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Lesson 24: Congruence Criteria for Triangles—ASA and SSS

Student Outcomes

- Students learn why any two triangles that satisfy the ASA or SSS congruence criteria must be congruent.

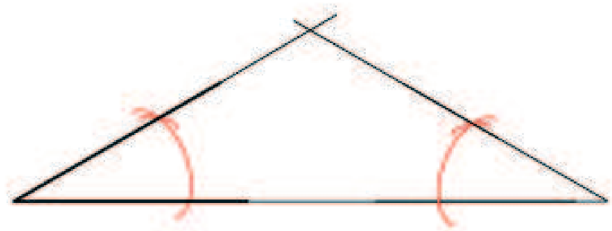
Lesson Notes

This is the third lesson in the Congruency topic. So far students have studied the SAS triangle congruence criteria and how to prove base angles of an isosceles triangle congruent. Students examine two more triangle congruence criteria in this lesson: ASA and SSS. Each proof assumes the initial steps from the proof of SAS; ask students to refer to their notes on SAS to recall these steps before proceeding with the rest of the proof. Exercises will require the use of all three triangle congruence criteria.

Classwork

Opening Exercise (7 minutes)

Use the provided 30˚ angle as one base angle of an isosceles triangle. Use a compass and straight edge to construct an appropriate isosceles triangle around it.



Compare your constructed isosceles triangle with a neighbor's. Does the use of a given angle measure guarantee that all the triangles constructed in class have corresponding sides of equal lengths?

No; side lengths may vary.

Discussion (25 minutes)

Today we are going to examine two more triangle congruence criteria, Angle-Side-Angle (ASA) and Side-Side-Side (SSS), to add to the SAS criteria we have already learned. We begin with the ASA criteria.

Angle-Side-Angle triangle congruence criteria (ASA): Given two triangles ABC and A'B'C'. If $\angle CAB = \angle C'A'B'$ (Angle), $AB = A'B'$ (Side), and $\angle CBA = \angle C'B'A'$ (Angle), then the triangles are congruent.

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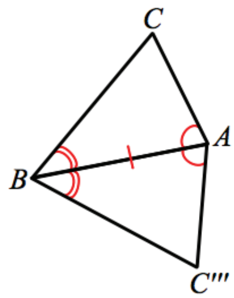
Proof

We do not begin at the very beginning of this proof. Revisit your notes on the SAS proof and recall that there are three cases to consider when comparing two triangles. In the most general of cases, when comparing two distinct triangles, we translate one vertex to another (choose congruent corresponding angles). A rotation brings congruent corresponding sides together. Since the ASA criteria allows for these steps, we begin here.



In order to map $\triangle ABC'''$ to $\triangle ABC$, we apply a reflection Λ across the line AB. A reflection will map A to A and B to B, since they are on line AB. However, we will say that $\Lambda(C''') = C^*$; though we know that $\Lambda(C''')$ is now in the same half-plane of line AB as C, we cannot assume that C''' maps to C. So we have $\Lambda(\triangle ABC''') = \triangle ABC^*$. To prove the theorem, we need to verify that C^* is C.

By hypothesis, we know that $\angle CAB = \angle C'''AB$ (recall that $\angle C'''AB$ is the result of two rigid motions of $\angle C'A'B'$, so must have the same angle measure as $\angle C'A'B'$). Similarly, $\angle CBA = \angle C'''BA$. Since $\angle CAB = \Lambda(\angle C'''AB) = \angle C^*AB$, and C and C^* are in the same half-plane of line AB, we conclude that the rays, $\overrightarrow{AC}$ and $\overrightarrow{AC^*}$, must actually be the same ray. Because the points A and C^* define the same ray as $\overrightarrow{AC}$, the point C^* must be a point on the ray $\overrightarrow{AC}$ somewhere. Using the second equality of angles, $\angle CBA = \Lambda(\angle C'''BA) = \angle C^*BA$, we can also conclude that the rays, $\overrightarrow{BC}$ and $\overrightarrow{BC^*}$, must be the same ray. Therefore, the point C^* must also be on the ray $\overrightarrow{BC}$. Since C^* is in both of the rays, $\overrightarrow{AC}$ and $\overrightarrow{BC}$, and the two rays only have one point in common, namely C, we conclude that $C = C^*$.

We have now used a series of rigid motions to map two triangles that meet the ASA criteria onto one another.

Side-Side-Side triangle congruence criteria (SSS): Given two triangles ABC and A'B'C'. If $AB = A'B'$ (Side), $AC = A'C'$ (Side), and $BC = B'C'$ (Side) then the triangles are congruent.

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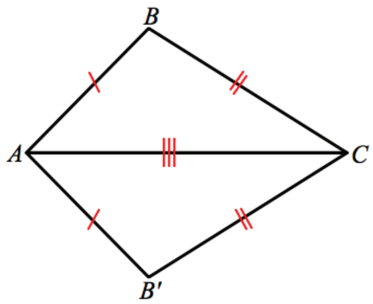
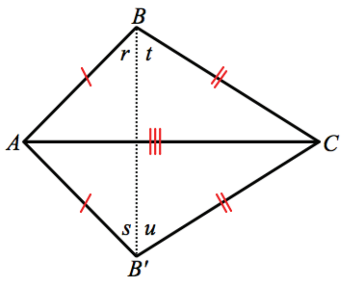
Proof

Again, we do not start at the beginning of this proof, but assume there is a congruence that brings a pair of corresponding sides together, namely the longest side of each triangle.



Without any information about the angles of the triangles, we cannot perform a reflection as we have in the proofs for SAS and ASA. What can we do? First we add a construction: draw an auxiliary line from B to B', labeling the angles created by the auxiliary line as r, s, t, and u.



Since $\mathrm{AB} = AB'$ and $CB = CB'$, $\triangle ABB'$ and $\triangle CBB'$ are both isosceles triangles respectively by definition. Therefore, $r = s$, because they are base angles of an isosceles triangle $\triangle ABB'$. Similarly, $t = u$, because they are base angles of $\triangle CBB'$. Hence, $\angle ABC = r + t = s + u = \angle AB'C$. Since $\angle ABC = \angle AB'C$, we say that $\triangle ABC \cong \triangle AB'C$ by SAS.

We have now used a series of rigid motions and a construction to map two triangles that meet the SSS criteria onto one another.

Now we have three triangle congruence criteria at our disposal: SAS, ASA, and SSS. We will use these criteria to determine whether or not pairs of triangles are congruent.

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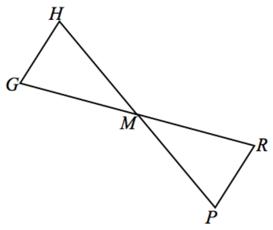
Exercises (6 minutes)

Based on the information provided, determine whether a congruence exists between triangles. If a congruence between triangles exists, or if multiple congruencies exist, state the congruencies and the criteria used to determine them.

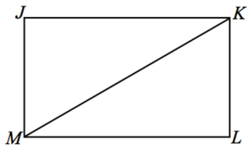
1. **Given:** M is the midpoint of HP, $\angle H = \angle P$.

   $\triangle GHM \cong \triangle RPM$, ASA



2. **Given:** Rectangle $JKLM$ with diagonal KM.

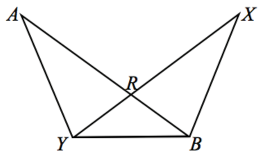
   $\triangle JKM \cong \triangle LMK$, SSS/SAS/ASA



3. **Given:** $RY = RB$, $AR = XR$.

   $\triangle ARY \cong \triangle XRB$, SAS

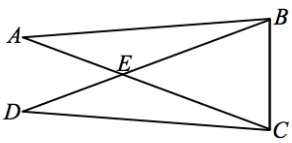
   $\triangle ABY \cong \triangle XBY$, SAS



4. **Given:** $\angle A = \angle D$, $AE = DE$.

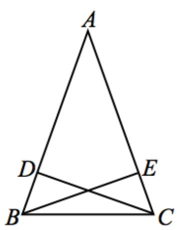
   $\triangle AEB \cong \triangle DEC$, SAS

   $\triangle DBC \cong \triangle ACB$, SAS/ASA



5. **Given:** $AB = AC$, $BD = \frac{1}{4}AB$, $CE = \frac{1}{4}AC$.

   $\triangle ABE \cong \triangle ACD$, SAS



Exit Ticket

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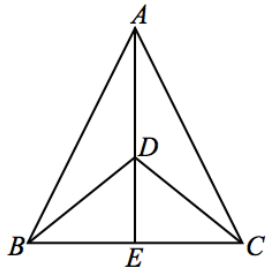
Exit Ticket

Based on the information provided, determine whether a congruence exists between triangles. If a congruence between triangles exists, or if multiple congruencies exist, state the congruencies and the criteria used to determine them.

Given: $BD = CD$, E is the midpoint of BC.



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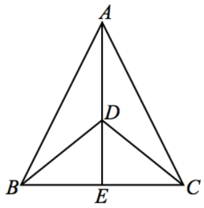
Exit Ticket Sample Solutions

The following solutions indicate an understanding of the objectives of this lesson:

Based on the information provided, determine whether a congruence exists between triangles. If a congruence between triangles exists, or if multiple congruencies exist, state the congruencies and the criteria used to determine them.

Given: $BD = CD$, E is the midpoint of BC.



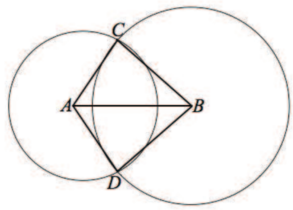
$\triangle ABE \cong \triangle ACD$, SAS

Problem Set Sample Solutions

Use your knowledge of triangle congruence criteria to write proofs for each of the following problems.

1. **Given:** Circles with centers A and B intersect at C and D.

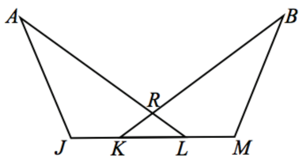
   Prove: $\angle CAB \cong \angle DAB$.



CA=DA	*Radius of Circle*
CB=DB	*Radius of Circle*
AB=AB	*Common Side*
$\Delta CAB \cong \Delta DAB$	*SSS*
$\angle CAB \cong \angle DAB$	*Corr.* $\angle s$ *of* Δ

2. **Given:** $\angle J = \angle M, JA = MB, JK = KL = LM$.

   Prove: $KR = LR$.



$\angle J = \angle M$	*Given*
$JA = MB$	*Given*
$JK = KL = LM$	*Given*
JL = JK +KL	*Segments Add*
KM = KL +LM	*Segments Add*
JL = KM	*Substitution*
$\Delta AJL \cong \Delta BMK$	*SAS*
$\angle RKL \cong \angle RLK$	*Corr.* $\angle s$ *of* $\cong \Delta$
KR =LR	*Base* $\angle s$ *converse*

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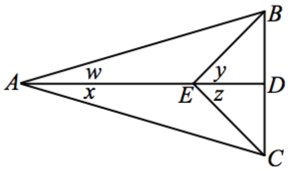
3. **Given:** $\angle w = \angle x$ and $\angle y = \angle z$.

Prove: (1) $\triangle ABE \cong \triangle ACE$

(2) $AB = AC$ and $AD \perp BC$



$\angle y = \angle z$	*Given*
$\angle AEB = \angle AEC$	*Supplements of = ∠s*
$AE = AE$	*Common Side*
$\angle w = \angle x$	*Given*
$\therefore \Delta ABE \cong \Delta ACE$	*ASA*
$\therefore AB \cong AC$	*corr. sides of ≅ Δ*
$AD = AD$	*Common Side*
$\Delta CAD \cong \Delta BAD$	*SAS*
$\angle ADC \cong \angle ADB$	*corr. ∠s of ≅ Δ*
$\angle ADC + \angle ADB = 180^\circ$	*∠s on a line*
$2(\angle ADC) = 180$	
$\angle ADC = 90^\circ$	
$AD \perp BC$	*Def. of ⊥*