LESSON 6: Fractions—Multiplication and Division

Lesson Summary: First, students will solve a word problem involving fractions. In Activity 1, they will practice multiplying fractions. In Activity 2, they do a real-life application problem. In Activity 3, they will practice dividing fractions and mixed numbers. In Activity 4, they will complete various word problems in the workbook. There are an exit ticket and an extra word problem at the end. Estimated time for the lesson is two hours.

Materials Needed for Lesson 6:
- 3 Worksheets with answers (attached)
- Video on dividing mixed numbers (length 4:52)
- *Mathematical Reasoning Test Preparation for the 2014 GED Test Workbook Pages 16-17*

- Teacher Note: You may decide to have students complete only part of the worksheet in class and assign the rest as homework or extra practice, depending on your students' needs.

Objectives: Students will be able to:

- Solve a problem involving addition and subtraction of fractions
- Multiply and divide fractions and mixed numbers
- Solve an application problem without being told what steps to take (as they would in real life)

ACES Skills Addressed: N, CT, SM
CCRS Mathematical Practices Addressed: Building Solution Pathways, Make Sense of Problems and Persevere in Solving Them, Use Appropriate Tools Strategically
Levels of Knowing Math Addressed: Intuitive, Abstract, Application, and Communication

Notes:
You can add more examples if you feel students need them before they work. Any ideas that concretely relates to their lives make good examples.

For more practice as a class, feel free to choose some of the easier problems from the worksheets to do together. The "easier" problems are not necessarily at the beginning of each worksheet. Also, you may decide to have students complete only part of the worksheets in class and assign the rest as homework or extra practice.

The GED Math test is 115 minutes long and includes approximately 46 questions. The questions have a focus on quantitative problem solving (45%) and algebraic problem solving (55%).

Students must be able to understand math concepts and apply them to new situations, use logical reasoning to explain their answers, evaluate and further the reasoning of others, represent real world problems algebraically and visually, and manipulate and solve algebraic expressions.

This computer-based test includes questions that may be multiple-choice, fill-in-the-blank, choose from a drop-down menu, or drag-and-drop the response from one place to another.

The purpose of the GED test is to provide students with the skills necessary to either further their education or be ready for the demands of today's careers.

Lesson 6 Warm-up: Solve the fractions problem	Time: 10 Minutes

Write on the board: Juan is driving his kids to their after-school activities. First, he drops off Pablo at soccer practice ¾ mile from home, then he drops off Alma at the gym 1 ½ miles further, and finally he brings Juanita to her job at Wendy's, which is another 3 1/3 mile further than the gym.

Basic Questions:
- How far did Juan drive from his house to Wendy's? (3/4 + 1 ½ = 2 ¼ miles + 3 1/3 = 5 7/12)

Have the students draw a map (number line) to help.

Extension Questions:
- Change the fractions to decimals and add. (0.75 + 1.5 = 2.25 + 3.33 = 5.58 miles)
- How far did he drive total if he returned home the same way?
  (5 7/12 one way, then doubled to go home 5 7/12 + 5 7/12= 10 14/12= 11 2/12 = 11 1/6.)

Lesson 6 Activity 1: Multiplying Fractions	Time: 25- 30 Minutes

Example A: Financial advisers generally recommend that you spend no more than 1/3 of your income on housing expenses. If you make $30,000 a year, how much is 1/3?

- Students may solve by dividing $30,000 by 3 to get $10,000. Is there another way? You can multiply 1/3 x 30,000. How does it look? $\frac{1}{3} \times \frac{30,000}{1} = \frac{1 x 30,000}{3 x 1} = \frac{30,000}{3}$ = 10,000. (Remind students that a whole number by itself is the same as a whole number over 1).

Example B: You give your child half a candy bar and tell her to split her half with her younger brother. How much of the whole candy bar did her brother get?

- Let the students solve this. Did they draw a picture? Just say it in words? Subtract half from the whole and then half of that (1/4)? What is the equation we write?   ½ x ½ = ¼. Why did we multiply? (Because multiplying by a half is the same as dividing by 2.)

- Why did the number get smaller even though we multiplied? Draw a picture to show why. Draw the candy bar, then cut out half, then show what half of that is. When we multiply a proper fraction (which is less than 1) by another number, the answer is less than the original number. We are asking what a smaller portion of that number is.

<u>Example C</u>: What is 1/3 x 4 ½? Have the students estimate first. Will it be more or less than 4? More or less than 2? Why or why not? (Since 1/3 is a smaller portion than 1/2, then the answer should be less than 2.)

1/3 x 4 ½ = 1/3 x 9/2 = 9/6 = 1 3/6 = 1 ½.

If there is time, you could have the students do these with decimals also.

Do **Worksheet Lesson 6.1** on multiplying mixed numbers.

Lesson 6 Activity 2 Application: Alan's Budget Problem	**Time: 20-25 Minutes**

You can read the problem to the students and have them tell you pertinent information:

> *Alan and Marisa have recently divorced. The judge told Alan that he needs to pay 1/10 of his gross income in alimony to his ex-wife as well as ¼ of his gross income for their daughter's child support. Alan's salary is $44,000 annually and he pays 15% in taxes.*
>
> *Now Alan wants to rent an apartment that costs $800 a month in rent plus about $150 for related household expenses. Following the rule of thumb about not spending more than a third of one's income on housing, do you think Alan has enough money for this apartment?*

Let the students work on this question and circulate to help. If they need hints, you may remind them of the difference between gross and net income. You can also help them start. They may be using whole numbers or decimals to solve the problem. That's fine, but if do they solve the problem this way, encourage them to go back and do it again with fractions.

Spend time discussing how they solved the problem. What did they do first? Does it matter what you do first? It doesn't matter in which order they deducted the taxes, the alimony, or the child support as long as the calculations are all based on gross income. Then the resulting net income should be divided by 12 months to determine how much he has net per month. That will determine if he has enough.

Make sure to include fractions when you (or students) do the answers on the board and to acknowledge that all ways of solving the problem are okay.
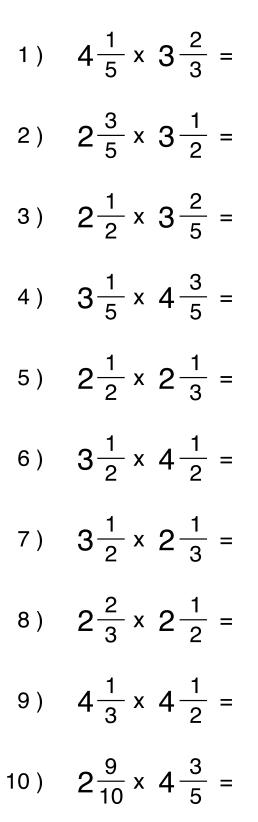
<u>Answer:</u> No. $44,000 - $6,600 in taxes - $4,400 in alimony - $11,000 in child support = $22,000 left in net annual income. $22,000 divided by 12 months = $1833.33 a month. 1/3 of his monthly net income is $611.11, not enough for the apartment he wants to rent.

Extension Questions:

- In planning ahead for retirement, Alan calculates that if he can save 5 ½ times his current gross income, he will have enough to live comfortably. How much money is that? ($242,000)

- How much should he pay for housing if he wants to pay no more than ¼ of his monthly net income? ($458.33)

Worksheet 6.1

1) $4\dfrac{1}{5} \times 3\dfrac{2}{3} =$

2) $2\dfrac{3}{5} \times 3\dfrac{1}{2} =$

3) $2\dfrac{1}{2} \times 3\dfrac{2}{5} =$

4) $3\dfrac{1}{5} \times 4\dfrac{3}{5} =$

5) $2\dfrac{1}{2} \times 2\dfrac{1}{3} =$

6) $3\dfrac{1}{2} \times 4\dfrac{1}{2} =$

7) $3\dfrac{1}{2} \times 2\dfrac{1}{3} =$

8) $2\dfrac{2}{3} \times 2\dfrac{1}{2} =$

9) $4\dfrac{1}{3} \times 4\dfrac{1}{2} =$

10) $2\dfrac{9}{10} \times 4\dfrac{3}{5} =$

Worksheet 6.1 Answers

1) $\quad 4\frac{1}{5} \times 3\frac{2}{3} = \dfrac{21 \times 11}{5 \times 3} = \dfrac{231}{15} = \dfrac{77}{5} = 15\frac{2}{5}$

2) $\quad 2\frac{3}{5} \times 3\frac{1}{2} = \dfrac{13 \times 7}{5 \times 2} = \dfrac{91}{10} = \qquad 9\frac{1}{10}$

3) $\quad 2\frac{1}{2} \times 3\frac{2}{5} = \dfrac{5 \times 17}{2 \times 5} = \dfrac{85}{10} = \dfrac{17}{2} = 8\frac{1}{2}$

4) $\quad 3\frac{1}{5} \times 4\frac{3}{5} = \dfrac{16 \times 23}{5 \times 5} = \dfrac{368}{25} = \qquad 14\frac{18}{25}$

5) $\quad 2\frac{1}{2} \times 2\frac{1}{3} = \dfrac{5 \times 7}{2 \times 3} = \dfrac{35}{6} = \qquad 5\frac{5}{6}$

6) $\quad 3\frac{1}{2} \times 4\frac{1}{2} = \dfrac{7 \times 9}{2 \times 2} = \dfrac{63}{4} = \qquad 15\frac{3}{4}$

7) $\quad 3\frac{1}{2} \times 2\frac{1}{3} = \dfrac{7 \times 7}{2 \times 3} = \dfrac{49}{6} = \qquad 8\frac{1}{6}$

8) $\quad 2\frac{2}{3} \times 2\frac{1}{2} = \dfrac{8 \times 5}{3 \times 2} = \dfrac{40}{6} = \dfrac{20}{3} = 6\frac{2}{3}$

9) $\quad 4\frac{1}{3} \times 4\frac{1}{2} = \dfrac{13 \times 9}{3 \times 2} = \dfrac{117}{6} = \dfrac{39}{2} = 19\frac{1}{2}$

10) $\quad 2\frac{9}{10} \times 4\frac{3}{5} = \dfrac{29 \times 23}{10 \times 5} = \dfrac{667}{50} = \qquad 13\frac{17}{50}$

Lesson 6 Activity 3: Dividing Fractions	**Time: 15 Minutes**

Example A: You're starting a new exercise program. You want to jog and stop every ½ mile to walk a bit. If you want to jog a total of 3 miles (not including your walking breaks), how many times will you stop to walk?

- Give students a few minutes to solve any way they want. They may add ½ + ½ until they get to 3. They make draw a number line or they may divide.
- Remind students that when dividing fractions, we invert the second fraction.
- $3 \div \frac{1}{2} = \frac{3}{1} \; x \; \frac{2}{1} = \frac{6}{1} = 6$ times

Example B: I have 2 ½ cups of oatmeal. How many ½ cup portions can I serve?
- Draw pictures to show 2 ½ cups and to make it easy to see the portions.
- $2 \frac{1}{2} \div \frac{1}{2} = \frac{5}{2} \; x \; \frac{2}{1} = \frac{10}{2} = 5$ portions of oatmeal.

3) Do a few examples of **Worksheet 6.2 Dividing Fractions and Worksheet 6.3 Dividing Mixed Numbers**. Assign the rest for homework.

Lesson 6 Activity 4: Workbook Problems	**Time: 20 Minutes**

Do the word problems on pages 16-17 of the workbook. Circulate to help. Have students (or you) explain any of the problems that were more challenging.

Worksheet 6.2 Dividing Fractions

1) $\dfrac{3}{4} \div \dfrac{5}{10} =$

2) $\dfrac{2}{4} \div \dfrac{1}{10} =$

3) $\dfrac{1}{2} \div \dfrac{3}{10} =$

4) $\dfrac{2}{4} \div \dfrac{2}{3} =$

5) $\dfrac{1}{2} \div \dfrac{2}{4} =$

6) $\dfrac{1}{3} \div \dfrac{1}{2} =$

7) $\dfrac{1}{2} \div \dfrac{6}{10} =$

8) $\dfrac{1}{10} \div \dfrac{4}{5} =$

9) $\dfrac{2}{5} \div \dfrac{6}{10} =$

10) $\dfrac{9}{10} \div \dfrac{1}{2} =$

Worksheet 6.2 Answers

1) $\dfrac{3}{4} \div \dfrac{5}{10} = \dfrac{3 \times 10}{4 \times 5} = \dfrac{30}{20} = \dfrac{3}{2} = 1\dfrac{1}{2}$

2) $\dfrac{2}{4} \div \dfrac{1}{10} = \dfrac{2 \times 10}{4 \times 1} = \dfrac{20}{4} = \dfrac{5}{1} = 5\dfrac{0}{1}$

3) $\dfrac{1}{2} \div \dfrac{3}{10} = \dfrac{1 \times 10}{2 \times 3} = \dfrac{10}{6} = \dfrac{5}{3} = 1\dfrac{2}{3}$

4) $\dfrac{2}{4} \div \dfrac{2}{3} = \dfrac{2 \times 3}{4 \times 2} = \dfrac{6}{8} = \dfrac{3}{4}$

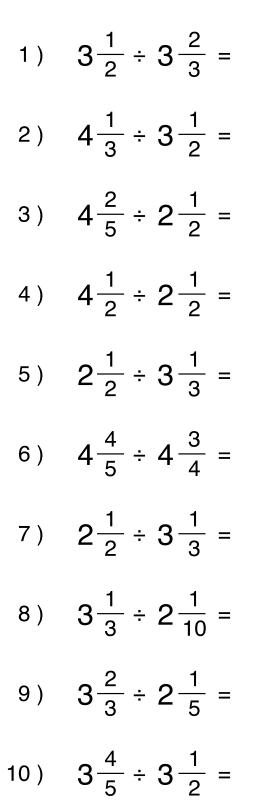5) $\dfrac{1}{2} \div \dfrac{2}{4} = \dfrac{1 \times 4}{2 \times 2} = \dfrac{4}{4} = 1$

6) $\dfrac{1}{3} \div \dfrac{1}{2} = \dfrac{1 \times 2}{3 \times 1} = \dfrac{2}{3}$

7) $\dfrac{1}{2} \div \dfrac{6}{10} = \dfrac{1 \times 10}{2 \times 6} = \dfrac{10}{12} = \dfrac{5}{6}$

8) $\dfrac{1}{10} \div \dfrac{4}{5} = \dfrac{1 \times 5}{10 \times 4} = \dfrac{5}{40} = \dfrac{1}{8}$

9) $\dfrac{2}{5} \div \dfrac{6}{10} = \dfrac{2 \times 10}{5 \times 6} = \dfrac{20}{30} = \dfrac{2}{3}$

10) $\dfrac{9}{10} \div \dfrac{1}{2} = \dfrac{9 \times 2}{10 \times 1} = \dfrac{18}{10} = \dfrac{9}{5} = 1\dfrac{4}{5}$

Worksheet 6.3 Dividing Mixed Numbers (Homework)

1) $\quad 3\dfrac{1}{2} \div 3\dfrac{2}{3} =$

2) $\quad 4\dfrac{1}{3} \div 3\dfrac{1}{2} =$

3) $\quad 4\dfrac{2}{5} \div 2\dfrac{1}{2} =$

4) $\quad 4\dfrac{1}{2} \div 2\dfrac{1}{2} =$

5) $\quad 2\dfrac{1}{2} \div 3\dfrac{1}{3} =$

6) $\quad 4\dfrac{4}{5} \div 4\dfrac{3}{4} =$

7) $\quad 2\dfrac{1}{2} \div 3\dfrac{1}{3} =$

8) $\quad 3\dfrac{1}{3} \div 2\dfrac{1}{10} =$

9) $\quad 3\dfrac{2}{3} \div 2\dfrac{1}{5} =$

10) $\quad 3\dfrac{4}{5} \div 3\dfrac{1}{2} =$

Worksheet 6.3 Homework Answers

1) $3\dfrac{1}{2} \div 3\dfrac{2}{3} = \dfrac{7 \times 3}{2 \times 11} = \dfrac{21}{22}$

2) $4\dfrac{1}{3} \div 3\dfrac{1}{2} = \dfrac{13 \times 2}{3 \times 7} = \dfrac{26}{21} = 1\dfrac{5}{21}$

3) $4\dfrac{2}{5} \div 2\dfrac{1}{2} = \dfrac{22 \times 2}{5 \times 5} = \dfrac{44}{25} = 1\dfrac{19}{25}$

4) $4\dfrac{1}{2} \div 2\dfrac{1}{2} = \dfrac{9 \times 2}{2 \times 5} = \dfrac{18}{10} = \dfrac{9}{5} = 1\dfrac{4}{5}$

5) $2\dfrac{1}{2} \div 3\dfrac{1}{3} = \dfrac{5 \times 3}{2 \times 10} = \dfrac{15}{20} = \dfrac{3}{4}$

6) $4\dfrac{4}{5} \div 4\dfrac{3}{4} = \dfrac{24 \times 4}{5 \times 19} = \dfrac{96}{95} = 1\dfrac{1}{95}$

7) $2\dfrac{1}{2} \div 3\dfrac{1}{3} = \dfrac{5 \times 3}{2 \times 10} = \dfrac{15}{20} = \dfrac{3}{4}$

8) $3\dfrac{1}{3} \div 2\dfrac{1}{10} = \dfrac{10 \times 10}{3 \times 21} = \dfrac{100}{63} = 1\dfrac{37}{63}$

9) $3\dfrac{2}{3} \div 2\dfrac{1}{5} = \dfrac{11 \times 5}{3 \times 11} = \dfrac{55}{33} = \dfrac{5}{3} = 1\dfrac{2}{3}$

10) $3\dfrac{4}{5} \div 3\dfrac{1}{2} = \dfrac{19 \times 2}{5 \times 7} = \dfrac{38}{35} = 1\dfrac{3}{35}$

Lesson 6 Exit Ticket:	**Time: 5 Minutes**

Write on the board:
If Minneapolis gets 1 1/3 inches of snowfall every day for 5 days, how much snow is it total?

1 1/3 x 5 = 4/3 x 5/1 = 20/3 = 6 2/3 inches.

Lesson 6 Extra Problem about Food Portions	**Time: 10 Minutes**

The federal government (choosemyplate.gov) recommends that we eat the following proportions of food for our meals: ½ as vegetables and fruit, ¼ as grains, and the rest as protein.

Basic Questions:
- What portion (fraction) of one's meal should be protein? *(1/4)*
- A 10 year-old male should consume 1800 calories a day. How many calories should come from grains? *(1/4 of 1800 = 450 calories)*

Extension Questions:
- Of the grains, half should be whole grains. What fraction of the total is it? *(1/2 x ¼ = 1/8)*
- A 20 year-old male should consume about 1 ½ times more calories than a 10-year old. How many calories is that? *(1 ½ x 1800 = 2700 calories)*
- A 40 year-old male should consume 1 2/5 times more calories than the 10 year old. How many calories is that? *(1 2/5 x 1800 = 7/5 x 1800 = 2520 calories)*

Encourage students to solve with both fractions and decimals if they finish quickly.