	0

At 0 seconds, when Aretha begins, her height is 90 feet.

Every 2 seconds, Aretha's height decreases by 10 feet. Her rate of descent is $\frac{10}{2} = 5$ feet per second.

DISCUSS IT

Ask: How did you use the information from the graph to solve the problem?

Share: I used the graph by . . .

DISCUSS IT SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *Which variable is dependent on the other? How do you know?*
- *Why does the graph slant downward from left to right?*
- *Why is only the first quadrant shown?*

Common Misconception Listen for students who think the initial height is 18 feet. As students share their strategies, emphasize that the x-axis represents time and the y-axis represents height. Ask students to discuss the meaning of the points (0, 90) and (18, 0) in the context of the problem. Students should see that (0, 90) shows that the height at 0 seconds is 90 feet, while (18, 0) shows that the height at 18 seconds is 0 feet.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- table of values completed and used to find initial value and rate of descent
- (misconception) 18 feet identified as initial height
- y-intercept and slope found directly from graph and interpreted as initial value and rate of descent
- linear equation written and interpreted in the context of problem

Facilitate Whole Class Discussion

Call on students to share selected strategies. Remind students that a good explanation describes what they did and why they decided to do it that way.

Guide students to **Compare and Connect** the representations. To prompt students to use precise academic language, call on volunteers to reword vague or unclear statements.

ASK *What do the strategies have in common?*

LISTEN FOR Each shows that the slope is -5, and this is interpreted as the constant decrease in height of 5 feet per second. Each finds that the y-intercept is 90, and this is interpreted as an initial height of 90 feet.

Model It & Analyze It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

ASK *How are Model It and Analyze It related?*

LISTEN FOR Model It shows how to find the equation for the line. Analyze It shows how to interpret the equation in the context of the problem.

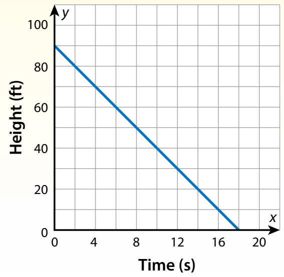
For modeling the function with a linear equation, prompt students to examine the x- and y-intercepts of the graph. Ask: *Why do you think the intercepts make good choices for finding the slope of this graph?*

For the analysis of each part of the equation, prompt students to focus on the numeric values in the equation. Ask: *What is it about the scenario that makes the rate of change negative?*

Explore different ways to interpret a linear function.

Aretha is rappelling down the side of a cliff. The graph shows her height above the ground in feet as a function of time in seconds as she descends.

What is Aretha's height above ground when she begins rappelling down? At what rate does she descend?



Model It

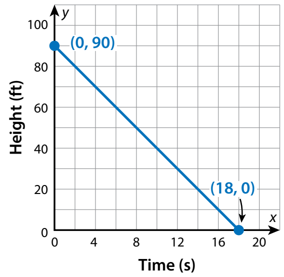
You can model the function with a linear equation.

Find the slope and y-intercept.

$$\text{slope} = \frac{90 - 0}{0 - 18}$$
$$= \frac{90}{-18}$$
$$= -5$$

The graph shows that the y-intercept is 90.

An equation of the function is $y = -5x + 90$.



Analyze It

You can answer the questions by analyzing each part of the equation.

$$y = \underbrace{-5}_{\text{rate of change}}x + \underbrace{90}_{\text{initial value}}$$

The rate of change shows how Aretha's height, y, changes over time, x.

The initial value of the function is Aretha's height at time 0.

DIFFERENTIATION | EXTEND

SMP 4

Deepen Understanding

Considering How Changing the Situation Affects the Graph and Equation

Prompt students to consider how changing the initial height and rate of descent affects the graph and equation.

ASK *Suppose Aretha starts at a height less than 90 feet, but descends at the same rate. How would the graph change?*

LISTEN FOR The graph would be a line parallel to the original line, but it would start at a value less than 90 on the y-axis. It would intersect the x-axis at a value less than 18.

ASK *How would the equation change?*

LISTEN FOR It would have a different value of b, but the same value of m.

ASK *Suppose Aretha starts at a height of 90 feet, but descends more slowly. How would the graph and equation change?*

LISTEN FOR The y-intercept of the graph would be the same, but the line would be less steep. It would intersect the x-axis at a value greater than 18. The equation would have the same value of b. The value of m would still be a negative, but would be a value greater than -5.

Develop Interpreting a Linear Function

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the rates of change and initial values are the same in each representation. Explain that they will now use those representations to reason about how to interpret a linear function.

Before students begin to record and expand on their work in Model It & Analyze It, tell them that problem 2 will prepare them to provide the explanations asked for in problem 3.

Monitor and Confirm Understanding 1

- Aretha's height above the ground is represented by y, and x represents the time she has been rappelling.
- Height, y, is a function of time, x, in this situation.

Facilitate Whole Class Discussion

2 Look for understanding of the connection between the equation, the graph, and the situation.

ASK *How is Aretha's starting height shown in the graph and equation?*

LISTEN FOR It is the initial value of the function. It is the y-intercept of the graph and the constant in the equation.

ASK *How is Aretha's rate of descent shown in the graph and equation?*

LISTEN FOR It is the rate of change for the function. It is the slope of the graph and the coefficient of x in the equation.

3 Look for understanding that the initial value and rate of change for a linear function are related to the graph and equation of the function.

ASK *What parts of the equation $y = mx + b$ represent the initial value and the rate of change of the linear function?*

LISTEN FOR The initial value is the constant term, b. The rate of change is the coefficient of the x-term, m.

ASK *What components of the graph represent the initial value and the rate of change of the linear relationship?*

LISTEN FOR The initial value is the y-intercept, and the rate of change is the slope.

4 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to interpret a linear function.**

1 How can you determine what x and y represent in the equation of the function?

Possible answer: The value of the output y is a function of the input x, so y represents Aretha's height above the ground and x represents the time she has been rappelling.

2 Look at **Model It** and **Analyze It**. What is Aretha's height above the ground when she starts rappelling? At what rate does she descend? How do you know?

90 ft; 5 ft/s; The y-intercept of the graph and the initial value for the function is 90. The slope and rate of change is -5.

3 **a.** Explain how to write an equation of a linear function when you know the rate of change and initial value of the function.

Possible answer: You can write the equation in $y = mx + b$ form. The rate of change is the value of m and the initial value is the value of b.

b. Describe how the rate of change and initial value of the linear function are represented in the graph of the function.

Possible answer: The rate of change is the slope of the graph of the function. The initial value is the y-intercept of the graph of the function.

4 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.

Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Visual Model

Recognize initial values and rates of change in real-world scenarios.

If students are unsure about how to determine which value represents the initial value and which value represents the rate of change, then use this activity to provide them with some additional scenarios to analyze.

- Display this scenario: *Jay plans to save $15 per week. He has $50 saved now.*
- Ask: *How can you use the terminology* is a function of *to describe the relationship?* [The amount Jay will save is a function of the number of weeks.]
- Ask: *What is the initial value? Why?* [50; It is the amount already saved.]
- Invite a student to underline this initial value.
- Ask: *What is the rate of change? Why?* [15; It is the amount saved per week.]
- Invite a student to circle this rate of change.
- Repeat this process with the descriptions below:

  A bakery has 300 muffins. It then sells 25 muffins every half hour after it opens.

  A bathtub contains 200 liters of water. It drains its water 50 liters per minute.

  A plant is 6 inches tall. It then grows 5 inches every 2 weeks.

Apply It

For all problems, encourage students to use a model to support their thinking. Allow some leeway in the precision of any graphs they make to represent the situations.

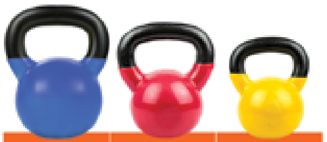
5 **a.** Students should recognize that the cost per class is the rate of change. They may find the rate of change for each gym by using the values in the table, or they may make graphs of the functions and find the slope of each line.

b. Students may also solve this problem by making graphs of the functions and analyzing the *y*-intercepts. The *y*-intercept for Gym A is 0, and the *y*-intercept for Gym B is 60.

6 Students may use the graph to determine the slope and *y*-intercept and write an equation. The part of this question that will be challenging is coming up with a real-world situation that lends itself to the units (time in seconds and distance in feet) that are provided.

Apply It

➤ **Use what you learned to solve these problems.**



5 The total monthly cost of exercise classes is a linear function of the number of classes attended. The table shows costs for two different gyms.

Classes Attended	Monthly Cost Gym A	Monthly Cost Gym B
5	\$75	\$110
10	\$150	\$160
15	\$225	\$210
20	\$300	\$260

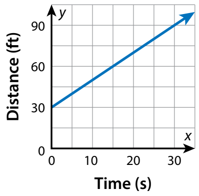
a. What is the cost per class at each gym?

It costs \$15 per class at Gym A and \$10 per class at Gym B.

b. Which gym charges a monthly fee? How do you know?

Gym B; Each class costs \$10 to attend, but the total cost is more than 10 times the number of classes attended.

6 The graph shows distance in feet as a function of time in seconds. Write an equation for the function and describe a situation that it could represent. Include the initial value and rate of change for the function and what each quantity represents in this situation.



$y = 2x + 30$; Possible answer: A bird flies from its nest into the air. The initial value is 30 and represents how many feet above ground the nest is. The rate of change is 2 and represents how many feet farther away from the ground the bird flies each second.

7 Mr. Seda plans a field trip for one of his classes. He rents one bus for the whole class and purchases a museum ticket for each student. The equation $y = 11x + 400$ gives the cost of the field trip as a function of the number of students who attend.

What is the initial value of the function? What is the rate of change? What do these values tell you about the field trip?

400; 11; It costs \$400 to rent the bus, and each museum ticket is \$11.

CLOSE EXIT TICKET

7 Students' solutions should show an understanding that:

- *x* represents the number of students, and *y* represents the cost.
- the constant in the equation, 400, is the initial value, which is the cost for 0 students; it represents the cost to rent the bus.
- the coefficient 11 is the rate of change for the function, so the cost increases by \$11 for each student; \$11 is the cost of each ticket.

Error Alert If students say the initial value is 11 and the rate of change is 400, then have them look back at the Analyze It, which shows that the coefficient of *x* is the rate of change and the constant is the initial value. Then, have them discuss with a partner what these values indicate about the situation.

Practice Interpreting a Linear Function

Problem Notes

Assign **Practice Interpreting a Linear Function** as extra practice in class or as homework.

1 **a.** Students may calculate the rate of change using any two pairs of values in the table. ***Basic***

b. Students should recognize when there is an initial value of 0, the *y*-intercept is also 0. ***Medium***

2 Students may use the graph to determine the slope and *y*-intercept in order to write the equation. Students should consider the axis labels, the initial value, and the value of the slope when describing a situation the linear function could reasonably represent. ***Challenge***

LESSON 16 | SESSION 2 Name:

Practice Interpreting a Linear Function

➤ **Study the Example showing how to interpret a linear function. Then solve problems 1–4.**

Example

Snow falls early in the morning and stops. Then at noon snow begins to fall again and accumulate at a constant rate. The table shows the number of inches of snow on the ground as a function of time after noon. What is the initial value of the function? What does this value represent?

Hours After Noon	Inches of Snow
0	6
1	8.5
2	11

The initial value is 6, the number of inches of snow at noon, when the time value is 0. It represents the amount of snow that was already on the ground before it began snowing again.

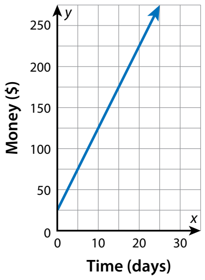
1 **a.** What is the rate of change of the function in the Example? What does this value represent?

2.5; It represents how much the depth of the snow increases each hour.

b. Suppose there was no snow on the ground before it began snowing at noon. What is the equation of this function?

$y = 2.5x$

2 The graph shows money in dollars as a function of time in days. Write an equation for the function, and describe a situation that it could represent. Include the initial value, rate of change, and what each quantity represents in the situation.



$y = 10x + 25$; Possible answer: Someone has some money saved. She earns more money each day babysitting. The initial value is 25 and represents the amount she has saved before she starts babysitting. The rate of change is 10 and represents the amount of her babysitting money she saves each day.

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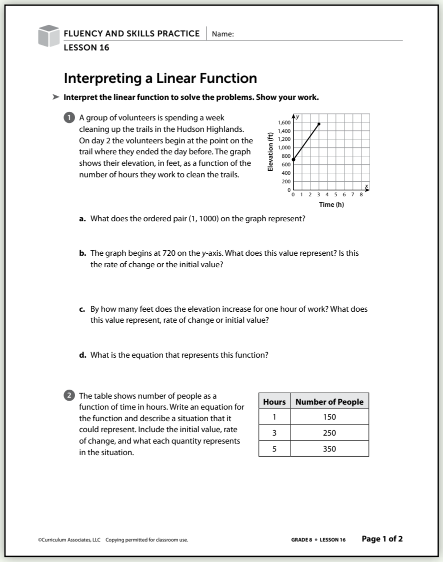
Fluency & Skills Practice

Interpreting a Linear Function

In this activity, students solve problems by examining a graph, a table, and an equation that each represent a linear function, and they interpret the initial value and rate of change in the context of each problem.



FLUENCY AND SKILLS PRACTICE | Name:
LESSON 16

Interpreting a Linear Function

➤ Interpret the linear function to solve the problems. Show your work.

1 A group of volunteers is spending a week cleaning up the trails in the Hudson Highlands. On day 2 the volunteers begin at the point on the trail where they ended the day before. The graph shows their elevation, in feet, as a function of the number of hours they work to clean the trails.

a. What does the ordered pair (1, 1000) on the graph represent?

b. The graph begins at 720 on the *y*-axis. What does this value represent? Is this the rate of change or the initial value?

c. By how many feet does the elevation increase for one hour of work? What does this value represent, rate of change or initial value?

d. What is the equation that represents this function?

2 The table shows number of people as a function of time in hours. Write an equation for the function and describe a situation that it could represent. Include the initial value, rate of change, and what each quantity represents in the situation.

Hours	Number of People
1	150
3	250
5	350

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Additional Practice

3 a. *Basic*

b. An x-value of 0 means the number of ounces of salad is zero; therefore, the value of the function represents the cost of just the soup. ***Medium***

c. The rate of change is 0.45, which represents the amount the cost increases each time the number of ounces of salad increases by 1. So, it is the cost per ounce of salad. ***Medium***

d. Students may substitute 8 for x into the equation $y = 0.45x$. ***Medium***

4 **C is correct.** The rate of change is the coefficient of the x-term, which is 15.

D is correct. The initial value is the cost for 0 events, which is the annual membership fee.

F is correct. The total cost depends on, or is a function of, the number of events Carmela attends.

A is not correct. The initial value is the constant in the equation.

B is not correct. x represents the number of events Carmela attends.

E is not correct. The independent variable, or x-variable, is the number of events. The dependent variable, or y-variable, is the total cost. The total cost is a function of the number of events.

Medium

3 Each day Kyle buys a cup of soup and a salad for lunch. The salad costs a certain amount per ounce. The equation below models the total cost of Kyle's lunch.

$y = 0.45x + 3.75$

a. What do the variables x and y represent? Use the phrase *is a function of* to describe how the equation relates these quantities to one another.

x represents the weight of Kyle's salad in ounces; y represents the cost of his lunch; The cost of Kyle's lunch is a function of the weight of his salad.

b. What does the value of the function for $x = 0$ represent?

The cost of a cup of soup, or what Kyle's lunch would cost if he did not purchase a salad

c. What does the rate of change represent?

The cost per ounce of salad

d. What is the cost of an 8-ounce salad without soup? How do you know?

\$3.60; Possible explanation: The cost of the salad is 8(0.45) = 3.60.

4 Carmela is a member of a social club. She pays an annual membership fee and \$15 for each event she attends. The equation $y = 15x + 25$ represents her total cost each year. Which statement about the function is true? Select all that apply.

A The initial value is 15.

B x represents the cost of each event.

C The rate of change is 15.

D The initial value represents the annual membership fee.

E The number of events she attends is a function of the total cost.

F The total cost is a function of the number of events she attends.

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 3** **Apply It**

Levels 1–3: Listening/Reading

Support students in reading and interpreting Apply It problem 6. Read the problem as students follow along. Use **Act It Out** with pictures or drawings to demonstrate the meaning of *embroidery floss*. Model pronunciation. Then help students identify and label the *skeins* in the image. Repeat for *cross-stitch*. Next, point to each row on the table and ask questions to help students read the amounts and the costs: *How many skeins are there in this row? What is the cost?* Help students answer in full sentences:

- *There are ____ skeins.*
- *The cost for ____ skeins is ____.*

Next, have students turn to partners to write and read the equation.

Levels 2–4: Listening/Reading

Support students in reading and interpreting Apply It problem 6. Read the problem with students. Use **Act It Out** with pictures or drawings to demonstrate the meaning of *embroidery floss*. Have students say the term. Correct pronunciation as needed. Then have students identify and label the *skeins* in the image. Repeat for *cross-stitch*. Next, ask questions to help students read the table: *What is the cost for (amount) skeins? How many skeins can be purchased for (cost)?* Encourage students to answer in full sentences.

Next, have students turn to partners to write and label the equation. Invite volunteers to read and explain the equation.

Levels 3–5: Listening/Reading

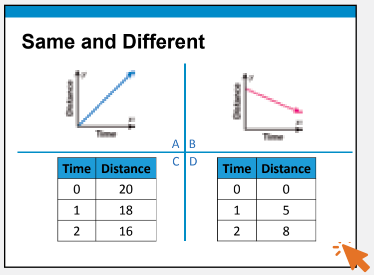
Support students in reading and interpreting Apply It problem 6. Modify **Three Reads**. Have students read the paragraph aloud. Invite other students to listen carefully and help with pronunciation. Call on volunteers to explain words other students may not know. Then, have students take turns reading and explaining rows from the table. For the next read, have students read the paragraph in silence. Allow think time for students to think about the problem and write the equation. Invite volunteers to tell how the equation relates to the table.

Develop Writing an Equation for a Linear Function from Two Points

Purpose

- **Develop** strategies for writing an equation for a linear function when given two ordered pairs.
- **Recognize** that you can choose the ordered pairs from a table of values or from a graph of the function.

START CONNECT TO PRIOR KNOWLEDGE



Possible Solutions

All show a relationship between distance and time.

A, B, and C represent constant rates of change.

A and D represent positive rates of change and also have initial value 0.

B and C represent negative rates of change.

WHY? Support students' facility with recognizing rates of change and initial values in graphs and tables.

DEVELOP ACADEMIC LANGUAGE

WHY? Unpack sentences with the phrase *is a linear function of.*

HOW? Read Try It. Help students analyze the third sentence. Underline *is a linear function of.* Guide students to think about the relationship and talk about the quantities. Ask: *What does the amount of land depend on? What is the independent quantity?* Offer these sentence frames:

- *The amount of land depends on ____.*
- *The independent quantity is ____.*

TRY IT SMP 1, 2, 4, 5, 6

Make Sense of the Problem

See ***Connect to Culture*** to support student engagement. Before students work on Try It, use **Co-Craft Questions** to help them make sense of the problem. Have students work in pairs to explain what the situation is about and to write questions that might be asked about the situation.

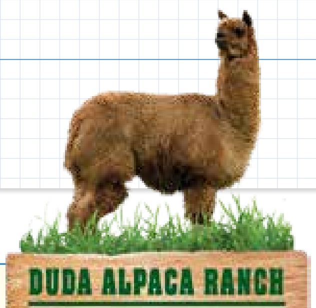
LESSON 16 | SESSION 3

Develop Writing an Equation for a Linear Function from Two Points

➤ **Read and try to solve the problem below.**

The Duda family owns 10 acres of land. They want to buy more land and start a ranch. The amount of land they need is a linear function of the number of grazing animals they plan to have. The family decides to raise alpaca. The table gives the number of acres they need for different numbers of alpaca.

Write an equation to model the data in the table.

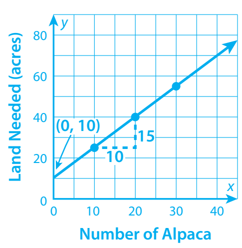


Number of Alpaca	Land Needed (acres)
10	25
20	40
30	55

TRY IT Math Toolkit graph paper, straightedges

Possible work:

SAMPLE A



slope $= \frac{\text{rise}}{\text{run}} = \frac{15}{10} = \frac{3}{2}$

The y-intercept is 10.

Substitute into $y = mx + b$:

$y = \frac{3}{2}x + 10$

SAMPLE B

Number of Alpaca	Land (acres)
0	10
10	25
20	40
30	55

-10, -10, -10 ; -15, -15, -15

Initial value:

When $x = 0$, $y = 10$.

Rate of change:

$\frac{55 - 40}{30 - 20} = \frac{15}{10} = 1.5$

Substitute into $y = mx + b$:

$y = 1.5x + 10$

DISCUSS IT

Ask: How did you use the information given in the table?

Share: I used the numbers in the table to . . .

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DISCUSS IT SMP 2, 3, 6

Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *Which values represent the inputs and outputs in this situation?*
- *Is the initial value given in the table? Does the description state the initial value? How can you find it?*

Common Misconception Listen for students who think the first data pair listed in the table represents the initial value and use 25 as the constant in their equation. As students share their strategies, ask them where the initial value would be shown on the graph. Students should see that the initial value is the y-intercept, so the x-coordinate (in this case the number of animals) of the ordered pair should be zero. Discuss how students can find the initial value based upon the information provided in the problem.

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- table extended backward to find initial value; two rows in table used to find rate of change
- graph used to find slope and y-intercept
- (misconception) strategy assumes initial value is 25, resulting in equation with constant 25
- table used to calculate rate of change; rate of change and one pair of values substituted into $y = mx + b$ to find b

Facilitate Whole Class Discussion

Call on students to share selected strategies. Remind students to project their voices while they explain the strategy they used to write the equation.

Guide students to **Compare and Connect** the representations. To emphasize key mathematical ideas as students share their comparisons, ask others to repeat and rephrase the statements.

ASK *What are some of the methods these strategies used to find the initial value?*

LISTEN FOR Some used the pattern in the table to work backward. Some made a graph and found where it crossed the y-axis. Some found the rate of change and then used it and one data point to solve for b in $y = mx + b$.

Picture It & Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

ASK *Why can the models use different points to find the rate of change?*

LISTEN FOR The slope of a line is the same between any two points on a line.

For the graph, prompt students to examine the initial value. Ask: *How can the slope and a point be used to count back to the y-axis and find the y-intercept?*

For the algebraic method, prompt students to focus on how to find m and b for $y = mx + b$.

- *Where do the values substituted for x and y come from?*
- *Would you get the same value of b if you substituted x- and y-values from a different point on the line?*

➤ **Explore different ways to write an equation for a linear function.**

The Duda family owns 10 acres of land. They want to buy more land and start a ranch. The amount of land they need is a linear function of the number of grazing animals they plan to have. The family decides to raise alpaca. The table gives the number of acres they need for different numbers of Alpaca.

Write an equation to model the data in the table.

Number of Alpaca	Land Needed (acres)
10	25
20	40
30	55

Picture It

You can use a graph to find the slope and y-intercept.

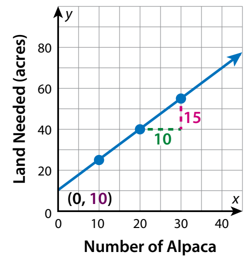
Plot the points given in the table and graph the line.

$\text{slope} = \frac{\text{rise}}{\text{run}}$

$= \frac{15}{10}$

$= 1.5$

y-intercept $= 10$



Model It

You can calculate the rate of change and initial value.

Use the values from any two rows of the table to find the rate of change.

Use (10, 25) and (30, 55): rate of change $= \frac{55 - 25}{30 - 10} = \frac{30}{20} = 1.5$

To find the initial value, substitute the rate of change and one pair of values from the table into the equation for a linear function.

Use (10, 25): $y = mx + b$

$25 = 1.5(10) + b$

$10 = b$

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DIFFERENTIATION | EXTEND

SMP 2

Deepen Understanding
Exploring the Meaning of the Initial Value

Prompt students to consider the meaning of the y-intercept in context.

ASK *What is the y-intercept for this linear relationship? What does it represent?*

LISTEN FOR 0; the amount of land needed for 0 animals.

ASK *Does it make sense that 0 animals require 10 acres of land?*

LISTEN FOR No; 0 animals require no land.

ASK *Reread the problem as it was first presented. How would you interpret the initial value?*

LISTEN FOR The initial value of 10 represents the 10 acres owned by the Duda family prior to buying more land to start the ranch.

ASK *How would the equation and graph be affected if the Duda family did not own any land prior to having any animals?*

LISTEN FOR The initial value, b, would be 0, and the rate of change, m, would be the same. The equation would be $y = 1.5x$. The line would have the same slope, but would pass through the origin.

Develop Writing an Equation for a Linear Function from Two Points

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the ordered pairs are the same in each representation. Explain that they will now use those ordered pairs to reason about how to write an equation for a linear function based on two points.

Before students begin to record and expand on their work in Picture It & Model It, tell them that problem 3 will prepare them to provide the explanation asked for in problem 4.

Monitor and Confirm Understanding 1 – 2

- The linear equation that models this situation is $y = 1.5x + 10$, which means that the family has 10 acres and needs 1.5 more acres for each alpaca they raise.
- Any pair of related *x*- and *y*-values will yield the same equation.
- The *x*- and *y*-values substituted to find the initial value must be coordinates of the same point; they must come from the same row of the table.

Facilitate Whole Class Discussion

3 Look for understanding that a linear function must have a constant rate of change.

> **ASK** *What would it mean if there were more than one value for the rate of change?*
>
> **LISTEN FOR** If there were different values for the rate of change depending on which pairs of points you used to calculate it, then the function could not be linear.

4 Look for understanding that the coordinates of two points on a line will provide the necessary information to write the equation of the line.

> **ASK** *Once you find the slope, how do you find the y-intercept?*
>
> **LISTEN FOR** Substitute the slope for *m* and the coordinates of one of the points for *x* and *y* in the equation $y = mx + b$. Then, solve for *b*.

> **ASK** *How can you check your equation to make sure it is correct?*
>
> **LISTEN FOR** Substitute the ordered pairs into the equation and see if they make a true statement.

5 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ **Use the problem from the previous page to help you understand how to write an equation for a linear function.**

1 Look at **Picture It** and **Model It**. What is the equation for the amount of land needed as a function of the number of alpaca on the ranch? What does the equation mean in this situation?

$y = 1.5x + 10$; Possible answer: The family has 10 acres and needs an additional 1.5 acres for each alpaca they raise.

2 **a.** Look at **Model It**. What other pair of *x*- and *y*-values could you use to find the initial value? Use these values to show you get the same answer.

I could use either of the other two pairs.

Possible work: $x = 20$ and $y = 40$; $\longrightarrow 40 = 1.5(20) + b \longrightarrow 10 = b$

b. Edward uses $x = 10$ and $y = 40$ to find the initial value. What is his mistake?

Possible answer: The *x*- and *y*-values he chose did not come from the same row of the table, so they are not coordinates of the same point. He should have used $x = 10$ with $y = 25$ or $x = 20$ with $y = 40$.

3 How do you know that there is only one possible value for the rate of change?

Possible explanation: A linear function has a constant rate of change, so there is only one rate of change for the function.

4 Explain how you can write the equation for a linear function when you know only two points on the line.

Possible explanation: You can write the equation of a line if you know its slope and *y*-intercept. You can use the two points to find the slope. Then you can use one of the points and the slope to find the *y*-intercept.

5 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.

Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Visual Model

Determine information needed to write a linear equation.

If students are unsure about how to use given information to write an equation in the form $y = mx + b$, work through these examples with the class.

- Display this description: *line with slope = −2 and y-intercept = 1.*
- Ask: *In the equation $y = mx + b$, what is m? b? What is the equation?* [−2; 1; $y = -2x + 1$]
- Display this description: *line passing through (−1, 4) and (0, 5).*
- Ask: *Is either the slope or y-intercept given?* [Yes, the *y*-intercept is 5.]
- Invite a student to show how to calculate the slope, which is 1.
- Ask: *What is an equation in slope-intercept form for this line?* [$y = x + 5$]
- Display this description: *line passing through (−2, −1) and (2, 5).*
- Ask: *What is the slope?* [$\frac{3}{2}$]
- Walk students through the steps of calculating the *y*-intercept, 2, by substituting $\frac{3}{2}$ for *m* and the coordinates of one of the points for *x* and *y* in $y = mx + b$.
- Ask: *What is an equation in slope-intercept form for this information?* [$y = \frac{3}{2}x + 2$]

Apply It

For all problems, encourage students to use a model to support their thinking.

6 See ***Connect to Culture*** to support student engagement. Students also may calculate the rate of change using a different combination of two points. For example, rate of change $= \frac{13.5 - 11.25}{8 - 5} = \frac{2.25}{3} = 0.75$.

7 Students may make a rough sketch of the line described to help them visualize the situation before writing their explanation.

Apply It

➤ **Use what you learned to solve these problems.**

Skeins of Embroidery Floss	Cost of Project
5	\$11.25
8	\$13.50
12	\$16.50

6 The cost of a cross-stitch project is a function of the number of skeins of embroidery floss it requires. The table shows the cost of projects that use different amounts of embroidery floss. What is the equation of the linear function that models this situation? Show your work.

Possible work:

rate of change $= \frac{16.5 - 13.5}{12 - 8}$ $= \frac{3}{4}$ $= 0.75$

$13.5 = \frac{3}{4}(8) + b$

$13.5 = 6 + b$

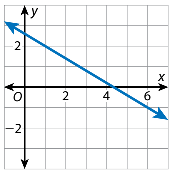
$7.5 = b$

SOLUTION $y = 0.75x + 7.5$

7 You are trying to find the equation of the line that passes through two points, *P* and *Q*. *P* is in Quadrant II and *Q* is in Quadrant IV. What do you know about the slope of the line? Explain.

The slope is negative. Possible explanation: When you move along the line from *P* to *Q*, the *x*-coordinate increases because it goes from negative to positive. The *y*-coordinate decreases because it goes from positive to negative. When one coordinate increases as the other decreases, the rate of change is negative.

8 What is the equation of the function shown by the graph? Show your work.



Possible work: Use points (1, 2) and (6, −1).

slope $= \frac{2 - (-1)}{1 - 6}$ $= -\frac{3}{5}$

$2 = -\frac{3}{5}(1) + b$

$2\frac{3}{5} = b$

SOLUTION $y = -\frac{3}{5}x + 2\frac{3}{5}$

CLOSE EXIT TICKET

8 Students' solutions should show an understanding that:

- the slope can be calculated using any two points on the line.
- the *y*-intercept can be found by substituting the slope for *m* and the coordinates of a point on the line for *x* and *y* in the equation $y = mx + b$ and solving for *b*.

Error Alert If students mistakenly calculate the slope as $\frac{3}{5}$, then point out that the line slants downward from left to right. Ask students if a positive slope makes sense in this situation. Then remind them of the importance of beginning with the same point each time they calculate the difference in the coordinates.

Practice Writing an Equation for a Linear Function from Two Points

Problem Notes

Assign **Practice Writing an Equation for a Linear Function from Two Points** as extra practice in class or as homework.

1. Students may also calculate slope and use the graph to determine the y-intercept, b, by looking at where the graph of the line crosses the y-axis. ***Medium***
2. Once students find that the slope is 2, they may substitute 2 for m and the point (5, 23) for x and y into $y = mx + b$ to find b. ***Basic***

LESSON 16 | SESSION 3 Name:

Practice Writing an Equation for a Linear Function from Two Points

➤ **Study the Example showing how to write an equation for a linear function from two points. Then solve problems 1–4.**

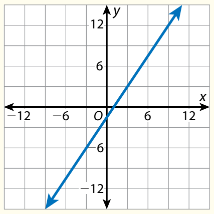
Example

What is the equation of the function shown by the graph? Show your work.

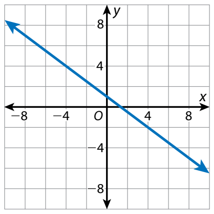
The lines passes through (9, 12) and (3, 3).

$$\text{slope} = \frac{12-3}{9-3} = \frac{9}{6} = \frac{3}{2}$$

$$y = mx + b$$
$$12 = \frac{3}{2}(9) + b$$
$$12 = \frac{27}{2} + b$$
$$-\frac{3}{2} = b$$
$$y = \frac{3}{2}x - \frac{3}{2}$$



1. What is the equation of the function shown by the graph? Show your work.



Possible work: (−4, 4) and (4, −2)

$$\text{slope} = \frac{4-(-2)}{-4-4} = \frac{6}{-8} = -\frac{3}{4}$$

$$-2 = -\frac{3}{4}(4) + b$$
$$-2 = -3 + b$$
$$1 = b$$

SOLUTION $y = -\frac{3}{4}x + 1$

2. The graph of a linear function passes through the points (3, 19) and (5, 23). Write an equation for the function. Show your work.

Possible work:

$$\text{slope} = \frac{23-19}{5-3} = \frac{4}{2} = 2$$

$$19 = 2(3) + b$$
$$19 = 6 + b$$
$$13 = b$$

SOLUTION $y = 2x + 13$

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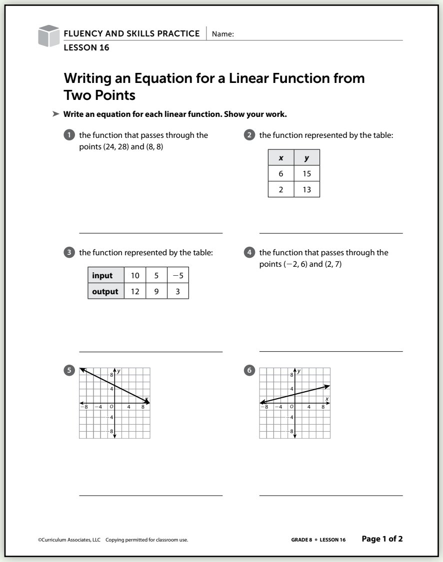
Fluency & Skills Practice

Writing an Equation for a Linear Function from Two Points

In this activity, students are given two points, a table, or the graph of a linear function and asked to write the equation for each function.



Additional Practice

3 **a.** Students may use a different pair of points to find the slope. ***Medium***

b. Students should recognize that the number of tickets determines the cost, so the cost is a function of the number of tickets. ***Medium***

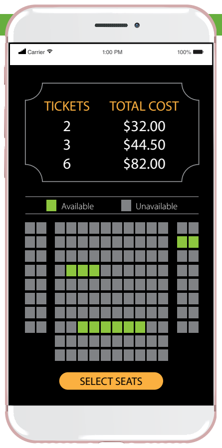
c. Students should realize that the cost of each movie ticket is the same as the constant rate of change, or the slope, m. ***Basic***

d. Students should realize that the convenience fee is the initial value, or b. ***Basic***

4 **a.** Students may use the points in reverse order from the example to find the slope. For example: $\frac{-39 - 26}{9 - (-6)} = \frac{-65}{15} = \frac{-13}{3}$. ***Medium***

b. Students should realize that since the value of b is 0, the graph of the line passes through the origin. ***Basic***

3 Vinh pays a convenience fee when he reserves movie tickets on his cell phone app. The app shows him the total cost of his purchase for different numbers of tickets.



a. What is the equation that models this linear function? Show your work.

Possible work:

$$m = \frac{82 - 32}{6 - 2} = \frac{50}{4} = 12.5$$

$$y = mx + b$$
$$32 = 12.5(2) + b$$
$$32 = 25 + b$$
$$7 = b$$

SOLUTION $y = 12.5x + 7$

b. Use the phrase *is a function of* to describe the situation represented by the equation you wrote in problem 3a.

Possible answer: The total cost of buying movie tickets with an app is a function of the number of tickets bought.

c. How much is each movie ticket? **Each ticket costs $12.50.**

d. How much is the convenience fee? **The convenience fee is $7.00.**

4 **a.** The graph of a linear function passes through the points $(-6, 26)$ and $(9, -39)$. Write an equation for the function. Show your work.

Possible work:

$$m = \frac{26 - (-39)}{-6 - 9} = \frac{65}{-15} = -\frac{13}{3}$$

$$y = mx + b$$
$$26 = -\frac{13}{3}(-6) + b$$
$$26 = 26 + b$$
$$0 = b$$

SOLUTION $y = -\frac{13}{3}x$

b. What is the y-intercept of the line? **0**

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 4** **Apply It**

Levels 1–3: Reading/Speaking

Prepare students to respond to Apply It problem 10. Read the first sentence and help students **Say It Another Way**. Explain that *constant* means it does not change. Then ask: *How is Aniyah driving?* Suggest students use *same* to answer. Then read the second and third sentences. Have students circle quantities and units. Then help them compare. Ask: *Are the units of time the same? What do you need to do?* Allow time for students to convert. Next, have them talk about the data:

- *After ____ minutes, Aniyah is ____.*
- *The distance is a function of ____.*

Have students turn to partners to write an equation that models the situation.

Levels 2–4: Reading/Speaking

Prepare students to respond to Apply It problem 10. Invite volunteers to explain the meaning of *constant*. Encourage them to share examples of things that can be constant. Then have them read the first sentence and **Say It Another Way**. Suggest students use *same* or *does not vary*. Then ask them to read the second and third sentences. Have students tell what they notice about the quantities and units. Then give them time to convert. Guide them to use the words *after* and *function* to talk about the data. Then have students turn to partners. Encourage them to use the conversion to say the third sentence another way. Next, have students write an equation that models the situation.

Levels 3–5: Reading/Speaking

Prepare students to respond to Apply It problem 10. Have students discuss what they notice about the quantities and units. Then have them reread the problem and **Say It Another Way.** After they paraphrase, have other students provide feedback, for example: *You can use* depends on *instead of* as a function of. *You said,* speed that does not vary, *but you can also say* invariable speed. *Instead of* 1 hour, *you could say* 60 minutes.

Then have students write an equation that models the situation.

Develop Writing an Equation for a Linear Function from a Verbal Description

Purpose

- **Develop** strategies for writing an equation for a linear function when given a verbal description of the situation.
- **Recognize** that a rate of change and initial value are needed to write the equation of a linear function.

START CONNECT TO PRIOR KNOWLEDGE

Always, Sometimes, Never

A The number of tickets sold to total sales represents a positive rate of change.

B A hiker's elevation over time represents a positive rate of change.

C The volume of water in a pool as it drains over time represents a positive rate of change.

Solutions

A is always true.

B is sometimes true.

C is never true.

WHY? Support students' facility with analyzing rates of change in real-world contexts.

DEVELOP ACADEMIC LANGUAGE

WHY? Unpack verbal descriptions that have introductory clauses.

HOW? Read Try It. Discuss the third sentence. Explain that the comma separates an introduction or explanation. Ask: *When is Kadeem on page 80?* [one hour after he . . .] Have students find other examples of introductory clauses in Apply It problem 10. Guide them to ask questions that can be answered with the introduction.

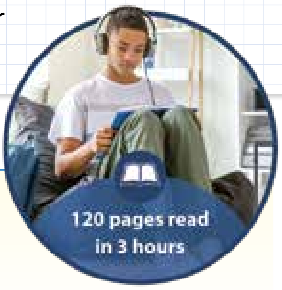
TRY IT SMP 1, 2, 4, 5, 6

Make Sense of the Problem

Before students work on Try It, use **Three Reads** to help them make sense of the problem. Begin by reading the verbal description aloud and discussing the scenario. Then ask a student to read the description a second time. Call on volunteers to explain what the problem is asking. Then have students work with a partner to read the problem a third time and determine the important quantities in the description that will be needed to solve the problem. Have students share ideas with the class.

LESSON 16 | SESSION 4 ■■■■□

Develop Writing an Equation for a Linear Function from a Verbal Description



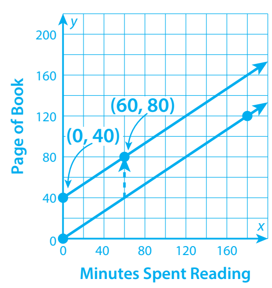
➤ **Read and try to solve the problem below.**

Kadeem spends the afternoon reading a book he started yesterday. He reads 120 pages in 3 hours. One hour after Kadeem begins reading, he is on page 80. Write an equation for the page he is on, *y*, as a function of minutes spent reading, *x*. What page number was he on when he started reading today?

TRY IT Math Toolkit graph paper, straightedges, tracing paper

Possible work:

SAMPLE A



If he started the book at 0 minutes, he would be on page 120 after 180 minutes. (60, 80) is not on this line, so translate up until it is.

The line looks like it passes through (0, 40), so the *y*-intercept is 40.

Slope: $\frac{80-40}{60-0} = \frac{40}{60} = \frac{2}{3}$. Equation: $y = \frac{2}{3}x + 40$.

He was on page 40 when he began reading today.

SAMPLE B

120 pages in 3 hours is $\frac{120}{3} = 40$ pages per hour

If he was on page 80 after 1 hour, then he must have started on page 40 because $80 - 40 = 40$.

120 pages in 180 minutes is $\frac{120}{180} = \frac{2}{3}$ page per minute.

The rate of change is $\frac{2}{3}$. The initial value is 40.

$y = \frac{2}{3}x + 40$

DISCUSS IT

Ask: How did you begin to solve the problem?

Share: The first thing I did was . . .

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DISCUSS IT SMP 1, 2, 3, 5, 6

Support Partner Discussion

After students work on Try It, encourage them to respond to Discuss It with a partner. If students need support in getting started, prompt them to ask each other questions such as:

- *Which quantities in the problem do you need in order to find the rate of change?*
- *Does the description give the initial value?*
- *If he reads 120 pages in 3 hours, how was he able to be on page 80 after the first hour?*

Common Misconception Listen for students who think that Kadeem was on page 120 after 3 hours (or 180 minutes). These students may use the points (60, 80) and (180, 120) to find the equation. As students share their strategies, help them see that Kadeem reads 40 pages every 60 minutes. So, if he is on page 80 after 60 minutes, he will be on page 120 after 120 minutes and on page 160 after 180 minutes. Therefore, the line goes through (180, 160), not (180, 120).

Select and Sequence Student Strategies

Select 2–3 samples that represent the range of student thinking in your classroom. Here is one possible order for class discussion:

- table to find rate of change and initial value
- rate of change found and used to work backward from (60, 80) to find initial value
- (misconception) solution incorrectly assumes line goes through (60, 80) and (180, 120)
- point and rate of change used to graph line; then line used to find initial value
- rate of change calculated; then rate of change substituted for m and (60, 80) substituted for x and y in $y = mx + b$ and equation solved for b

Facilitate Whole Class Discussion

Call on students to share selected strategies. After each strategy, allow individual think time for students to process the ideas.

Guide students to **Compare and Connect** the representations. Review that one way to connect representations is to explain how each shows the rate of change and initial value for the situation.

ASK *What are some different ways of finding the initial value?*

LISTEN FOR Graph the line and use it to find the y-intercept. Substitute the slope and coordinates of a point into $y = mx + b$ and solve for b. Use the rate of change to work backward from (60, 80).

Picture It & Model It

If students presented these models, have students connect these models to those presented in class.

If no student presented at least one of these models, have students first analyze key features of the models, and then connect them to the models presented in class.

ASK *How does each representation find the rate of change? How does each representation find the initial value?*

LISTEN FOR Each uses 120 pages in 3 hours to find the rate of change. Picture It uses the graph to find the initial value and Model It uses the equation.

For the graph, prompt students to identify how the axes are labeled to represent the problem. Ask: *What are the input and output variables?*

For the equation, prompt students to connect the quantities and units in the description to the equation. Ask: *Why is the rate of change $\frac{2}{3}$?*

LESSON 16 | SESSION 4

➤ Explore different ways to write an equation for a linear function from a verbal description.

Kadeem spends the afternoon reading a book he started yesterday. He reads 120 pages in 3 hours. One hour after Kadeem begins reading, he is on page 80. Write an equation for the page he is on, y, as a function of minutes spent reading, x. What page number was he on when he started reading today?

Picture It

You can graph the function.

Plot the point (60, 80).

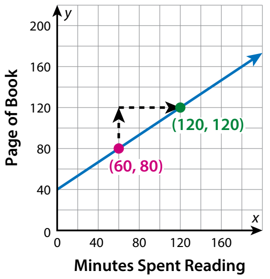
Find the rate of change, which is the slope of the line.

120 pages in 3 hours is 120 pages in 180 minutes.

slope $= \frac{120}{180} = \frac{40}{60} = \frac{2}{3}$

Use the slope to plot another point at (120, 120).

Draw a line through the points and identify the y-intercept.



Model It

You can calculate the rate of change and initial value.

$\frac{120 \text{ pages}}{3 \text{ hours}}$ can be written as $\frac{120 \text{ pages}}{180 \text{ minutes}}$.

The rate of change is $\frac{2}{3}$ page per minute.

To find the initial value, use the equation for a linear function. Then substitute the rate of change and the point (60, 80).

$y = mx + b$

$80 = \frac{2}{3}(60) + b$

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DIFFERENTIATION | EXTEND

SMP 7

Deepen Understanding

Using Structure to Determine How Changing Units Affects the Graph

Prompt students to think about the two units of time presented in this verbal description. Have them think about how considering the page Kadeem is on as a function of reading time in hours, rather than minutes, would affect the graph.

ASK *How would the scale and labels of the x-axis change if the graph showed the page number as a function of hours spent reading?*

LISTEN FOR Each grid space along the x-axis represents 20 minutes, which is the same as $\frac{1}{3}$ hour. So, the x-axis numbers would change from 40, 80, 120, 160 to $\frac{2}{3}$, $1\frac{1}{3}$, 2, $2\frac{2}{3}$. The label would change to Hours Spent Reading.

ASK *How would the scale and labels of the y-axis change?*

LISTEN FOR The y-axis scale and labels would remain the same.

ASK *How would the graphs compare to one another?*

LISTEN FOR The graphs would look the same but with a different scale and labels on the x-axis. Point (60, 80) would be $(1, 1\frac{1}{3})$ and point (120, 120) would be (2, 2).

Develop Writing an Equation for a Linear Function from a Verbal Description

CONNECT IT

SMP 2, 4, 5, 6

Remind students that the quantities and the relationships between them are the same in each representation. Explain that they will now use those relationships to reason about how to write the equation of a linear function from a verbal description.

Before students begin to record and expand on their work in Picture It & Model It, tell them that problem 5 will prepare them to provide the explanation asked for in problem 6.

Monitor and Confirm Understanding 1 – 3

- The point (60, 80) shows that Kadeem is on page 80 after reading for 60 minutes.
- The initial value, 40, means that Kadeem was on page 40 when he started reading today.
- The equation for the page number, y, as a function of time in minutes, x, is $y = \frac{2}{3}x + 40$.

Facilitate Whole Class Discussion

4 Look for understanding of how the equation changes when the time is measured in hours.

> **ASK** *Why is the rate of change 40?*
>
> **LISTEN FOR** He reads 120 pages in 3 hours, which is 120 ÷ 3, or 40 pages per hour.

> **ASK** *Why is the initial value still 40?*
>
> **LISTEN FOR** The starting page number is not affected by the unit of time.

5 Look for understanding that the equations give the same answers as long as the values substituted for x are in the correct units.

> **ASK** *Into which equation will you substitute 3? 180? Why?*
>
> **LISTEN FOR** I will substitute 3 into $y = 40x + 40$ and 180 into $y = \frac{2}{3}x + 40$ because they are written in terms of hours and minutes, respectively.

6 Look for understanding that the equations represent the same function. Both give the page number for any time. The time must be expressed in minutes for one equation and hours for the other.

7 **Reflect** Have all students focus on the strategies used to solve the Try It. If time allows, have students discuss their ideas with a partner.

CONNECT IT

➤ Use the problem from the previous page to help you understand how to write an equation for a linear function from a verbal description.

1 What does the point (60, 80) represent?
After reading for 60 minutes, Kadeem has read up to page 80.

2 Use the graph in **Picture It** to estimate the y-intercept. Check your estimate by solving the equation in **Model It** to find the value of b. What does this value represent in this situation?
40; The page number Kadeem was on when he started reading today

3 Write the equation for the page Kadeem is on as a function of minutes he spends reading.
$y = \frac{2}{3}x + 40$

4 What is the rate of change in pages per hour? Write an equation for the page Kadeem is on as a function of hours spent reading.
40; $y = 40x + 40$

5 Use the equations you wrote in problems 3 and 4 to find the page that Kadeem was on after 3 hours, or 180 minutes, of reading. Are your answers the same?
160; Yes

6 Do the different equations in problems 3 and 4 represent the same function? Explain.
Yes; They both represent the page Kadeem is on as a function of time. One equation measures the time in hours and the other measures the time in minutes.

7 **Reflect** Think about all the models and strategies you have discussed today. Describe how one of them helped you better understand how to solve the **Try It** problem.
Responses will vary. Check student responses.

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DIFFERENTIATION | RETEACH or REINFORCE

Visual Model

Write an equation for a linear function from a verbal description.

If students are unsure about how to write an equation for a linear function from a verbal description, then use this activity to walk them through the process.

- Display the following: *Tressa is stocking cans on shelves. She stocks 100 cans in 2 hours. One hour after she starts, there are 60 cans on the shelves.*
- Write $y = mx + b$. Explain that you want to write an equation of this form, where x is the number of hours Tressa works and y is the number of cans on the shelves.
- Underline m. Ask: *At what rate does Tressa stock the shelves? How do you know?* [50 cans per hour; 100 cans in 2 hours is the same as 50 cans per hour] Replace m with 50.
- Underline b. Ask: *If Tressa stocks 50 cans per hour, how many cans did she stock the first hour?* [50]
- Ask: *According to the problem, how many cans were on the shelf 1 hour after she started?* [60] *So, how many cans must have been on the shelf when she started?* [10] Replace b with 10.
- To assess understanding, change one of the numbers in the problem and ask students to explain how the equation would change.

Apply It

For all problems, encourage students to use a model to support their thinking. Allow some leeway in precision with regard to their graphs, should they decide to make a graph to help them visualize the scenarios.

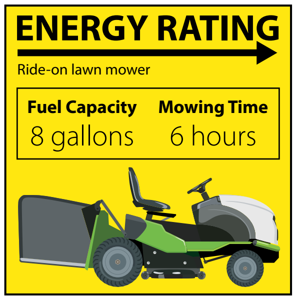
8 Students should recognize that the initial value is 8 gallons, which is the amount of gas in a full tank. Because the gas is being consumed and its amount is decreasing as the mower is running, the slope should be negative.

9 Students should understand that the change in the value of the function refers to the change in *y* (the output value), and the change in the input refers to the change in *x*.

Apply It

➤ **Use what you learned to solve these problems.**

8 A lawn mower has the given energy rating. A full tank of gas can power the lawn mower for about 6 hours. What equation can be used to find the amount of gas left in the tank, *y*, as a function of the mowing time, *x*? Show your work.



Possible work:

When the mower has run 0 hours, there are 8 gallons left. So the initial value is 8.

The rate of change is negative because the amount of gas left decreases as time increases.

rate of change $= -\frac{8 \text{ gallons}}{6 \text{ hours}} = -\frac{4}{3}$ gallons per hour

SOLUTION $y = -\frac{4}{3}x + 8$

9 The graph of a linear function passes through the point (–2, –7). When the input increases by 3, the value of the function increases by 8. What is the equation that models the function?

$y = \frac{8}{3}x - \frac{5}{3}$

10 Aniyah is driving home at a constant speed. After 20 minutes, she is 70 miles from home. After 1 hour, she is 40 miles from home. What equation models her distance from home, *y*, as a function of time, *x*? Show your work.

Possible work and answer for time in minutes:

One hour is 60 minutes. Use (20, 70) and (60, 40).

The rate of change is $\frac{70 - 40}{20 - 60} = \frac{30}{-40} = -\frac{3}{4}$.

$70 = -\frac{3}{4}(20) + b$

$70 = -15 + b$

$85 = b$

SOLUTION $y = -\frac{3}{4}x + 85$

CLOSE **EXIT TICKET**

10 Students' solutions should show an understanding that:

- the points (20, 70) and (60, 40) satisfy the function if using minutes, or the points $\left(\frac{1}{3}, 70\right)$ and (1, 40) satisfy the function if using hours.
- the two points can be used to find the rate of change.
- to find the initial value, the rate of change can be substituted for *m* and the coordinates of one point for *x* and *y* in $y = mx + b$, and then the equation can be solved for *b*.

Error Alert If students use (20, 70) and (1, 40) as the coordinates of their points, then remind them the unit of time needs to be the same for both points.

Practice Writing an Equation for a Linear Function from a Verbal Description

Problem Notes

Assign **Practice Writing an Equation for a Linear Function from a Verbal Description** as extra practice in class or as homework.

1. **a.** Students should realize that situations described as having a constant rate of change can always be modeled with a linear function. ***Medium***

   b. Students know slope $= \frac{\text{rise}}{\text{run}}$. In this situation, as time increases, height decreases. In other words, the run is positive, and the rise is negative So, the slope, or rate of change, is negative. ***Medium***

   c. Students may graph the function by plotting points at (16, 0) and (6, 0), drawing a line through them, and then using $\frac{\text{rise}}{\text{run}}$ to find the slope of the line. ***Medium***

LESSON 16 | SESSION 4 Name:

Practice Writing an Equation for a Linear Function from a Verbal Description

➤ **Study the Example showing how to write an equation for a linear function from a verbal description. Then solve problems 1–4.**

Example

Dolores is making a music video using a drone. She sets the drone on a platform 1 meter above the ground. Then she uses the controls to make it rise at a constant rate. The drone reaches a height of 16 meters in 5 seconds. What is the equation for the drone's height, *y*, as a function of time, *x*?

At 0 seconds, the drone is 1 meter above the ground.

At 5 seconds, the drone is 16 meters above the ground.

rate of change: $\frac{16 - 1}{5 - 0} = \frac{15}{5} = 3$ initial value: 1

Use the equation for a linear function, $y = mx + b$.

$y = 3x + 1$

1. The drone in the Example hovers at 16 meters for a few minutes before being lowered at a constant rate. It reaches the ground after 6 seconds.

   a. Why can the drone's descent be modeled by a linear function?

   The rate of change during the descent is constant, so it can be modeled by a linear function.

   b. The linear model of the drone's descent gives its height as a function of time. Is the rate of change positive or negative? Explain.

   Negative; Possible explanation: As the drone descends, its height decreases as time increases.

   c. What equation models the drone's descent as time increases? Show your work.

   Possible work: 0 seconds: 16 m; 6 seconds: 0 m

   rate of change: $\frac{16 - 0}{0 - 6} = \frac{16}{-6} = -\frac{8}{3}$; initial value: 16

   SOLUTION $y = -\frac{8}{3}x + 16$

Vocabulary

initial value
in a linear function, the value of the output when the input is 0.

linear function
a function that can be represented by a linear equation.

rate of change
in a linear relationship between *x* and *y*, it tells how much *y* changes when *x* changes by 1.

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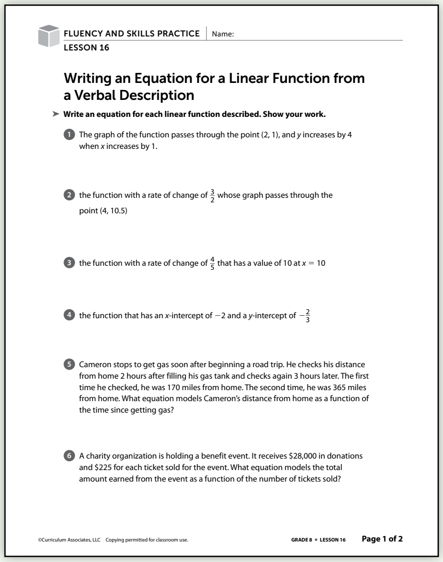
Fluency & Skills Practice

Writing an Equation for a Linear Function from a Verbal Description

In this activity, students are given problems and asked for the linear equation that represents each situation. Problems are given both with and without context.


FLUENCY AND SKILLS PRACTICE | Name:
LESSON 16

Writing an Equation for a Linear Function from a Verbal Description

➤ **Write an equation for each linear function described. Show your work.**

1. The graph of the function passes through the point (2, 1), and y increases by 4 when x increases by 1.
2. the function with a rate of change of $\frac{3}{2}$ whose graph passes through the point (4, 10.5)
3. the function with a rate of change of $\frac{4}{5}$ that has a value of 10 at $x = 10$
4. the function that has an x-intercept of -2 and a y-intercept of $-\frac{2}{3}$
5. Cameron stops to get gas soon after beginning a road trip. He checks his distance from home 2 hours after filling his gas tank and checks again 3 hours later. The first time he checked, he was 170 miles from home. The second time, he was 365 miles from home. What equation models Cameron's distance from home as a function of the time since getting gas?
6. A charity organization is holding a benefit event. It receives $28,000 in donations and $225 for each ticket sold for the event. What equation models the total amount earned from the event as a function of the number of tickets sold?

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Additional Practice

2 **a.** Students should only consider the amount spent. So, they should not include the $10 they receive for selling each shirt. ***Medium***

b. The club does not collect any money if they sell 0 shirts, so the initial value is 0. ***Medium***

c. The profit is the amount the club earns minus the amount they spend, so the equation is $y = 10x - (2.5x + 60)$, which simplifies to $y = 7.5x - 60$. ***Challenge***

3 ***Basic***

4 **a.** The given information describes two points on the line, $(-2, 0)$ and $(8, -25)$. Students can use these points to find the slope, and then use the slope and one of the points to find the *y*-intercept. ***Medium***

b. The given information describes the *y*-intercept and the slope. ***Basic***

c. The given information indicates that $(0, 18)$ and $(-15, 0)$ are on the line. This means that the *y*-intercept is 18. Students may use these points to find the slope. ***Medium***

d. Because the function represents a proportional relationship, it passes through $(0, 0)$. So, its *y*-intercept is 0. The points $(0, 0)$ and $(3, 7)$ may be used to find the slope. ***Medium***

2 The Drama Club is selling tie-dye T-shirts as a fundraiser. They buy the dyeing materials for $60 and white T-shirts for $2.50 each. They sell the finished shirts for $10 each.

a. Write an equation for the money they spend, *y*, as a function of the number of T-shirts they buy, *x*.

$y = 2.5x + 60$

b. Write an equation for the money they collect, *y*, as a function of the number of T-shirts they sell, *x*.

$y = 10x$

c. Write an equation for their profit, *y*, as a function of the number of T-shirts they sell, *x*.

$y = 7.5x - 60$

3 On his first birthday, Tomás was 30 inches tall. For the next year, he grew half an inch each month. What equation models his height during that year, *y*, as a function of the number of months, *x*?

$y = 0.5x + 30$

4 Write an equation for each linear function described below.

a. The value of the function at $x = -2$ is 0. The value of the function at $x = 8$ is -25.

$y = -2.5x - 5$

b. The graph of the function has a *y*-intercept of 13. When *x* increases by 1, *y* decreases by 4.

$y = -4x + 13$

c. The graph of the function intersects the *y*-axis at $y = 18$ and intersects the *x*-axis at $x = -15$.

$y = \frac{6}{5}x + 18$

d. The function describes a proportional relationship. Its graph passes through the point (3, 7).

$y = \frac{7}{3}x$

DIFFERENTIATION | ENGLISH LANGUAGE LEARNERS

Use with **Session 5** **Apply It**

Levels 1–3: Speaking/Listening

Support students to understand and use the language in Apply It problem 1 and in Pair/Share. Read the problem and have students circle words they can use to talk about the problem. Make a **Co-Constructed Word Bank** with relevant words. Encourage students to brainstorm cognates for *temperature*, *different*, *degrees*, *point*, and *function*.

Next, have some small groups write an equation that shows the temperature in degrees Fahrenheit as a function of degrees Celsius. Have other groups write an equation that shows the temperature in degrees Celsius as a function of degrees Fahrenheit. Then monitor as different groups meet and compare.

Levels 2–4: Speaking/Listening

Support students to understand and use the language in Apply It problem 1 and in Pair/Share. Read the problem and have students suggest words they can use to talk about the problem. Make a **Co-Constructed Word Bank.** Then invite volunteers to talk about countries that use Celsius. Next, have students work in small groups. Have some groups write an equation that shows the temperature in degrees Fahrenheit as a function of degrees Celsius. Have other groups write an equation that shows the temperature in degrees Celsius as a function of degrees Fahrenheit. Monitor as different groups meet and compare. Have students use the words *input* and *output* to describe each case.

Levels 3–5: Speaking/Listening

Support students to understand and use the language in Apply It problem 1 and in Pair/Share. Read the problem and have students make a **Co-Constructed Word Bank** with words and cognates they can use to talk about the problem. Next, have students work in small groups. Have some groups write an equation that shows the temperature in degrees Fahrenheit as a function of degrees Celsius. Have other groups write an equation that shows the temperature in degrees Celsius as a function of degrees Fahrenheit. Have groups meet and compare their work. Ask students to explain their equations and suggest situations in which they could use the equations to convert scales.

Refine Using Functions to Model Linear Relationships

Purpose

- **Refine** strategies for using functions to model linear relationships.
- **Refine** understanding of the meaning and interpretation of rate of change and initial value in a variety of contexts.

START CONNECT TO PRIOR KNOWLEDGE

What is an equation of the linear function that models the situation?

Hours	Cost
2	$200
4	$350
6	$500

Solution

$y = 75x + 50$

WHY? Confirm students' understanding of writing a function to model a linear relationship, identifying common errors to address as needed.

MONITOR & GUIDE

Before students begin to work, use their responses to the **Start** to determine those who will benefit from additional support. Use the **Error Analysis** table below to guide remediation.

Have all students complete the Example and problems 1–3, using Consider This and Pair/Share as appropriate. Observe and monitor their reasoning and guide or redirect students as needed.

LESSON 16 | SESSION 5

Refine Using Functions to Model Linear Relationships

➤ **Complete the Example below. Then solve problems 1–9.**

Example

A swimming pool is being filled at a constant rate of 6 gallons per minute. After 1 minute, there are 8 gallons of water in the pool. What equation represents this function?

Look at how you could use the rate of change and a point to write the equation.

The rate of change is 6.

The 8 gallons of water in the pool after 1 minute is represented by the point (1, 8) on the graph of the function.

$y = mx + b$

$8 = 6(1) + b$

$2 = b$

SOLUTION $y = 6x + 2$

CONSIDER THIS . . .
The information given represents the slope and a point on the line when the function is graphed.

PAIR/SHARE
What does 2 represent in this situation?

Apply It

1 Celsius (C) and Fahrenheit (F) are two different scales for measuring temperature. The freezing point of water is 0°C, or 32°F. The boiling point of water is 100°C, or 212°F. Write an equation that shows the temperature in degrees Fahrenheit as a function of the temperature in degrees Celsius. Show your work.

Possible work: Use (0, 32) and (100, 212).

The initial value is 32.

Rate of change: $\frac{212 - 32}{100 - 0}$

$= \frac{180}{100}$

$= \frac{9}{5}$

SOLUTION $y = \frac{9}{5}x + 32$

CONSIDER THIS . . .
Are the Celsius temperatures the input values or output values?

PAIR/SHARE
How can you write an equation for temperature in degrees Celsius as a function of temperature in degrees Fahrenheit?

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START ERROR ANALYSIS

If the error is . . .	Students may . . .	To support understanding . . .
$y = 50x + 75$	have switched the rate of change and initial value.	Encourage students to use substitution to check that their equation is correct. Remind students that the initial value is the constant in the equation.
$y = 75x + 200$	have thought the first cost in the table was the initial value.	Prompt students to extend the table to include $x = 0$ to find the initial value of the function.
$y = 75x$	have assumed the initial value is 0 or that the relationship is proportional.	Prompt students to plot the points and draw a line through them. Ask them to identify the slope and then use the rise over run pattern to plot a point at $x = 0$.

Example

Guide students in understanding the Example. Ask:

- *How can you use the fact that after 1 hour, there are 8 gallons of water in the pool?*
- *How can you use the rate of change and a point to write an equation of the function?*
- *What do the values of m and b in the equation of the function represent in this context?*

Help all students focus on the Example and responses to the questions by suggesting they ask their classmates to clarify statements they do not understand.

Look for understanding that the rate of change of 6 was substituted for m and the point (1, 8) was substituted for (x, y) in the equation $y = mx + b$, and then the equation was solved for b.

Apply It

1 See ***Connect to Culture*** to support student engagement. Students may identify the initial value by noticing that the freezing point of water is 0°C or 32°F. If students do not notice that the initial value is given, they may substitute the rate of change and one pair of values into the equation for a linear function, $y = mx + b$, to calculate the initial value. *DOK 2*

2 Students may check their equations by substituting a value for x, and then simplifying the right side of their equation to find a corresponding value for y. Then, they may check that the ordered pair (x, y) is on the line shown in the graph. *DOK 1*

3 **B is correct.** The initial value is the amount in the account after 0 weeks, which is the amount used to open the account.

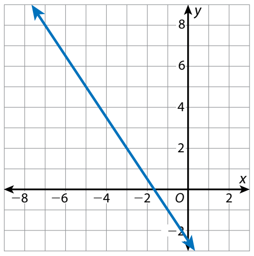
A is not correct. This answer is the rate of change of the function.

C is not correct. This answer is the value of y when $x = 1$.

D is not correct. This answer is the value of y after x weeks.

DOK 3

2 What is the equation of the function shown by the graph? Show your work.



Possible work:

Use (−5, 5) and (−1, −1).

$m = \frac{5 - (-1)}{-5 - (-1)}$ $\quad y = mx + b$

$= \frac{6}{-4}$ $\quad -1 = -\frac{3}{2}(-1) + b$

$= -\frac{3}{2}$ $\quad -1 = \frac{3}{2} + b$

$-\frac{5}{2} = b$

CONSIDER THIS . . . How can you find the rate of change for the function?

SOLUTION $y = -\frac{3}{2}x - \frac{5}{2}$

PAIR/SHARE Can you use the graph to check your answer? Explain.

3 Lillie uses some birthday money to open a bank account. Then she deposits the same amount into the account each week. The equation $y = 35x + 100$ represents the total amount in the account after x weeks. What does the initial value of the function represent?

A The amount deposited each week

B The amount used to open the bank account

C The total amount in the account after the first week of saving

D The total amount in the account after x weeks

Chloe chose A as the correct answer. How might she have gotten that answer?

Possible answer: Chloe may have confused the initial value of the function with the rate of change.

CONSIDER THIS . . . What point on the graph of the function represents the initial value?

PAIR/SHARE How much did Lillie use to open the bank account? How much does she deposit each week?

GROUP & DIFFERENTIATE

Identify groupings for differentiation based on the **Start** and problems 1–3. A recommended sequence of activities for each group is suggested below. Use the resources on the next page to differentiate and close the lesson.

Approaching Proficiency
- **RETEACH** Visual Model
- **REINFORCE** Problems 5, 7, 8

Meeting Proficiency
- **REINFORCE** Problems 4–8

Extending Beyond Proficiency
- **REINFORCE** Problems 4–8
- **EXTEND** Challenge

Have all students complete the **Close: Exit Ticket**.

Resources for Differentiation are found on the next page.

Refine Using Functions to Model Linear Relationships

Apply It

4 **a.** Students should represent the amount of gas with x and cost with y. Students may use the given information to find two points on the line. Then they may use the points to find the rate of change and initial value. ***DOK 2***

b. Students may reason that in problem 4a, the phrase before *as a function of* is represented by y. The phrase after *as a function of* is represented by x. ***DOK 1***

c. The initial value is the value of y when $x = 0$, or the cost of a car wash. The rate of change is the cost of a gallon of gas. ***DOK 2***

5 **C is correct.** In the equation, 25 is the starting amount and 50 is the amount saved each week; $5(50) + 25 = 275$.

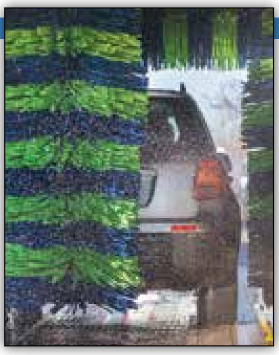
A is not correct. This answer switches the rate of change and initial value.

B is not correct. This answer gives the correct amount saved in 5 weeks but does not give the correct total amount saved.

D is not correct. This answer confuses the initial amount, $x = 0$, with the amount saved in the first week, $x = 1$.

DOK 2

6 The y-values increase by 10, but the x-values do not increase by a constant amount. So, the rate of change is not constant. ***DOK 3***



4 Javier and Ellema both get their cars washed and fill their gas tanks at the same gas station. Javier pays \$26.96 for a car wash and 5.6 gallons of gas. Ellema pays \$48.62 for a car wash and 13.2 gallons of gas.

a. What is an equation for the cost of gas and a car wash as a function of the amount of gas bought? Show your work.

Possible work:

rate of change $= \frac{48.62 - 26.96}{13.2 - 5.6} = \frac{21.66}{7.6} = 2.85$

$y = mx + b$

$26.96 = 2.85(5.6) + b$

$11 = b$

SOLUTION $y = 2.85x + 11$

b. What does each variable represent?

x represents the number of gallons of gas bought and y represents the total cost of the car wash and gas combined.

c. What are the initial value and the rate of change of the function? What does each one represent?

The initial value is \$11, the cost of a car wash without buying any gas. The rate of change is \$2.85, the cost of 1 gallon of gas.

5 Which savings plan can be modeled by $y = 50x + 25$?

A Start with \$50. Save \$25 each week.

B Save \$250 in 5 weeks for a total of \$300.

C Start with \$25. The total saved after 5 weeks is \$275.

D The total saved is \$25 the first week and \$50 the second week.

6 Does the table show a linear function? Explain.

x	0	2	5	10
y	20	30	40	50

No; Possible explanation: The rate of change is not constant.

$\frac{30 - 20}{2 - 0} = \frac{10}{2} = 5$; $\frac{50 - 40}{10 - 5} = \frac{10}{5} = 2$

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DIFFERENTIATION

RETEACH

Visual Model

Find the rate of change and initial value of a linear function using its graph.

Students approaching proficiency with using equations to model linear functions will benefit from finding the equation of a line using a graph.

Materials For display: large four-quadrant coordinate plane

- Invite a student to plot points at (4, 5) and (8, 7) and then draw a line through them.
- Ask: *How can you use the graph to find the slope of this line?* [Divide the rise by the run.]
- Invite a student to mark and label the rise and the run and calculate the slope.
- Ask: *What is the slope? How does the slope of the line relate to the rate of change for the related linear function?* [$\frac{1}{2}$; It has the same value.]
- Ask: *How can you use the slope of the line to help identify the y-intercept?* Demonstrate how to start at (4, 5) and then move 1 unit down and 2 units left until you reach the y-axis at (0, 3).
- Ask: *What is the y-intercept? How does the y-intercept of the line relate to the initial value for the related linear function?* [3; It has the same value.]
- Ask: *What is the equation for the linear function modeled by the line?* [$y = \frac{1}{2}x + 3$]
- Ask: *Describe a real-life function this equation could represent.* [Sample answer: A spider starts at a height of 3 meters off the ground. It climbs up a wall 1 meter every 2 minutes. The function $y = \frac{1}{2}x + 3$ gives its height in meters, y, as a function of the number of minutes, x.]
- Repeat, as time allows, with points on a line with negative slope.

7 *DOK 1*

8 **a.** The rate of change is m and the initial value is b in the equation $y = mx + b$.

b. The slope is the rate of change.

c. Substitute the slope and the point into the equation $y = mx + b$ and solve for b.

d. The y-intercept and the value of the function at $x = 5$ give two points to find the slope.

e. The slope or another point is needed.

DOK 2

CLOSE EXIT TICKET

9 **Math Journal** Look for understanding that the rate of change is 50 and the initial value is 150.

Error Alert If students describe a situation that does not match the graph, then have them use the axis labels to help them understand the meaning of the initial value and rate of change.

End of Lesson Checklist

INTERACTIVE GLOSSARY Support students by suggesting they look at the use of *initial value* in Session 2. Connect the value of 90 in the equation under Analyze It to the point (0, 90) on the graph in Model it.

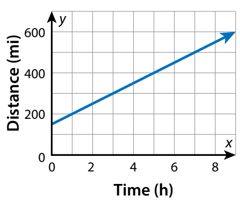
SELF CHECK Have students review and check off any new skills on the Unit 4 Opener.

7 The equation $y = 0.15x + 0.40$ represents the cost of mailing a letter weighing 1 ounce or more. In the equation, x represents the weight of the letter in ounces and y represents the cost in dollars of mailing the letter. In this situation, the cost of mailing a letter is a function of the weight of the letter.

8 Tell whether the information given is enough to write an equation for the linear function.

	Yes	No
a. The initial value and the rate of change of the function	●	○
b. The slope of the line and the rate of change of the function	○	●
c. The slope of the line and one point on the line that is not the y-intercept	●	○
d. The y-intercept of the line and the value of the function at $x = 5$	●	○
e. The y-intercept of the line and the value of the function at $x = 0$	○	●

9 **Math Journal** The graph shows distance as a function of time. Write an equation for the line. Then describe a situation that could be represented by the graph. Include the initial value and rate of change for the function. Then tell what each quantity represents in this situation.



$y = 50x + 150$; Possible answer: A family is taking a road trip. The initial value is 150 and represents how many miles they are from home when they begin driving for the day. The rate of change is 50 and represents how many miles farther from home they are each hour.

End of Lesson Checklist

- [ ] **INTERACTIVE GLOSSARY** Find the entry for *initial value*. Give an example of a situation that can be modeled by a linear function with an initial value of 30.
- [ ] **SELF CHECK** Go back to the Unit 4 Opener and see what you can check off.

REINFORCE

Problems 4–8

Solve problems using functions to model linear relationships.

Students meeting proficiency will benefit from additional work with using functions to model linear relationships by solving problems in a variety of formats.

- Have students work on their own or with a partner to solve the problems.
- Encourage students to show their work.

EXTEND

Challenge

Write an equation using variable points.

Students extending beyond proficiency will benefit from writing equations for two functions to see where those functions intersect.

- Have students work with a partner to solve this problem: *One line passes through the points (0, 3) and (5, 13). Another line passes through the points (0, 9) and (2, 10). At what point do the lines intersect?*
- Students may use each pair of points to find the slope and y-intercept of each line. Then they can solve the system of equations to find the point of intersection.

PERSONALIZE

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