Lesson 7: Applications of uniform circular motion, Nonuniform circular motion (Sections 5.3-5.7)

Last time we began discussing rotational motion. There is a close correspondence between the familiar linear parameters and their angular counterparts.

Description	Linear	Angular
position	x	θ
displacement	Δx	$\Delta\theta$
rate of change of position	v_x	ω
average rate of change of position	$v_{x,av} = \dfrac{\Delta x}{\Delta t}$	$\omega_{av} = \dfrac{\Delta\theta}{\Delta t}$
instantaneous rate of change of position	$v_x = \lim\limits_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t}$	$\omega = \lim\limits_{\Delta t \to 0} \dfrac{\Delta\theta}{\Delta t}$

Problem-Solving Strategy for an Object in Uniform Circular Motion (page 163)

1. Begin as for any Newton's second law problem: identify all the forces acting on the object and draw a FBD.
2. Choose perpendicular axes at the point of interest so that one is radial and the other is tangent to the circular path.
3. Find the radial component of each force.
4. Apply Newton's second law as follows:

$$\sum F_r = ma_r$$

where ΣF_r is the radial component of the net force and the radial component of the acceleration is

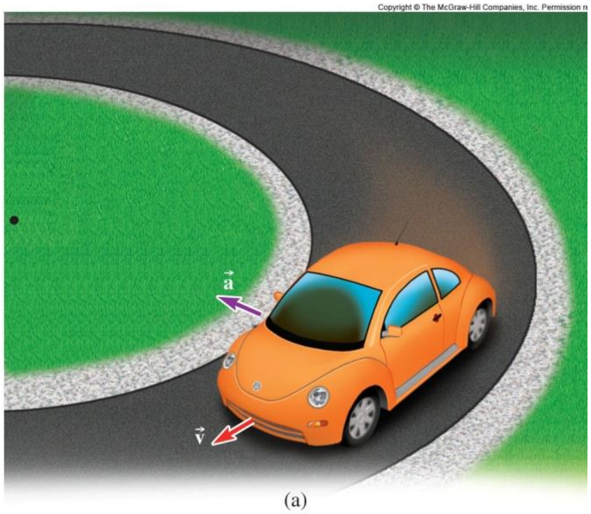
$$a_r = \frac{v^2}{r} = \omega^2 r$$

(For uniform circular motion, neither the net force nor the acceleration has a tangential component since the speed is constant.)

Banked and Unbanked curves

A car turning through a curve at constant speed is accelerated towards the center of the curve. If the road is flat, the radial force is supplied by friction.

If the wheels of the car are rolling without slipping, the radial force is static friction. If the wheels are sliding, it is kinetic friction. From the diagram below (note the direction of *x*-axis is contrary to our convention)



(a)

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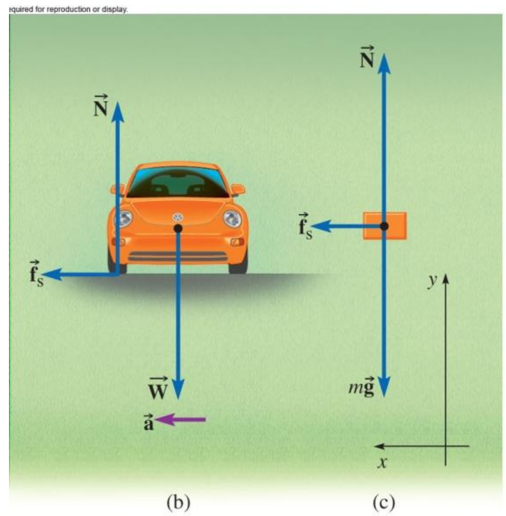
$$\sum F_x = ma_x \qquad\qquad \sum F_y = ma_y$$

$$f_s = m\frac{v^2}{r} \qquad\qquad N - mg = 0$$



What is the fastest safe speed for a curve? That occurs when the car needs the largest possible static frictional force on the curve.

$$f_{s,\max} = \mu_s N$$

$$m\frac{v^2}{r} = \mu_s mg$$

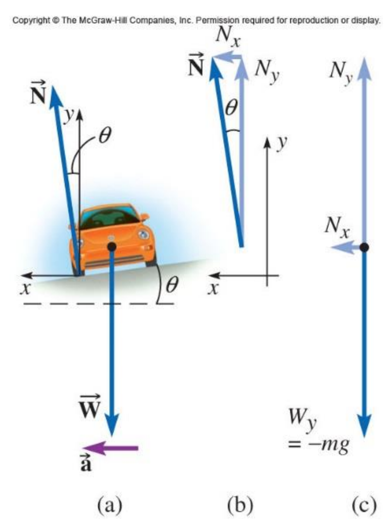
$$v_{\max} = \sqrt{\mu_s rg}$$

The maximum safe speed depends on

- the coefficient of friction. The lower μ_s, the slower the car must go. (Slow down on wet streets!)
- the radius of the curve. Go slow around sharp curves (where r is small)!
- the acceleration due to gravity. (Be careful driving on the moon!!!)

The situation can be improved by **banking the curve**.

The needed radial force is supplied by a component of the normal force. It is important to notice that the x-axis does not point down the incline. It points in the direction of the radial acceleration to the left. The car does not slide down the incline. It is accelerated towards the center of the curve.



$$\sum F_x = ma_x \qquad\qquad \sum F_y = ma_y$$

$$N_x = m\frac{v^2}{r} \qquad\qquad N_y - W = 0$$

$$N\sin\theta = m\frac{v^2}{r} \qquad\qquad N\cos\theta = mg$$

Dividing the two equations to eliminate the normal force

$$\frac{N\sin\theta}{N\cos\theta} = \frac{m\frac{v^2}{r}}{mg}$$

$$\tan\theta = \frac{v^2}{rg}$$

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A car traveling this speed around a curve banked at angle θ with radius r, will not require any friction to safely travel around the curve. What happens if the car goes too fast? What happens if the car goes too slowly?

Circular orbits

The Earth remains in orbit around the sun because of the gravitational pull of the sun. Gravitation supplies the radial force needed to keep the Earth in its orbit. The same physics occurs with satellites in orbit around the Earth.

After working for 20 years with the observations of Tycho Brahe, Johannes Kepler stated his **three laws of planetary motion**:

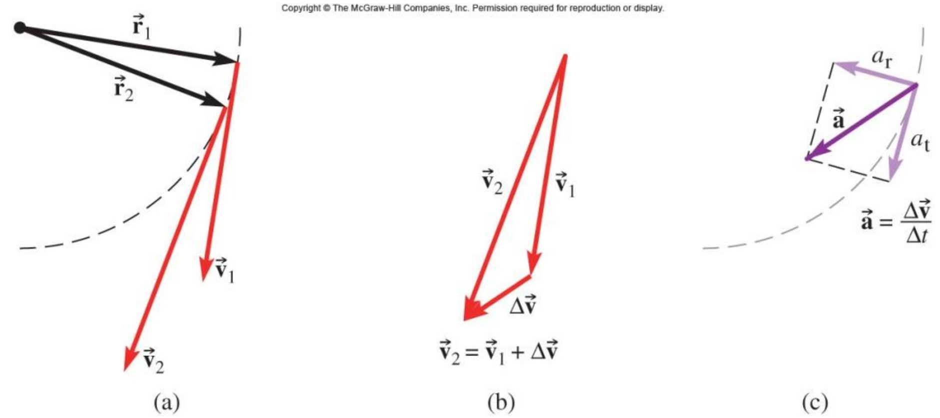
- The planets travel in elliptical orbits with the Sun at one focus of the ellipse.
- A line drawn from a planet to the Sun sweeps out equal areas in equal time intervals.
- The square of the orbital period is proportional to the cube of the average distance from the planet to the Sun.

In a remarkable verification of his law of gravitation, Newton was able to derive Kepler's laws.

http://galileoandeinstein.physics.virginia.edu/lectures/tycho.htm

Nonuniform Circular Motion

Suppose the rotating wheel changes its angular speed.



This time $\Delta\vec{\mathbf{v}}$ does not point towards the center of the circle!

There is now a tangential acceleration as well as the radial acceleration we have studied. Since the radial and tangential directions are perpendicular to each other, the overall acceleration is

$$a = \sqrt{a_r^{\ 2} + a_t^{\ 2}}$$

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Problem-Solving Strategy for an Object in Nonuniform Circular Motion (page 165)

1. Begin as for any Newton's second law problem: Identify all the forces acting on the object and draw an FBD.
2. Choose perpendicular axes at the point of interest so that one axis is radial and the other is tangent to the circular path.
3. Find the radial component of each force.
4. Apply Newton's second law along the radial direction:

$$\sum F_r = ma_r$$

where

$$a_r = \frac{v^2}{r} = \omega^2 r$$

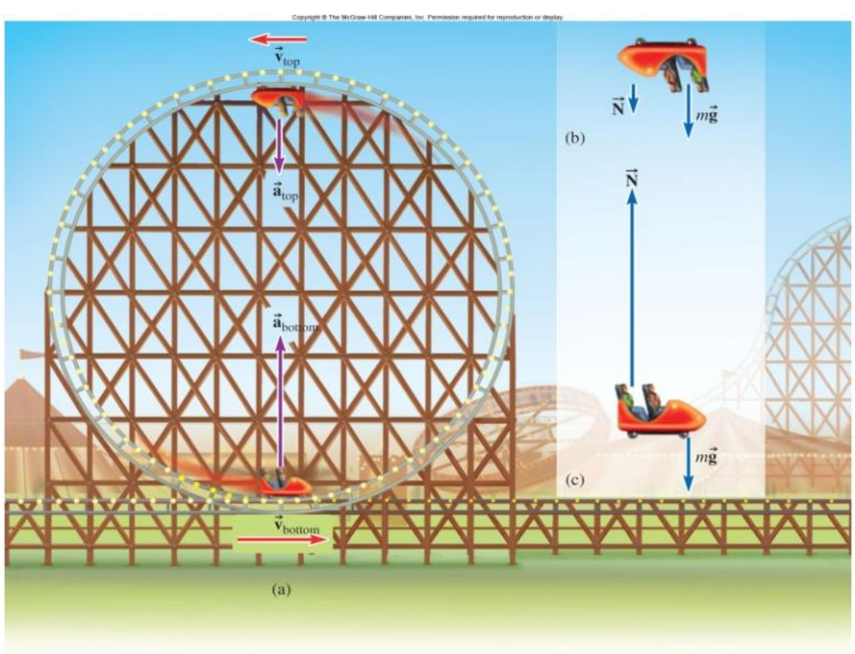
5. If necessary, apply Newton's second law to the tangential force components:

$$\sum F_t = ma_t$$

The tangential acceleration component a_t determines how the speed of the object changes.

Apparent Weight

At what point of the ride would you feel heaviest? Lightest?



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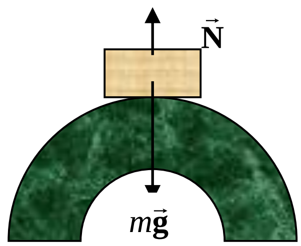
To travel around a circle, there must be an acceleration towards the center of the circle. At the top, some of the radial acceleration is supplied by the weight. The rest is supplied by the normal force, the force of the seat on the passenger. At the bottom, the normal force must overcome the weight. Weight does not change, but the normal force does. The reaction to the normal force (the seat pushing on the passenger) is the passenger pushing on the seat. That force is largest at the bottom, where you feel the heaviest.

Problem 5.42 A car approaches the top of a hill that is shaped like a vertical circle with a radius of 55.0 m. What is the fastest speed that the car can go over the hill without losing contact with the ground?



The radial direction is in the –*y*-direction. Using Newton's second law,

$$\sum F_r = ma_r$$
$$mg - N = m\frac{v^2}{r}$$

When the car is just about to leave the ground $N = 0$.

$$mg - 0 = m\frac{v^2}{r}$$
$$v = \sqrt{rg} = \sqrt{(55.0\,\mathrm{m})(9.8\,\mathrm{m/s^2})} = 23.2\,\mathrm{m/s}$$

Tangential and Angular Acceleration
We can add rows to our table

Description	**Linear**	**Angular**
position	x	θ
displacement	Δx	$\Delta\theta$
Rate of change of position	v_x	ω
Average rate of change of position	$v_{x,av} = \frac{\Delta x}{\Delta t}$	$\omega_{av} = \frac{\Delta\theta}{\Delta t}$
Instantaneous rate of change of position	$v_x = \lim_{\Delta t\to 0}\frac{\Delta x}{\Delta t}$	$\omega = \lim_{\Delta t\to 0}\frac{\Delta\theta}{\Delta t}$

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Average rate of change of speed	$a_{x,av} = \dfrac{\Delta v_x}{\Delta t}$	$\alpha_{av} = \dfrac{\Delta \omega}{\Delta t}$
Instantaneous rate of change of speed	$a_x = \lim\limits_{\Delta t \to 0} \dfrac{\Delta v_x}{\Delta t}$	$\alpha = \lim\limits_{\Delta t \to 0} \dfrac{\Delta \omega}{\Delta t}$

The components of acceleration are

$$a_r = \frac{v^2}{r} = \omega^2 r \qquad a_t = \alpha r$$

Using similar reasoning to what we used for uniform linear motion, we can create equations for uniform angular motion.

Uniform Linear Motion	**Uniform Angular Motion**
$a_x = \text{constant}$	$\alpha = \text{constant}$
$\Delta v_x = v_{fx} - v_{ix} = a_x \Delta t$	$\Delta \omega = \omega_f - \omega_i = \alpha \Delta t$
$\Delta x = \frac{1}{2}(v_{fx} + v_{ix})\Delta t$	$\Delta \theta = \frac{1}{2}(\omega_f + \omega_i)\Delta t$
$\Delta x = v_{ix}\Delta t + \frac{1}{2} a_x (\Delta t)^2$	$\Delta \theta = \omega_i \Delta t + \frac{1}{2}\alpha(\Delta t)^2$
$v_{fx}^2 - v_{ix}^2 = 2a_x \Delta x$	$\omega_f^2 - \omega_i^2 = 2\alpha\Delta\theta$

Problem 5.52 A disk rotates with constant angular acceleration. The initial speed of the disk is 2π rad/s. After the disk rotates through 10π radians, the angular speed is 7π rad/s. (a) What is the magnitude of the angular acceleration? (b) How much time did it take for the disk to rotate through 10π radians? (c) What is the tangential acceleration of a point located at a distance of 5.0 cm from the center of the disk?

(a)

$$\begin{aligned}
\omega_f^2 - \omega_i^2 &= 2\alpha\Delta\theta \\
\alpha &= \frac{\omega_f^2 - \omega_i^2}{2\Delta\theta} \\
&= \frac{(7\pi \text{ rad/s})^2 - (2\pi \text{ rad/s})^2}{2(10\pi \text{ rad})} \\
&= 2.25\pi \text{ rad/s}^2 \\
&= 7.07 \text{ rad/s}^2
\end{aligned}$$

(b)

$$\Delta\theta = \tfrac{1}{2}(\omega_f + \omega_i)\Delta t$$

$$\Delta t = \frac{2\Delta\theta}{\omega_f + \omega_i} = \frac{2(10\pi \text{ rad})}{7\pi \text{ rad/s} + 2\pi \text{ rad/s}} = 2.22 \text{ s}$$

(c)

$$a_t = \alpha r = (2.25\pi \text{ rad/s}^2)(0.05\text{m}) = 0.353\text{m/s}^2$$

Weightlessness
Are these people weightless?

Recall,

$$g = \frac{GM_E}{r^2}$$

Even when 200 miles above the Earth's surface, g is not zero. In fact, it is still close to 9.8 m/s^2 since the 200 mile altitude is tiny compared to the radius of the earth (3,960 miles).

Why do things float in the space station? Both of the accelerations of the space station and the astronauts are the same. It is similar to the weightlessness experienced by a person falling in a severed elevator.

A video of weightlessness

https://www.youtube.com/watch?v=LWGJA9i18Co

To make artificial gravity, spin the space station. This is the principle of a centrifuge.