$\qquad$

## Solving Math Problems

## Four steps

1. Read the question.
2. Write down the formula. $\qquad$
3. Substitute the numbers in the problem into the formula.
4. Calculate the answer.

$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Solving Math Problems Using formulas

Each of these choices expresses the same formula, but in a way that lets you solve it for $A, B$, or $C$ :

$$
\begin{aligned}
& A=B \times C \\
& B=A \div C \\
& C=A \div B
\end{aligned}
$$


$\qquad$
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## Solving Math Problems

## Using formulas

Isolate the unknown.

- The unknown is the element that you're trying to determine.
- The unknown should always sit alone on one side of the equals sign.
- All the information that you already know should be on the other side.


## Solving Math Problems

 Using formulasExample: What is the length of a property that is 9,000 square feet and 100 feet wide?

- The formula for area is $A=L \times W$.
- $L$ is the unknown, so switch the formula to $\mathrm{L}=\mathrm{A} \div \mathrm{W}$.

$$
L=9,000 \div 100
$$

$$
90=9,000 \div 100
$$

## Decimal Numbers

 Converting fraction to decimalCalculators use only decimals, not fractions.
If a problem contains a fraction, convert it to
a decimal:

- Divide the top number (the numerator) by the bottom number (the denominator).

$$
\begin{aligned}
& 1 / 4=1 \div 4=0.25 \\
& 1 / 3=1 \div 3=0.333 \\
& 5 / 8=5 \div 8=0.625
\end{aligned}
$$

## Decimal Numbers

Converting decimal to percentage
To convert a decimal to a percentage, move the decimal point two places to the right and add a percent sign.

$$
\begin{aligned}
& 0.02=2 \% \\
& 0.80=80 \% \\
& 1.23=123 \%
\end{aligned}
$$

## Decimal Numbers

## Converting percentage to decimal

To convert a percentage to a decimal, reverse the process:

- Move the decimal point two places to the left and remove the percent sign.

$$
\begin{aligned}
& 2 \%=0.02 \\
& 80 \%=0.8 \\
& 123 \%=1.23
\end{aligned}
$$

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## Area Problems <br> Formula: $\mathrm{A}=\mathrm{L} \times \mathrm{W}$

To determine
the area of a
rectangular or
square space,
use this formula:
$A=L \times W$

$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Area Problems

You might also be asked to factor other
elements into an area problem, such as:

* cost per square foot,
*rental rate, or
\& the amount of the broker's commission.


## Area Problems

## Example

An office is 27 feet wide by 40 feet long.
It rents for $\$ 2$ per square foot per month. How
much is the monthly rent?

- Part 1: Calculate area
$A=27$ feet $\times 40$ feet
$\mathrm{A}=1,080$ square feet
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
- Part 2: Calculate rent

Rent $=1,080 \times \$ 2$
Rent $=\$ 2,160$
559

## Area Problems

## Square yards

Some problems express area in square yards rather than square feet.

Remember: 1 square yard $=9$ square feet

- 1 yard is 3 feet
- 1 square yard measures 3 feet on each side $\qquad$
-3 feet $\times 3$ feet $=9$ square feet


## Area Problems

 Miles / AcresD 1 mile $=5,280$ feet
D 1 acre $=43,560$ square feet

## Area Problems

Triangle formula: $A=1 / 2 B \times H$

To determine the
area of a right
triangle, use this formula:
$A=1 / 2 B \times H$

Right triangle:
a triangle with a
$90^{\circ}$ angle


## Area of a Triangle

Visualize a rectangle, then cut it in half diagonally. What's left is a right triangle.

- If you're finding the area of a right triangle, it doesn't matter at what point in the formula you cut the rectangle in half-any of these variations will reach the same result:

$$
\begin{aligned}
& A=1 / 2 B \times H \\
& A=B \times 1 / 2 H \\
& A=(B \times H) \div 2
\end{aligned}
$$


$\qquad$
$\qquad$
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$\qquad$

Triangles

## Example, continued

A triangular lot is
140 feet long and 50
feet wide at its base.
What is the area?
Variation 1:
A $=(1 / 2 \times 50) \times 140$
$A=25 \times 140$
$A=3,500$ sq. feet

## Triangles

## Example, continued

A triangular lot is 140 feet long and 50
feet wide at its base.
What is the area?
Variation 2:
$A=50 \times(1 / 2 \times 140)$
$\mathrm{A}=50 \times 70$
$A=3,500$ sq. feet


## Triangles

Example, continued

A triangular lot is
140 feet long and 50
feet wide at its base.
What is the area?
Variation 3:
$A=(50 \times 140) \div 2$
$A=7,000 \div 2$
$A=3,500$ sq. feet


## Area Problems <br> Odd shapes

To find the area of an irregular shape:

1. Divide the figure up into squares, rectangles, and right triangles.
2. Find the area of each of the shapes that make up the figure.
3. Add the areas together.

## Odd Shapes

Example
The lot's western side
is 60 feet long. Its northern side is 100 feet long, but its southern side is 120 feet long.

To find the area of this

lot, break it into a
rectangle and a
triangle.

## Odd Shapes <br> Example, continued

Area of rectangle
A $=60 \times 100$
$A=6,000$ sq. feet


## Odd Shapes <br> Example, continued

To find the length of the
triangle's base, subtract
length of northern
boundary from length of
southern boundary.
$120-100=20$ feet


Area of triangle:
$A=(1 / 2 \times 20) \times 60$
A $=600$ sq. feet

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## Odd Shapes

Avoid counting same section twice

A common mistake when working with odd shapes is to calculate the area of part of the figure twice.

This can happen
with a figure like
this one.

## Odd Shapes

Avoid counting same section twice
Here's the wrong way
to calculate the area of
this lot.
$25 \times 50=1,250$
$40 \times 20=800$
$1,250+800=2,050$
By doing it this way, you measure the middle of the shape

twice.
$\qquad$
$\qquad$
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$\qquad$

## Odd Shapes

## Avoid counting same section twice

Avoid the problem by
breaking the shape
down like this instead.

Find height of smaller
rectangle by subtracting
height of top rectangle
(25 feet) from height of

the whole shape (40
feet).
$40-25=15$ feet

## Odd Shapes

Avoid counting same section twice

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## Odd Shapes

Avoid counting same section twice

Now calculate
the area of each
rectangle and add
them together:
$40 \times 20=800$ sq. ft.
$30 \times 25=750$ sq. ft.
$800+750=1,550$ sq. ft.


## Odd Shapes <br> Narrative problems

Some area problems are expressed only in narrative form, without a visual.

In that case, draw the shape yourself and then break the shape down into rectangles and triangles.

## x

## Odd Shapes

Example

A lot's boundary begins at a certain point and runs due south for 319 feet, then east for 426 feet, then north for 47 feet, and then back to the point of beginning.

To solve this problem, first draw the shape.

## Odd Shapes

Example
A lot's boundary begins at a certain point and runs due south for 319 feet, then east for 426 feet, then north for 47 feet, and then back to the point of beginning.


## Odd Shapes

Example, continued

Break it down into a rectangle and a triangle as shown.

Subtract 47 from 319
to find the height of the
triangular portion.
$319-47=272$ feet

$\qquad$

## Odd Shapes <br> Example, continued

Calculate the area of
the rectangle.
$426 \times 47=$
20,022 sq. ft.

 $\qquad$

## Odd Shapes

Example, continued

Calculate the area of the triangle.
$(1 / 2 \times 426) \times 272=$
57,936 sq. feet


## Odd Shapes

Example, continued
Add together the area of the rectangle and the triangle to find the lot's total square footage.
$20,022+57,936=$
77,958 sq. feet


## Volume Problems

Area: A measurement of two-dimensional space.

Volume: A measurement of three-

dimensional space.

- Width, length, and height
- Cubic feet instead of square feet

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## Volume Problems

Formula: V = L $\times \mathrm{W} \times \mathrm{H}$

To calculate volume, use this formula: $V=L \times W \times H$

Volume $=$ Length $\times$ Width $\times$ Height


## Volume Problems

Cubic yards

If you see a problem that asks for cubic yards, remember that there are 27 cubic feet in a cubic yard:

3 feet $\times 3$ feet $\times 3$ feet $=27$ cubic feet

## Volume Problems

## Example

A trailer is 40 feet long, 9 feet wide, and
7 feet high. How many cubic yards does it contain?
$40 \times 9 \times 7=2,520$ cubic feet
$2,520 \div 27=93.33$ cubic yards

## Summary Area and Volume

D Area of a square or rectangle: $A=L \times W$
D Area of a right triangle: $A=1 / 2 B \times H$
D Divide odd shapes into squares, rectangles, and triangles
D Volume: V $=\mathrm{L} \times \mathrm{W} \times \mathrm{H}$
D Square feet, square yards, cubic feet, cubic yards, miles, acres

## Percentage Problems

Many math problems ask you to find a certain percentage of another number.

This means that you will need to multiply $\qquad$
the percentage by that other number.


## Percentage Problems

## Working with percentages

Percentage problems usually require you to change percentages into decimals and/or decimals into percentages.

Example: What is $85 \%$ of $\$ 150,000$ ?
$.85 \times \$ 150,000=\$ 127,500$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Percentage Problems

## Example

One common example of a percentage problem is calculating a commission.

D Example: A home sells for $\$ 300,000$. The listing broker is paid a $6 \%$ commission on the sales price. The salesperson is entitled to $60 \%$ of that commission. How much is the salesperson's share?
$\$ 300,000 \times .06=\$ 18,000$
$\$ 18,000 \times .60=\$ 10,800$

## Percentage Problems

Formula: $\mathrm{W} \times \%=\mathrm{P}$

Basic formula for solving percentage problems:

Whole $\times$ Percentage $=$ Part
$W \times \%=P$ $\qquad$

$\qquad$
$\qquad$
$\qquad$

## Percentage Problems

Formula: $\mathrm{W} \times \%=\mathrm{P}$

The "whole" is the larger figure, such as the property's sale price. $\qquad$
The "part" is the smaller figure, such as the commission owed.

Depending on the problem, the "percentage" may be referred to as the "rate."

- Examples: a 7\% commission rate, a 5\% interest rate, a 10\% rate of return


## Percentage Problems

 Interest and profit problemsNote that you'll also use the percentage formula when you're asked to calculate interest or profit.

Example: A lender makes an interest-only loan of $\$ 140,000$. The interest rate is $6.5 \%$. How much is the annual interest?

$$
\begin{gathered}
\mathrm{W} \times \%=\mathrm{P} \\
\$ 140,000 \times .065=\$ 9,100
\end{gathered}
$$

## Percentage Problems

Interest and profit problems

Example: An investor makes an \$85,000 investment. She receives a 12\% annual return on her investment. What is the amount of her profit?
$\mathrm{W} \times \%=\mathrm{P}$
$\$ 85,000 \times .12=\$ 10,200$

## Percentage Problems Isolating the unknown

If you need to determine the percentage (the rate) or the amount of the whole, rearrange the formula to isolate the unknown on one side of the equals sign. $\qquad$
$A=B \times C \quad P=W \times \%$
$A \div B=C \quad P \div W=\%$
$A \div C=B \quad P \div \%=W$

## Percentage Problems

Finding the percent

Example: An investor makes an \$85,000 investiment and receives al $\$ 10,200$ return. $\qquad$

What is the rate of return?

$$
P \div W=\%
$$

$\qquad$
$\$ 10,200 \div \$ 85,000=.12($ or $12 \%)$

## Percentage Problems

 Finding the wholeExample: An investor receives a \$10,200 return on her investment. This is a 12\% return on her investment. How much did she invest? $\qquad$
$\qquad$ $\$ 10,200 \div .12=\$ 85,000$
$\qquad$

## Percentage Problems Multiply or divide?

Knowing when to divide or to multiply can be the hardest part of solving percentage problems. Rule of thumb:

- If the missing element is the part (the smaller number), it's a multiplication problem. $\qquad$
- If the missing element is either the whole (the larger number) or the percentage, it's a division problem.


## Multiply or Divide?

 Finding the partYou know the whole (the sale price) and the rate. The part (the commission) is the missing element, so it's a multiplication problem.
$\mathrm{W} \times \mathrm{\%}=\mathrm{P}$
$\$ 300,000 \times .06=\$ 18,000$


## Multiply or Divide?

Finding the part
Example: A home sells for $\$ 300,000$. The listing broker is paid a 6\% commission on the sales price. How much is the broker's commission?


## Multiply or Divide?

Finding the percentage or rate
Example: A lender makes an interest-only loan of $\$ \mathbf{1 4 0 , 0 0 0}$. The annual interest is $\$ 9,100$. What is the interest rate?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Multiply or Divide?

Finding the percentage or rate
You know the part (the interest) and the whole (the loan amount). The percentage (the interest rate) is the missing element, so this is a division problem.

$$
P \div W=\%
$$

$\$ 9,100 \div \$ 140,000=.07=7 \%$

## Multiply or Divide?

Finding the whole

You know the part (the interest) and the percentage (rate of return). The missing element is the whole, so this is a division problem.

$$
P \div \%=W
$$

$\$ 10,200 \div \% 12=\$ 85,000$

## Multiply or Divide? <br> Finding the whole

Example: An investor receives a \$10,200 return on her investment. This is a 12\% return on her investment. How much did she invest?


## Summary

## Percentage Problems

D Percentage formula:
Whole $\times$ Percentage (Rate) $=$ Part
$W \times \%=P$
$\mathrm{P} \div \mathrm{W}=\%$
$\mathrm{P} \div \%=W$
D Types of percentage problems: commission problems, interest problems, and profit problems.

## Loan Problems

 InterestYou've learned how to solve interest problems where the interest is given as an annual figure.
Now let's look at problems where interest is given in semiannual, quarterly, or monthly installments.

- In each case, the first step is to convert the interest into an annual figure.


## Loan Problems Semiannual interest

Example: A real estate loan calls for semiannual interest-only payments of $\$ 3,250$. The interest rate is $9 \%$. What is the loan amount?

## Loan Problems

Semiannual interest
Semiannual: two payments per year.
$\$ 3,250 \times 2=\$ 6,500$ annual interest

You know the part (the interest) and the
rate.

You need to find the whole (the loan amount). $\qquad$ $\mathrm{P} \div \%=W$
$\$ 6,500 \div .09=\$ 72,222.22$ $\qquad$

## Loan Problems

Quarterly interest
Example: A real estate loan calls for quarterly interest-only payments of $\$ 2,371,88$. The loan balance is $\$ 115,000$. What is the interest rate? $\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Loan Problems <br> Quarterly interest

Quarterly: 4 payments per year $\qquad$ $\$ 2,371.88 \times 4=\$ 9,487.52$ (annual interest)
You know the part (the interest) and the whole (the loan amount). You need to find the rate.

$$
P \div W=\%
$$

$\$ 9,487,52 \div \$ 115,000=.0825$ or $8,25 \%$

$\qquad$
$\qquad$
$\qquad$
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$\qquad$

## Loan Problems <br> Monthly interest

Monthly: 12 payments per year
$\$ 517.50 \times 12=\$ 6,210$ (annual interest)
You know the part (the interest) and the
$\qquad$
$\qquad$
$\qquad$ whole(the loan amount). You need to find the rate.
$P \div W=\%$
$\$ 6,210 \div \$ 92,000=.0675$ or $6.75 \%$

## Loan Problems <br> Amortization

Some problems will tell you the interest $\qquad$
portion of a monthly payment and ask you to
determine the loan's current principal balance.

Solve these in the same way as the problems just discussed.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Loan Problems

 AmortizationExample: The interest portion of a loan's monthly payment is $\mathbf{\$ 2 5 6 . 6 7}$. The interest rate is $7 \%$. What is the loan balance prior to the fifth payment? $\qquad$


## Loan Problems <br> Amortization

$\$ 256.67 \times 12=\$ 3,080.04$ (annual interest)
You know the part (the interest) and the rate, and you need to find the whole (the loan balance).

$$
\begin{gathered}
P \div \%=W \\
\$ 3,080 \div .07=\$ 44,000
\end{gathered}
$$

## Loan Problems <br> Amortization

Some problems may
tell you the monthly
principal and interest payment (instead
of just the interest
portion of the monthly payment).

These require several additional steps.

## Loan Problems

## Amortization

Example: The balance of a loan is $\$ 96,000$. The interest rate is $8 \%$. The monthly principal and interest payment for a loan is $\$ 704.41$.

How much will this payment reduce the loan balance? $\qquad$
$\qquad$

## Loan Problems

 AmortizationLoan balance: \$96,000 Monthly P\&l: \$704.41
Interest rate: 8\%

D Step 1: Calculate the annual interest. $\qquad$

$$
W \times \%=P
$$

$\$ 96,000 \times .08=\$ 7,680$ (annual interest)

D Step 2: Calculate the monthly interest.
$\$ 7,680 \div 12=\$ 640$

## Loan Problems

Amortization
$\begin{array}{ll}\text { Loan balance: } \$ 96,000 & \text { Monthly P\&l: } \$ 704.41 \\ \text { Interest rate: } 8 \% & \text { Monthly interest: } \$ 640\end{array}$

- Step 3 : Subtract monthly interest from total monthly payment to determine monthly principal.

$$
\$ 704.41-\$ 640=\$ 64.41
$$

D Step 4: Subtract monthly principal from loan balance.
$\$ 96,000-\$ 64.41=\$ 95,935.59$

## Loan Problems

Amortization

You might see a question like this in which you're asked how much the second or third $\qquad$ payment will reduce the loan balance.

In that case, you would calculate the first payment's effect and then repeat the four steps again, using the new balance.

## Loan Problems

## Amortization

Step 1: $\$ 95,935.59 \times .08=\$ 7,674.85$
Step 2: $\$ 7,674.85 \div 12=\$ 639.57$
Step 3: $\$ 704.41-\$ 639.57=\$ 64.84$
Step 4: $\$ 95,935.59-\$ 64.84=\$ 95,870.75$

- The second payment would reduce the loan balance to $\$ 95,870.75$.
- To see how much the third payment would reduce the loan balance, you'd repeat the four steps yet again.


## Summary

## Loan Problems

D Use the percentage formula for loan problems. Whole $\times$ Percentage (Rate) $=$ Part

D Convert semiannual, quarterly, or monthly interest into annual interest before substituting numbers into formula.

D Amortization problems ask you to find a loan's principal balance.

## Profit or Loss Problems

Another common type of percentage problem involves a property owner's profit or loss over a period of time.

- Here the "whole" is the property's value at an earlier point (which we'll call Then).
- The "part" is the property's value at a later point (which we'll call Now). $\qquad$


## Profit or Loss Problems

"Then" and "Now" formula

The easiest way to approach these problems is by using this modification of the percentage formula:

Then $\times$ Percentage $=$ Now

Of course, this can be changed to:
Now $\div$ Percentage $=$ Then
Now $\div$ Then $=$ Percentage

## Profit or Loss Problems

Calculating a loss

Example: A seller sells her house for $\$ 220,000$, which represents a $30 \%$ loss. How much did she originally pay for the house?

## Profit or Loss Problems

## Calculating a loss

You know the Now value and the percentage of the loss.

- You need to find the Then value (the original value of the house).
- Rearrange the basic formula to isolate Then:

Now $\div$ Percentage $=$ Then

## Profit or Loss Problems

 Calculating a lossNow $\div$ Percentage $=$ Then
\$220,000
D The key to solving this problem is choosing the correct percentage to put into the formula. $\qquad$

- Here the correct percentage is $70 \%$, not $30 \%$.
- The house didn't sell for $30 \%$ of its original value. It sold for $30 \%$ less than its original value.

$$
100 \%-30 \%=70 \%
$$

## Profit or Loss Problems Calculating a loss

When dealing with a loss, you can determine the rate using this formula:

```
Percentage
```

It's the percentage received that must be used in the formula.

## Profit or Loss Problems

 Calculating a gainTo calculate a gain in value, add the percentage $\qquad$ gained to $100 \%$ find the percentage received:

## $100 \%+$ Percentage Gained =

$\qquad$
Percentage Received
Returning to the example, if the sale had resulted in a 30\% profit instead of a 30\% loss, that would mean the house sold for $130 \%$ of
$\qquad$ what the seller originally paid for it:

100\%

## Profit or Loss Problems Calculating al gain

Example: A seller sells her house for $\$ 220,000$ which represents a 30\% gain. How much did $\qquad$ she originally pay for the house?

$$
\$ 220,000 \div 1.30=\$ 169,231
$$

$\qquad$
Now $\div$ Percentage Received $=$ Then
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Profit or Loss Problems

 Calculating a gainNote that if a seller sells a house for $130 \%$ of what she paid for it, she didn't make a 130\% profit.

She received 100\% of what she paid, plus 30\% She received a 309 $\qquad$
$\qquad$

## Profit or Loss Problems

Appreciation and depreciation
A profit or loss problem may also be expressed in terms of appreciation or depreciation.
$\qquad$
$\qquad$

If so, the problem is solved the same way as
an ordinary profit and loss problem.

$\qquad$
$\qquad$
$\qquad$

## Profit or Loss Problems <br> Compound depreciation

You may see problems in which you're told 9 how much a property appreciated or depreciated per year over several years.

This requires you to repeat the same calculation for each year.
$\div 5$
16


## Profit or Loss Problems

Compound depreciation

Example: A property is currently worth $\$ 220,000$. It has depreciated four and a half percent per year for the past five years. What was the property worth five years ago?
$\qquad$
$\qquad$
$\qquad$
$\qquad$
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$\qquad$
$\qquad$

## Profit or Loss Problems

 Compound depreciationThe house is losing value, so first subtract the $\qquad$ rate of loss from $100 \%$.

$$
100 \%-4.5 \%=95.5 \%, \text { or } .955
$$

You know the Now value and the rate.
The missing element is the Then value:
Now $\div$ Percentage $=$ Then
$\$ 220,000 \div .955=\$ 230,366.49$
The house was worth $\$ 230,366$ one year ago ${ }_{568}$

## Profit or Loss Problems <br> Compound depreciation

Now repeat the calculation four more times, to $\qquad$ determine how much the house was worth five years ago:
$\$ 230,366 \div .955=\$ 241,221$ (value 2 years ago)
$\$ 241,221 \div .955=\$ 252,587$ (value 3 years ago)
$\$ 252,587 \div .955=\$ 264,489$ (value 4 years ago)
$\$ 264,489 \div .955=\$ 276,952$ (value 5 years ago)

## Profit or Loss Problems Compound appreciation

If you're told that a property gained value at a particular rate over several years, you'll use the same process.

The difference is that you'll need to add
the rate of change to $100 \%$, instead of subtracting it from 100\%.

## Profit or Loss Problems

Compound appreciation

Example: A property is currently worth
$\$ 380,000$. It has appreciated in value $4 \%$ per
$\qquad$
year for the last four years. What was it worth
four years ago? $\qquad$

$\qquad$
$\qquad$

## Profit or Loss Problems Compound appreciation

Add the rate of appreciation to $100 \%$. $\qquad$ $100 \%+4 \%=104 \%$, or 1.04
You know the Now value and the rate of change, so use the formula Now $\div$ Percentage $=$ Then.
$\$ 380,000 \div 1.04=\$ 365,385$ (value 1 year ago)
$\$ 365,385 \div 1.04=\$ 351,332$ (value 2 years ago) $\qquad$
$\$ 351,332 \div 1.04=\$ 337,819$ (value 3 years ago)
$\$ 337,819 \div 1.04=\$ 324,826$ (value 4 years ago)
$\qquad$

## Summary

## Profit or Loss Problems

D Then $\times$ Percentage $=$ Now
D To find the percentage received:

- If there's been a loss in value, subtract the rate of change from 100\%.
- If there's been a gain (a profit), add the rate of change to $100 \%$.
D Compound appreciation and depreciation: repeat the profit or loss calculation as needed.
$\qquad$


## Capitalization Problems

Capitalization: The process used to convert a property's income into the property's value.

- In the appraisal of income property, the
property's value depends on its income.
- The value is the price an investor would be willing to pay for the property. $\qquad$
- The property's annual net income is the return on the investment.


## Capitalization Problems

Formula: $\mathrm{V} \times \mathrm{\%}=\mathrm{I}$
Capitalization problems are another type of percentage problem.

$$
\text { Whole } \times \text { Percentage }=\text { Part }
$$

Here the "part" is the property's income, and the "whole" is the property's value:

Value $\times$ Capitalization Rate $=$ Income
or
Income $\div$ Rate = Value or
Income $\div$ Value $=$ Rate

## Capitalization Problems

Capitalization rate
The capitalization rate represents the rate of return an investor would likely want on this investment.

An investor who wants a higher rate of return would not be willing to pay as much for the property as an investor who's willing to accept a lower rate of return.
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$\qquad$
$\qquad$

## Capitalization Problems

Calculating value

Example: A property generates an annual net income of $\$ 48,000$. An investor wants a $12 \%$ rate of return on his investment. How much could he pay for the property and realize his desired rate of return?

## Capitalization Problems

Calculating value
Income $\div$ Rate $=$ Value $\$ 48,000 \div .12=\$ 400,000$

The investor could pay $\$ 400,000$ for this $\qquad$ property and realize a $12 \%$ return.

## Capitalization Problems

Calculating value

Example: An investment property has a net income of \$40,375. An investor wants a $10.5 \%$ rate of return. What would the value of the property be for her?

## Capitalization Problems

Calculating value

Income $\div$ Rate = Value
$\$ 40,375 \div .105=\$ 384,524$
She could pay $\$ 384,524$ for this property and realize a 10.5\% return.


## Capitalization Problems

Finding the cap rate

Example: An investment property is valued at \$425,000 and its net income is $\$ 40,375$. What is the capitalization rate?


Capitalization Problems
Finding the cap rate
Income $\div$ Value $=$ Rate
$\$ 40,375 \div \$ 425,000=.095$, or $9.5 \%$


## Capitalization Problems

Changing the cap rate
The capitalization rate is up to the investor. It depends on how much risk he or she is willing to absorb.

- One investor might be satisfied with a 9.5\% cap rate.
- Another, more aggressive, investor might want a $10.5 \%$ return on the same property.

Some problems ask how a property's value will change if a different cap rate is applied. 569

## Capitalization Problems

Changing the cap rate

Example: Using a capitalization rate of 10\%, a property is valued at $\$ 150,000$. What would its value be using an $11 \%$ capitalization rate?


## Capitalization Problems

Changing the cap rate
Step 1: Calculate the property's net income. You
know the value and the rate, so use the formula
Value $\times$ Rate $=$ Income.

$$
\$ 450,000 \times .10=\$ 45,000
$$

Step 2: Calculate the value at the higher cap rate.
Income $\div$ Rate $=$ Value
$\$ 45,000 \div .11=\$ 409,091$
The property would be worth $\$ 40,909$ less at the higher cap rate.

## Capitalization Problems

Changing the cap rate
Example: A property with a net income of $\$ 16,625$ is valued at $\$ 190,000$. If its cap rate is increased by $1 \%$, what would its new value be?


## Capitalization Problems

Changing the cap rate
Step 1: Find the current capitalization rate.
Income $\div$ Value $=$ Rate
\$16,625 $\div$ \$190,000 $=.0875$
Step 2: Increase the cap rate by $1 \%$.
$8.75 \%+1 \%=9.75 \%$, or .0975
Step 3: Calculate the new value.
Income $\div$ Rate $=$ Value.
$\$ 16,625 \div .0975=\$ 170,513$

## Capitalization Problems Calculating net income

In some problems, you'll be given the property's annual gross income and a list of the operating expenses instead of the annual net income.

Before you can use the capitalization formula, you'll have to subtract the expenses from the gross income to get the net income.

## Capitalization Problems

Calculating net income

Example: A six-unit apartment building rents three units for $\$ 650$ a month and three units for $\$ 550$ a month. The annual operating expenses are $\$ 4,800$ for utilities, $\$ 8,200$ for property taxes, $\$ 1,710$ for insurance, $\$ 5,360$ for maintenance, and $\$ 2,600$ for management fees. If the capitalization rate is $83 / 4 \%$, what is the property's value?

## Capitalization Problems

Calculating net income

Step 1: Calculate the gross annual income.

$$
\begin{aligned}
& \$ 550 \times 3 \times 12=\$ 19,800 \\
& \$ 650 \times 3 \times 12=\$ 23,400
\end{aligned}
$$

$\$ 19,800+\$ 23,400=\$ 43,200$ (gross income)

## Capitalization Problems

Calculating net income
Step 2: Subtract expenses from gross income.
$\qquad$
$\qquad$ \$43,200
-\$4,800
-\$8,200
-\$1,710
-\$5,360
-\$2,600

\$20,530 (net income)

Capitalization Problems
Calculating net income

Step 3: Calculate the value. You know the net income and the rate, so use the formula Income $\div$ Rate $=$ Value .
$\$ 20,530 \div .0875=\$ 234,629$


## Capitalization Problems

Calculating net income: OER
Some problems give you the property's operating expense ratio (OER) rather than a list of the operating expenses.

- OER is the percentage of the gross income that goes to pay operating expenses.
- Multiply the gross income by the OER to determine the annual operating expenses. Then subtract the expenses from the gross income to determine the net income.


## Capitalization Problems

Calculating net income: OER

Example: A store grosses \$758,000 annually. It has an operating expense ratio of $87 \%$. With a capitalization rate of $91 / 4 \%$, what is its value?


## Capitalization Problems

## Calculating net income: OER

Step 1: Multiply the gross income by the OER $\$ 758,000 \times .87=\$ 659,460$ (operating expenses)
Step 2: Subtract the expenses from gross income. \$758,000 - \$659,460 = \$98,540 (net income) $\qquad$
Step 3: Use the capitalization formula to find the property's value. $\qquad$
Income $\div$ Rate = Value
$\$ 98,540 \div .0925=\$ 1,065,297$ $\qquad$

$\qquad$
$\qquad$

Tax Assessment Problems

Tax assessment problems are another type of percentage problem.

Whole $\times \%=$ Part
Assessed Value $\times$ Tax Rate $=$ Tax


Tax Assessment Problems
Assessment ratio
Some problems simply give you the assessed value.

Others give you the market value and the assessment ratio, and you have to calculate the assessed value.

Example: The property's market value is $\$ 100,000$ and the assessment ratio is $80 \%$. $\$ 100,000 \times .80=\$ 80,000$
The assessed value is $\$ 80,000$.

## Tax Assessment Problems

Assessment ratio

Example: The property's market value is $\$ 200,000$. It is subject to a $25 \%$ assessment ratio and an annual tax rate of $2.5 \%$. How much is the annual tax the property owner must pay?


## Tax Assessment Problems

Assessment ratio

Step 1: Calculate the assessed value by multiplying the market value by the ratio. $\$ 200,000 \times .25=\$ 50,000$ (assessed value)

Step 2: Calculate the tax.

$$
\text { Assessed Value } \times \text { Tax Rate }=\text { Tax }
$$

$$
\$ 50,000 \times .025=\$ 1,250(\operatorname{tax})
$$

The property owner is required to pay $\$ 1,250$.

## Tax Assessment Problems Tax rate per $\$ 100$ or $\$ 1,000$

In some questions, the tax rate will not be expressed as a percentage, but as a dollar amount per hundred dollars or per thousand dollars of assessed value.

Divide the value by 100 or 1,000 to find the number of \$100 or \$1,000 increments. Then multiply that number by the tax rate.

## Tax Assessment Problems

Tax rate per \$100

Example: A property is assessed at $\$ 125,000$.
The tax rate is $\$ 2.10$ per hundred dollars of
assessed value. What is the annual tax?


## Tax Assessment Problems

Tax rate per \$100

Step 1: Determine how many hundred-dollar
increments are in the assessed value.
$\$ 125,000 \div 100=1,250$ (\$100 increments)
Step 2: Multiply the number of increments by
the tax rate.
$1,250 \times \$ 2.10=\$ 2,625($ annual tax)

## Tax Assessment Problems

 Tax rate per \$1,000Example: A property is assessed at \$396,000.
The tax rate is $\$ 14.25$ per thousand dollars of $\qquad$
assessed value. What is the annual tax?
$\qquad$

$\qquad$
$\qquad$

## Tax Assessment Problems

## Tax rate per \$1,000

Step 1: Determine how many thousand-dollar increments are in the assessed value. $\qquad$
$\$ 396,000 \div 1,000=396$ (\$1,000 increments)
Step 2: Multiply the number of increments by the tax rate.
$396 \times \$ 14.25=\$ 5,643($ annual $\operatorname{tax})$

## Tax Assessment Problems Tax rate in mills

One other way in which a tax rate may be $\qquad$ expressed is in terms of mills per dollar of assessed value. $\qquad$

- A mill is one-tenth of a cent, or one-thousandth of a dollar.
- To convert mills to a percentage rate, multiply by . 001
$\qquad$
$\qquad$
$\qquad$


## Tax Assessment Problems

Tax rate in mills

Example: A property is assessed at $\$ 290,000$
and the tax rate is 23 mills per dollar of $\qquad$
assessed value. What is the annual tax?
$\qquad$

$\qquad$
$\qquad$
$\qquad$

## Tax Assessment Problems

Tax rate in mills

Step 1: Convert mills to a percentage rate. 23 mills/dollar x . 001 = . 023 or 2.3\%

Step 2: Multiply the assessed value by the tax rate to determine the tax. $\$ 290,000 \times .023=\$ 6,670$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Summary

Tax Assessment Problems
Assessed Value $\times$ Tax Rate $=$ Tax $\qquad$

- To find assessed value, you may have to multiply market value by the assessment ratio. $\qquad$
- Tax rate may be given as a percentage, as a dollar amount per $\$ 100$ or $\$ 1,000$ of value, or in mills. $\qquad$
D Multiply mills by .001 to get a percentage rate.


## Seller's Net Problems

This type of problem asks how much a seller will have to sell the property for to get a specified net
amount from the sale.

In the basic version of this type of problem, you're told the seller's desired net and the

costs of sale.

## Seller's Net Problems

## Basic version

$\qquad$
Start with the desired net proceeds, then: $\qquad$

1. add the costs of the sale, except for the commission; $\qquad$
2. subtract the commission rate from $100 \%$;
3. divide the results of Step 1 by the results $\qquad$ of Step 2. $\qquad$
$\qquad$

## Seller's Net Problems

## Basic version

Example: A seller wants to net \$220,000 from the sale of his property. He will pay $\$ 1,650$ in attorney's fees, $\$ 700$ for the escrow fee, $\$ 550$ for repairs, and a $6 \%$ brokerage commission. How much will he have to sell the property for?

## Seller's Net Problems

## Basic version

1. Add the costs of the sale to the desired net:
$\$ 220,000+\$ 1,650+\$ 700+\$ 550=\$ 222,900$
2. Subtract the commission rate from $100 \%$ :

$$
100 \%-6 \%=94 \%, \text { or } .94
$$

3. Calculate the necessary sales price:
$\$ 222,900 \div .94=\$ 237,127.66$
The sales price will have to be at least
$\$ 237,127.66$ for the seller to get his desired net.

## Seller's Net Problems

## Variations

Variations on this type of problem:
D Variation 1: You're told the original purchase price and the percentage of profit the seller wants from the sale.

- This requires an additional step, calculating the seller's desired net.


## Seller's Net Problems <br> Variation 1

Example: A seller bought land two years ago for $\$ 72,000$ and wants to sell it for a $25 \%$ profit. She'll have to pay a $7 \%$ brokerage fee, $\$ 250$ for a survey, and $\$ 2,100$ in other closing costs. For what price will she have to sell the property?

## Seller's Net Problems

## Variation 1

1. Use the "Then and Now" formula to calculate the desired net.

Then $\times$ Rate $=$ Now
$\$ 72,000 \times 1.25=\$ 90,000$ desired net
Or calculate the profit and add it to the original value to get the desired net:
$\$ 72,000 \times 25 \%=\$ 18,000+\$ 72,000=\$ 90,000$

## Seller's Net Problems

## Variation 1

$\qquad$
2. Next, add the costs of sale, except for the commission.
$\$ 90,000+\$ 250+\$ 2,100=\$ 92,350$
3. Subtract the commission rate from $100 \%$. $100 \%-7 \%=93 \%$, or .93
4. Finally, calculate the necessary sales price. $\qquad$ $\$ 92,350 \div .93=\$ 99,301$

## Seller's Net Problems

## Variation 2

In another variation on this type of problem, you're asked to factor in the seller's mortgage balance.

- This is more realistic, since most sellers have a loan to pay off.
- Just add the loan balance as one of the closing costs.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$


## Seller's Net Problems

## Variation 2

Example: A seller wants to net $\$ 24,000$ from selling his home. He will have to pay \$3,300 in closing costs, $\$ 1,600$ in discount points, \$1,475 for repairs, \$200 in attorney's fees, and a $6 \%$ commission. He will also have to pay off the mortgage balance, which is $\$ 46,050$. How much does he need to sell his home for?

## Seller's Net Problems

## Variation 2

1. Add the costs of sale and the mortgage
balance to the desired net.
\$24,000 + \$3,300 + \$1,600 +
$\$ 1,475+\$ 200+\$ 46,050=\$ 76,625$
2. Subtract the commission rate from $100 \%$. $100 \%-6 \%=94 \%$, or .94
3. Finally, calculate the necessary sales price. $\$ 76,625 \div .94=\$ 81,516$

Summary Seller's Net Problems

1. Desired Net + Costs of Sale + Loan Payoff
2. Subtract commission rate from $100 \%$
3. Divide Step 1 total by Step 2 rate. Result is how much property must sell for.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Proration Problems

Prorating an expense means dividing it proportionally, when someone is responsible for only part of it.
$\qquad$
$\qquad$

Items often prorated in real estate transactions include: $\qquad$

- property taxes
- insurance premiums
- mortgage interest



## Proration Problems

Closing date is proration date
Seller's responsibility for certain expenses
$\qquad$ includes closing date.

Buyer's responsibility for certain expenses
$\qquad$
$\qquad$ begins on the day after closing date.

$\qquad$
$\qquad$

## Proration Problems

In advance or In arrears
If seller is in arrears on a particular expense, seller will be charged (or debited) for a share of the expense at closing.

- Buyer may be credited with same amount.

If seller has paid an expense in advance,
$\qquad$
seller will be refunded a share of the overpaid amount at closing.

- Buyer may be debited for same amount.


## Proration Problems

## 3 Steps

Prorating an expense is a three-step process: $\qquad$

1. Calculate the per diem (daily) rate of the $\qquad$ expense.
2. Determine the number of days the party is responsible for.
3. Multiply per diem rate by number of days.

## Proration Problems <br> 365 days or 360 days

Proration calculations for mortgage interest should be based on a 360-day year; all other calculations should be based on a 365-day year-unless you're instructed otherwise by the question.


## Proration Problems <br> Property taxes

Remember that in some counties, the property tax year is different from the calendar year.

Also, payments are sometimes divided into installments.

$\qquad$
$\qquad$
$\qquad$

Proration Problems
Property taxes

Example: The closing date is Feb. 3 and the seller has not yet paid the annual property taxes of $\$ 2,045$. Taxes are due for the entire calendar year on July 1. At the closing, the tax proration will be a debit for the seller and a credit to the buyer. How much will the seller owe the buyer?

## Proration Problems

Property taxes
Step 1: Calculate the per diem rate.

$$
\$ 2,045 \div 365=\$ 5.60
$$

Step 2: Count the number of days.
31 (Jan) +3 (Feb) $=34$ Days
Step 3: Multiply rate by number of days.

$$
\$ 5.60 \times 34=\$ 190.40
$$

The seller will be debited $\$ 190.40$ at closing.
The buyer will be credited for the same amount.

## Proration Problems

## Insurance

Example: The sellers of a house have a oneyear prepaid hazard insurance policy with an annual premium of $\$ 1,350$. The policy has been paid for through March of next year, but the sale of their house will close on November 12 of this year. The buyer's responsibility for insuring the property begins after the day of closing. How much will be refunded to the sellers at closing?

## Proration Problems

Insurance

```
Step 1: Calculate the per diem rate.
```

$\$ 1,350 \div 365=\$ 3.70$
Step 2: Count the number of days.
18 (Nov.) +121 (Dec. - March) = 139 days
Step 3: Multiply per diem rate by number of days.
$\$ 3.70 \times 139=\$ 514.30$
Sellers will be credited $\$ 514.30$
(Buyer will not be debited for this amount, unless she is assuming seller's policy.)

## Proration Problems

Mortgage interest
For interest prorations, don't forget that mortgage interest is almost always paid:

- on a monthly basis
- in arrears (at end of the month in which it accrues)

If you aren't given the amount of annual interest, first use the loan amount and interest rate to calculate it.

- Then do the other proration steps.


## Proration Problems <br> Mortgage interest

Two types of mortgage interest usually must be prorated at closing:

- seller's final interest payment
- buyer's prepaid interest
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$


## Prorating Mortgage Interest

## Seller's final interest payment

Example: A seller is selling her home for $\$ 275,000$. She has a mortgage at $7 \%$ interest with a balance of $\$ 212,500$. The sale closes on May 14, and the seller will owe interest for the day of closing. At closing, how much will the seller's final interest payment be? $\qquad$
$\qquad$

## Prorating Mortgage Interest

Seller's final interest payment
Step 1: Calculate the annual interest. $\qquad$
$\$ 212,500 \times .07=\$ 14,875$
Step 2: Calculate the per diem rate.

$$
\$ 14,875 \div 360=\$ 41.32
$$

$\qquad$
Step 3: Count the number of days. May 1 through May 14 = 14 days
Step 4: Multiply per diem by number of days.

## Prorating Mortgage Interest Buyer's prepaid interest

Prepaid interest: At closing, the buyer is charged interest for closing date through the end of the month in which closing occurs. Also called interim interest.

- Example: Sale is closing on April 8.
- Buyer's first loan payment, due June 1, will include May interest, but not April interest.
- At closing, buyer will pay interest for April 8 through April 30.


## Prorating Mortgage Interest

## Buyer's prepaid interest

Example: A buyer purchased a house with a $\$ 350,000$ loan at $5.5 \%$ annual interest. The transaction closes Jan. 17. The buyer is responsible for the day of closing. How much prepaid interest will the buyer have to pay?

## Prorating Mortgage Interest

 Buyer's prepaid interestStep 1: Calculate the annual interest. $\qquad$ $\$ 350,000 \times .055=\$ 19,250$
Step 2: Calculate the per diem rate. $\$ 19,250 \div 360=\$ 53.47$
Step 3: Count the number of days. $\qquad$
Jan. 17 through Jan. 31 = 15 days
Step 4: Multiply per diem rate by days.
$\$ 53.47 \times 15=\$ 802.05$
Buyer will owe $\$ 802.05$ in prepaid interest at

## Summary

 Proration Problems1. Calculate per diem rate.
(360-day year for mortgage interest or
365-day year for everything else.)
2. Count number of days.
3. Multiply per diem rate by number of days.
