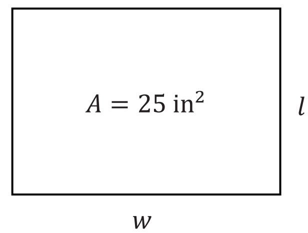
Lesson 15: Rearranging Formulas

Exploratory Challenge – Rearranging Familiar Formulas



1. The area A of a rectangle is 25 in^2. The formula for area is $A = lw$.

   A. If the width w is 10 inches, what is the length l?

   $A = l \times w$ $w = 10$
   $l = 2.5$

   B. If the width w is 15 inches, what is the length l?

   $A = l \times w$ $w = 15$
   $l = ?\ 5/3$

   $\frac{A}{w}$ $\frac{25}{15} = \frac{5}{3}$

2. A. Joey rearranged the area formula to solve for l. His beginning work is shown below. Finish his work to isolate l.

$$A = lw$$

$$\frac{A}{w} = \frac{lw}{w}$$

$$l = \frac{A}{w}$$

   B. Verify that the area formula, solved for l, will give the same results for l as having solved for l in the original area formula. Use both w is 10 inches and w is 15 inches with an area of 25 in^2.

   $l = \frac{25}{10}$ $l = \frac{25}{15}$

   $l = 2.5$ $l = \frac{5}{3}$

3. In the first column solve each equation for x. Then follow the same steps to solve the "formula" for x in the second column. Remember a variable symbol, like a, b, c, and d, represents a number.

Equation	**"Formula"**
A. $2x-6=10$ $+6 \quad +6$ $\frac{2x}{2}=\frac{16}{2}$ $x=8$	$ax-b=c$ $+b \quad +b$ $\frac{ax}{a}=\frac{b+c}{a}$ $x=\frac{b+c}{a}$
B. $-3x-3=-12$ $+3 \quad +3$ $\frac{-3x}{-3}=\frac{-9}{-3}$ $x=3$	$-ax-b=-c$ $+b \quad +b$ $\frac{-ax}{-a}=\frac{b-c}{-a}$ $x=\frac{b-c}{-a}$ or $\frac{-c+b}{-a}$
C. $9-4x=21$ $-9 \quad -9$ $\frac{-4x}{-4}=\frac{12}{-4}$ $x=-3$	$a-bx=c$ $-a \quad -a$ $\frac{-bx}{-b}=\frac{c-a}{-b}$ $x=\frac{c-a}{-b}$
D. $\cancel{2}\cdot\frac{3x-1}{\cancel{2}}=10\cdot 2$ $3x-1=20$ $+1 \quad +1$ $\frac{3x}{3}=\frac{21}{3}$ $x=7$	$\cancel{c}\cdot\frac{ax-b}{\cancel{c}}=d\cdot c$ $ax-b=cd$ $+b \quad +b$ $\frac{ax}{a}=\frac{cd+b}{a}$ $x=\frac{cd+b}{a}$
E. $\frac{x}{2}+5=15$ $-5 \quad -5$ $2\cdot\frac{x}{2}=10\cdot 2$ $x=20$	$\frac{x}{a}+b=c$ $-b \quad -b$ $\cancel{a}\cdot\frac{x}{\cancel{a}}=(c-b)\cdot a$ $x=a(c-b)$ $x=ac-ab$

4. Solve the equation $ax - b = c$ for a. The variable symbols x, b, and c, represent numbers.

$+b \quad +b$

$\frac{ax}{x} = \frac{c+b}{x}$

$a = \frac{c+b}{x}$

5. Complete the chart below.

Formula	Use the Given Values and Solve	Solve the Formula for One Variable	Use the Given Values and the Equation from the Previous Column then Solve
The perimeter formula for a rectangle is $\boldsymbol{p = 2(l + w)}$, where p represents the perimeter, l represents the length, and w represents the width.	Calculate l when $p = 70$ and $w = 15$.	Solve $p = 2(l + w)$ for l.	Calculate l when $p = 70$ and $w = 15$.
The area formula for a triangle is $A = \frac{1}{2}bh$, where A represents the area, b represents the length of the base, and h represents the height.	Calculate b when $A = 100$ and $h = 20$.	Solve $A = \frac{1}{2}bh$ for b.	Calculate b when $A = 100$ and $h = 20$.

6. Rearrange each formula to solve for the specified variable. Assume no variable is equal to 0.

A. Given $A = P(1 + rt)$, solve for P.

$\frac{A}{1+rt} = P$

B. Given $K = \frac{1}{2}mv^2$, solve for m.

$2 \cdot K = \cancel{2} \cdot \frac{1}{\cancel{2}} \cdot m \cdot v^2$

$\frac{2k}{v^2} = \frac{m \cdot v^2}{v^2}$

$\frac{2k}{v^2} = m$

Linear equation: pt/slope → slope/intercept
$y-y_1=m(x-x_1)$ $y=mx+b$

Solving for *y*

Linear equations written in **standard form**, $Ax + By = C$, are not as useful as linear equations written in slope-intercept form, $y = mx + b$. Solve for y in each standard equation. Then give the slope and y-intercept.

$Ax+By=C$ $y=mx+b$

	Standard Form → Slope-Intercept Form (show your work in this space)	Slope-Intercept Form	Slope	*y*-intercept
7.	$-2x + y = 5$ $+2x \quad +2x$ $y=2x+5$	$y=2x+5$	$\frac{2}{1}$	5
8.	$3x + 4y = 12$ $-3x \quad -3x$ $\frac{\cancel{4}y}{\cancel{4}}=\frac{-3x}{4}+\frac{12}{4}$	$y=\frac{-3}{4}x+3$	$-\frac{3}{4}$ or $\frac{3}{-4}$	3
9.	$x - 5y = 10$ $-x \quad -x$ $\frac{-5y}{-5}=\frac{-x}{-5}+\frac{10}{-5}$	$y=\frac{x}{5}-2$	$\frac{1}{5}$	-2
10.	$8x - 4y = 2$ $-8x \quad -8x$ $\frac{-4y}{-4}=\frac{-8x}{-4}+\frac{2}{-4}$	$y=2x-\frac{1}{2}$	$\frac{2}{1}$	$-\frac{1}{2}$
11.	$-x+\frac{1}{2}y=7$ $+x \quad +x$ $2\cdot\frac{1}{2}y=2\cdot x+2\cdot 7$	$y=2x+14$	$\frac{2}{1}$	14

12. **Looking for Patterns** Explain a way you can get the slope from standard form without rewriting the equation. How about the y-intercept?

$Ax+By=C$
$-Ax \qquad -Ax$
$\frac{By}{B}=\frac{-Ax}{B}+\frac{C}{B} \longrightarrow$

$y=\boxed{m}x+\boxed{b}$

$y=\boxed{\frac{-A}{B}}x+\boxed{\frac{C}{B}}$

Ex: $2x+3y=9$

$A=2 \quad B=3 \quad C=9$

Lesson Summary

The properties and reasoning used to solve equations apply regardless of how many variables appear in an equation or formula. Rearranging formulas to solve for a specific variable can be useful when solving applied problems.

$m=-\frac{2}{3} \quad b=\frac{9}{3}=3$

Standard form of a linear equation $Ax + By = C$

Slope-intercept form of a linear equation $y = mx + b$, m = slope, b = y-intercept

Point- Intercept form of a linear equation $y - y_1 = m(x - x_1)$, m = slope (x_1, y_1) is the point on the line

Homework Problem Set

For Problems 1–8, solve for x. Assume no variables equal 0

1. $ax + 3b = 2f$	2. $rx + h = -k$	3. $3px = 2q(r - 5)$	4. $\frac{x+b}{4} = c$
5. $\frac{x}{5} - 7 = 2q$	6. $\frac{2x}{7} - \frac{x}{7} = ab$	7. $\frac{3x}{m} - \frac{x}{m} = p$	8. $\frac{3ax+2b}{c} = 4d$

Rewrite each linear equation in slope-intercept form.

9. $x = 5y - 1$

10. $-4x + y = 17$

11. $3x + 6y = 7$

12. $4y = 8x - 14$

13. $-y = 2x$

14. $9x - 7y = 23$

15. The science teacher wrote three equations on a board that relate velocity, v, distance traveled, d, and the time to travel the distance, t, on the board.

$$v = \frac{d}{t} \qquad t = \frac{d}{v} \qquad d = vt$$

Would you need to memorize all three equations? Explain your reasoning.

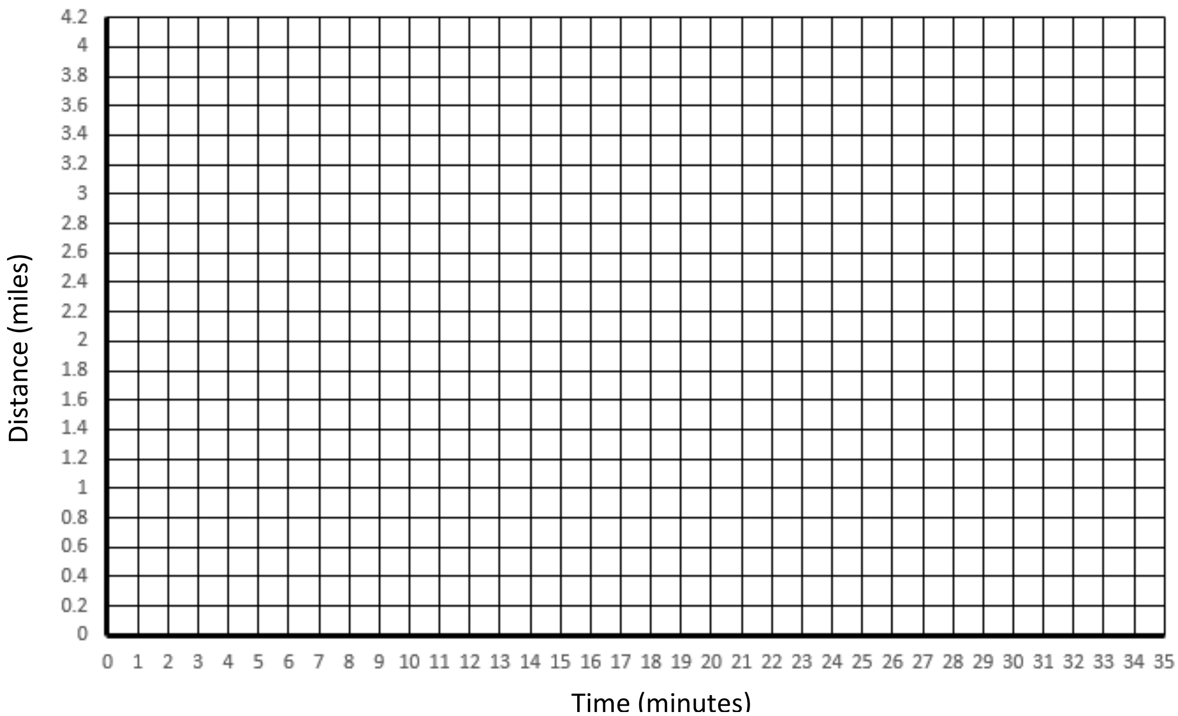
Solve for *x* in each equation. You may want to start with the equations on the right and then solve the equations on the left, using the same patterns.

Equation Containing More Than One Variable	Related Equation
16. Solve $ax + b = d - cx$ for x.	17. Solve $3x + 4 = 6 - 5x$ for x.
18. Solve for x. $$\frac{ax}{b} + \frac{cx}{d} = e$$	19. Solve for x. $$\frac{2x}{5} + \frac{x}{7} = 3$$

Spiral Review – Writing Equations and Finding Solutions

20. May and June were running at the track. May started first and ran at a steady pace of 1 mile every 11 minutes. June started 5 minutes later than May and ran at a steady pace of 1 mile every 9 minutes.

A. Sketch May and June distance-versus-time graphs on a coordinate plane at the right. Put a title on your graph, and include a legend.



B. **Challenge** - Write linear equations that represent each girl's mileage in terms of time in minutes.

C. Who was the first person to run 3 mi.?

D. Estimate when did June pass May?