

Weekly Focus: rational expressions Weekly Skill: computation

LESSON 28: Rational Expressions and Equations part 1

Lesson Summary: For the warm-up, students will solve a problem about a patio. In Activity 1, they will simplify rational expressions. In Activity 2, students will add and subtract rational expressions, and in Activity 3, they will multiply and divide them. There is an exit ticket at the end. Estimated time for the lesson is 2 hours.

Materials Needed for Lesson 28:

- Video (length 6:37) on simplifying expressions
- Video (length 3:11) on adding and subtracting rational expressions. The videos are required for teachers and optional for students.
- 3 Notes sheets (28A, 28B, 28C) to be used for students and/or for teaching notes. These come from algebra2go.
- 4 Worksheets (28.1, 28.2, 28.3, 28.4) with answers (attached)

Objectives: Students will be able to:

- Solve the patio word problem with a quadratic equation
- Simplify rational expressions
- Add, subtract, multiply and divide rational expressions

ACES Skills Addressed: N, CT, LS

CCRS Mathematical Practices Addressed: Reason Abstractly and Quantitatively, Use Appropriate Tools Strategically

Levels of Knowing Math Addressed: Intuitive, Abstract

<u>Notes:</u>

You can add more examples if you feel students need them before they work. Any ideas that concretely relate to their lives make good examples.

For more practice as a class, feel free to choose some of the easier problems from the worksheets to do together. The "easier" problems are not necessarily at the beginning of each worksheet. Also, you may decide to have students complete only part of the worksheets in class and assign the rest as homework or extra practice.

The GED Math test is 115 minutes long and includes approximately 46 questions. The questions have a focus on quantitative problem solving (45%) and algebraic problem solving (55%).

Students must be able to understand math concepts and apply them to new situations, use logical reasoning to explain their answers, evaluate and further the reasoning of others, represent real world problems algebraically and visually, and manipulate and solve algebraic expressions.

This computer-based test includes questions that may be multiple-choice, fill-in-the-blank, choose from a drop-down menu, or drag-and-drop the response from one place to another.

The purpose of the GED test is to provide students with the skills necessary to either further their education or be ready for the demands of today's careers.



Lesson 28 Warm-up: Solve the patio problem	Time: 5-10 Minutes
Write on the board: Felicia wants to increase the size of her or to each side to make it 49 ft ² .	outdoor patio by adding 2 feet
Basic Questions:	
 What is the shape of her patio? It is square because she added the same length to B What was the area of her patio before? It was 25 square feet because it was 2 feet shorter o 	
 Extension Questions: Solve the problem with expressions, multiplying, and factorie Let x = length of one side of original patio (x + 2)(x + 2) = 49 x² + 4x + 4 = 49 x² + 4x - 45 = 0 (x - 5)(x + 9) = 0 x = 5 or x = -9. Since length can't be negative, the original content of the content of the second of the content of the content	

Lesson 28 Activity 1: Simplifying Rational Expressions Time: 20-25 Minutes

- 1. **Rational expressions** are like rational numbers (can't have a zero denominator) except that they have variables in the numerator and/or denominator.
- 2. Some examples are: $\frac{5}{x}$ and $\frac{n+1}{n-1}$ and $\frac{r^2+5r}{r^2}$
- 3. Rational expressions are in their simplest form if the numerator and denominator cannot be simplified further just like with regular fractions. The last example above can be simplified to $\frac{r+5}{r}$.
- 4. Examples:
 - 18

1.

60 can reduce by dividing both numerator and denominator by 6 to get 3/10

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	7x - 21
2.	28 can be reduced by taking 7 out of every term to get $\frac{x-3}{4}$
	$30a^4b^2$
3.	$\overline{40a^2b^5}$ can be reduced by dividing both numerator and denominator by 10a ² b ² to get $\frac{3a^2}{4b^3}$
	3s - 12t
	$(s-4t)^2$ can be reduced by diving both by $(s-4t)$ to get $\frac{3}{(s-4t)}$
	Give students Notes 28A for more examples and explanations.
	Practice with Worksheet 28.1 . Have volunteers do some answers on the board.

esson 28 Activity 2: Adding and Subtracting Rational xpressions	Time: 30-35 Minutes
 Like regular fractions, rational expressions can be added common denominator. 	and subtracted once they have a
2. Do the examples on Notes 28B Adding and Subtracting R the notes for your students or have them take notes as yo common denominator).	• • • • •
3. Do Worksheet 28.2	
4. Have volunteers do a few problems on the board.	

esson 28 Activity 3: Multiplying and Dividing Rational	Time: 35-40 Minutes
Expressions	
1. Like regular fractions, rational expressions can be multiplie common denominator.	d or divided without needing a
2. Do the examples on Notes 28C Multiplying and Dividing Re	ational Expressions. You can copy
the notes for your students or have them take notes as you	u explain.
3. Do Worksheet 28.3 Multiplying Rational Expressions	
4. Have volunteers do a few problems on the board.	
5. Do Worksheet 28.4 Dividing Rational Expressions	
6. Have volunteers do a few problems on the board.	



Time: 5 Minutes



Notes 28A Simplifying Expressions

simplifying rational expressions
consider the expression; Can we reduce?
2X+4
2
we cannot cancel the 2's, but we
can factor the numerator
2X+4 = 2(X+2)
So our expression can be written
$2(\chi + 2)$
2
Now the 2 in the numerator can be
cancelled with the 2 in the
denominator.
2(x+2) = x+2
2
EXAMPLE 1: SIMPLIFY EACH EXPRESSION:
A) 3X+12
2x+8
We must factor the top and bottom first to
see if any factors will cancel:
3X+12 = 3(x+4) = 3 [the binomials
2×+8 2(×+4) 2 cancel!]



B) $(x+3)(x+5)$
(x - 3)(x + 5)
Here the numerator and denominator
are already factored. Since there
is an (X+5) in TOP AND in the bottom,
they will cancel.
(x+3)(x+5) = x+3
(X-3)(X+5) X-3
c) $4\chi^2 - 4\chi$
51-5
we need to factor the top and bottom
first to see if any factors will cancel:
$4X^2 - 4X = 4X(X-1) = 4X$
5x - 5 $5(x - 1)$ 5
D) $\chi^2 + 2\chi - 15$
$\chi^2 + \ell \chi + 5$
we need to factor the top and bottom
first to see if any factors will cancel.
Let's start with the top:
$\chi^2 + 2\chi - 15$
we need to use the abc method.





$\chi^2 + 2\chi - 15$
a=1
b=2
C = -15
a.c = -15
we need factors of -15 that add
to 2 (the b term)
-15 sum -15 -14
3 -5 -2
-3 5 (2)
so we get
$\frac{30 \text{ WC 9Cl}}{\chi^2 + 2\chi - 15} = (\chi - 3)(\chi + 5)$
$1 - \frac{121}{15 - (1 - 3)(1 + 5)}$
NIDINI Ila desetucio elect
Now the denuminator:
$\chi^2 + \ell \chi + 5$
<u> </u>
6=6
C=5
a·c=5
We need factors of 5 that add to
6 Lthe 6 term)
The factors are 5 and 1.

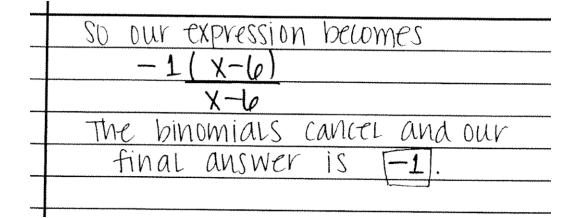




	so we get
	$\chi^{2} + \chi \chi + 5 = (\chi + 5)(\chi + 1)$
	Therefore our expression can be written:
	$\chi^{2} + 2\chi - 15 = (\chi - 3)(\chi + 5)$
******	$\chi^{2} + \ell \chi + 5 \qquad (\chi + 5)(\chi + 1)$
	WILL anything cancel?
⊳	what's the final solution?
	ANSWER:
-	
	E) (u-X)
-	$(\chi - \ell e)$
	we are hoping to factor the top or
	bottom so that something will cancel.

	If we factor out a negative 1 from
Paramatan	the top we get
	the top we get u-x = -1(-u+x)
	the top we get
	the top we get U-X = -1(-U+X) X-U = X-U Using the commutative property of
	the top we get U-X = -1(-U+X) X-U = X-U Using the commutative property of
	the top we get $\frac{10-X}{X-1e} = -1(-1e+x)$ $\frac{10-X}{X-1e} = -1(-1e+x)$





Worksheet 28.1 Simplifying Expressions

1)
$$\frac{q^2 + 10q + 21}{q^2 + 7q + 12}$$
 6) $\frac{3n + 12}{21}$

2)
$$\frac{h-4}{8h-32}$$
 7) $\frac{12x^6}{16x}$



4)
$$\frac{72b^3}{45b^5}$$
 9) $\frac{54k^4}{27k^4}$

5)
$$\frac{18s^4}{12s^4}$$
 10) $\frac{z^2 - 14z + 45}{z^2 - z - 20}$

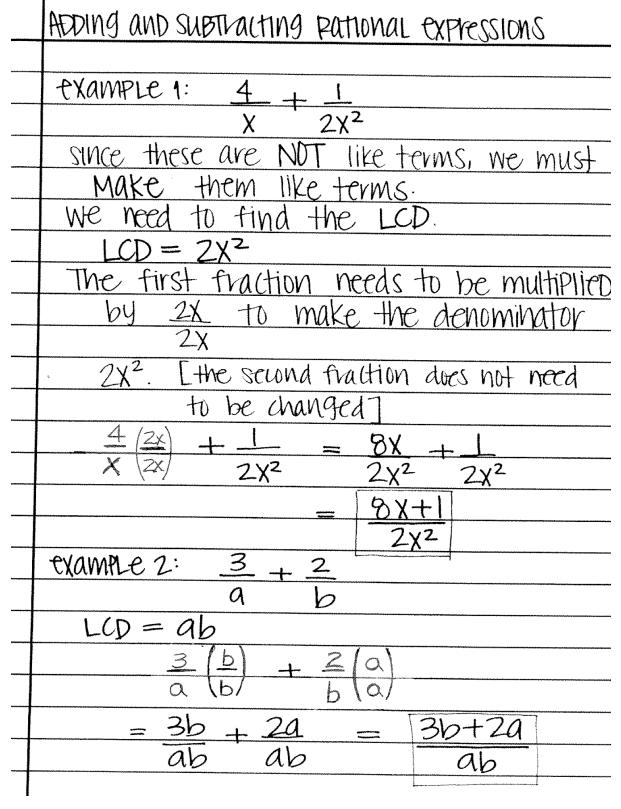


Worksheet 28.1 Answers

1)	$\frac{q^2 + 10q + 21}{q^2 + 7q + 12}$	6)	<u>3n + 12</u> 21
	$\frac{q+7}{q+4}$		$\frac{n+4}{7}$
2)	<u>h - 4</u> 8h - 32	7)	<u>12x⁶</u> 16x
	$\frac{1}{8}$		$\frac{3x^5}{4}$
3)	$\frac{12}{6p+30}$	8)	<u>5r - 5</u> 10
	2 p + 5		$\frac{r-1}{2}$
4)	$\frac{72b^3}{45b^5}$	9)	$\frac{54k^4}{27k^4}$
	$\frac{8}{5b^2}$		2
5)	<u>18s</u> ⁴ 12s⁴	10)	$\frac{z^2 - 14z + 45}{z^2 - z - 20}$
	$\frac{3}{2}$		$\frac{z-9}{z+4}$



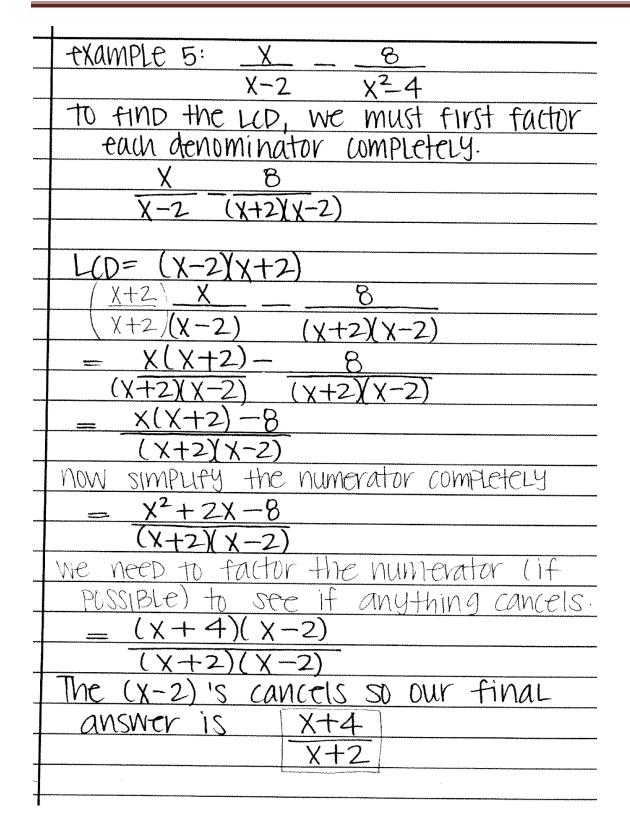
Notes 28B Adding and Subtracting Rational Expressions





txamPLt 3: 3 + 2 - 1
$\chi^2 y \chi y^2 \chi y$
$L(D = \chi^2 y^2)$
(y) = -(xy)
$(y) x^2 y (x / x y^2 (x y / x y))$
$= \underline{34} + \underline{2x} - \underline{x4}$
$\frac{\chi^2 y^2}{\chi^2 y^2} \chi^2 y^2 \qquad \chi^2 y^2$
= 3y + 2x - xy
$example 4: \underline{1} + \underline{2}$
X+4 X
LCD = X(X+4)
(X) - + (X+4) 2
(x) x+4 $(x+4)$ x
= X + 2(X+4)
$X(x+4)$ $\overline{X(x+4)}$
$= \chi + 2(\chi + 4)$
X(x+4)
NOW SIMPLIFY the numerator completely.
Note: Theres no need to multiply the
penominator out.
= X + 2X + 8 = 3X + 8
$X(X+4)$ $\overline{X(X+4)}$





Worksheet 28.2 Adding and Subtracting Rational Expressions

Add or Subtract the two expressions in each problem.

1)
$$\frac{7d}{d+6} - \frac{6}{d+3}$$

6) $\frac{7y-7n}{5y^{5}} - \frac{4y-3n}{5y^{5}}$
2) $\frac{6p^{2} + 3x^{2}}{5p^{4}} - \frac{2p^{2} + 4x^{2}}{5p^{4}}$
7) $\frac{6n + 8y}{2n^{2}y^{2}} - \frac{6n - 9y}{2n^{2}y^{2}}$
3) $\frac{7n^{3} - 8}{5n^{5} - 2} - \frac{8n}{5n^{5} - 2}$
8) $\frac{8c}{3c - 4} - \frac{2}{5c - 7}$
4) $\frac{5s^{2} - 6}{8s^{4} - 7} - \frac{3s}{8s^{4} - 7}$
9) $\frac{7r}{4} - \frac{2r + 2}{3r + 6}$
5) $\frac{5k}{7} - \frac{6k + 6}{3k + 4}$
10) $\frac{8b - 4}{7b^{4} - 5b} - \frac{2b - 3}{7b^{4} - 5b}$

Worksheet 28.2 Answers

Add or Subtract the two expressions in each problem.

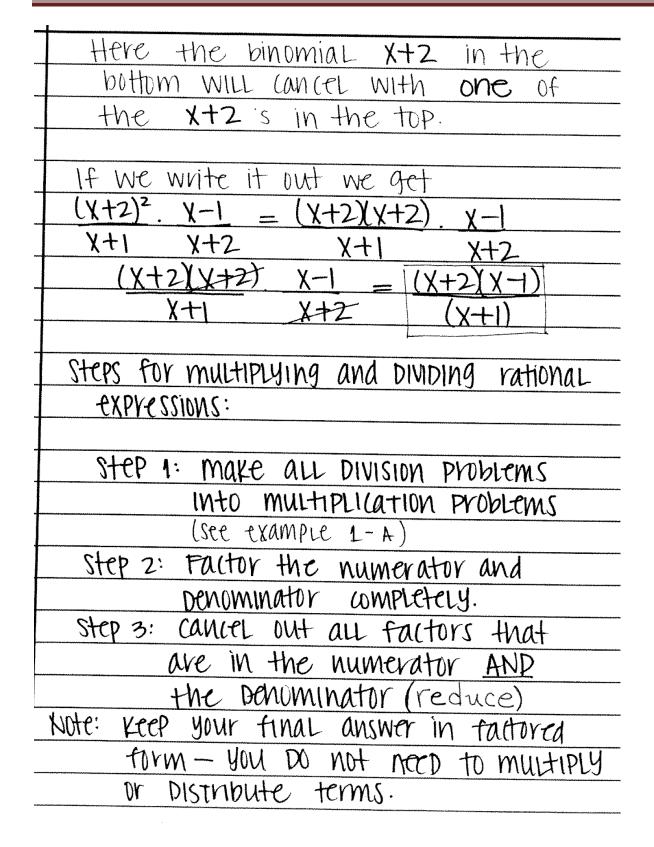
1)
$$\frac{7d}{d+6} - \frac{6}{d+3}$$

 $\frac{7d^2 + 15d - 36}{(d+6)(d+3)}$
2) $\frac{6p^2 + 3x^2}{5p^4} - \frac{2p^2 + 4x^2}{5p^4}$
 $\frac{4p^2 - x^2}{5x^4}$
3) $\frac{7h^3 - 8}{5h^5 - 2} - \frac{8h}{5h^5 - 2}$
 $\frac{4h^3 - 8}{5h^5 - 2}$
4) $\frac{5s^2 - 6}{8s^4 - 7} - \frac{3s}{8s^4 - 7}$
 $\frac{2s^2 - 6}{8s^4 - 7}$
5) $\frac{5k}{7} - \frac{6k + 6}{3k + 4}$
 $\frac{15k^2 - 22k - 42}{7(3k + 4)}$
6) $\frac{7y - 7n}{5y^5} - \frac{4y - 3n}{5y^5}$
7) $\frac{6n + 8y}{2n^2y^2} - \frac{6n - 9y}{2n^2y^2}$
7) $\frac{6n + 8y}{2n^2y^2} - \frac{6n - 9y}{2n^2y^2}$
7) $\frac{6n + 8y}{2n^2y^2} - \frac{6n - 9y}{2n^2y^2}$
8) $\frac{8c}{3c - 4} - \frac{2}{5c - 7}$
9) $\frac{7r}{4} - \frac{2r + 2}{3r + 6}$
10) $\frac{8b - 4}{7b^4 - 5b} - \frac{2b - 3}{7b^4 - 5b}$



Notes 28C Multiplying and Dividing Rational Expressions multiplying and dividing rational expressions Recall: 3 В -----З 8 3.2 6 14 1.7 example 1: A) X в 4 8 8 4 2 2χ 1) B) X+3 X - I÷ X-2 X-2 $\chi + 3$ ÷ Х X+3 X-2 Х X-2 2 X-2 X٠ 1 X+3X+31 Х $(\chi + 2)^{2}$ Ċ X — I X+2 $\chi + 1$



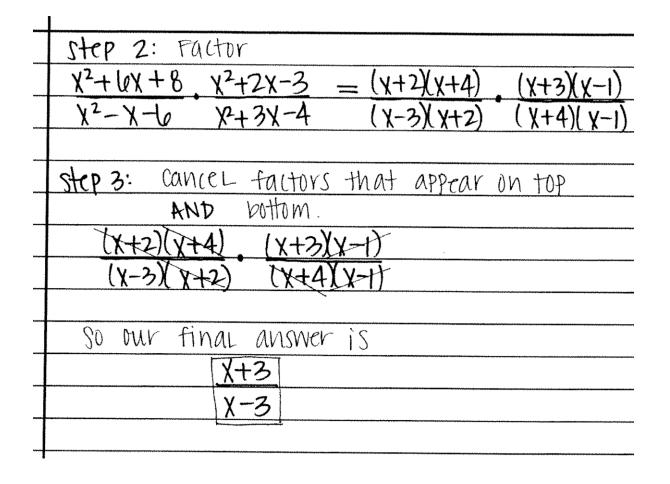






Example 2:
$x^2 + 6x + 9$, $x - 2$
x ² -4 x+3
step 1: This step is not needed since it is
already a multiplication problem.
step 2: Factor.
$\chi^{2} + l_{0}\chi + q$, $\chi - 2 = (\chi + 3)(\chi + 3)$, $(\chi - 2)$
$\chi^2 - 4$ $\chi + 3$ $(\chi + 2)(\chi - 2)$ $(\chi + 3)$
step 3: cancel factors that appear on top
AND brttom.
(x+3)(x+3), (x-2)
(X+2)(X-2) (X+3)
so our final answer is X+3
X+2
Example 3:
$\frac{\chi^2 + \iota x + 8}{2} \div \frac{\chi^2 + 3\chi - 4}{2}$
$\chi^2 - \chi - 6$ $\chi^2 + 2\chi - 3$
step 1: x2+ 6x+8, x2+2x-3
$\chi^2 - \chi - 6 \qquad \chi^2 + 3\chi - 4$







FINDING THE LCD OF RATIONAL EXPressions Handout

STEPS to FINDING the LCD of Rational expressions: step 1: Factor each Denominator completely step 2: LIST Each Different factor from

step 1, the greatest number of

times it appears in a

Denominator.

step 3: MULTIPLY The factors from step 2 - This is the LCD

Example: FIND the LCD of the terms 2 $X^{2}+2X+1$ x2+3x+2

There are Four Denominators: X, X+1, X²+2X+1, X²+3X+2 Step 1: Factoring taun Denominator we get X, X+1, (X+1)², (X+1)(X+2)

step 2: The DIFFERENT factors are X, X+1 and X+2

the highest power of x is 1, the highest power of (X+1) is 2 and the highest power of (X+2) is 1 Step 3: $LCD = X(X+1)^2(X+2)$



Worksheet 28.3 Multiplying Rational Expressions Simplify each expression.

1)
$$\frac{8}{4} \cdot \frac{6}{10c}$$
 6) $\frac{45y^2 - 45y}{55y^2 - 55y} \cdot \frac{10y}{10}$

2)
$$\frac{(d+12)(d-3)}{d+12} \cdot \frac{2}{(d+9)(d+12)}$$
 7) $\frac{8(q-6)}{6} \cdot \frac{4q}{8(q-6)}$

3)
$$\frac{8}{(h+11)} \cdot \frac{4h-48}{(h-12)}$$

8) $\frac{(p-8)(p-5)}{p-8} \cdot \frac{12}{(p+2)(p-8)}$

4)
$$\frac{45z^2 + 45z}{18z^2 + 18z} \cdot \frac{8z}{8}$$
 9) $\frac{2(r-4)}{(r-4)} \cdot \frac{5r}{2(r+6)}$

5)
$$\frac{9}{8} \cdot \frac{10}{12x}$$
 10) $\frac{12(b-6)}{6} \cdot \frac{8b}{12(b-6)}$

Worksheet 28.3 Answers

1)	$\frac{8}{4} \cdot \frac{6}{10c}$	6)	$\frac{45y^2 - 45y}{55y^2 - 55y} \cdot \frac{10y}{10}$
	<u>6</u> 5c		<u>9y</u> 11
2)	$\frac{(d+12)(d-3)}{d+12} \cdot \frac{2}{(d+9)(d+12)}$ $\frac{2(d-3)}{(d+9)(d+12)}$	7)	$\frac{8(q-6)}{6} \cdot \frac{4q}{8(q-6)}$ $\frac{2q}{3}$
3)	$\frac{8}{(h+11)} \cdot \frac{4h-48}{(h-12)}$ $\frac{32}{h+11}$	8)	$\frac{(p-8)(p-5)}{p-8} \cdot \frac{12}{(p+2)(p-8)}$ $\frac{12(p-5)}{(p+2)(p-8)}$
4)	$\frac{45z^{2} + 45z}{18z^{2} + 18z} \cdot \frac{8z}{8}$ $\frac{5z}{2}$	9)	$\frac{2(r-4)}{(r-4)} \cdot \frac{5r}{2(r+6)}$ $\frac{5r}{r+6}$
5)	$\frac{9}{8} \cdot \frac{10}{12x}$ $\frac{15}{16x}$	10)	$\frac{12(b-6)}{6} \cdot \frac{8b}{12(b-6)}$ $\frac{4b}{3}$

Worksheet 28.4 Dividing Rational Expressions

1)
$$\frac{11p}{2} \div \frac{7}{4}$$
 6) $\frac{5x}{x+4} \div \frac{5x}{7x+28}$

2)
$$\frac{33y^2 + 19y - 60}{27y^2 + 75y + 50} \div \frac{y^2}{36y^2 + 112y + 80}$$
 7) $\frac{d+7}{d+12d+35} \div \frac{6d}{d+9}$

3)
$$\frac{10k^2}{3} \div \frac{9k}{8}$$
 8) $\frac{6g}{g+5} \div \frac{6g}{10g+50}$

4)
$$\frac{84b - 77}{8} \div \frac{108b - 99}{8b}$$
 9) $\frac{5r}{10} \div \frac{12}{11}$

5)
$$\frac{q^2 + 12q + 27}{q^2 + 14q + 33} \div \frac{1}{q + 11}$$
 10) $\frac{6n}{11} \div \frac{5}{4}$

Worksheet 28.4 Answers

1)
$$\frac{11p}{2} \div \frac{7}{4}$$

22p
7
7
2) $\frac{33y^2 + 19y - 60}{27y^2 + 75y + 50} \div \frac{y^2}{36y^2 + 112y + 80}$
 $\frac{4(11y - 12)(1y + 2)}{y^2}$
3) $\frac{10k^2}{3} \div \frac{9k}{8}$
6) $\frac{5x}{x + 4} \div \frac{5x}{7x + 28}$
7) $\frac{d + 7}{d + 12d + 35} \div \frac{6d}{d + 9}$
 $\frac{d + 9}{6d(d + 5)}$
8) $\frac{6g}{g + 5} \div \frac{6g}{10g + 50}$

4)
$$\frac{84b - 77}{8} \div \frac{108b - 99}{8b}$$

9) $\frac{5r}{10} \div \frac{12}{11}$
 $\frac{7b}{9}$
 $\frac{11r}{24}$

5)
$$\frac{q^2 + 12q + 27}{q^2 + 14q + 33} \div \frac{1}{q + 11}$$
 10) $\frac{6n}{11} \div \frac{5}{4}$
q + 9 $\frac{24n}{55}$