Lesson 2: Review of AC circuit Analysis

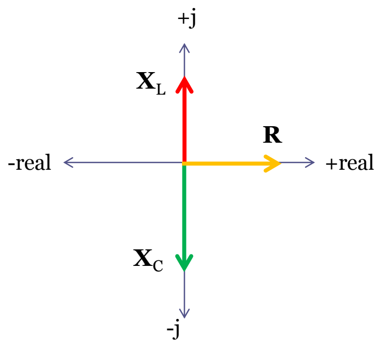
ET 332b
Ac Motors, Generators and Power Systems

Learning Objectives

After this presentation you will be able to:

- Compute impedances for circuit elements given values of capacitance, inductance and resistance
- Simplify series and parallel impedances
- Use the current divider and voltage divider rules in ac circuits
- Solve series and parallel ac circuits using phasors

Impedance Diagrams



The resistor has zero phase shift.
It is represented by:

$\mathbf{R} = R\angle 0^\circ$

Reactance of inductors and capacitors

<u>Inductive reactance</u>

$$\overline{X}_L = 2\pi \cdot f \cdot L \cdot j$$
$$\overline{X}_L = \omega \cdot L \cdot j$$

Phase shift of 90 degrees

$$\mathbf{X}_L = X_L\angle 90^\circ$$

<u>Capacitive reactance</u>

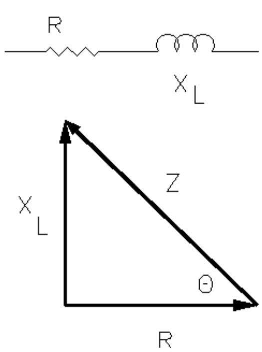
$$\overline{X}_C = \frac{1}{2\pi \cdot f \cdot C \cdot j} = \frac{1}{\omega \cdot C \cdot j}$$

$$\frac{1}{j} = -j \quad \text{so} \quad \overline{X}_C = -j \cdot \left[\frac{1}{2\pi \cdot f \cdot C}\right]$$

Phase shift of -90 degrees $\mathbf{X}_C = X_C\angle -90^\circ$

Series Impedances: R-L circuits

The R-L impedance triangle



Series R-L circuits

$$\mathbf{Z} = R + j \cdot X_L$$

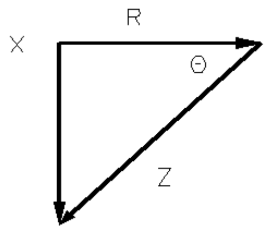
$$|\mathbf{Z}| = \sqrt{R^2 + X_L^2}$$

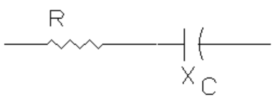
$$\theta = \tan^{-1}\left(\frac{X_L}{R}\right)$$

Phase angle will always be positive for series R-L circuit.

Series Impedances: R-C circuits

The R-C impedance triangle





Series R-C circuits

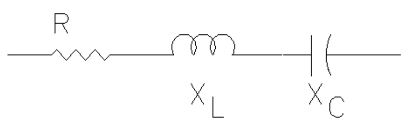
$$\mathbf{Z} = R - j \cdot X_C$$

$$|\mathbf{Z}| = \sqrt{R^2 + X_C{}^2}$$

$$\theta = \tan^{-1}\left(\frac{X_C}{R}\right)$$

Phase angle will always be negative for series R-C circuit.

General Series Impedances: R-L-C circuits



The net reactive effect can either be inductive (positive) or capacitive (negative). The reactive component with larger reactance magnitude with dominate the series combination

$$\mathbf{Z} = R + j \cdot X_L - j \cdot X_C$$

$$|\mathbf{Z}| = \sqrt{R^2 + (X_L - X_C)^2}$$

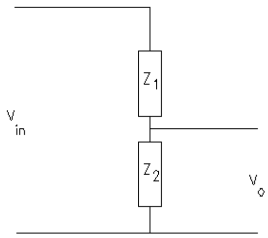
$$\theta = \tan^{-1}\left(\frac{(X_L - X_C)}{R}\right)$$

Ac Circuit Analysis With Phasors

All circuit laws from dc analysis hold for phasor analysis of ac circuits

Ohm's Law $\overline{V} = \overline{I} \cdot \overline{Z}$ $\quad \overline{Z} = \dfrac{\overline{V}}{\overline{I}}$ $\quad \overline{I} = \dfrac{\overline{V}}{\overline{Z}}$

All theorems for dc circuit analysis are valid for phasors

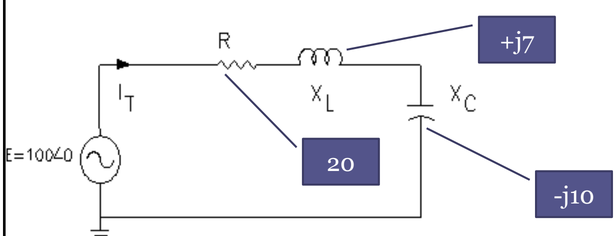


Voltage divider rule for ac circuits- same formula as dc but with phasor quantities

$$\overline{V}_o = \left(\frac{\overline{Z}_2}{\overline{Z}_1 + \overline{Z}_2} \right) \cdot \overline{V}_{in}$$

AC Circuit Analysis With Phasors

Example 2-1 Series Circuits: a) Find Z_T and I_T for the circuit shown with the following values: $X_L = 7\ \Omega$, $X_C = 10\ \Omega$,and R=20 Ω.
b) find the voltage across the series combination of X_L and X_C using the voltage divider rule.



Example 2-1 Solution (1)

$$\bar{Z}_T = R + jX_L - jX_C$$

$$\bar{Z}_T = 20 + j7 - j10\ \Omega$$

$$\bar{Z}_T = 20 - j3 = 20.22\angle{-8.53^\circ}\ \Omega$$

Answer

$$\bar{E} = 100\angle{0^\circ}$$

$$\bar{I}_T = \frac{100\angle{0^\circ}}{20.22\angle{-8.53^\circ}} = 4.946\angle{8.53^\circ}\ A$$

Answer

Example 2-1 Solution (2)

Component voltage drops

$$\bar{V}_R = \bar{I}_T R = 4.946\angle{8.53^\circ}(20\angle{0^\circ}) = 98.92\angle{8.53^\circ}\ V$$

$$\bar{V}_L = \bar{I}_T \bar{X}_L = (4.946\angle{8.53^\circ})(7\angle{90^\circ}) = 34.622\angle{98.53^\circ}\ V$$

$$\bar{V}_C = \bar{I}_T \bar{X}_C = (4.946\angle{8.53^\circ})(10\angle{-90^\circ}) = 49.46\angle{-81.47^\circ}\ V$$

b) find the voltage across the series combination of X_L and X_C using the voltage divider rule.

$$\bar{Z}_{LC} = 7j - 10j = -3j = 3\angle{-90^\circ}$$

$$\bar{V}_Z = \bar{E}\left[\frac{\bar{Z}_{LC}}{\bar{R} + \bar{X}_L + \bar{X}_C}\right]$$

$$R + jX_L - jX_C = 20 + j7 - j10$$

$$R + jX_L - jX_C = 20.22\angle{-8.53^\circ}\ \Omega$$

Example 2-1 Solution (3)

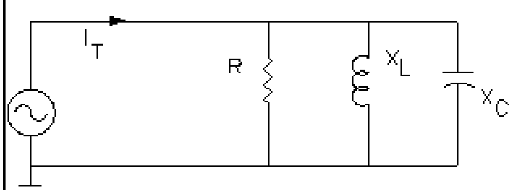
$$\bar{V}_Z = 100\angle 0^\circ \left[\frac{3\angle -90^\circ}{20.22\angle -8.53^\circ}\right]$$

$$\bar{V}_Z = 100\angle 0^\circ \left(0.14836\angle -81.47^\circ\right)$$

$$\bar{V}_Z = 14.836\angle -81.47^\circ$$

Answer

Parallel Ac Circuits



Admittance defined as:

$$\bar{Y} = \frac{1}{\bar{Z}}$$

$$\bar{Y} = \frac{1}{\bar{Z}} = G + j\cdot B$$

For single elements in parallel

$$\frac{1}{\bar{Z}_T} = \frac{1}{R} + \frac{1}{j\cdot X_L} + \frac{1}{-j\cdot X_C}$$

$$\bar{Y}_T = \frac{1}{R} + \frac{1}{j\cdot X_L} + \frac{1}{-j\cdot X_C}$$

Units on admittance: Siemens (S)
G= conductance
B= susceptance

Also

$$\bar{Z}_T = \frac{1}{\frac{1}{R} + \frac{1}{j\cdot X_L} + \frac{1}{-j\cdot X_C}}$$

Parallel Ac Circuits

From Ohm's Law $\bar{I}_T = \dfrac{\bar{E}}{\bar{Z}_T}$

Substitute in definition of admittance to get... $\bar{I} = \bar{E} \cdot \bar{Y}_T$

When parallel elements are made up of series-connected components, take the sum of these elements before inverting to get the admittance.

Do this

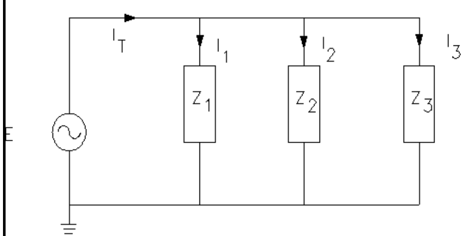
$$\bar{Y}_T = \frac{1}{R + j \cdot X}$$

Not this

$$\cancel{\bar{Y}_T = \frac{1}{R} + \frac{1}{j \cdot X}}$$

Parallel Ac Circuits

Current Division Holds For Phasor Analysis



In the most general form, Z_1 through Z_3 can have series connected elements. If these are converted to admittances....

$$\bar{Y}_T = \frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2} + \frac{1}{\bar{Z}_3} = \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3$$

Current in any branch can be found by using the general current division formula

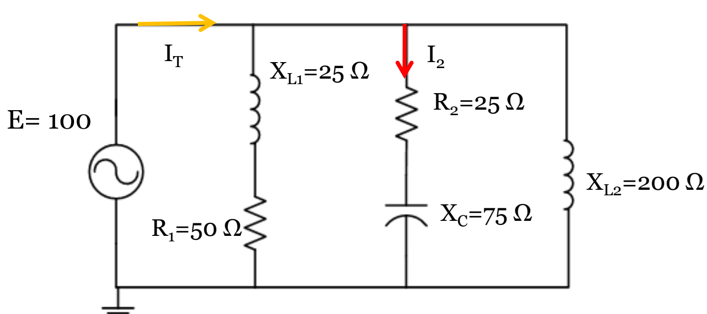
$$\bar{I}_T = \bar{E} \cdot \bar{Y}_T \qquad \bar{I}_k = \left(\frac{\bar{Y}_k}{\bar{Y}_T}\right) \cdot \bar{I}_T$$

Where k is the branch where the current is to be found

Admittances and Current Division

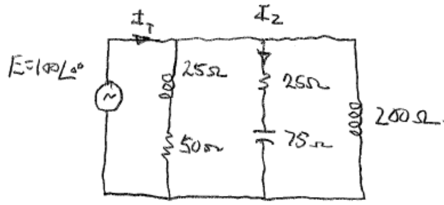
If impedances are used with current division, only two branches must be in parallel. There is not limit to the number of parallel branches with admittance. Using admittances note that the branch where the current is flowing is in the numerator.

Parallel Circuit Example



Example 2-2: Find the total admittance, Y_T, Z_T and I_T for the circuit above. Also find the current I_2 using the current divider rule.

Example 2-2 Solution (1)



$$\bar{Z}_1 = 50 + j25\ \Omega$$

$$\bar{Z}_2 = 25 - j75\ \Omega$$

$$Z_3 = -j200\ \Omega$$

$$\frac{1}{\bar{Z}_1} = \frac{1}{50+j25} = \frac{1}{55.9\angle 26.56°} = 0.01789\angle -26.56°$$

$$\frac{1}{\bar{Z}_2} = \frac{1}{25-j75} = \frac{1}{79.056\angle -71.6°} = 0.01265\angle 71.6°$$

$$\frac{1}{\bar{Z}_3} = \frac{1}{200\angle 90°} = 0.005\angle -90°$$

Example 2-2 Solution (2)

$$\bar{Y}_T = \frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2} + \frac{1}{\bar{Z}_3}$$

$$\bar{Y}_T = 0.01789\angle -26.56° + 0.01265\angle 71.6° + 0.005\angle -90°$$

$$\bar{Y}_T = (0.016 - j0.008) + (0.004 + j0.012) + (0.005j) = 0.02 + 0.009j$$

$$\boxed{\bar{Y}_T = 0.0219\angle 24.22°}$$

Ans

Example 2-2 Solution (3)

$$\bar{Z}_T = \frac{1}{\bar{Y}_T} = \frac{1}{0.0219\angle 24.22^\circ} = \boxed{45.66\angle -24.22^\circ}$$

Ans

$$\bar{I}_T = \frac{\bar{E}}{\bar{Z}_T} = \bar{E}\,\bar{Y}_T = 100\angle 0^\circ\,(0.0219\angle 24.22^\circ)$$

$$\boxed{\bar{I}_T = 2.19\angle 24.22^\circ\ A}$$

Ans

Example 2-2 Solution (4)

$$\bar{I}_2 = \frac{\bar{Y}_2}{\bar{Y}_T}(\bar{I}_T) = \frac{0.01265\angle 71.56^\circ}{0.0219\angle 24.22^\circ}(2.19\angle 24.22^\circ)$$

$$\bar{I}_2 = 0.57763\angle 47.34^\circ\,(2.19\angle 24.22^\circ)$$

$$\boxed{\bar{I}_2 = 1.265\angle 71.56^\circ\ A}$$

Ans

End Lesson 2: Review of AC circuit Analysis

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