Lesson 3.1 Reteach — *Constant Rate of Change*

Relationships that have straight-line graphs are called **linear relationships**. The rate of change between any two points in a linear relationship is the same, or constant. A linear relationship has a **constant rate of change**.

Example

The height of a hot air balloon after a few seconds is shown. Determine whether the relationship between the two quantities is linear. If so, find the constant rate of change. If not, explain your reasoning.

Time (s)	Height of Hot Air Balloon (ft)
1	9
2	18
3	27
4	36

+1 +1 +1

+9 +9 +9

As the number of seconds increase by 1, the height of the balloon increases by 9 feet.

Since the rate of change is constant, this is a linear relationship. The constant rate of change is $\frac{9}{1}$ or 9 feet per second. This means that the balloon is rising 9 feet per second.

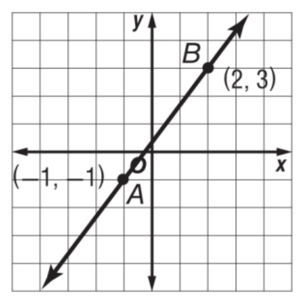
Lesson 3.2 Reteach — *Slope*

The slope m of a line passing through points, (x_1, y_1) and (x_2, y_2) is the ratio of the difference in the y-coordinates to the corresponding difference in the x-coordinates. As an equation, the slope is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 1

Find the slope of the line that passes through *A*(–1, –1) and *B*(2, 3).



$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

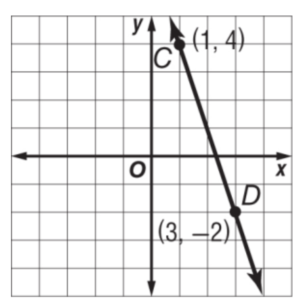
$m = \frac{3-(-1)}{2-(-1)}$ $(x_1, y_1) = (–1, –1)$, $(x_2, y_2) = (2, 3)$

$m = \frac{4}{3}$ Simplify.

Check When going from left to right, the graph of the line slants upward. This is correct for a positive slope.

Example 2

Find the slope of the line that passes through *C*(1, 4) and *D*(3, –2).



$m = \frac{y_2 - y_1}{x_2 - x_1}$ Slope formula

$m = \frac{-2-4}{3-1}$ $(x_1, y_1) = (–1, 4)$, $(x_2, y_2) = (3, –2)$

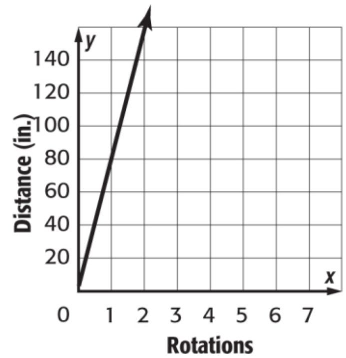
$m = \frac{-6}{2}$ or -3 Simplify.

Check When going from left to right, the graph of the line slants downward. This is correct for a negative slope.

Lesson 3.3 Reteach *Equations in $y = mx$ Form*

Some verbal sentences translate into two-step equations.

Example 1
The distance that a bicycle travels varies directly with the number of rotations that its tires make. Determine the distance that the bicycle travels for each rotation.



Since the graph of the data forms a line, the rate of change is constant. Use the graph to find the constant ratio.

$\frac{\text{distance traveled}}{\text{\# of rotations}} \longrightarrow \frac{80}{1} \quad \frac{160}{2} \text{ or } \frac{80}{1} \quad \frac{240}{3} \text{ or } \frac{80}{1} \quad \frac{320}{4} \text{ or } \frac{80}{1}$

The bicycle travels 80 inches for each rotation of the tires.

Example 2
The number of trading cards varies directly as the number of packages. If there are 84 cards in 7 packages, how many cards are in 12 packages?

Let x = the number of packages and y = the total number of cards.

$y = mx$	Direct variation equation
$84 = m(7)$	$y = 84$, $x = 7$
$12 = m$	Simplify.
$y = 12x$	Substitute for $m = 12$.

Use the equation to find y when $x = 12$.

$y = 12x$	
$y = 12(12)$	$x = 12$
$y = 144$	Multiply.

There are 144 cards in 12 packages.

Lesson 3.4 Reteach *Slope-Intercept Form*

Linear equations are often written in the form $y = mx + b$. This is called the **slope-intercept form**. When an equation is written in this form, m is the slope and b is the y-intercept.

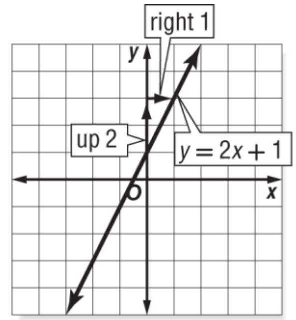
Example 1
State the slope and the *y*-intercept of the graph of $y = x - 3$.

$y = x - 3$	Write the original equation.
$y = 1x + (-3)$	Write the equation in the form $y = mx + b$.
$\uparrow \quad \uparrow$	
$y = mx + b$	$m = 1$, $b = -3$

The slope of the graph is 1, and the y-intercept is -3.

You can use the slope intercept form of an equation to graph the equation.

Example 2
Graph $y = 2x + 1$ using the slope and y-intercept.



Step 1 Find the slope and y-intercept.
$y = 2x + 1$ slope = 2, y-intercept = 1

Step 2 Graph the y-intercept 1.

Step 3 Write the slope 2 as $\frac{2}{1}$. Use it to locate a second point on the line.

$m = \frac{2}{1}$ ← change in y : up 2 units
← change in x : right 1 unit

Step 4 Draw a line through the two points.

Lesson 3.5 Reteach *Graph a Line Using Intercepts*

Standard form is when an equation is written in the form $Ax + By = C$.

Example
State the x- and y-intercepts of $3x + 2y = 6$. Then graph the function.

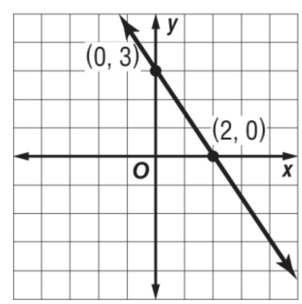
Step 1 Find the x-intercept.

To find the x-intercept, let $y = 0$.

$3x + 2y = 6$	Write the equation.
$3x + 2(0) = 6$	Replace y with 0.
$3x + 0 = 6$	Multiply.
$3x = 6$	Simplify.
$x = 2$	Divide each side by 3.

The x-intercept is 2.

Step 2 Find the y-intercept.



To find the y-intercept, let $x = 0$.

$3x + 2y = 6$	Write the equation.
$3(0) + 2y = 6$	Replace x with 0.
$0 + 2y = 6$	Multiply.
$2y = 6$	Simplify.
$y = 3$	Divide each side by 2.

The y-intercept is 3.

Step 3 Graph the points (2, 0) and (0, 3) on a coordinate plane. Then connect the points.

Lesson 3.6 Reteach *Write Linear Equations*

Point-slope form is when an equation is written in the form $y - y_1 = m(x - x_1)$, where (x_1, y_1) is a given point on a non-vertical line and m is the slope of the line.

Example
Write an equation in point-slope form and slope-intercept form for a line that passes through (2, –5) and has a slope of 4.

Step 1	$y - y_1 = m(x - x_1)$	Point-slope form
	$y - (-5) = 4(x - 2)$	$(x_1, y_1) = (2, -5)$, $m = 4$
	$y + 5 = 4(x - 2)$	Simplify.
Step 2	$y + 5 = 4(x - 2)$	Write the equation.
	$y + 5 = 4x - 8$	Distributive Property
	$-5 = \quad -5$	Addition Property of Equality
	$y = 4x - 13$	Simplify.

Check: Substitute the coordinates of the given point in the equation.

$y = 4x - 13$

$-5 \stackrel{?}{=} 4(2) - 13$

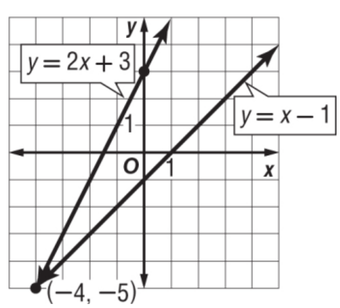
$-5 = -5$ ✓

Lesson 3.7 Reteach *Solve Systems of Equations by Graphing*

Graph the **system of equations**.		
No Solution – Parallel Lines	**One Solution –** Lines cross at one place	**Infinite Solutions –** Same Line

Example
Solve the system $y = 2x + 3$ and $y = x - 1$ by graphing.

Graph each equation on the same coordinate plane.



The graphs appear to intersect at (–4, –5).

Check this estimate by replacing x with –4 and y with –5.

Check	$y = 2x + 3$	$y = x - 1$
	$-5 \stackrel{?}{=} 2(-4) + 3$	$-5 \stackrel{?}{=} -4 - 1$
	$-5 = -5$ ✓	$-5 = -5$ ✓

The solution of the system is (–4, –5).

Lesson 3.8 Reteach

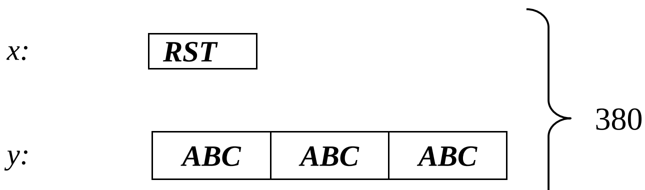
Solve Systems of Equations Algebraically

Solve the system of equations using **substitution**, replace a variable with the other equation.		
No Solution – $a = b$	**One Solution** – $x = a$	**Infinite Solutions** – $a = a$ or $x = x$

Example
You own three times as many shares of *ABC* stock as you do of *RST* stock. Altogether you have 380 shares of stock.

a. Write a system of equations to represent this situation.

Draw a bar diagram.



Use the diagram to write the system.

$y = 3x$	There are 3 times as many shares *ABC* stocks as *RST* stocks.
$x + y = 380$	The total number of stocks owned is 380.

b. Solve the system algebraically. Interpret the solution.

Since y is equal to $3x$, you can replace y with $3x$ in the second equation.

$x + y = 380$	Write the equation.
$x + 3x = 380$	Replace y with $3x$.
$4x = 380$	Simplify.
$\frac{4x}{4} = \frac{380}{4}$	Division Property of Equality
$x = 95$	Simplify.

Since $x = 95$ and $y = 3x$, then $y = 285$ when $x = 95$. The solution of this system of equations is (95, 285). This means that you own 95 shares of *RST* stock and 285 shares of *ABC* stock.