## Lesson 3: Vanishing Points and Looking at Art



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You've probably heard the term "vanishing point" before, and we mentioned it in Lesson 1 when we discussed the masking tape drawing of a library (see Figure 1 below).


Figure 1. Locating vanishing points.
In the drawing on bottom of Figure 1 are two vanishing points $V_{1}$ and $V_{2}$. The three lines which converge to $V_{1}$, for example, represent lines in the real world (architectural lines of the building) which are actually parallel to one another. Clearly, however, the images of these lines are not parallel, because they intersect. The reason we say "vanishing point" instead of "intersection point" is that a single line, all by itself, can have a vanishing point; the explanation is illustrated in Figure 2.


TOP VIEW


SIDE VIEW

Figure 2. The vanishing point is where the line appears to vanish.
In Figure 2 a viewer looks along various lines of sight (dashed lines) at a line $L$ in the real world. The viewer looks at farther and farther points on the line, and keeps seeing
the line as long as his line of sight intersects it. At a certail moment, however, his line of sight becomes exactly parallel to $L$ and no longer intersects it. That's the precise moment at which the line $L$ seems to vanish, simply because the viewer isn't looking at it anymore. For this reason, the intersection of this special line of sight with the picture plane is called the vanishing point of the line $L$. A vanishing point always lies in the picture plane: its height is determined by the side view in Figure 2, and its right-left location is determined by the top view. Notice that the special line of sight parallel to $L$ would also be parallel to any other line $M$ which was also parallel to $L$; that is, if any line is parallel to $L$, then it has the same vanishing point as $L$.

To state these results in a theorem, recall that a line is parallel to a plane if it does not intersect the plane. Clearly the line $L$ in Figure 2 is not parallel to the picture plane. Thus we have

## Theorem 2: The Vanishing Point Theorem

If two or more lines in the real world are parallel to one another, but not parallel to the picture plane, then they have the same vanishing point. The perspective images of these lines will not be parallel. If fully extended in a drawing, the image lines will intersect at the vanishing point.


Figure 3. Parallel lines in the real world whose images converge to a vanishing point.

Notice that the drawing in Figure 3 apparently has only one vanishing point. It would seem reasonable to call this "one-point perspective," but there is usually one more requirement before this term is used. This requirement is illustrated in Figure 4. Figure 4 is basically the same setup as in Figure 2, except the line $L$ is orthogonal (perpendicular)
to the picture plane. Thus the special line of sight parallel to $L$ (the sight line which goes through the vanishing point) is also orthogonal to the picture plane. This means that the correct location for the viewer's eye is directly in front of the vanishing point, and this is the situation which is usually referred to as one-point perspective.


Figure 4. In one-point perspective, the only lines with vanishing points are orthogonal to the picture plane.

A perspective drawing is in one-point perspective if (i) only one vanishing point $V$ is used, and (ii) image lines which converge to $V$ represent lines in the real world which are orthogonal to the picture plane.

It's often possible to tell by looking that a drawing is in true one-point perspective. In this case it may be easy to find the exact viewing position for the viewer's eye. One such example is the drawing of a rectangular box in Figure 5.


Figure 5. A box in one-point perspective.

First, let's see how we can tell that Figure 5 exhibits true one-point perspective. Clearly there is one vanishing point $V$ (conveniently located at the base of a tree), but we must also verify that the image lines (dashed) which converge to $V$ represent lines in the real world (the edges of the real box) which are orthogonal to the picture plane (the plane of the page). This will be true if the front face of the box is parallel to the picture plane. But this must be the case, because the image lines of the edges of the front face appear to be parallel; if the front face of the actual box were not parallel to the picture plane, the at least one opposing pair of its edges would not be parallel to the picture plane, and by Theorem 2, their images in the drawing would converge to a vanishing point. In other words, since the front face of the box appears undistorted, the drawing is in true one-point perspective.

It therefore follows that the correct viewing position is somewhere directly opposite the vanishing point $V$-but how far from the page? To determine that, we need the top view of the perspective setup for the box (see Figure 6).


Figure 6. The shaded triangles are similar.
In Figure 6 we see the vanishing point $V$ for the edges of the box which are orthogonal to the picture plane, and we also see the vanishing point $V^{\prime}$ for the diagonal of the top face of the box. Since the indicated pairs of lines are parallel, the two shaded triangles are similar. Thus the ratios of corresponding sides are equal:

$$
\frac{d}{a}=\frac{D}{A} .
$$

We can easily solve for the viewing distance $d$ to get

$$
\begin{equation*}
d=a\left(\frac{D}{A}\right) \tag{1}
\end{equation*}
$$

Obviously we need more information to find $d$, but often this can be gleaned from the context of the artwork. Suppose, for instance, we know that the box in Figure 5 is a cube. This may seem strange, because the box doesn't look like a cube - it looks too elongated (more like a dumpster), but the picture will look better when we determine $d$. If the box is a cube, then the top is a square (even though it wasn't drawn as a square in Figure 6), and $A$ and $D$ in Figure 6 must be equal. In this case, $(D / A)=1$, so by Equation (1) we have

$$
d=a .
$$

But $a$ is the distance between $V$ and $V^{\prime}$, so the viewing distance for a cube is the same as the distance between the main vanishing point $V$ and the vanishing point $V^{\prime}$ of the diagonal of the top face (see Figure 7). This distance can be measured directly on the drawing!


Figure 7. The viewing distance for a cube.
In the drawing (Figure 7) we locate $V^{\prime}$ (base of the other tree) by drawing the dashed diagonal line of the top face of the box. How do we know that $V^{\prime}$ is on the same horizontal line as $V$ ? Because the dashed lines are images of real lines which are level with the ground, so the sight lines of the viewer to their vanishing points must be level also.

To test out our determination of $d$, use the large drawing in Figure 8 as follows:

- Close your right eye.
- Hold the page vertically and place your left eye directly in front of the point $V$ ( $n o t$ in the center of the page!).
- Move the page until your left eye is $d$ units away from $V$. (You may want to use a thumb and forefinger to measure the distance between the trees.)
- Without changing your position, let your eye roll down and to the left to look at the box. Although it may be a bit close for comfortable viewing, it should look much more like a cube!


Figure 8. To see a cube, look with one eye, directly opposite $V$, at a distance $d$.

We have only used a little mathematics, but we have accomplished a lot. For one thing, we see the importance of the unique, correct perspective viewpoint. If we view art from the wrong viewpoint, it can appear distorted - a cube can look like a dumpster. For another thing, the majority of perspective works in museums are done in one-point perspective, with clues that can help determine the viewing distance. Thus our simple trick can actually be used in viewing and enjoying many paintings in museums and galleries. In Figure 9 we see the trick applied to finding the viewpoint for the painting, Interior of Antwerp Cathedral. Since the floor tiles are squares, they serve the same purpose as the square top of the cube in the previous discussion. The viewing distance is as indicated, with the correct viewpoint directly in front of the main vanishing point $V$.


Figure 9. Peter Neeffs the Elder, Interior of Antwerp Cathedral, 1651. (Courtesy of the Indianapolis Museum of Art.)

Although it's not possible to tell by viewing this small reproduction of Antwerp Cathedral, the effect of viewing the actual painting in the Indianapolis Museum of Art gives a surprising sensation of depth, of being "in" the cathedral. The viewing distance is only about 24 inches, so most viewers never view the painting from the best spot for the sensation of depth!

Of course you can't draw lines on the paintings and walls of an art museum, so some other method is needed to find the main vanishing point and the viewing distance. A good solution is to hold up a pair of wooden shish kebab skewers, aligning them with lines in the painting to find the location of their intersection points. First, the main vanishing point $V$ is located. Then one skewer is held horizontally so that it appears to go through $V$, and the other is held aligned with one of the diagonals of the square tiles; the intersection point of the skewers is then $V^{\prime}$. Figure 10 shows workshop participants at the Indianapolis Museum of Art using their skewers to determine the viewpoint of a perspective painting. Then, one by one, the viewers assume the correct viewpoint, looking with one eye to enjoy
the full perspective effect. If shish kebab skewers aren't practical, any pair of straight edges, such as the edges of credit cards, will work almost as well for discovering viewpoints of perspective works.


Figure 10. Viewing art with shish kebab skewers at the Indianapolis Museum of Art.

Of course there are other important ways to view a painting. It's good to get very close to examine brushwork, glazes, and fine details. It's also good to get far away to see how the artist arranged colors, balanced lights and darks, etc. Our viewpoint-finding techniques add one more way to appreciate, understand, and enjoy many wonderful works of art.

## Exercises for Lesson 3

1. (Drawing your own cube.) In Figure 11 a start has been made on the drawing of a cube in one-point perspective. The front face ia a square, $V$ is the vanishing point, and the dashed lines are guide lines for drawing receding edges of the cube. Suppose you want to choose the viewing distance first. Let's say the viewing distance should be 7 inches. Finish the drawing of the cube. (HINT: For help in thinking about it, look at Figure 7. The idea is to draw the same lines, but in a different order!)


Figure 11. How do you finish the cube if the viewing distance is 7 inches?
2. Suppose the box in Figure 12 is not a cube. Let's say its front face is a square, but its top face is in reality twice long as it is wide from left to right. In this case, the viewing distance is not equal to the distance between the two trees. What is the viewing distance? (Figure 6 can help you think about this problem.) What if the top is three times as long as it is wide from left to right?


Figure 12. What if the box is not a cube?.
3. Now do the real thing: go to a gallery or museum and practice your viewing techniques!

