LESSON 21

• Prime and Composite Numbers
• Prime Factorization

Power Up

Building Power

facts

Power Up E

mental math

a. **Number Sense:** \$1.25 + 99¢

b. **Decimals:** \$6.50 ÷ 10

c. **Number Sense:** \$20.00 − \$15.75

d. **Calculation:** 6×34

e. **Calculation:** $1\frac{2}{3} + 2\frac{1}{3}$

f. **Fractional Parts:** $\frac{1}{3}$ of 36

g. **Measurement:** Which is greater 3 pints or 1 quart?

h. **Calculation:** Start with the number of sides of a hexagon, × 5, + 2, ÷ 8, + 1, ÷ 5.

problem solving

The first even counting number is 2; the sum of the first two even counting numbers is 6; the sum of the first three even counting numbers is 12. Add to this list the sums of first four, five, and six even counting numbers. Does this list of the sums of even counting numbers have a pattern? Can you describe a rule for continuing the sequence?

Understand We are given the sums of the first one, two, and three even counting numbers. We are asked to find the sums of the first four, five, and six even counting numbers and to find a pattern in the sums.

Plan We will *make a chart* to help us record our work in an organized way. Then we will use our chart to *find a pattern* in the sums of sequences of even counting numbers.

Solve We write the first six sequences, the number of terms in each sequence, and the sum of each sequence on our chart:

Sequence	Number of Terms	Sum
2	1	2
2 + 4	2	6
2 + 4 + 6	3	12
2 + 4 + 6 + 8	4	20
2 + 4 + 6 + 8 + 10	5	30
2 + 4 + 6 + 8 + 10 + 12	6	42

When we look at the number of terms and the resulting sums, we see several numbers that belong to the same fact families: 1 is a factor of 2, 2 is a factor of 6, 3 is a factor of 12, etc. We rewrite each sum as a multiplication problem using the number of terms as one of the factors:

Number of Terms		Sum
1	$1 \times 2 =$	2
2	$2 \times 3 =$	6
3	$3 \times 4 =$	12
4	$4 \times 5 =$	20
5	$5 \times 6 =$	30
6	$6 \times 7 =$	42

To find each sum, we can multiply the number of terms in the sequence by the next whole number.

Check We found a pattern in the sums of the sequences of even counting numbers. We can verify our solution by finding the sum of the seventh sequence using our multiplication method, then check the sum by adding the numbers one-by-one.

New Concepts

Increasing Knowledge

prime and composite numbers

We remember that the counting numbers (or natural numbers) are the numbers we use to count. They are

$$1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \ldots$$

Counting numbers greater than 1 are either **prime numbers** or **composite numbers.** A prime number has exactly two different factors, and a composite number has three or more factors. In the following table, we list the factors of the first ten counting numbers. The numbers 2, 3, 5, and 7 each have exactly two factors, so they are prime numbers.

Factors of Counting Numbers 1–10

Number	Factors
1	1
2	1, 2
3	1, 3
4	1, 2, 4
5	1, 5
6	1, 2, 3, 6
7	1, 7
8	1, 2, 4, 8
9	1, 3, 9
10	1, 2, 5, 10

We see that the factors of each of the prime numbers are 1 and the number itself. So we define a prime number as follows:

> A **prime number** is a counting number greater than 1 whose only factors are 1 and the number itself.

From the table we can also see that 4, 6, 8, 9, and 10 each have three or more factors, so they are composite numbers. Each composite number is divisible by a number other than 1 and itself.

Discuss The number 1 is neither a prime number nor a composite number. Why do you think that is true?

Example 1

Make a list of the prime numbers that are less than 16.

Solution

First we list the counting numbers from 1 to 15.

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15

A prime number must be greater than 1, so we cross out 1. The next number, 2, has only two divisors (factors), so 2 is a prime number. However, all the even numbers greater than 2 are divisible by 2, so they are not prime. We cross these out.

$\cancel{1}$, 2, 3, $\cancel{4}$, 5, $\cancel{6}$, 7, $\cancel{8}$, 9, $\cancel{10}$, 11, $\cancel{12}$, 13, $\cancel{14}$, 15

The numbers that are left are

2, 3, 5, 7, 9, 11, 13, 15

The numbers 9 and 15 are divisible by 3, so we cross them out.

2, 3, 5, 7, $\cancel{9}$, 11, 13, $\cancel{15}$

The only divisors of each remaining number are 1 and the number itself. So the prime numbers less than 16 are **2, 3, 5, 7, 11,** and **13.**

Example 2

List the factor pairs for each of these numbers:

16 17 18

Classify **Which of these numbers is prime?**

Solution

The factor pairs for 16 are **1 and 16, 2 and 8, 4 and 4.**

The factor pair for 17 is **1 and 17.**

The factor pairs for 18 are **1 and 18, 2 and 9, 3 and 6.**

Note that perfect squares have one pair of identical factors. Therefore they have an odd number of different factors. Also note that prime numbers have only one factor pair since they have only two factors.

Example 3

List the composite numbers between 40 and 50.

Solution

First we write the counting numbers between 40 and 50.

41, 42, 43, 44, 45, 46, 47, 48, 49

Any number that is divisible by a number besides 1 and itself is composite. All the even numbers in this list are composite since they are divisible by 2. That leaves the odd numbers to consider. We quickly see that 45 is divisible by 5, and 49 is divisible by 7. So both 45 and 49 are composite. The remaining numbers, 41, 43, and 47, are prime. So the composite numbers between 40 and 50 are **42, 44, 45, 46, 48,** and **49.**

prime factorization

Every composite number can be *composed* (formed) by multiplying two or more prime numbers. Here we show each of the first nine composite numbers written as a product of **prime** factors,

$4 = 2 \cdot 2$	$6 = 2 \cdot 3$	$8 = 2 \cdot 2 \cdot 2$
$9 = 3 \cdot 3$	$10 = 2 \cdot 5$	$12 = 2 \cdot 2 \cdot 3$
$14 = 2 \cdot 7$	$15 = 3 \cdot 5$	$16 = 2 \cdot 2 \cdot 2 \cdot 2$

Thinking Skill

Generalize

The prime factorization of a number is: $2 \cdot 3 \cdot 3$. What is the number?

Notice that we **factor** 8 as $2 \cdot 2 \cdot 2$ and not $2 \cdot 4$, because 4 is not prime.

When we write a composite number as a product of prime numbers, we are writing the **prime factorization** of the number.

Example 4

Write the prime factorization of each number.

a. **30**   **b.** **81**   **c.** **420**

Solution

We will write each number as the product of two or more prime numbers.

a. $30 = \mathbf{2 \cdot 3 \cdot 5}$ — We do not use $5 \cdot 6$ or $3 \cdot 10$, because neither 6 nor 10 is prime.

b. $81 = \mathbf{3 \cdot 3 \cdot 3 \cdot 3}$ — We do not use $9 \cdot 9$, because 9 is not prime.

c. $420 = \mathbf{2 \cdot 2 \cdot 3 \cdot 5 \cdot 7}$ — Two methods for finding this are shown after example 5.

Explain How can you quickly tell that these three numbers are composite without writing the complete prime factorization of each number?

Example 5

Write the prime factorization of 100 and of $\sqrt{100}$.

Solution

The prime factorization of 100 is $\mathbf{2 \cdot 2 \cdot 5 \cdot 5}$. We find that $\sqrt{100}$ is 10, and the prime factorization of 10 is $\mathbf{2 \cdot 5}$. Notice that 100 and $\sqrt{100}$ have the same prime factors, 2 and 5, but that each factor appears half as often in the prime factorization of $\sqrt{100}$.

There are two commonly used methods for factoring composite numbers. One method uses a factor tree. The other method uses division by primes. We will factor 420 using both methods.

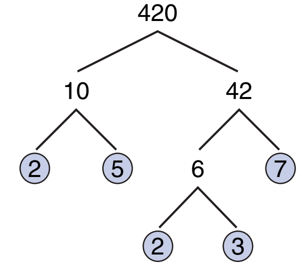
Thinking Skill

Represent

You can begin a factor tree with any pair of factors. Find the prime factorization of 420 starting with 2 and 210. Then find it starting with 6 and 70. Is the result the same?

To factor a number using a **factor tree,** we first write the number. Below the number we write any two whole numbers greater than 1 that multiply to equal the number. If these numbers are not prime, we continue the process until there is a prime number at the end of each “branch” of the factor tree. These numbers are the prime factors of the original number. We write them in order from least to greatest.

Factor Tree



$$420 = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 7$$

To factor a number using **division by primes,** we write the number in a division box and divide by the smallest prime number that is a factor. Then we divide the resulting quotient by the smallest prime number that is a factor. We repeat this process until the quotient is 1.[1] The divisors are the prime factors of the number.

Division by Primes

$$\begin{array}{r} 1 \\ \mathbf{7}\overline{)7} \\ \mathbf{5}\overline{)35} \\ \mathbf{3}\overline{)105} \\ \mathbf{2}\overline{)210} \\ \mathbf{2}\overline{)420} \end{array}$$

$$420 = 2 \cdot 2 \cdot 3 \cdot 5 \cdot 7$$

We can use prime factorization to help us find the greatest common factor (GCF) of two or more numbers.

Step 1: List the prime factors for each number.

Step 2: Identify the shared factors.

Step 3: Multiply the shared factors to find the GCF.

[1] Some people prefer to divide until the quotient is a prime number. In this case, the final quotient is included in the list of prime factors.

Example 6

Write the prime factorization of 36 and 60. Use the results to find the greatest common factor of 36 and 60.

Solution

1. Using a factor tree or division by primes, we find the prime factorization of 36 and 60.
2. Identify the shared factors.

$$\begin{aligned} 36 &= 2 \cdot 2 \cdot 3 \cdot 3 \\ 60 &= 2 \cdot 2 \cdot 3 \cdot 5 \end{aligned}$$

   We see 36 and 60 share two 2s and one 3.
3. We multiply the shared factors: $2 \cdot 2 \cdot 3 = 12$. The GCF of 36 and 60 is **12.**

Practice Set

a. List the first ten prime numbers.

b. *Classify* If a whole number greater than 1 is not prime, then what kind of number is it?

c. Write the prime factorization of 81 using a factor tree.

d. Write the prime factorization of 360 using division by primes.

e. *Generalize* Write the prime factorization of 64 and of $\sqrt{64}$.

f. Use prime factorization to find the GCF of 18 and 81.

Written Practice

Strengthening Concepts

1. (14) Two thirds of the students wore green on St. Patrick's Day. What fraction of the students did not wear green on St. Patrick's Day?

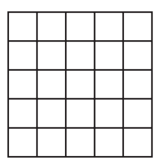
2. (13) Three hundred forty-three quills were carefully placed into 7 compartments. If each compartment held the same number of quills, how many quills were in each compartment?

3. (19, 20) Choose the formula for the perimeter of a rectangle.

A $P = 2L + 2W$ **B** $P = 4s$

C $A = LW$ **D** $A = s^2$

4. (20) Write a squaring fact and a square root fact illustrated by this square.



5. (15) Write each number as a reduced fraction or mixed number:

a. $3\frac{12}{21}$ **b.** $\frac{12}{48}$ **c.** 12%

*** 6.** (21) List the prime numbers between 50 and 60.

*** 7.** (21) Write the prime factorization of each number:

a. 50 **b.** 60 **c.** 300

8. (4) **Justify** Which point could represent 1610 on this number line? How did you decide?

9. (15) Complete each equivalent fraction:

a. $\frac{2}{3} = \frac{?}{15}$ **b.** $\frac{3}{5} = \frac{?}{15}$ **c.** $\frac{?}{3} = \frac{8}{12}$

d. What property of multiplication do we use to rename fractions?

10. (9) **a.** How many $\frac{1}{3}$s are in 1?

b. How many $\frac{1}{3}$s are in 3?

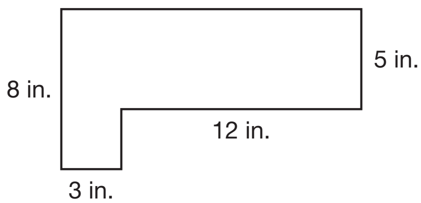
*** 11.** (20) The perimeter of a regular quadrilateral is 12 inches. What is its area?

*** 12.** (8, 19) **Represent** Use a ruler to draw a rectangle that is $\frac{3}{4}$ in. wide and twice as long as it is wide.

a. How long is the rectangle?

b. What is the perimeter of the rectangle?

*** 13.** (19) Find the perimeter of this hexagon:



14. (14) A number cube is rolled once. What is the probability of getting an odd number greater than 5?

Solve:

15. (9) $p + \frac{3}{5} = 1$ **16.** (9) $\frac{3}{5}q = 1$ **17.** (3) $\frac{w}{25} = 50$

18. (9, 15) $\frac{1}{6} + f = \frac{5}{6}$ **19.** (10) $m - 3\frac{2}{3} = 1\frac{2}{3}$ **20.** (3) $51 = 3c$

Simplify:

21. (9) $\frac{2}{3} + \frac{2}{3} + \frac{2}{3}$ *** 22.** (20) $\left(\frac{2}{3}\right)^3$

* **23.** (21) **a.** Write the prime factorization of 225.

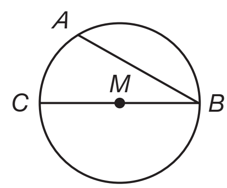
b. Generalize Find $\sqrt{225}$ and write its prime factorization.

24. (15) Describe how finding the greatest common factor of the numerator and denominator of a fraction can help reduce the fraction.

25. (17) Draw $\overline{AB}$ $2\frac{1}{2}$ inches long. Then draw $\overline{BC}$ $2\frac{1}{2}$ inches long perpendicular to $\overline{AB}$. Complete the triangle by drawing $\overline{AC}$. Use a protractor to find the measure of $\angle A$.

26. (9, 10) Write $1\frac{3}{4}$ as an improper fraction. Multiply the improper fraction by the reciprocal of $\frac{2}{3}$. Then write the product as a mixed number.

* **27.** (Inv. 2) Classify Refer to the circle at right with center at point M to answer **a–d.**



a. Which segment is a diameter?

b. Which segment is a chord but not a diameter?

c. Which two segments are radii?

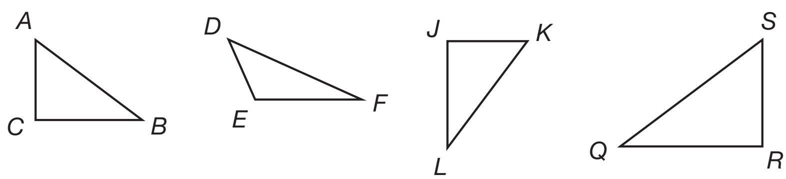
d. Which angle is an inscribed angle?

28. (16) Alicia's father asked her to buy a gallon of milk at the store. The store had milk only in quart-sized containers. What percent of a gallon is a quart? How many quart containers did Alicia have to buy?

29. (2) **a.** Compare: $a + b \bigcirc b + a$

b. What property of operations applies to part **a** of this problem?

* **30.** (18) Analyze Refer to the triangles below to answer **a–c.**



a. Which triangle appears to be congruent to $\triangle ABC$?

b. Which triangle is not similar to $\triangle ABC$?

c. Which angle in $\triangle QRS$ corresponds to $\angle A$ in $\triangle ABC$?

LESSON 22

Problems About a Fraction of a Group

Power Up

Building Power

facts Power Up E

mental math

a. Number Sense: \$1.54 + 99¢

b. Decimals: 8¢ × 100

c. Calculation: \$10.00 – \$7.89

d. Calculation: 7 × 53

e. Calculation: $3\frac{3}{4} + 1\frac{1}{4}$

f. Fractional Parts: $\frac{1}{4}$ of 24

g. Measurement: Which is greater a gallon or 2 quarts?

h. Calculation: Start with the number of years in half a century. Add the number of inches in half a foot; then divide by the number of days in a week. What is the name of the polygon with this number of sides?

problem solving Yin has 25 tickets, Bobby has 12 tickets, and Mary has 8 tickets. How many tickets should Yin give to Bobby and to Mary so that they all have the same number of tickets?

New Concept

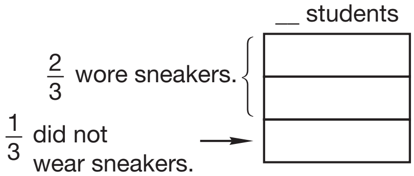
Increasing Knowledge

In Lesson 13 we looked at problems about equal groups. In Lesson 14 we considered problems about parts of a whole. In this lesson we will solve problems that involve both equal groups and parts of a whole. Many of the problems will require two or more steps to solve.

Consider the following statement:

Two thirds of the students in the class wore sneakers on Monday.

We can draw a diagram for this statement. We use a rectangle to represent all the students in the class. Next we divide the rectangle into three equal parts. Then we describe the parts.

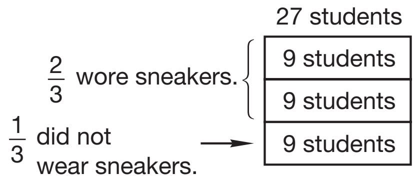


If we know how many students are in the class, we can figure out how many students are in each part.

Two thirds of the 27 students in the class wore sneakers on Monday.

There are 27 students in all. If we divide the group of 27 students into three equal parts, there will be 9 students in each part. We write these numbers on our diagram.

Analyze Why do we divide the rectangle into 3 equal parts rather than any other number of equal parts?



Since $\frac{2}{3}$ of the students wore sneakers, we add two of the parts and find that 18 students wore sneakers. Since $\frac{1}{3}$ of the students did not wear sneakers, we find that 9 students did not wear sneakers.

Example 1

Diagram this statement. Then answer the questions that follow.

Two fifths of the 30 students in the class are boys.

a. How many boys are in the class?

b. How many girls are in the class?

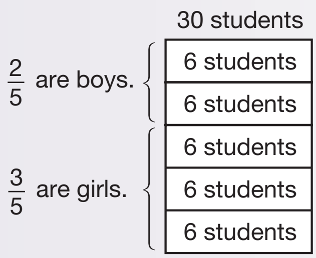
Solution

Thinking Skill

Model

How could you use colored counters to model the solution to example 1?

We draw a rectangle to represent all 30 students. Since the statement uses fifths to describe a part of the class, we divide the class of 30 students into five equal parts. Since 30 ÷ 5 is 6, there are 6 students in each part.



Now we can answer the questions.

a. Two of the five parts are boys. Since there are 6 students in each part, there are **12 boys.**

b. Since two of the five parts are boys, three of the five parts must be girls. Thus there are **18 girls.**

Another way to find the answer to **b** after finding the answer to **a** is to subtract. Since 12 of the 30 students are boys, the rest of the students (30 − 12 = 18) are girls.

Predict How many girls would there be in the class if $\frac{1}{5}$ were boys?

Example 2

In the following statement, change the percent to a fraction. Then diagram the statement and answer the questions.

Britt read 80% of a 40-page book in one day.

a. What fraction of the book did Britt read in one day?

b. How many pages did Britt read in one day?

Math Language
Remember that a **percent** can be expressed as a fraction with a denominator of 100.

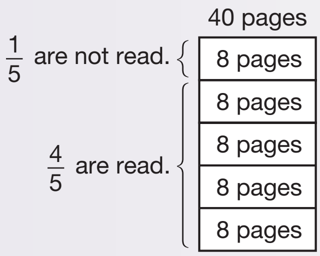
Solution

This problem is about a fraction of a group, but the fraction is expressed as a percent. We write 80% as 80 over 100 and reduce.

$$\frac{80}{100} \div \frac{20}{20} = \frac{4}{5}$$

So 80% is equivalent to the fraction $\frac{4}{5}$.

Now we draw a rectangle to represent all 40 pages, dividing the rectangle into five equal parts. Since 40 ÷ 5 is 8, there are 8 pages in each part.



Now we can answer the questions.

a. Britt read $\frac{4}{5}$ **of the book** in one day.

b. Britt read 4 × 8 pages, which is **32 pages in one day.**

Represent Write an equation you could use to find the number of pages Britt did not read yet. Use the answer to **b** above to help you write the equation. Then solve the equation.

Practice Set

Model Diagram each statement. Then answer the questions.

First statement: *Three fourths of the 60 marbles in the bag were red.*

a. How many marbles were red?

b. How many marbles were not red?

Second statement: *Sixty percent of the 20 tomatoes were green.*

c. What fraction of the tomatoes were not green?

d. How many tomatoes were green?

e. **Formulate** For the following statement, write and answer two questions: *Three fifths of the thirty students were girls.*

Written Practice *Strengthening Concepts*

1. *(11)* In Room 7 there are 28 students. In Room 9 there are 30 students. In Room 11 there are 23 students. How many students are in all three rooms?

2. *(13)* If the total number of students in problem 1 were equally divided among three rooms, how many students would be in each room?

3. *(11)* The largest state is Alaska. It has an area of about 663,000 square miles. The smallest state, Rhode Island, has an area of about 1,500 square miles. About how many more square miles is Alaska than Rhode Island?

* **4.** *(19, 20)* **a.** Write the formula for the perimeter of a square.

b. A landscape planner designed a square garden that is 24 feet long per side. How many feet of border are needed to surround the garden?

* **5.** *(22)* **Model** Diagram this statement. Then answer the questions that follow.

Five ninths of the 36 spectators were happy with the outcome.

a. How many spectators were happy with the outcome?

b. How many spectators were not happy with the outcome?

* **6.** *(22)* In the following statement, change the percent to a reduced fraction. Then diagram the statement and answer the questions.

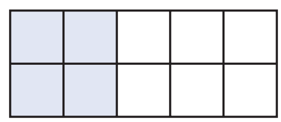
Twenty-five percent of three dozen plants are blooming.

a. What fraction of the total number of plants are not blooming?

b. How many plants are not blooming?

7. *(15)* **a.** What fraction of the rectangle is shaded?

b. What percent of the rectangle is not shaded?



8. *(9)* **a.** How many $\frac{1}{4}$s are in 1?

b. **Explain** Tell how you can use the answer to part **a** to find the number of $\frac{1}{4}$s in 3.

9. *(2)* **a.** Multiply: $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 0$

b. **Analyze** What property is illustrated by the multiplication in part **a?**

10. (9) *Analyze* Simplify and compare: $\frac{3}{3} - \left(\frac{1}{3} \cdot \frac{3}{1}\right) \bigcirc \left(\frac{3}{3} - \frac{1}{3}\right) \cdot \frac{3}{1}$

*** 11.** (19, 20) Draw a rectangle *ABCD* so that *AB* is 2 in. and *BC* is 1 in.

a. What is the perimeter of rectangle *ABCD*?

b. What is the area of the rectangle?

c. What is the sum of the measures of all four angles of the rectangle?

*** 12.** (21) *Generalize* Write the prime factorization of each number:

a. 32 **b.** 900 **c.** $\sqrt{900}$

13. (15) For each fraction, write an equivalent fraction that has a denominator of 60.

a. $\frac{5}{6}$ **b.** $\frac{3}{5}$ **c.** $\frac{7}{12}$

14. (10) Add the three fractions with denominators of 60 from problem **13,** and write their sum as a mixed number.

15. (4, 10) **a.** Arrange these numbers in order from least to greatest:

$$0, -\frac{2}{3}, 1, \frac{3}{2}, -2$$

b. Which of these numbers are positive?

*** 16.** (14, 15) *Predict* If one card is drawn from a regular deck of cards, what is the probability the card will be a heart?

Evaluate Find the value of each variable.

17. (9, 15) $\frac{5}{12} + a = \frac{11}{12}$

18. (3) $121 = 11x$

19. (10) $2\frac{2}{3} = y - 1\frac{1}{3}$

*** 20.** (20) $10^2 \cdot 10^5 = 10^n$

Simplify:

21. (15) $\frac{5}{6} + \frac{5}{6} + \frac{5}{6}$

22. (10) $\frac{15}{2} \cdot \frac{10}{3}$

*** 23.** (20) $\left(\frac{5}{6}\right)^2$

*** 24.** (20) $\sqrt{30^2}$

*** 25.** (2,9) *Justify* Give reasons for the steps used to simplify the following expression by using the commutative, associative, inverse, and identity properties of multiplication.

$$\frac{3}{1} \cdot \frac{2}{3} \cdot \frac{1}{3}$$

26. (10,15) Write $1\frac{1}{2}$ and $1\frac{2}{3}$ as improper fractions. Then multiply the improper fractions, and write the product as a mixed number.

*** 27.** (16) A package that weighs 1 lb 5 oz weighs how many ounces?

*** 28.** (17) Use a protractor to draw a 45° angle.

29. (4, 9) *Justify* Find the next number in this sequence and explain how you found your answer.

$$\ldots, 100, 10, 1, \frac{1}{10}, \ldots$$

30. (4) Write an odd negative integer greater than -3.

Early Finishers

Real-World Application

This weekend workers will cut the grass on the high school football field and repaint the white outline around the field.

a. The field is 360 feet long and 160 feet wide. Find the perimeter and area of the field.

b. One quart of paint is enough to paint a 200 ft stripe. How many quarts of paint should be purchased to paint a stripe around the entire field? Show your work.

c. If it takes a large mower 25 seconds to mow 800 ft^2, how long will it take to mow the whole field? Show your work. (Assume the paths are cut with no overlap.)

LESSON 23

Subtracting Mixed Numbers with Regrouping

Power Up

Building Power

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mental math

a. **Number Sense:** \$3.65 + 98¢

b. **Decimals:** \$25.00 ÷ 100

c. **Positive/Negative:** 449 − 500

d. **Calculation:** 8×62

e. **Calculation:** $1\frac{1}{2} + 2\frac{1}{2}$

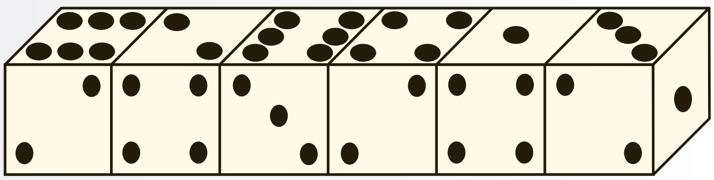
f. **Fractional Parts:** $\frac{1}{2}$ of 76

g. **Measurement:** What fraction of a minute is one second?

h. **Calculation:** 8×8, $-\ 1$, $\div\ 9$, $\times\ 4$, $-\ 1$, $\div\ 3$, $\times\ 2$, $+\ 2$, $\div\ 4$

problem solving

Altogether, how many dots on the six number cubes are not visible in the illustration?



New Concept

Increasing Knowledge

Math Language
In *regrouping,* we exchange a value for an equal amount. For example, 1 ten for 10 ones, or 1 whole for $\frac{4}{4}$.

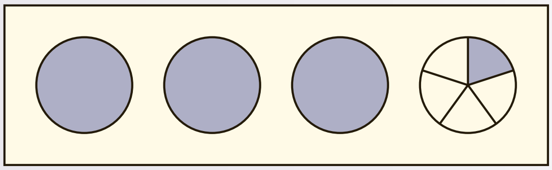
In this lesson we will practice subtracting mixed numbers that require regrouping. Regrouping that involves fractions differs from regrouping with whole numbers. When regrouping with whole numbers, we know that each unit equals ten of the next-smaller unit. However, when regrouping from a whole number to a fraction, we need to focus on the denominator of the fraction to determine how to regroup. We will use illustrations to help explain the process.

Example 1

There are $3\frac{1}{5}$ pies on the shelf. If the baker takes away $1\frac{2}{5}$ pies, how many pies will be on the shelf?

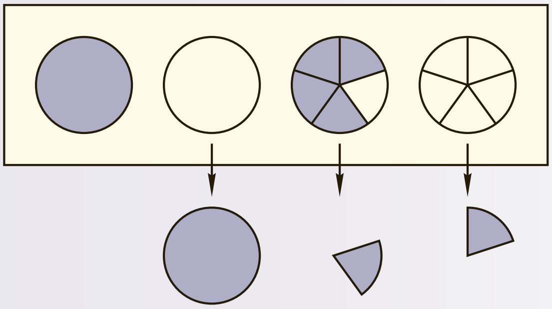
Solution

To answer this question, we subtract $1\frac{2}{5}$ from $3\frac{1}{5}$. Before we subtract, however, we will draw a picture to see how the baker solves the problem.



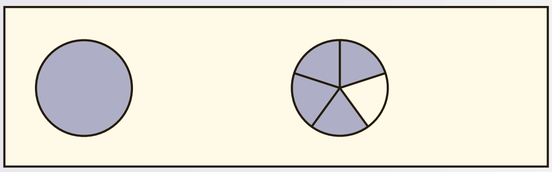
$3\frac{1}{5}$

In order for the baker to remove $1\frac{2}{5}$ pies, it will be necessary to slice one of the whole pies into fifths. After cutting one pie into fifths, there are 2 whole pies plus $\frac{5}{5}$ plus $\frac{1}{5}$, which is $2\frac{6}{5}$ pies. Then the baker can remove $1\frac{2}{5}$ pies, as we illustrate.



$2\frac{6}{5}$

$1\frac{2}{5}$ removed



$1\frac{4}{5}$ remain

As we can see from the picture, **$1\frac{4}{5}$ pies** will be left on the shelf.

To perform the subtraction on paper, we first rename $3\frac{1}{5}$ as $2\frac{6}{5}$, as shown below. Then we can subtract.

$$\begin{array}{r} 3\frac{1}{5} \\ -\ 1\frac{2}{5} \\ \hline \end{array} \xrightarrow{2 + \frac{5}{5} + \frac{1}{5}} \begin{array}{r} 2\frac{6}{5} \\ -\ 1\frac{2}{5} \\ \hline 1\frac{4}{5} \end{array}$$

Example 2

Thinking Skill

Analyze

How can we tell we will need to regroup just by looking at the example?

Simplify: $3\frac{5}{8} - 1\frac{7}{8}$

Solution

We need to regroup in order to subtract. The mixed number $3\frac{5}{8}$ equals $2 + 1 + \frac{5}{8}$, which equals $2 + \frac{8}{8} + \frac{5}{8}$. Combining $\frac{8}{8}$ and $\frac{5}{8}$ gives us $\frac{13}{8}$, so we use $2\frac{13}{8}$. Now we can subtract and reduce.

$$\begin{array}{r} 3\frac{5}{8} \\ -\ 1\frac{7}{8} \\ \hline \end{array} \xrightarrow{2 + \frac{8}{8} + \frac{5}{8}} \begin{array}{r} 2\frac{13}{8} \\ -\ 1\frac{7}{8} \\ \hline 1\frac{6}{8} = \mathbf{1\frac{3}{4}} \end{array}$$

Example 3

Simplify: $83\frac{1}{3}\% - 41\frac{2}{3}\%$

Solution

The fraction in the subtrahend is greater than the fraction in the minuend, so we rename $83\frac{1}{3}\%$.

$$\begin{array}{r} 83\frac{1}{3}\% \\ -\ 41\frac{2}{3}\% \\ \hline \end{array} \xrightarrow{\left(82 + \frac{3}{3} + \frac{1}{3}\right)\%} \begin{array}{r} 82\frac{4}{3}\% \\ -\ 41\frac{2}{3}\% \\ \hline \mathbf{41\frac{2}{3}\%} \end{array}$$

Example 4

Simplify: $6 - 1\frac{3}{4}$

Solution

We rewrite 6 as a mixed number with a denominator of 4. Then we subtract.

$$\begin{array}{r} 6 \\ -\ 1\frac{3}{4} \\ \hline \end{array} \longrightarrow \begin{array}{r} 5\frac{4}{4} \\ -\ 1\frac{3}{4} \\ \hline \mathbf{4\frac{1}{4}} \end{array}$$

Example 5

Simplify: $100\% - 16\frac{2}{3}\%$

Solution

Thinking Skill

Explain

How has 100% regrouped to $99\frac{3}{3}\%$?

We rename 100% as $99\frac{3}{3}\%$ and subtract.

$$\begin{array}{r} 100\ \ \% \\ -\ 16\frac{2}{3}\% \\ \hline \end{array} \longrightarrow \begin{array}{r} 99\frac{3}{3}\% \\ -\ 16\frac{2}{3}\% \\ \hline \mathbf{83\frac{1}{3}\%} \end{array}$$

Practice Set Simplify:

a. There were seven pies on the shelf. If the server removes $2\frac{1}{3}$ pies, how many pies will be on the shelf?

b. $6\frac{2}{5} - 1\frac{4}{5}$

c. $5\frac{1}{6} - 1\frac{5}{6}$

d. $100\% - 12\frac{1}{2}\%$

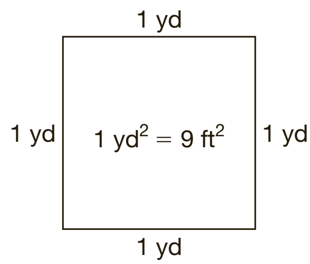
e. $83\frac{1}{3}\% - 16\frac{2}{3}\%$

Written Practice

Strengthening Concepts

1. (13) Willie shot eighteen rolls of film for the school annual. If there were thirty-six exposures in each roll, how many exposures were there in all?

2. (16) Carpeting is usually sold by the square yard. The number of square feet of area a carpet will cover is a function of the number of square yards installed. Create a function table that shows the number of square feet in 1, 2, 3, 4, and 5 square yards.



*** 3.** (19, 20) **a.** Write the formula for the area of a rectangle.

b. A professional basketball court is a rectangle that is 50 feet wide and 94 feet long. What is the area of the basketball court?

4. (13) The 16-pound turkey cost $14.24. What was the price per pound?

*** 5.** (22) **Model** Draw a diagram of the statement. Then answer the questions that follow.

Three eighths of the 56 restaurants in town were closed on Monday.

a. How many of the restaurants in town were closed on Monday?

b. How many of the restaurants in town were open on Monday?

*** 6.** (22) **Evaluate** In the following statement, write the percent as a reduced fraction. Then draw a diagram of the statement and answer the questions.

Forty percent of the 30 students in the class were boys.

a. How many boys were in the class?

b. How many girls were in the class?

Math Language
A **spheroid** is a three-dimensional object with the shape of a sphere, such as a ball.

7. (16) After contact was made, the spheroid sailed four thousand, one hundred forty inches. How many yards did the spheroid sail after contact was made?

8. (9) **a.** How many $\frac{1}{5}$s are in 1?

b. How many $\frac{1}{5}$s are in 3?

c. **Explain** How can you use the answer to **a** to solve **b?**

9. (9, 10) **Explain** Describe how to find the reciprocal of a mixed number.

10. (15) Replace each circle with the proper comparison symbol:

a. $\frac{2}{3} \cdot \frac{3}{2} \bigcirc \frac{5}{5}$ **b.** $\frac{12}{36} \bigcirc \frac{12}{24}$

11. (10, 15) Write $2\frac{1}{4}$ and $3\frac{1}{3}$ as improper fractions. Then multiply the improper fractions, and write the product as a reduced mixed number.

12. (15) Complete each equivalent fraction:

a. $\frac{3}{4} = \frac{?}{40}$ **b.** $\frac{2}{5} = \frac{?}{40}$ **c.** $\frac{?}{8} = \frac{15}{40}$

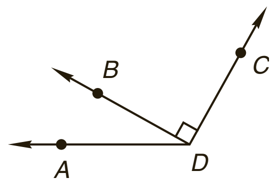
* **13.** (21) **Generalize** The prime factorization of 100 is $2 \cdot 2 \cdot 5 \cdot 5$. We can write the prime factorization of 100 using exponents this way:

$$2^2 \cdot 5^2$$

a. Write the prime factorization of 400 using exponents.

b. Write the prime factorization of $\sqrt{400}$ using exponents.

14. (7) Refer to this figure to answer **a–d:**



a. What type of angle is $\angle ADB$?

b. What type of angle is $\angle BDC$?

c. What type of angle is $\angle ADC$?

d. Which ray is perpendicular to $\overrightarrow{DB}$?

15. (15) Find fractions equivalent to $\frac{3}{4}$ and $\frac{2}{3}$ with denominators of 12. Then subtract the smaller fraction from the larger fraction.

Solve:

16. (3) $\frac{105}{w} = 7$

* **17.** (3, 20) $2x = 10^2$

18. (9, 15) $x + 1\frac{1}{4} = 6\frac{3}{4}$

19. (9, 15) $m - 4\frac{1}{8} = 1\frac{5}{8}$

* **20.** (23) **Analyze** There were five yards of fabric on the bolt of cloth. Fairchild bought $3\frac{1}{3}$ yards of the fabric. Then how many yards of fabric remained on the bolt?

Simplify:

* **21.** (23) $83\frac{1}{3}\% - 66\frac{2}{3}\%$

22. (9, 15) $\frac{7}{12} + \left(\frac{1}{4} \cdot \frac{1}{3}\right)$

23. (9, 15) $\frac{7}{8} - \left(\frac{3}{4} \cdot \frac{1}{2}\right)$

* **24.** (19) Draw $\overline{AB}$ $1\frac{3}{4}$ inches long. Then draw $\overline{BC}$ 1 inch long perpendicular to $\overline{AB}$. Complete the triangle by drawing $\overline{AC}$. Use a ruler to find the approximate length of $\overline{AC}$. Use that length to find the perimeter of $\triangle ABC$.

25. (17) Use a protractor to find the measure of $\angle A$ in problem 24. If necessary, extend the sides to measure the angle.

* **26.** (19) **Evaluate** Mary wants to apply a strip of wallpaper along the walls of the dining room just below the ceiling. If the room is a 14-by-12-ft rectangle, then the strip of wallpaper needs to be at least how long?

27. (9, 15) Multiply $\frac{3}{4}$ by the reciprocal of 3 and reduce the product.

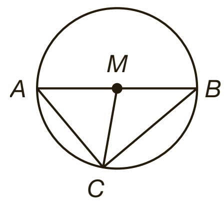
28. (18) **Model** Draw an octagon. (A stop sign is a physical example of an octagon.)

* **29.** (20) **Predict** A sequence of perfect cubes ($k = n^3$) may be written as in **a** or as in **b.** Find the next two terms of both sequences.

a. $1^3, 2^3, 3^3, \ldots$

b. 1, 8, 27, ...

* **30.** (Inv. 2) The figure shows a circle with the center at point M.



a. Which chord is a diameter?

b. Which central angle appears to be obtuse?

c. Name an inscribed angle that appears to be a right angle.

Early Finishers
Real-World Application

Zachary surveyed the 30 students in his class to find out how they get home. He found that 60% of the students ride the bus.

a. How many students ride the bus?

b. Half the students who do not ride the bus walk home. How many students walk home?

c. Based on his survey, what fraction of the students in the school might Zachary conclude walk to school?

LESSON 24

Reducing Fractions, Part 2

Power Up

Building Power

facts Power Up D

mental math

a. Number Sense: $5.74 + 98¢

b. Decimals: $1.50 × 10

c. Number Sense: $1.00 − 36¢

d. Calculation: 4 × 65

e. Calculation: $3\frac{1}{3} + 1\frac{2}{3}$

f. Fractional Parts: $\frac{1}{3}$ of 24

g. Measurement: What fraction represents 15 minutes of an hour?

h. Calculation: What number is 3 more than half the product of 4 and 6?

problem solving

Huck followed the directions on the treasure map. Starting at the big tree, he walked six paces north, turned left, and walked seven more paces. He turned left and walked five paces, turned left again, and walked four more paces. He then turned right, and took one pace. In which direction was Huck facing, and how many paces was he from the big tree?

New Concepts

Increasing Knowledge

using prime factorization to reduce

We have been practicing reducing fractions by dividing the numerator and the denominator by a common factor. In this lesson we will practice a method of reducing that uses prime factorization to find the common factors of the terms. If we write the prime factorization of the numerator and of the denominator, we can see how to reduce a fraction easily.

Math Language

The **greatest common factor** of two numbers is the greatest whole number that divides both numbers evenly.

Example 1

a. Use prime factorization to reduce $\frac{420}{1050}$.

b. Find the greatest common factor of 420 and 1050.

Solution

a. We rewrite the numerator and the denominator as products of prime numbers.

$$\frac{420}{1050} = \frac{2 \cdot 2 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 3 \cdot 5 \cdot 5 \cdot 7}$$

Next we look for pairs of factors that form a fraction equal to 1. A fraction equals 1 if the numerator and denominator are equal. In this fraction there are four pairs of numerators and denominators that equal 1. They are $\frac{2}{2}$, $\frac{3}{3}$, $\frac{5}{5}$, and $\frac{7}{7}$. Below we have indicated each of these pairs.

$$\frac{2 \cdot 2 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 3 \cdot 5 \cdot 5 \cdot 7}$$

Each pair reduces to $\frac{1}{1}$.

$$\frac{\overset{1}{\cancel{2}} \cdot 2 \cdot \overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{7}}}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{3}} \cdot 5 \cdot \underset{1}{\cancel{5}} \cdot \underset{1}{\cancel{7}}}$$

The reduced fraction equals $1 \cdot 1 \cdot 1 \cdot 1 \cdot \frac{2}{5}$, which is $\mathbf{\frac{2}{5}}$.

Math Language

A **prime factor** is a factor that is a prime number.

b. In **a** we found the common prime factors of 420 and 1050. The common prime factors are 2, 3, 5, and 7. The product of these prime factors is the greatest common factor of 420 and 1050.

$$2 \cdot 3 \cdot 5 \cdot 7 = \mathbf{210}$$

Explain How could you have used this greatest common factor to reduce $\frac{420}{1050}$?

Example 2

A set of alphabet cards includes one card for each letter of the alphabet. If one card is drawn from the set of cards, what is the probability of drawing a vowel, including y?

Solution

The vowels are a, e, i, o, u, and we are told to include y. So the probability of drawing a vowel card is 6 in 26, which we can reduce.

$$\frac{6}{26} = \frac{\overset{1}{\cancel{2}} \cdot 3}{\underset{1}{\cancel{2}} \cdot 13} = \mathbf{\frac{3}{13}}$$

reducing before multiplying

When multiplying fractions, we often get a product that can be reduced even though the individual factors could not be reduced. Consider this multiplication:

$$\frac{3}{8} \cdot \frac{2}{3} = \frac{6}{24} \qquad \frac{6}{24} \text{ reduces to } \frac{1}{4}$$

We see that neither $\frac{3}{8}$ nor $\frac{2}{3}$ can be reduced. The product, $\frac{6}{24}$, can be reduced. We can avoid reducing after we multiply by reducing before we multiply. Reducing before multiplying is also known as **canceling.** To reduce, any numerator may be paired with any denominator. Below we have paired the 3 with 3 and the 2 with 8.

$$\frac{3}{8} \cdot \frac{2}{3}$$

Then we reduce these pairs: $\frac{3}{3}$ reduces to $\frac{1}{1}$, and $\frac{2}{8}$ reduces to $\frac{1}{4}$, as we show below. Then we multiply the reduced terms.

$$\frac{\overset{1}{\cancel{3}}}{\underset{4}{\cancel{8}}} \cdot \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} = \frac{1}{4}$$

Discuss Why did we pair the 3 with the 3 and the 2 with the 8?

Example 3

Simplify: $\frac{9}{16} \cdot \frac{2}{3}$

Solution

Before multiplying, we pair 9 with 3 and 2 with 16 and reduce these pairs. Then we multiply the reduced terms.

$$\frac{\overset{3}{\cancel{9}}}{\underset{8}{\cancel{16}}} \cdot \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} = \mathbf{\frac{3}{8}}$$

Example 4

Simplify: $\frac{8}{9} \cdot \frac{3}{10} \cdot \frac{5}{4}$

Solution

We mentally pair 8 with 4, 3 with 9, and 5 with 10 and reduce.

$$\frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{9}}} \cdot \frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{10}}} \cdot \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{4}}}$$

We can still reduce by pairing 2 with 2. Then we multiply.

$$\frac{\overset{\overset{1}{\cancel{2}}}{\cancel{8}}}{\underset{3}{\cancel{9}}} \cdot \frac{\overset{1}{\cancel{3}}}{\underset{\underset{1}{\cancel{2}}}{\cancel{10}}} \cdot \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{4}}} = \mathbf{\frac{1}{3}}$$

Example 5

Simplify: $\frac{27}{32} \cdot \frac{20}{63}$

Solution

To give us easier numbers to work with, we factor the terms of the fractions before we reduce and multiply.

$$\frac{3 \cdot \overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{3}}}{2 \cdot 2 \cdot 2 \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}}} \cdot \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot 5}{\underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{3}} \cdot 7} = \mathbf{\frac{15}{56}}$$

Explain What is another method we could have used to simplify the expression in Example 5? Which method do you prefer and why?

Practice Set

Generalize Use prime factorization to reduce each fraction:

a. $\frac{48}{144}$

b. $\frac{90}{324}$

c. Find the greatest common factor of 90 and 324.

Reduce before multiplying:

d. $\frac{5}{8} \cdot \frac{3}{10}$

e. $\frac{8}{15} \cdot \frac{5}{12} \cdot \frac{9}{10}$

f. $\frac{8}{3} \cdot \frac{6}{7} \cdot \frac{5}{16}$

Math Language

To **factor** a number means to write it as the product of factors. For example: $\frac{24}{28} = \frac{4 \cdot 6}{4 \cdot 7}$.

g. Factor and reduce before multiplying: $\frac{36}{45} \cdot \frac{25}{24}$

h. Of the 900 students at Columbia Middle School, there are 324 seventh graders. If one student is chosen at random to lead the pledge at a school assembly, what is the probability the person chosen will be a seventh grader? Show how to use prime factorization to reduce the answer.

i. **Justify** How can you use reducing fractions to demonstrate that the product of a fraction and its reciprocal is 1?

Written Practice

Strengthening Concepts

1. (12) From Hartford to Los Angeles is two thousand, eight hundred ninety-five miles. From Hartford to Portland is three thousand, twenty-six miles. The distance from Hartford to Portland is how much greater than the distance from Hartford to Los Angeles?

2. (13) Hal ordered 15 boxes of microprocessors. If each box contained two dozen microprocessors, how many microprocessors did Hal order?

* **3.** (22) In the following statement, write the percent as a fraction. Then draw a diagram and answer the questions.

Ashanti went to the store with $30.00 and spent 75% of the money.

a. What fraction of the money did she spend?

b. How much money did she spend?

* **4.** (16, Inv. 2) **a.** If the diameter of a wheel is one yard, then its radius is how many inches?

b. **Justify** Write a statement telling how you know your solution to **a** is correct.

* **5.** (22) Nancy descended the 30 steps that led to the floor of the cellar. One third of the way down she paused. How many more steps were there to the cellar floor?

6. (9) **a.** How many $\frac{1}{8}$s are in 1?

b. How many $\frac{1}{8}$s are in 3?

7. (9) **a.** Write the reciprocal of 3.

b. What fraction of 3 is 1?

* **8.** (24) **a.** Use prime factorization to reduce $\frac{540}{600}$.

b. What is the greatest common factor of 540 and 600?

9. (17) What type of angle is formed by the hands of a clock at

a. 2 o'clock? **b.** 3 o'clock? **c.** 4 o'clock?

10. (15) **a.** *Explain* Describe how to complete this equivalent fraction.

$$\frac{3}{5} = \frac{?}{30}$$

b. *Justify* Name the property we use to find equivalent fractions.

* **11.** (21) *Generalize* The prime factorization of 1000 using exponents is $2^3 \cdot 5^3$.

a. Write the prime factorization of 10,000 using exponents.

b. Write the prime factorization of $\sqrt{10{,}000}$ using exponents.

12. (7) **a.** *Model* Draw two parallel lines that are intersected by a third line perpendicular to the parallel lines.

b. What type of angles are formed?

* **13.** (20) The perimeter of a square is one yard.

a. How many inches long is each side of the square?

b. What is the area of the square in square inches?

14. (2) This equation illustrates that which property does not apply to division?

$$10 \div 5 \neq 5 \div 10$$

15. (7) The front and back covers of a closed book represent two planes that are

A parallel **B** skew **C** intersecting **D** perpendicular

16. (13) *Formulate* Write and solve a word problem about equal groups that fits this equation.

$$12p = \$3.36$$

Solve:

17. (9, 15) $4\frac{7}{12} = x + 1\frac{1}{12}$

18. (9, 15) $w - 3\frac{3}{4} = 2\frac{3}{4}$

Simplify:

* **19.** (20) $10^5 \div 10^2$

* **20.** (20) $\sqrt{9} - \sqrt{4^2}$

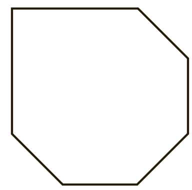
* **21.** (23) $100\% - 66\frac{2}{3}\%$

* **22.** (23) $5\frac{1}{8} - 1\frac{7}{8}$

* **23.** (20) $\left(\frac{5}{6}\right)^2$

* **24.** (24) $\frac{3}{4} \cdot \frac{1}{2} \cdot \frac{8}{9}$

25. (18) Kevin clipped three corners from a square sheet of paper. What is the name of the polygon that was formed?

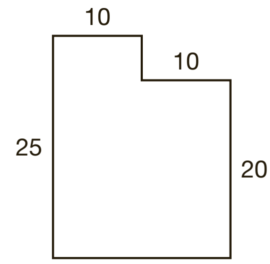


26. (1, 4) *Evaluate* Evaluate the following expressions for $a = 10$ and $b = 100$:

a. ab **b.** $a - b$ **c.** $\frac{a}{b}$

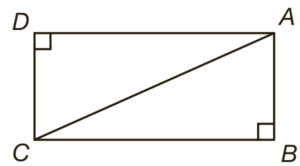
* **27.** (19) **a.** *Generalize* Find the perimeter of the figure at right. Dimensions are in yards. All angles are right angles.

b. *Justify* How did you find the missing measures of two sides of the figure?



28. (15) Find equivalent fractions for $\frac{1}{4}$ and $\frac{1}{6}$ that have denominators of 12. Then add them.

* **29.** (18) *Generalize* Segment *AC* divides rectangle *ABCD* into two congruent triangles. Angle *ADC* corresponds to $\angle CBA$. Name two more pairs of corresponding angles.



30. (4, 9) **a.** Arrange these numbers in order from least to greatest:

$$0, 1, -1, \frac{1}{2}, -\frac{1}{2}$$

b. *Predict* The ordered numbers in the answer to part **a** form a sequence. What are the next three positive numbers in the sequence?

LESSON 25

Dividing Fractions

Power Up — *Building Power*

facts Power Up F

mental math

a. Number Sense: $2.65 + $1.99

b. Decimals: $60.00 ÷ 10

c. Number Sense: $2.00 − $1.24

d. Calculation: 7×36

e. Calculation: $1\frac{3}{4} + 4\frac{1}{4}$

f. Fractional Parts: $\frac{1}{4}$ of 36

g. Measurement: What fraction represents 30 seconds of a minute?

h. Calculation: What number is 3 less than half the sum of 8 and 12?

problem solving Copy the problem and fill in the missing digits:

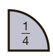
```
   3 6
 × _ _
 -----
   _ _ 0
 + _ 6
 -----
   _ _ _
```

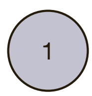
New Concept — *Increasing Knowledge*

"How many quarters are in a dollar?" is a way to ask, "How many $\frac{1}{4}$s are in 1?" This question is a division question:

$$1 \div \frac{1}{4}$$

We can model the question with the fraction manipulatives we used in Investigation 1.

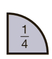
How many 

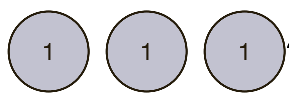
are in 

?

We see that the answer is 4. Recall that 4 (or $\frac{4}{1}$) is the reciprocal of $\frac{1}{4}$. Likewise, when we ask the question, "How many quarters ($\frac{1}{4}$s) are in three dollars (3)?" we are again asking a division question.

$$3 \div \frac{1}{4}$$

How many 

are in 

?

We can use the answer to the first question to help us answer the second question. There are four $\frac{1}{4}$s in 1, so there must be *three times as many* $\frac{1}{4}$s in 3. Thus, there are twelve $\frac{1}{4}$s in 3. We found the answer to the second question by multiplying 3 by 4, the answer to the first question. We will follow this same line of thinking in the next few examples.

Analyze your own thinking about this question: How many quarters are in five dollars? Our thinking probably takes two steps: (1) There are 4 quarters in a dollar, (2) So there are $5 \times 4 = 20$ quarters in five dollars.

Summarize How could you use this same thinking to find out how many dimes are in 5 dollars?

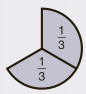
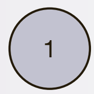
Example 1

a. How many $\frac{2}{3}$s are in 1? $(1 \div \frac{2}{3})$

b. How many $\frac{2}{3}$s are in 3? $(3 \div \frac{2}{3})$

Solution

a. We may model the question with manipulatives.

How many  are in  ?

We see from the manipulatives that the answer is more than 1 but less than 2. If we think of the two $\frac{1}{3}$ pieces as one piece, we see that another *half* of the $\frac{2}{3}$ piece would make a whole. Thus there are $\mathbf{\frac{3}{2}}$ (or $\mathbf{1\frac{1}{2}}$) $\frac{2}{3}$s in 1. Notice that the answer to question **a** is the reciprocal of $\frac{2}{3}$.

b. We use the answer to **a** to help us answer **b.** There are $\frac{3}{2}$ (or $1\frac{1}{2}$) $\frac{2}{3}$s in 1, so there are three times as many $\frac{2}{3}$s in 3. Thus, we answer the question by multiplying 3 by $\frac{3}{2}$ (or 3 by $1\frac{1}{2}$).

$$3 \times \frac{3}{2} = \frac{9}{2} = 4\frac{1}{2} \qquad 3 \times 1\frac{1}{2} = 1\frac{1}{2} + 1\frac{1}{2} + 1\frac{1}{2} = 4\frac{1}{2}$$

The number of $\frac{2}{3}$s in 3 is $\mathbf{4\frac{1}{2}}$. We found the answer by multiplying 3 by the reciprocal of $\frac{2}{3}$.

Thinking Skill

Summarize

What rule can we state about the division of the number 1 by a fraction?

Example 2

a. $\mathbf{1 \div \frac{2}{5}}$ **b.** $\mathbf{\frac{3}{4} \div \frac{2}{5}}$

Solution

a. The problem $1 \div \frac{2}{5}$ means, "How many $\frac{2}{5}$s are in 1?" The answer is the reciprocal of $\frac{2}{5}$, which is $\mathbf{\frac{5}{2}}$.

$$1 \div \frac{2}{5} = \frac{5}{2}$$

b. We use the answer to **a** to help us answer **b.** There are $\frac{5}{2}$ (or $2\frac{1}{2}$) $\frac{2}{5}$s in 1, so there are $\frac{3}{4}$ times as many $\frac{2}{5}$s in $\frac{3}{4}$. Thus we multiply $\frac{3}{4}$ by $\frac{5}{2}$.

$$\frac{3}{4} \times \frac{5}{2} = \frac{15}{8}$$

$$= 1\frac{7}{8}$$

The number of $\frac{2}{5}$s in $\frac{3}{4}$ is $\mathbf{1\frac{7}{8}}$. We found the answer by multiplying $\frac{3}{4}$ by the reciprocal of $\frac{2}{5}$.

Example 3

$$\mathbf{\frac{2}{3} \div \frac{3}{4}}$$

Solution

To find how many $\frac{3}{4}$s are in $\frac{2}{3}$, we take two steps. First we find how many $\frac{3}{4}$s are in 1. The answer is the reciprocal of $\frac{3}{4}$.

$$1 \div \frac{3}{4} = \frac{4}{3}$$

Then we use this reciprocal to find the number of $\frac{3}{4}$s in $\frac{2}{3}$. The number of $\frac{3}{4}$s in $\frac{2}{3}$ is $\frac{2}{3}$ times as many $\frac{3}{4}$s as are in 1. So we multiply $\frac{2}{3}$ by $\frac{4}{3}$.

$$\frac{2}{3} \times \frac{4}{3} = \mathbf{\frac{8}{9}}$$

This means there is slightly less than one $\frac{3}{4}$ in $\frac{2}{3}$. We found the answer by multiplying $\frac{2}{3}$ by the reciprocal of $\frac{3}{4}$.

Conclude Complete this sentence to create a rule for dividing fractions: To find the quotient of two fractions, ______ the dividend by the ______ of the ______.

Example 4

Sam walks $\frac{9}{10}$ of a mile to school. On his way to school he passes a bank which is $\frac{3}{4}$ of a mile from his home. What fraction of his walk has Sam completed when he reaches the bank?

Solution

The whole walk is the distance to school. The part is the distance to the bank. The fraction is "part over whole."

$$\frac{\text{part}}{\text{whole}} \quad \frac{\frac{3}{4}}{\frac{9}{10}}$$

Math Language

A **compound fraction** is a fraction whose numerator and/or denominator are also fractions.

This **compound fraction** means $\frac{3}{4}$ divided by $\frac{9}{10}$. We perform the division to find the answer.

$$\frac{3}{4} \div \frac{9}{10}$$

$$\frac{3}{4} \times \frac{10}{9} = \frac{5}{6}$$

When Sam reaches the bank he has completed $\mathbf{\frac{5}{6}}$ of his walk.

Find the number of $\frac{9}{10}$s in 1.

$$1 \div \frac{9}{10} = \frac{10}{9}$$

Use the number of $\frac{9}{10}$s in 1 to find the number of $\frac{9}{10}$s in $\frac{3}{4}$.

$$\frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{4}}} \times \frac{\overset{5}{\cancel{10}}}{\underset{3}{\cancel{9}}} = \mathbf{\frac{5}{6}}$$

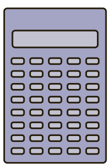
Working on paper, we often move from the original problem directly to step 2 by multiplying the dividend, the first fraction, by the reciprocal of the divisor, the second fraction.

$$\frac{3}{4} \div \frac{9}{10}$$

$$\frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{4}}} \times \frac{\overset{5}{\cancel{10}}}{\underset{3}{\cancel{9}}} = \mathbf{\frac{5}{6}}$$

When Sam reaches the bank, he is $\frac{5}{6}$ of the way to school.

Extend How far is the bank from Sam's school?



The reciprocal function on a calculator is the [1/x] key. Pressing this key changes the previously entered number to its reciprocal (in decimal form). If we press [2] then [1/x], the calculator display changes from 2 to 0.5, which is the reciprocal of 2 in decimal form ($\frac{1}{2} = 0.5$). The [1/x] key can be helpful when dividing. Consider this division problem.

$$144\overline{)\$10{,}461.60}$$

The divisor is 144. You could choose to divide \$10,461.60 by 144 or to multiply \$10,461.60 by the reciprocal of 144. Since multiplication is commutative, using the reciprocal allows you to enter the numbers in either order. The following multiplication yields the answer even though the entry begins with the divisor. Notice that we drop the terminal zero from \$10,461.60, since it does not affect the value.

[1] [4] [4] [1/x] [×] [1] [0] [4] [6] [1] [.] [6] [=]

Whether we choose to divide \$10,461.60 by 144 or to multiply by the reciprocal of 144, the answer is \$72.65.

Practice Set

a. How many $\frac{2}{3}$s are in 1? How many $\frac{2}{3}$s are in $\frac{3}{4}$?

b. How many $\frac{3}{4}$s are in 3?

c. Describe the two steps for finding the number of quarters in six dollars.

d. **Explain** Tell how to use the reciprocal of the divisor to find the answer to a division problem.

e. Describe the function of the $1/x$ key on a calculator.

Generalize Use the two-step method described in this lesson to find each quotient:

f. $\frac{3}{5} \div \frac{2}{3}$ **g.** $\frac{7}{8} \div \frac{1}{4}$ **h.** $\frac{5}{6} \div \frac{2}{3}$

i. Amanda has a ribbon $\frac{3}{4}$ of a yard long. She used $\frac{1}{2}$ of a yard of ribbon for a small package. What fraction of her ribbon did Amanda use? Write a fraction division problem for this story and show the steps. Then write the answer in a sentence.

Written Practice *Strengthening Concepts*

1. (13) Three hundred twenty-four students were given individual boxes of apple juice at lunch in the school cafeteria. If each pack of apple juice contained a half dozen individual boxes of juice, how many packages of juice were used?

2. (17, 19) Use a ruler to draw square *ABCD* with sides $2\frac{1}{2}$ in. long. Then divide the square into two congruent triangles by drawing $\overline{AC}$.

a. What is the perimeter of square *ABCD?*

b. What is the measure of each angle of the square?

c. What is the measure of each acute angle in $\triangle ABC$?

d. What is the sum of the measures of the three angles in $\triangle ABC$?

*** 3.** (11, 13, 22) **Evaluate** Use this information to answer questions **a–c.**

The family reunion was a success, as 56 relatives attended. Half of those who attended played in the big game. However, the number of players on the two teams was not equal since one team had only 10 players.

a. How many relatives played in the game?

b. If one team had 10 players, how many players did the other team have?

c. If the teams were rearranged so that the number of players on each team was equal, how many players would be on each team?

*** 4.** (16, Inv. 2) **Represent** The diameter of a circle is a function of the radius of the circle. Make a function table that shows the diameters of circles with radii that are $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ and 1 unit long.

5. (21) Use prime factorization to find the greatest common factor of 72 and 54.

*** 6.** (22) In the following statement, write the percent as a reduced fraction. Then diagram the statement and answer the questions.

Jason has read 75% of the 320 pages in the book.

a. How many pages has Jason read?

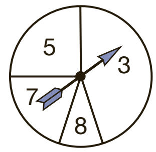
b. How many pages has Jason not read?

* **7.** (25) **a.** How many $\frac{3}{4}$s are in 1?

b. How many $\frac{3}{4}$s are in $\frac{7}{8}$?

8. (8, 14) **Estimate** Which is the best estimate of the probability of spinning a 3? Why?

A $\frac{2}{3}$ **B** $\frac{2}{4}$ **C** $\frac{2}{5}$



* **9.** (24) **a.** Write 84 and 210 as products of prime numbers. Then reduce $\frac{84}{210}$.

b. What is the greatest common factor of 84 and 210?

10. (9, 10) Write the reciprocal of each number:

a. $\frac{9}{10}$ **b.** 8 **c.** $2\frac{3}{8}$

d. What is the product of $2\frac{3}{8}$ and its reciprocal?

e. **Generalize** What rule do you know about reciprocals that could have helped you answer **d?**

11. (15) Find fractions equivalent to $\frac{3}{4}$ and $\frac{4}{5}$ with denominators of 20. Then add the two fractions you found, and write the sum as a mixed number.

* **12.** (21) **a.** The prime factorization of 40 is $2^3 \cdot 5$. Write the prime factorization of 640 using exponents.

b. Tell how you can use a calculator to verify your answer to **12a.** Then follow your procedure.

* **13.** (10, 24) Write $2\frac{2}{3}$ and $2\frac{1}{4}$ as improper fractions. Then find the product of the improper fractions.

14. (8, 15) **a.** Points *A* and *B* represent what mixed numbers on this number line?

b. Find the difference between the numbers represented by points *A* and *B.*

15. (17) **a.** Draw line *AB*. Then draw ray *BC* so that angle *ABC* measures 30°. Use a protractor.

b. What type of angle is angle *ABC?*

Solve:

* **16.** (23) $1\frac{7}{12} + y = 3$

17. (9, 15) $5\frac{7}{8} = x - 4\frac{5}{8}$

18. (3) $8n = 360°$

19. (9, 20) $\frac{4}{3}m = 1^3$

Simplify:

20. (10) $6\frac{1}{6} + 1\frac{5}{6}$

***21.** (24) $\frac{3}{4} \cdot \frac{5}{9} \cdot \frac{8}{15}$

* **22.** (25) $\frac{4}{5} \div \frac{2}{1}$

* **23.** (25) $\frac{8}{5} \div \frac{6}{5}$

24. (25) $\frac{3}{7} \div \frac{5}{6}$

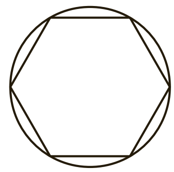
25. (10) $\frac{100\%}{8}$

* **26.** (26) **Generalize** In the division $5 \div \frac{3}{5}$, instead of dividing 5 by $\frac{3}{5}$, we can find the answer by multiplying 5 by what number?

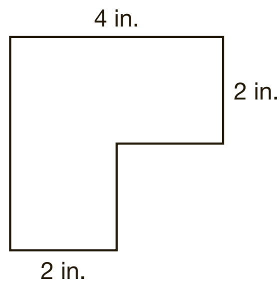
27. (20) **a.** Simplify and compare: $2^2 \cdot 2^3 \bigcirc 2^3 \cdot 2^2$

b. Simplify: $\sqrt{2^2}$

28. (19) A regular hexagon is inscribed in a circle. If one side of the hexagon is 6 inches long, then the perimeter of the hexagon is how many feet?



29. (19) A 2-in. square was cut from a 4-in. square as shown in the figure. What is the perimeter of the resulting polygon?



* **30.** (4, 21) Which negative integer is the opposite of the third prime number?