

Lesson 24: Two-Variable Linear Equations

LESSON 24: Two-Variable Linear Equations part 1

Weekly Focus: two-variable equations
Weekly Skill: solve

Lesson Summary: For the warm-up, students will solve a problem about a gym membership. In Activity 1, they will write equations with one variable. In Activity 2, they will solve equations by the substitution method. In Activity 3, they will solve equations by the combination method. There is an exit ticket at the end. Estimated time for the lesson is 2 hours.

Materials Needed for Lesson 24:

- Video A (length 9:58) on Method of Substitution
- Video B (length 8:03) on Method of Elimination (Combination). The videos are required for teachers and recommended for students.
- 2 Notes Handouts for students (24.2 and 24.4)
- 3 Worksheets (24.1, 24.3, 24.5) with answers (embedded links or attached)

Objectives: Students will be able to:

- Write equations for one variable word problems
- Solve two-variable equations with the substitution method
- Solve two-variable equations with the elimination/combination method

ACES Skills Addressed: N, CT, LS, ALS

CCRS Mathematical Practices Addressed: Attend to Precision, Look for and Make Use of Structure
Mathematical Fluency

Levels of Knowing Math Addressed: Intuitive, Abstract, and Application

Notes:

You can add more examples if you feel students need them before they work. Any ideas that concretely relate to their lives make good examples.

For more practice as a class, feel free to choose some of the easier problems from the worksheets to do together. The “easier” problems are not necessarily at the beginning of each worksheet. Also, you may decide to have students complete only part of the worksheets in class and assign the rest as homework or extra practice.

The GED Math test is 115 minutes long and includes approximately 46 questions. The questions have a focus on quantitative problem solving (45%) and algebraic problem solving (55%).

Students must be able to understand math concepts and apply them to new situations, use logical reasoning to explain their answers, evaluate and further the reasoning of others, represent real world problems algebraically and visually, and manipulate and solve algebraic expressions.

This computer-based test includes questions that may be multiple-choice, fill-in-the-blank, choose from a drop-down menu, or drag-and-drop the response from one place to another.

The purpose of the GED test is to provide students with the skills necessary to either further their education or be ready for the demands of today's careers.

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Lesson 24 Warm-up: Solve the gym membership question

Time: 5-10 Minutes

Write on the board: Jacob joined a gym last year. His total bill for the year was \$410 including a monthly membership fee and a one-time \$50 enrollment fee.

Basic Questions:

- How much was his monthly fee?
 - $\$410 - \$50 = \$360 / 12 \text{ months} = \30 a month
- Jacob's employer will reimburse him 20% of his monthly membership. How much will his reimbursement be?
 - $0.2 (30) = \$6$

Extension Questions:

- Use the information in the question to write an equation. Let m = monthly fee.
 - $50 + 12m = 410$
- Solve the problem by solving the equation.
 - $50 - 50 + 12m = 410 - 50$
 - $12m = 360$
 - $m = 30$

Lesson 24 Activity 1: Write Equations for Word Problems

Time: 15-20 Minutes

1. You will practice writing basic algebraic equations with one variable. When might this be useful? Examples of real life situations when you may need this skill are if you're trying to figure out if you got paid properly for a job or if you've been charged correctly for a bill. Algebra helps us solve a problem when we don't know all of the numbers yet.
2. Example A: The cell phone company charges \$44.95 a month and \$0.25 per text message. The bill for the month is \$60.45. How many texts (x) were sent?
 - a. $\$60.45 = \$44.95 + \$0.25x$
 - b. $60.45 - 44.95 = 44.95 - 44.95 + 0.25x$
 - c. $15.50 = 0.25x$
 - d. $x = 62 \text{ texts}$

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3. Example B: The sum of two consecutive even numbers is 30. Find the numbers.

- a. Point out to students that *is means equal*
- b. $x =$ first number
- c. $x+2 =$ second number
- d. $x + x + 2 = 30$
- e. $2x + 2 = 30$
- f. $2x = 28$
- g. $x= 14 =$ first number, so the second number is 16

Note: students might solve differently by trial and error but they also need to know how to set up equation.

4. Do **Worksheet 24.1**. Do one together first.

Lesson 24 Activity 2: Substitution Method

Time: 40 Minutes

1. Students just wrote and solved equations with one variable. Sometimes they also need to solve equations with two variables, two unknowns. When there are two unknowns, they can be solved if we have two equations. That is called a **system of equations**.
2. Two equations mean that there are 2 lines on a graph. Solving the system of equations gives us a point on the graph, written as an **ordered pair** (x, y) , where the 2 lines intersect. We will do graphing in a later lesson.
3. When there are two variables, you solve for one at a time and then use that answer to find the second variable.
4. There are two different ways to solve for two variables. The first method is called **substitution**.
5. Copy **Notes handout 24.2** for your students. Do each step on the board and explain as you go along. Do the 2 practice problems together.
6. Have students practice independently with [Worksheet 24.3](#). Have volunteers do some on the board.
7. Note: the handout notes are from the website algebra2go.

Lesson 24 Activity 3: Combination of Equations

Time: 40 Minutes

1. The second method of solving for two variables is called the combination or elimination method.
2. Copy the **Notes handout 24.4** for students or have them take notes as you write on the board.
3. Explain both examples carefully.
4. Do one or two problems from [Worksheet 24.5](#) together and then let students practice independently.
5. Have volunteers solve some problems at the board.

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Lesson 24 Exit Ticket: Write an Equation and Solve

Time: 5 Minutes

The cost of a pencil is 25 cents more than the cost of an eraser. If the cost of 8 pencils and 10 erasers is \$12.80, find the cost of each. Let e = cost of an eraser.

Answer:

$$10e + 8(e + 0.25) = \$12.80$$

$$10e + 8e + 2.00 = \$12.80$$

$$18e + 2.00 = \$12.80$$

$$18e = \$10.80$$

Eraser = 60 cents, pencil = 85 cents

Extra Time? Finish Early?

Students who finish early can go back to each worksheet and choose problems to solve by the other method. For example, solve problems on the substitution worksheet by the elimination method. They will get the same answers.

Worksheet 24.1 Write Equations for Word Problems

Directions: Complete the following problems on a separate sheet of paper.

1. Convert the following statements into equations.
 - a. 5 added to a number is 9.
 - b. 3 subtracted from a number is equal to 12.
 - c. 5 times a number decreased by 2 is 4.
 - d. 2 times the sum of the number x and 7 is 13.
2. A number is 12 more than the other. Find the numbers if their sum is 48.
3. Twice the number decreased by 22 is 48. Find the number.
4. Seven times the number is 36 less than 10 times the number. Find the number.
5. The sum of two consecutive even numbers is 38. Find the numbers.
6. The sum of three consecutive odd numbers is 51. Find the numbers.
7. Rene is 6 years older than her younger sister. After 10 years, the sum of their ages will be 50 years. Find their present ages. (Hint: Make x younger sister's age now)
8. The length of a rectangle is 10 cm more than its width. If the perimeter of rectangle is 80 cm, find the dimensions of the rectangle.

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Worksheet 24.1 Write Equations for Word Problems Answers

Directions: Complete the following problems on a separate sheet of paper.

1. Convert the following statements into equations.
 - a. 5 added to a number is 9. $n + 5 = 9$
 - b. 3 subtracted from a number is equal to 12. $n - 3 = 12$
 - c. 5 times a number decreased by 2 is 4. $5n - 2 = 4$
 - d. 2 times the sum of the number x and 7 is 13. $2(x + 7) = 13$
2. A number is 12 more than the other. Find the numbers if their sum is 48.

$$n + n + 12 = 48$$

$$2n + 12 = 48$$

$$2n = 36$$

$$n = 18$$
 So, $n + 12 = 30$
3. Twice the number decreased by 22 is 48. Find the number.

$$2n - 22 = 48$$

$$2n = 70$$

$$n = 35$$
4. Seven times the number is 36 less than 10 times the number. Find the number.

$$7n = 10n - 36$$

$$36 = 3n$$

$$12 = n$$
5. The sum of two consecutive even numbers is 38. Find the numbers.

$$n + n + 2 = 38$$

$$2n + 2 = 38$$

$$2n = 36$$

$$n = 18$$

$$n + 2 = 20$$
6. The sum of three consecutive odd numbers is 51. Find the numbers.

$$n + n + 2 + n + 4 = 51$$

$$3n + 6 = 51$$

$$3n = 45$$

$$n = 15$$

$$n + 2 = 17$$

$$n + 4 = 19$$
7. Rene is 6 years older than her younger sister. After 10 years, the sum of their ages will be 50 years. Find their present ages. (Hint: Make x younger sister's age now)

$$x + 10 + x + 6 + 10 = 50$$

$$2x + 26 = 50$$

$$2x = 24$$

$$x = 12 \text{ (younger sister)}$$

$$x + 6 = 18 \text{ (Rene)}$$
8. The length of a rectangle is 10 cm more than its width. If the perimeter of rectangle is 80 cm, find the dimensions of the rectangle.

$$2w + 2(w + 10) = 80$$

$$2w + 2w + 20 = 80$$

$$4w + 20 = 80$$

$$4w = 60$$

$$w = 15 \text{ (width), } w + 10 = 25 \text{ (length)}$$

Notes Handout 24.2 on Substitution Method

Solving Systems of Equations- Substitution

Recall: Solving a system of equations means finding the intersection of the lines.

example 1: $x - y = 2$ $x + y = 6$

Method of Substitution:

(i) Take one equation and solve for x OR y (it doesn't matter)

(ii) Substitute into the other equation

(iii) Solve for the variables

(i) Using the first equation, let's solve for x :

$$\begin{array}{r} x - y = 2 \\ + y \quad + y \\ \hline x = 2 + y \end{array}$$

(ii) Now we substitute this equation into the second equation

$$\begin{array}{r} x + y = 6 \\ (2 + y) + y = 6 \end{array}$$

(iii) And now we can solve for y

$$\begin{array}{r} 2 + y + y = 6 \\ 2 + 2y = 6 \end{array}$$

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$$\begin{array}{r} 2 + 2y = 6 \\ -2 \quad -2 \\ \hline 2y = 4 \\ 2 \quad 2 \\ \hline y = 2 \end{array}$$

Now that we know that $y=2$, we can easily solve for x since from part (i) we found that

$$\begin{array}{r} x = 2 + y \\ \quad \quad \downarrow \\ x = 2 + 2 \\ x = 4 \end{array}$$

Note: we could have plugged in $y=2$ in either original equation and still obtained $x=4$

So what does this mean?

Since $x=4$ and $y=2$, this means that the point $(4, 2)$ satisfies both equations.

In other words, these two lines intersect at the point $(4, 2)$.

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example 2: $4x + 7y = 13$
 $x + y = 1$

Using the substitution method, we want to choose one equation and solve for either x or y . Since the second equation is a bit less complicated than the first, we'll choose to use the second.

(i) using the second equation, let's solve for y - remember, it doesn't matter which variable you choose to solve for!

$$\begin{array}{r} x + y = 1 \\ -x \quad -x \\ \hline y = 1 - x \end{array}$$

(ii) Now we substitute this equation into the FIRST equation

$$\begin{array}{l} 4x + 7y = 13 \\ 4x + 7(1 - x) = 13 \end{array}$$

(iii) And now we can solve for x .

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$$4x + 7(1 - x) = 13$$

$$4x + 7 - 7x = 13$$

$$-3x + 7 = 13$$

$$\underline{-7 \quad -7}$$

$$-3x = 6$$

$$\underline{-3 \quad -3}$$

$$x = -2$$

Now that we know that $x = -2$, we can easily solve for y since from part (i) we found that

$$y = 1 - x$$

↓

$$y = 1 - (-2)$$

$$y = 1 + 2$$

$$y = 3$$

This means that the two lines intersect at the point

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Solving Systems - Substitution Practice Problems

Solve each system of equations:

1. $x + y = 2$
 $x - y = -4$

2. $x + y = 5$
 $2x - 3y = 0$

Notes Handout 24.4 on Combination of Equations

The first step in the combination method of solving any 2 variable systems is to look for the easiest way to eliminate a variable.

Example A:

$$\begin{aligned} 2x + 2y - 3 &= 3 \\ 5x - 2y + 9 &= 10 \end{aligned}$$

We can eliminate 'y' with one step by adding the equations together.

$$7x + 6 = 13$$

Then simply solve for 'x'

$$\begin{aligned} 7x + 6 &= 13 \\ -6 \quad -6 & \\ \hline 7x &= 7 \\ x &= 1 \end{aligned}$$

Now, insert the 'x' back into one of the original equations

$$\begin{aligned} 2(1) + 2y - 3 &= 3 \\ 2 + 2y - 3 &= 3 \\ 2y - 1 &= 3 \\ +1 \quad +1 & \\ \hline 2y &= 4 \\ y &= 2 \end{aligned}$$

Answer = (1,2)

Example B:

$$\begin{aligned} 17x - 5y &= 34 \\ 20x + 20y - 9 &= 31 \end{aligned}$$

The easiest variable to eliminate in this one is also the 'y'.

First, we have to make the 'y's in each equation opposite to one another so they can be eliminated. To do this, we'll multiply the top equation by four (the WHOLE equation).

$$\begin{aligned} 17x - 5y &= 34 \\ *4 \quad \quad *4 & \\ \hline 68x - 20y &= 136 \end{aligned}$$

Add the new equation and the bottom equation together.

$$\begin{aligned} 20x + 20y &= 40 \\ +68x - 20y &= 136 \\ \hline 88x &= 176 \\ x &= 2 \end{aligned}$$

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Insert 'x' into the first equation

$$17(2) - 5y = 34$$

$$34 - 5y = 34$$

$$-34 \quad -34$$

$$-5y = 0$$

$$y = 0$$

Solution: (2,0)

That is how you solve two variable systems of equations.

1. Multiply the top, bottom, or both equations to get one of the variables to cancel out
2. Add the equations together
3. Solve for the remaining variable
4. Substitute the solved variable into one of the original equations
5. Solve for the remaining variable
6. Write solution as a coordinate point (x , y)

Notes:

1. If both variables cancel out and the equation isn't equal ($3 = 0$ or $3 = 6$, etc.), there is no solution because the lines are parallel, in which case the 'solution' is "parallel, no solution".
2. If both variables cancel out and the equation IS equal ($2 = 2$ or $5 = 5$, etc.), there are infinite solutions because they are the same line, in which case the answer would be "same line, many/infinite solutions".
3. If it isn't coming out right, don't be afraid to start the equation over. You might have made a mistake early on, something hard to detect. Starting over can save you a lot of time!