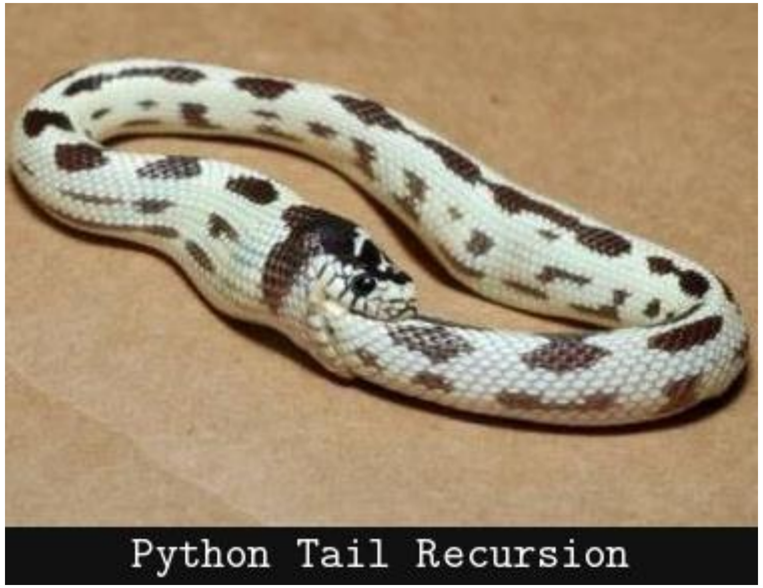
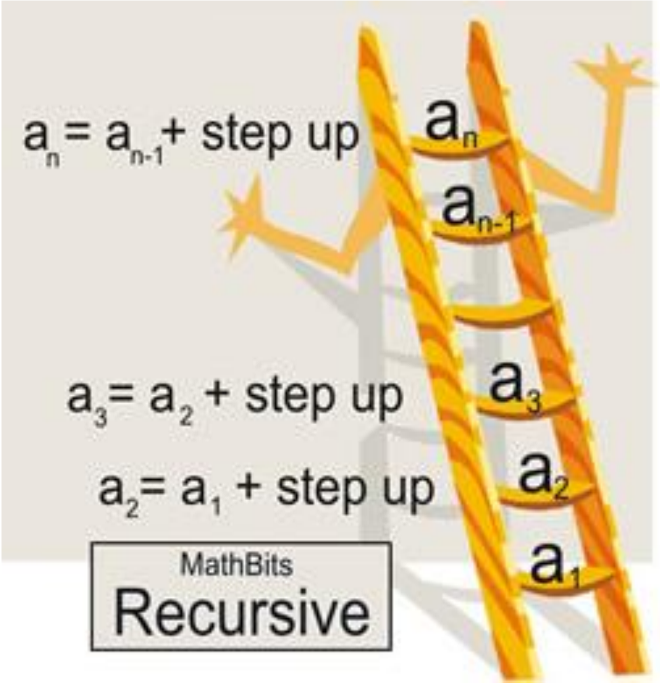
Lesson 2: Recursive Sequences

Thursday, May 25, 2017.



Learning Goals

- Create rules for generating sequences that are neither arithmetic nor geometric
- Explore patterns in sequences in which a term is related to the previous two terms
- Investigate patterns in Pascal's triangle, and use one of these patterns to **<u>expand binomials efficiently</u>**



Warm-Up Example:

Given the sequence 1, 8, 16, 26, 39, 56, 78,… determine the next three terms. Explain your reasoning.

Warm-Up Example:

Term		
1		
8		
16		
26		
39		

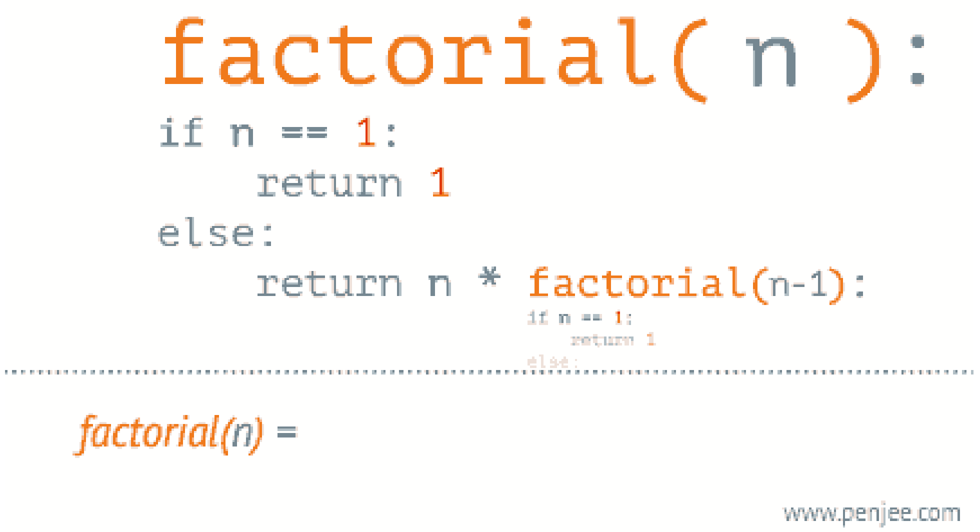
Warm-Up: Example

This example showed us that the sequence was neither arithmetic nor geometric, but there was a pattern that related the terms to each other to get the general term.

Sequences

- **Sometimes it is easier to calculate one term in a sequence using the <u>previous terms</u>.**

- Computer programmers use sequences of code to create instructions for computers. Often these sequences tell the computer to use a previous value to find the next one. This is known as a **recursive procedure.**

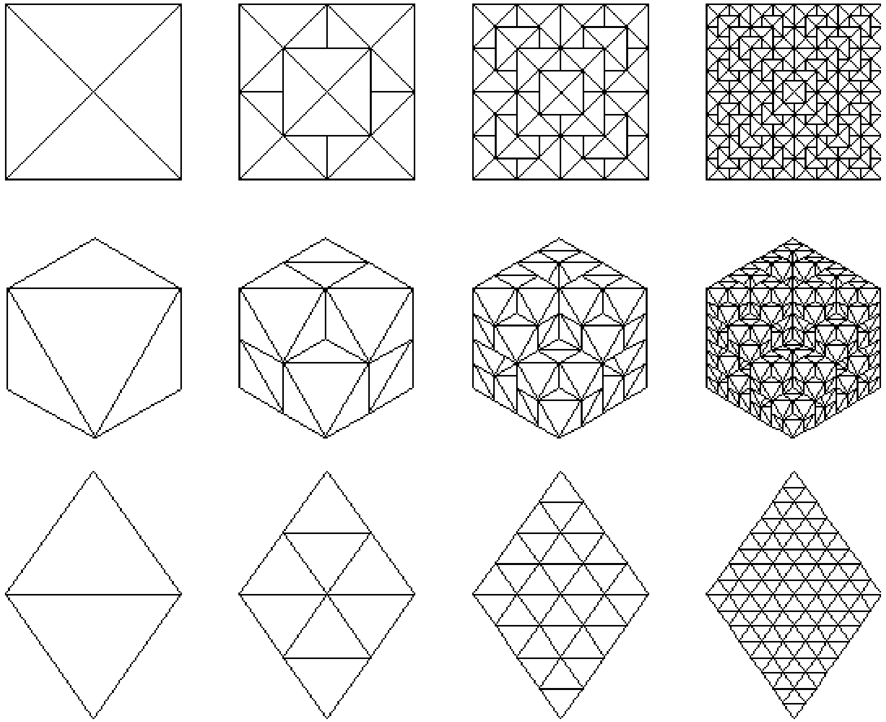


Definition: Recursive Sequence

- A sequence for which one term (or more) is given and each successive term is determined form the previous term(s).

Definition: Recursion Formula

- A recursion formula generates terms in a sequence by a formula using the previous term or terms.



Ex. 1: Model the Pattern

The first three diagrams in a pattern are shown.

Complete the table.

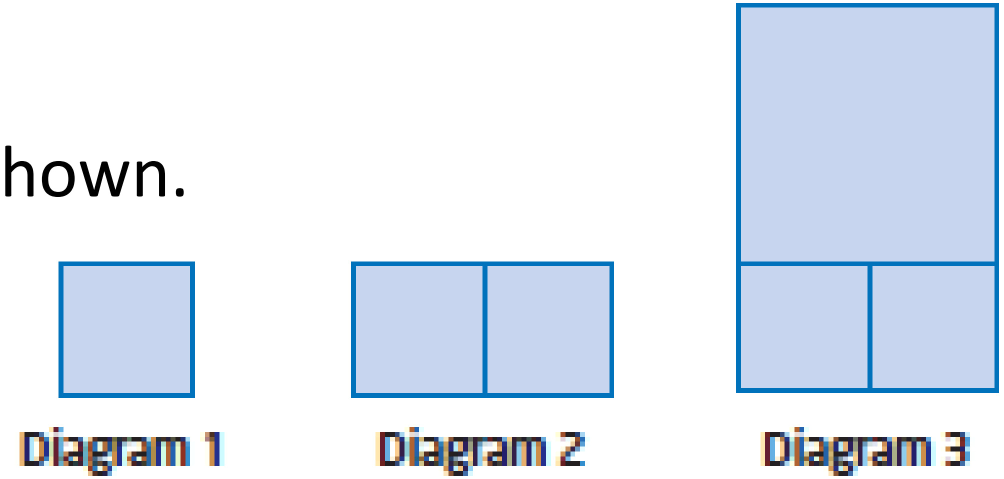


Diagram #	**1**	**2**	**3**	**4**	**5**	**6**	**…**
Side Length of New **Square** (Units)	1	1	2				

Ex. 1: Model the Pattern

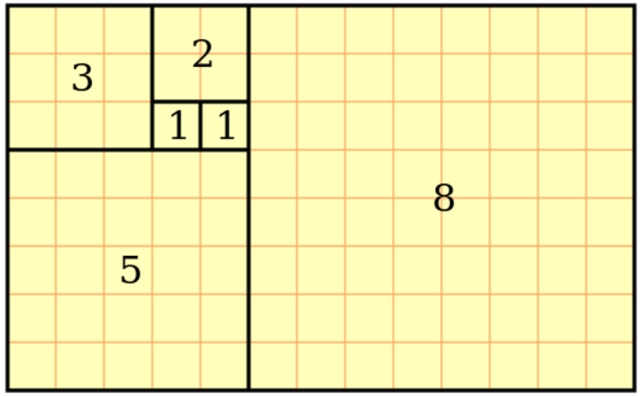


Diagram #	**1**	**2**	**3**	**4**	**5**	**6**	**…**
Side Length of New **Square** (Units)	1	1	2				

a) Write the side lengths of the squares as a sequence.

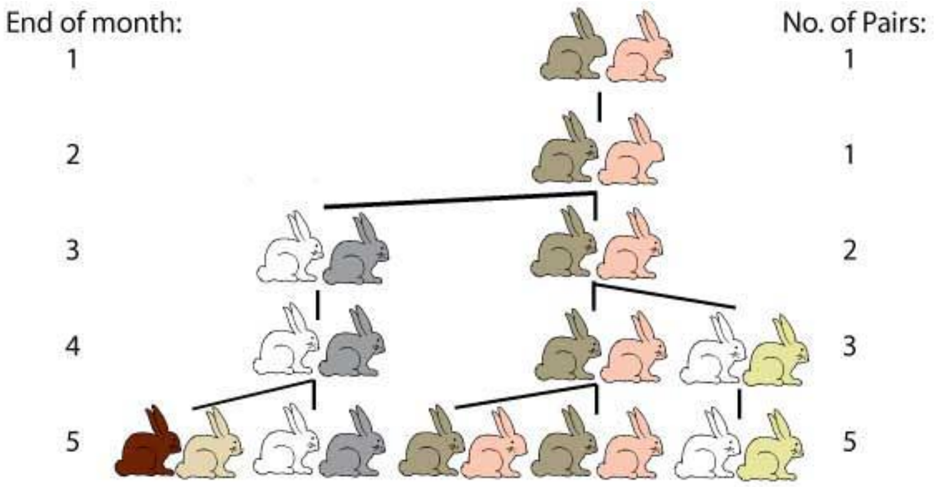
b) Determine the relationship between consecutive terms in the sequence and write a formula for the nth term, t_n, in terms of the $(n-1)$ th term, t_{n-1}, and the $(n-2)$th term, t_{n-2}.

The Fibonacci Sequence

The Fibonacci sequence is a recursive sequence because each term **beginning with the third term** is the result of adding the twc previous terms.

The **recursion formula** for this sequence can be written as

$$\boldsymbol{t_1 = 1, t_2 = 1,\ t_n = t_{n-1} + t_{n-2}}$$



Recursion Formula

A recursion formula consists of **at least two values**.

The first part(s) give the values of the first term(s) in the sequence.

(this is the value that's given…think about it…how would you calculate a recursion, which depends on previous values, without at least one of it's values?)

The last part is an equation that can be used to calculate each of the other terms from the term(s) before it.

Example 2

Write the first five terms of the sequence determined by the recursion formula

$$t_1 = 11$$

$$t_n = t_{n-1} - 4$$

Example 3

Write the first five terms of the sequence determined by the recursion formula

$$f(1) = -\frac{1}{2}, \qquad f(n) = f(n-1) + \frac{3}{2}$$

Writing a Recursive Formula – Arithmetic Sequences

To summarize the process of writing a recursive formula for an arithmetic sequence:

1. Determine if the sequence is arithmetic (*Do you add, or subtract, the same amount from one term to the next?*)
2. Find the common difference. (*The number you add or subtract.*)
3. Create a recursive formula by **stating the first term**, and **then stating the formula to be the previous term plus the common difference.**

$$t_n = a + (n-1)d, where\ n \in \mathbb{N}$$

Writing a Recursive Formula – Geometric Sequences

To summarize the process of writing a recursive formula for a geometric sequence:

1. Determine if the sequence is geometric (*Do you multiply, or divide, the same amount from one term to the next?*)
2. Find the common ratio. (*The number you multiply or divide.*)
3. Create a recursive formula by **stating the first term**, and then **stating the formula to be the common ratio times the previous term.**

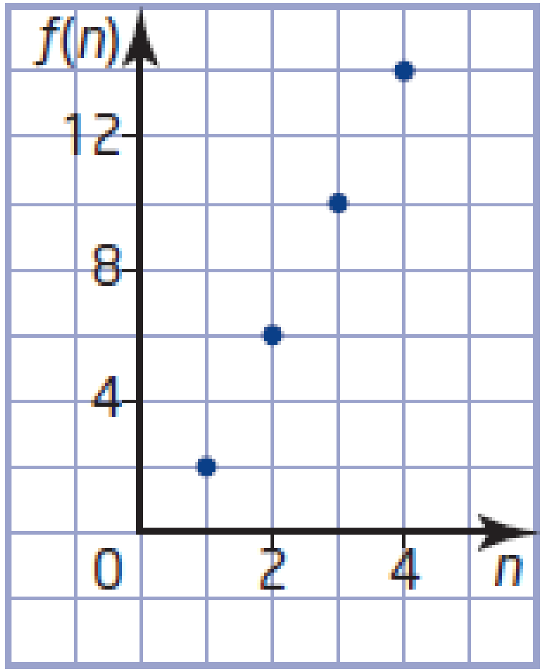
$t_n = ar^{n-1}$ *where* $r \neq 0$ *and* $n \in \mathbb{N}$

Example 4: Write a Recursion Formula

Determine a recursion formula for each sequence.

a) -3, 6, -12, 24, …

b)



c) 3, 5, 8, 12, …

Example 4: Write a Recursion Formula

a) -3, 6, -12, 24, …

$$t_1 = -3$$
$$t_2 = t_1 \times (-2)$$
$$t_3 = t_2 \times (-2)$$
$$t_4 = t_3 \times (-2)$$

$$\therefore \boxed{t_1 = -3 \ , t_n = (t_{n-1}) \times (-2)}$$

Example 5: **Medication in the Body**

A runner injures her knee in a race. Her doctor prescribes physiotherapy along with 500 mg of an anti-inflammatory medicine every 4 h for 3 days. The half-life of the anti-inflammatory medicine is approximately 4 h. This means that after 4 h, about half of the medicine is still in the body.

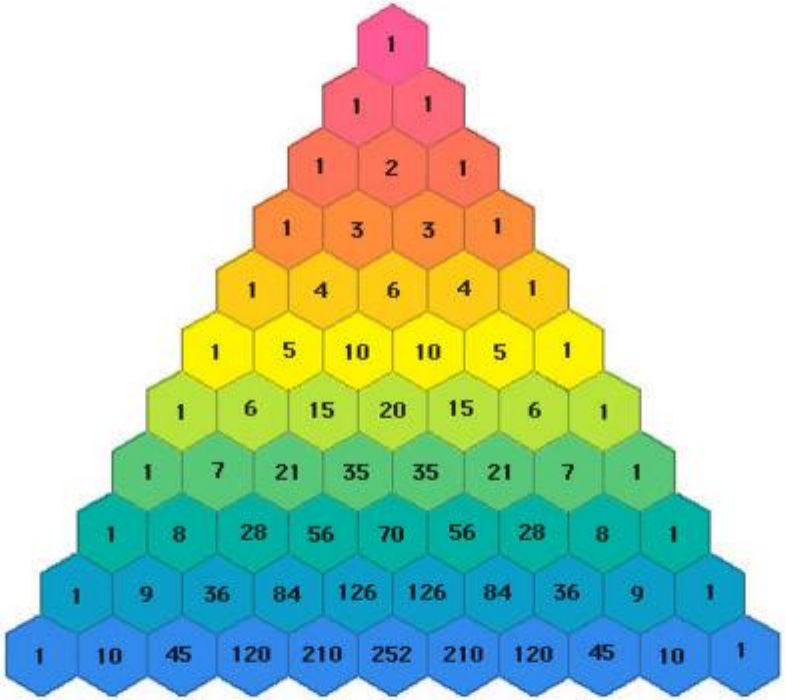
a) Make a table of values showing the amount of medicine remaining in the body after each 4-h period of time.

b) Write the amount of medicine remaining after each 4-h period as a sequence. Write a recursion formula for the sequence.

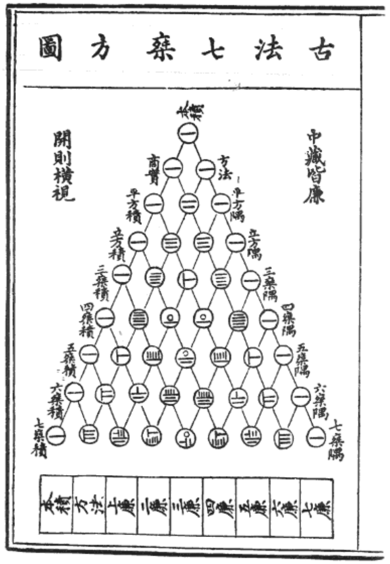
c) Graph the sequence.

d) Describe what happens to the medicine in the runner's body over time.

4-h Interval	Amount of Medicine (mg)
0	500
1	Medicine still left + New Pill =
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	

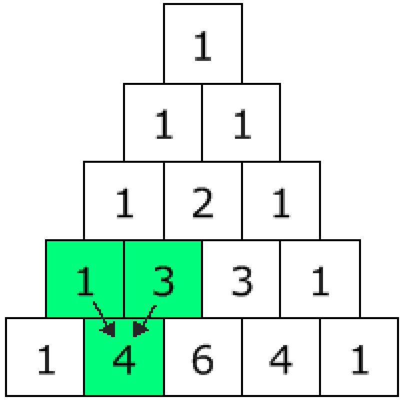


Pascal’s Triangle



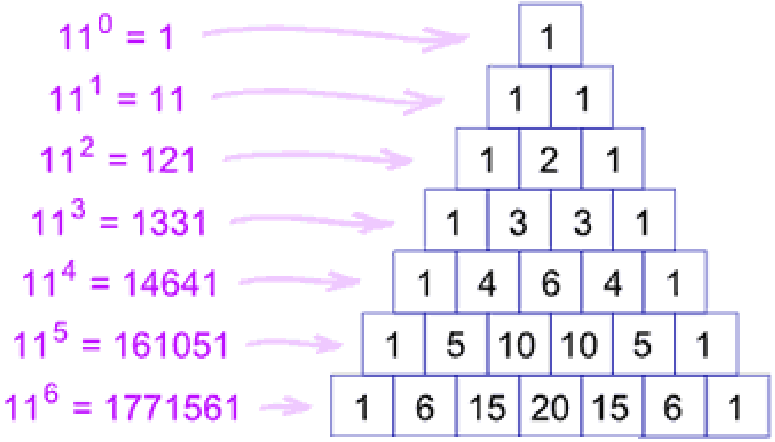
Pascal’s Triangle

The arrangement of numbers shown is called Pascal’s triangle. Each row is generated by calculating the sum of pairs of consecutive terms in the precious row.

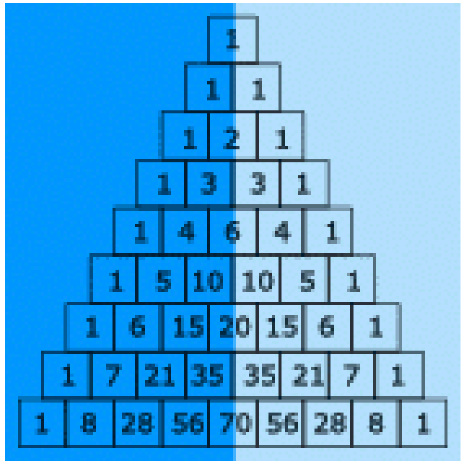


To build the triangle, start with "1" at the top, then continue placing numbers below it in a triangular pattern.

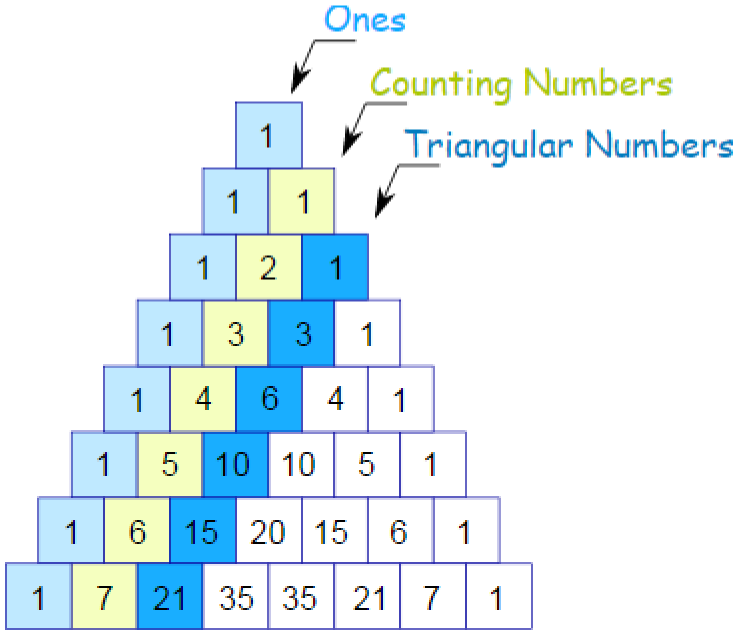
Each number is the numbers directly above it added together.



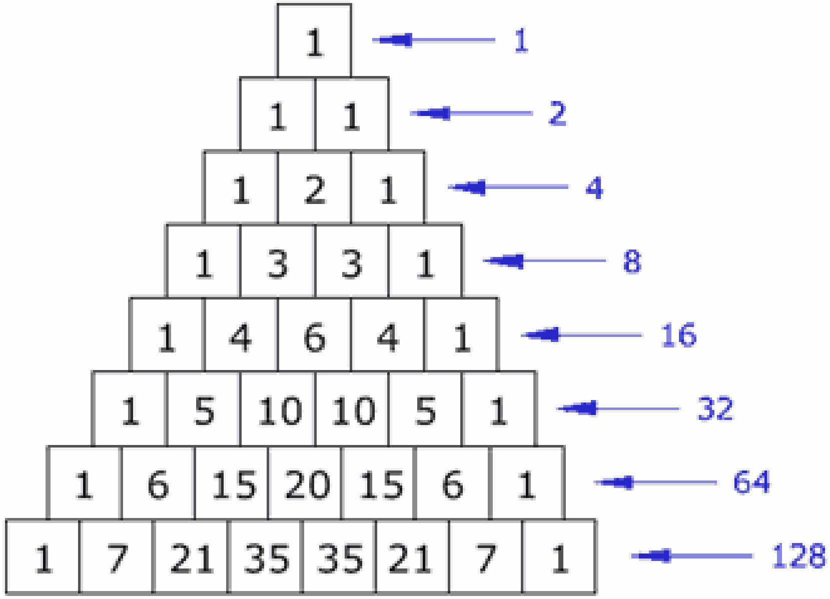
$11^0 = 1$
$11^1 = 11$
$11^2 = 121$
$11^3 = 1331$
$11^4 = 14641$
$11^5 = 161051$
$11^6 = 1771561$
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1



1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1



Ones
Counting Numbers
Triangular Numbers
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

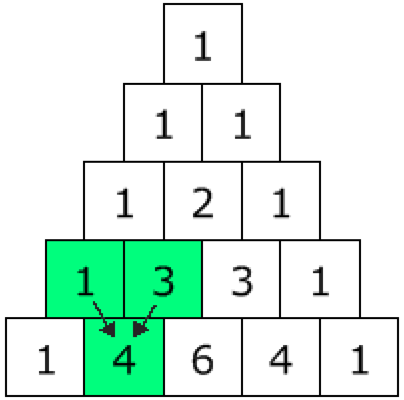


1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1
2
4
8
16
32
64
128

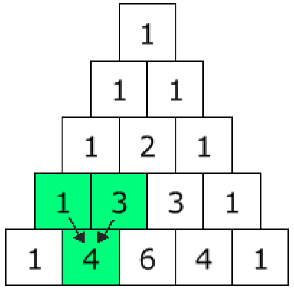
Pascal's Triangle

****The numbers in Pascal's Triangle correspond to **the coefficients in the expansion of binomials raised to whole-number exponents**. ****

The coefficients in the expansion of $(a + b)^n$ can be found in row n of Pascal's triangle.



Pascal's Triangle



Value of n	$(a + b)^n$
0	
1	
2	
3	

Example: Use Pascal's Triangle to expand each power of a binomial

$a)$ $(a+b)^7$

$b)$ $(m-n)^5$

$c)$ $(2x+1)^6$

$d)$ $(\frac{y}{2}-y^2)^4$

Homework

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Page 433 – 438 – **Read** & Work Through Examples

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