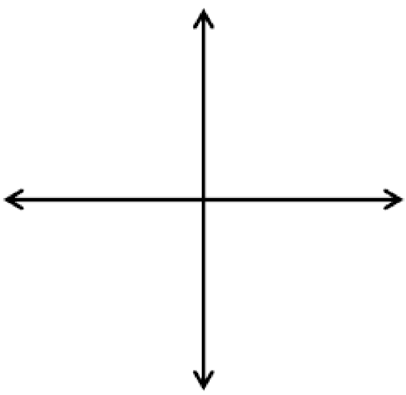
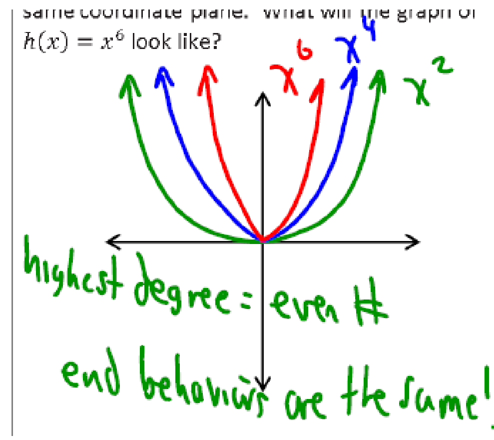
Lesson 8.1: Key Features of Polynomial Graphs

Learning Goals: What are the key features of the graph of a polynomial function?

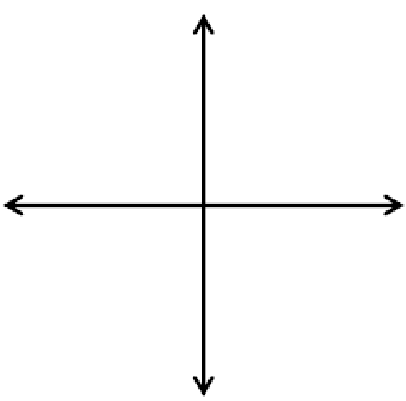
Discovery: How does the highest degree of the polynomial determine the shape of a graph? Degree = exponent

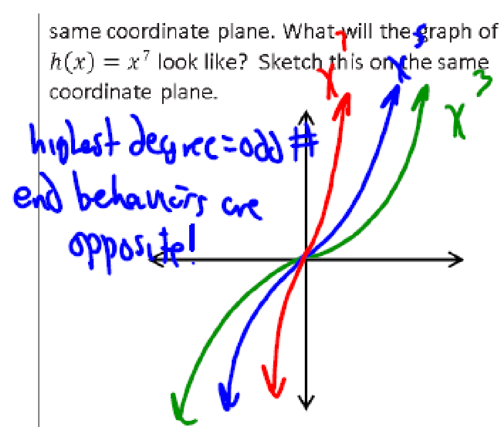
1. Sketch the graph of $f(x) = x^2$. What will the graph of $g(x) = x^4$ look like? Sketch it on the same coordinate plane. What will the graph of $h(x) = x^6$ look like?





2. Sketch the graph of $f(x) = x^3$. What will the graph of $g(x) = x^5$ look like? Sketch this on the same coordinate plane. What will the graph of $h(x) = x^7$ look like? Sketch this on the same coordinate plane.

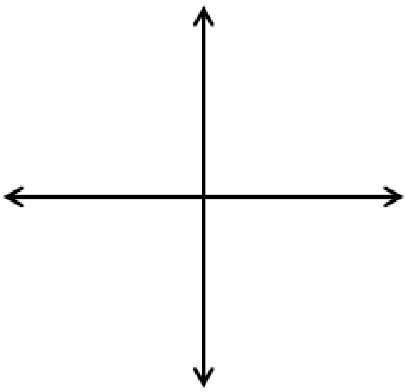


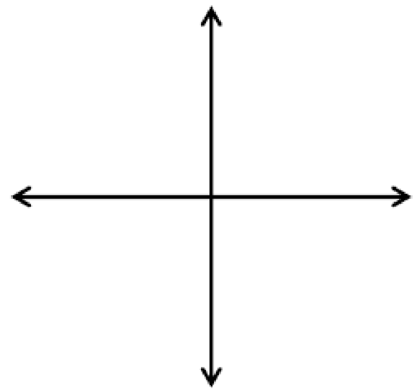


3. Consider the following functions $f(x)$ and $g(x)$, with a mixture of odd and even degree terms. Predict whether its end behavior will be like the functions in Example 1 or Example 2.

Graph the function f and g using a graphing utility to check your prediction.

$f(x) = 2x^4 + x^3 - x^2 + 5x + 3$ $\qquad\qquad$ $g(x) = 2x^5 - x^4 - 2x^3 + 4x^2 + x + 3$

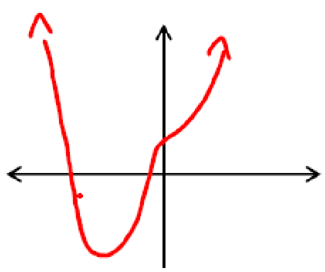




$f(x) = 2x^4 + x^3 - x^2 + 5x + 3$

example 1 even!

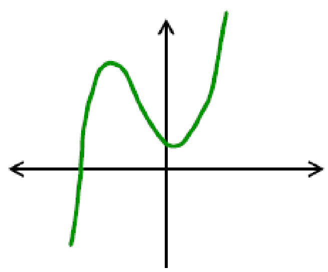
"end" of graph are the Same!



$g(x) = 2x^5 - x^4 - 2x^3 + 4x^2 + x + 3$

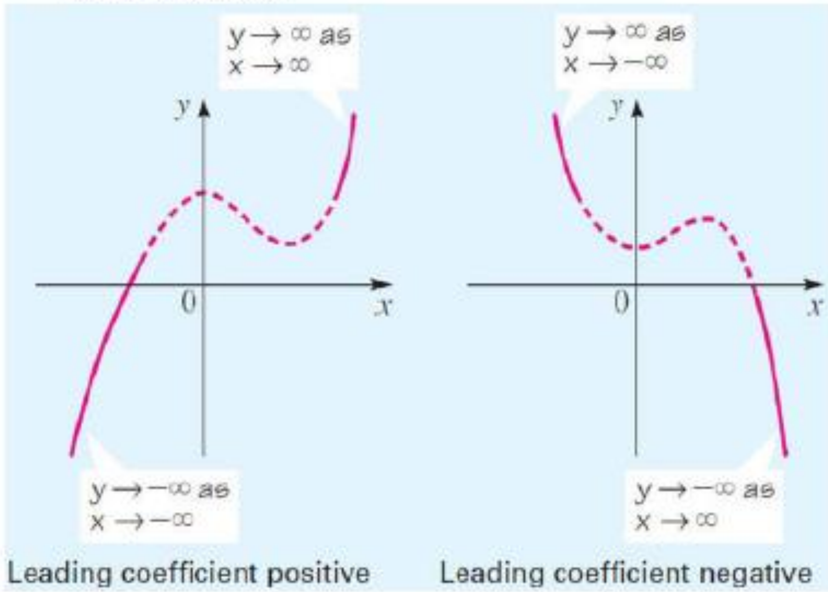
example 2 odd!

"ends" of graphs are the same!

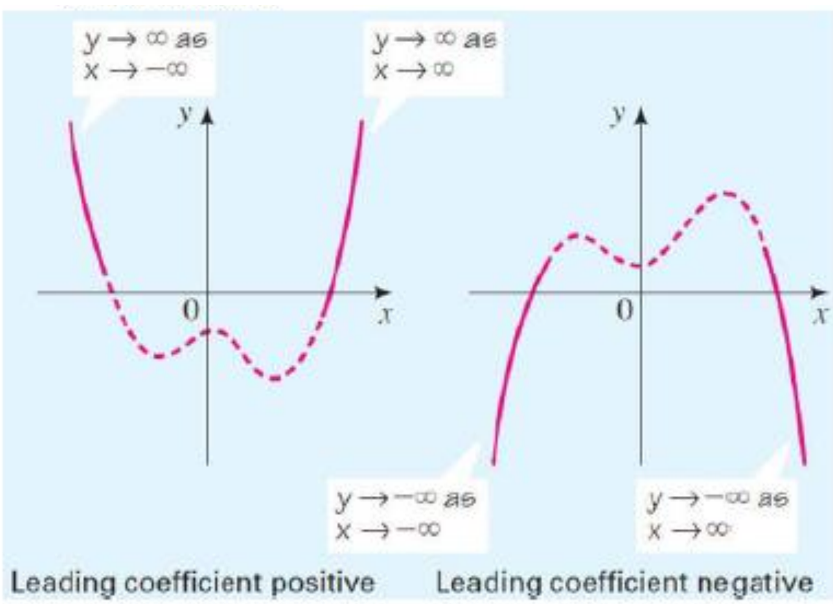


<u>**Summary of Odd/Even**</u>

If the highest degree of the polynomial is odd, the general shape of the graph will be as follows:

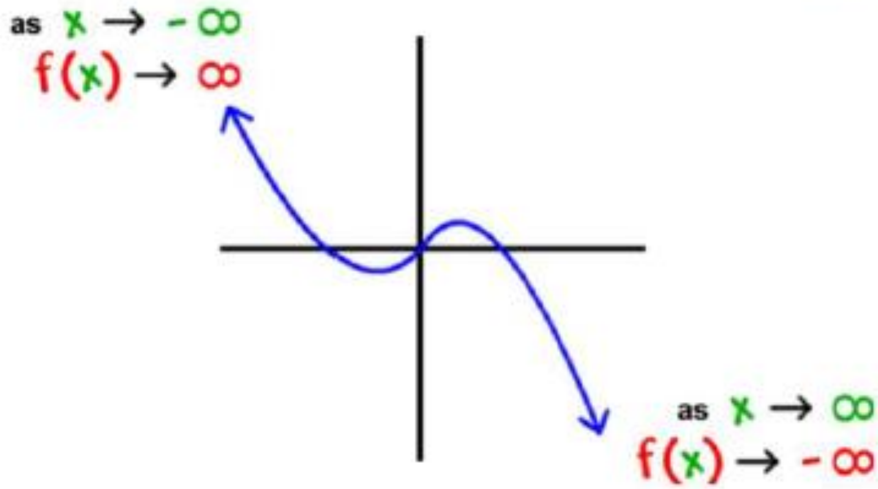


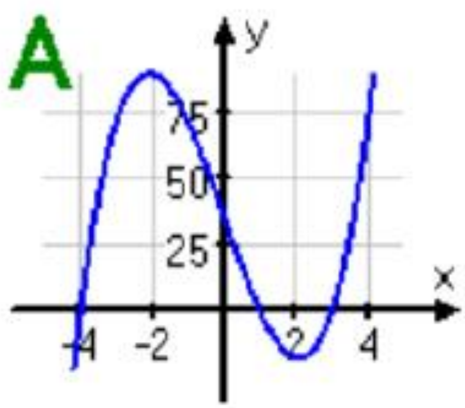
If the highest degree of the polynomial is even, the general shape of the graph will be as follows:

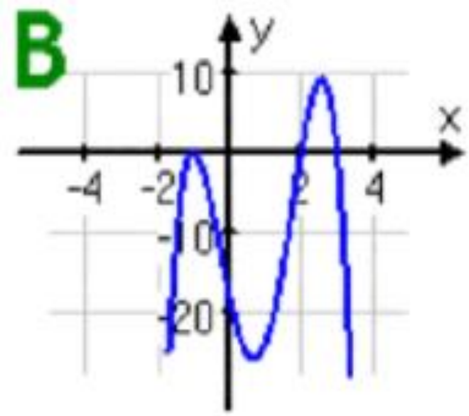


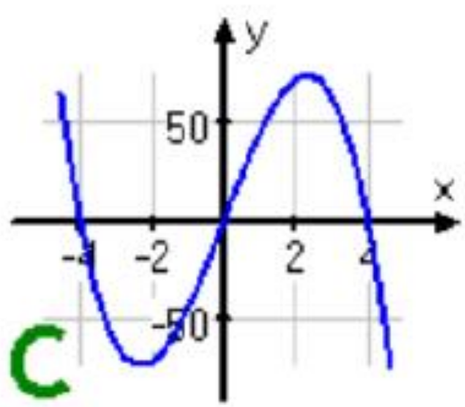
End Behavior: (Let f be a function whose domain and range are subsets of real numbers. The end behavior of a function f is a description of what happens to the values of the function

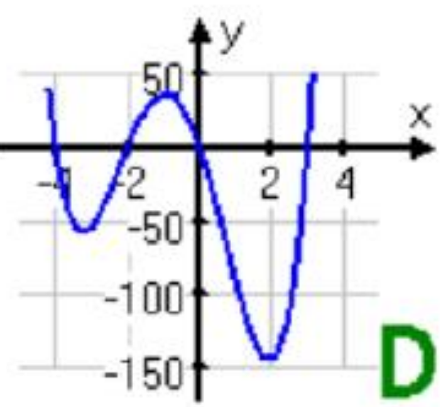
- as x approaches positive infinity, what do the y-values approach?
- as x approaches negative infinity, what do the y-values approach?

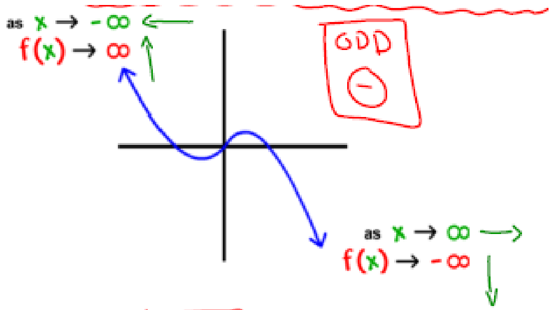

as $x \to -\infty$
$f(x) \to \infty$
as $x \to \infty$
$f(x) \to -\infty$


A
y
75
50
25
x
-4
-2
2
4


B
y
10
x
-4
-2
2
4
-10
-20


y
50
x
-4
-2
2
4
-50
C

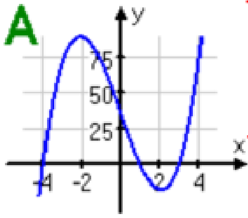

y
50
x
-4
-2
2
4
-50
-100
-150
D



ODD

−

A

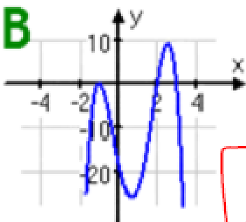


ODD

+

x → ∞ f(x) → ∞	x → -∞ f(x) → -∞

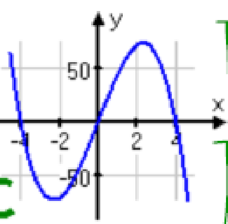
B



EVEN

−

x → ∞ f(x) → -∞	x → -∞ f(x) → -∞

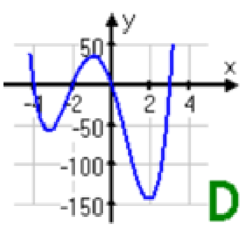


C

ODD

−

x → ∞ f(x) → -∞	x → -∞ f(x) → ∞



D

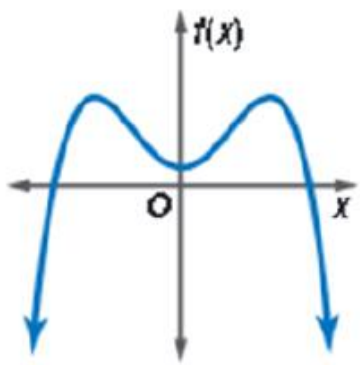
EVEN

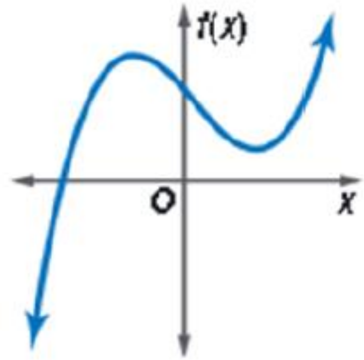
+

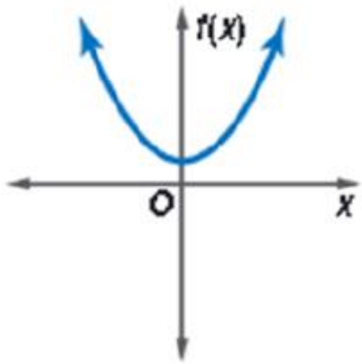
x → ∞ y → ∞	x → -∞ y → ∞

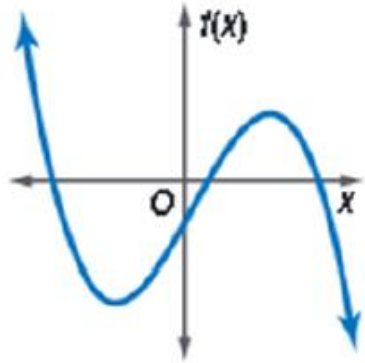
Exercise: For each graph given below:

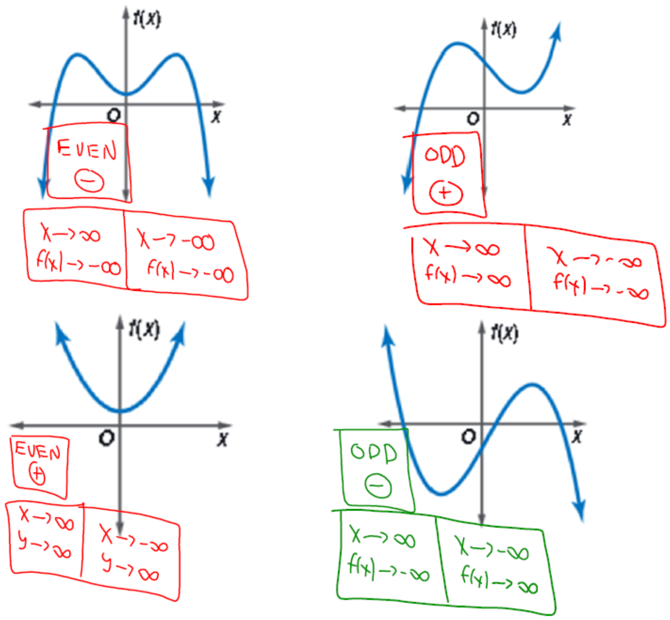
1) determine whether it represents an odd-degree or an even-degree polynomial;
2) determine the sign of the leading coefficent (positive or negative);
3) describe the end behavior of each graph.









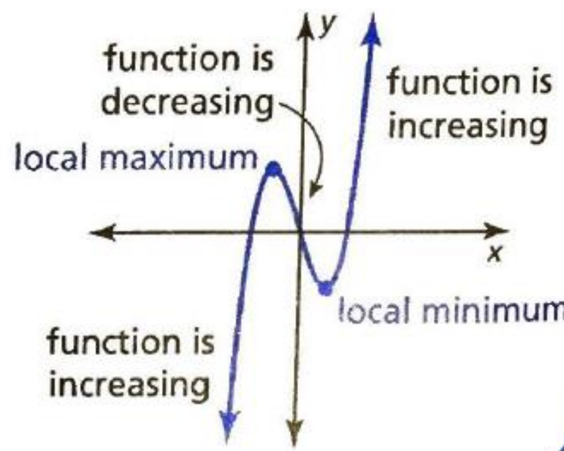


Relative (Local) Max and Min of a Polynomial

Another important characteristic of graphs of polynomial functions is that they have *turning points* corresponding to local maximum and minimum values.

- The y-coordinate of a turning point is a **local maximum** of the function when the point is higher than all nearby points.
- The y-coordinate of a turning point is a **local minimum** of the function when the point is lower than all nearby points.

The turning points of a graph help determine the intervals for which a function is increasing or decreasing.

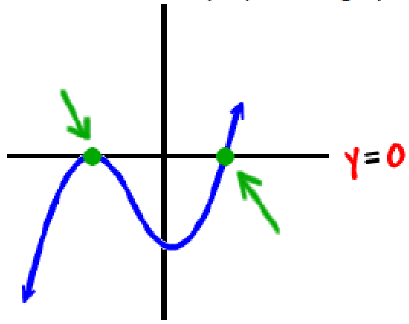


On what intervals is the function above increasing and decreasing?

Decreasing: $-2 < x < 1$ Increasing: $-\infty < x < -2$ and $1 < x < \infty$

Real Zeros of a Polynomial (x-intercepts)

- The zeros of a polynomial function are the solutions to the polynomial equation when the polynomial equals zero (when f(x) = 0)
- The ***real zeros*** of a polynomial are where the polynomial graph crosses the x-axis.

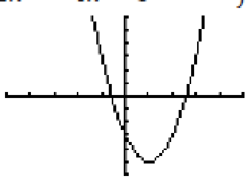


y-intercept of a Polynomial

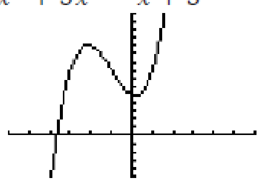
The y-intercepts of a polynomial is where the graph cross the y-axis.

- Consider the following examples of polynomial functions and their graphs to determine an easy way to find the y-intercept of the graph of a polynomial function.

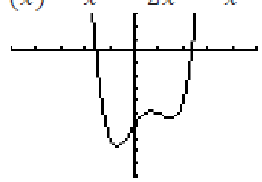
$f(x) = 2x^2 - 4x - 3$

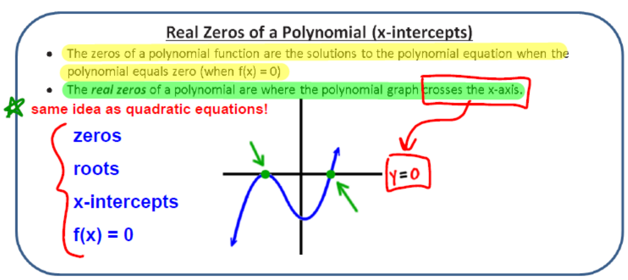


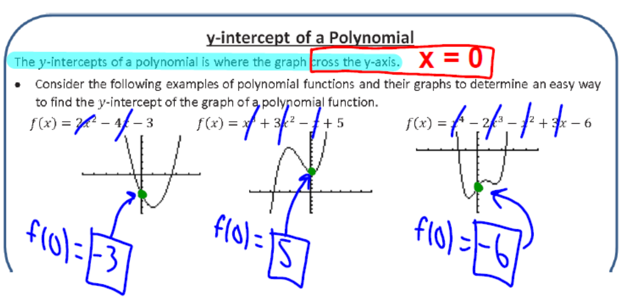
$f(x) = x^3 + 3x^2 - x + 5$



$f(x) = x^4 - 2x^3 - x^2 + 3x - 6$




Real Zeros of a Polynomial (x-intercepts)
The zeros of a polynomial function are the solutions to the polynomial equation when the polynomial equals zero (when f(x) = 0)
The real zeros of a polynomial are where the polynomial graph crosses the x-axis.
same idea as quadratic equations!
zeros
roots
x-intercepts
f(x) = 0
y=0


y-intercept of a Polynomial
The y-intercepts of a polynomial is where the graph cross the y-axis.
Consider the following examples of polynomial functions and their graphs to determine an easy way to find the y-intercept of the graph of a polynomial function.
$x = 0$

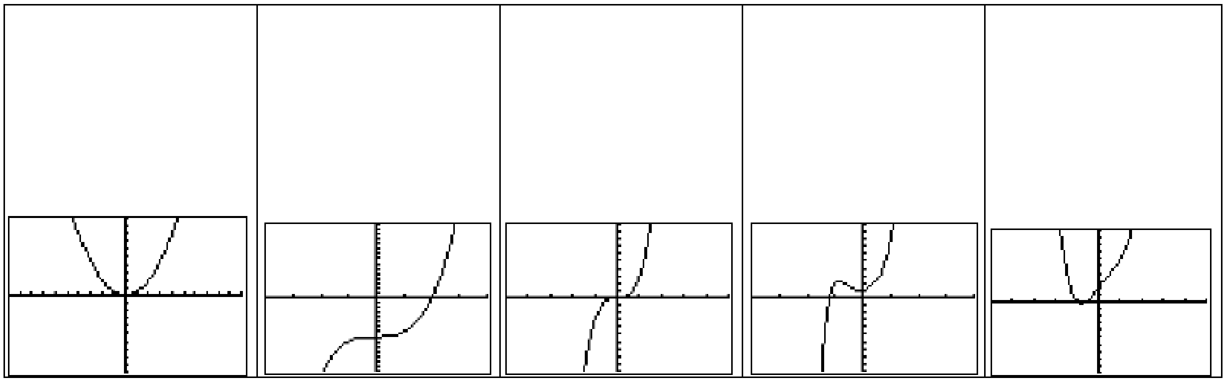
$f(0) = \boxed{-3}$

$f(0) = \boxed{5}$

$f(0) = \boxed{-6}$

Exercise: Match each graph below with the given functions that it represents (no calculators).

a. $y = 3x^3$ b. $y = \frac{1}{2}x^2$ c. $y = x^3 - 8$ d. $y = x^4 - x^3 + 4x + 2$ e. $3x^5 - x^3 + 4x + 2$

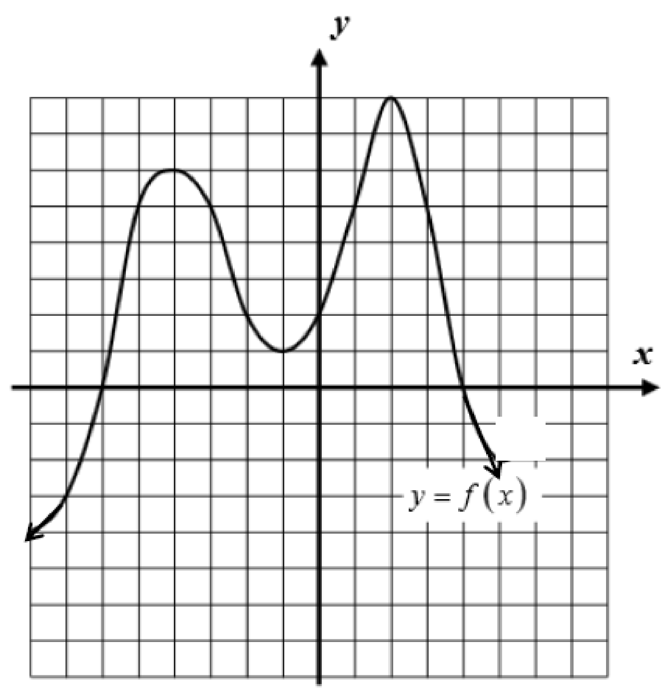


Use these to help you:

- Highest degree
- End behavior
- Leading coefficient (positive or negative)
- y-intercept

Putting It All Together

1. The function $y = f(x)$ is shown below.



Answer the following questions based on the graph:

a) State the x-intercepts and y-intercepts of the function.

x-intercepts: -6 & 4 y-intercepts: 2

b) State the domain and range of the function.

Domain: $(-\infty, +\infty)$ Range: $y \leq$ or $(-\infty, 8]$

c) State all the x-coordinates of the relative maximums and relative minimums.

Relative Maximums: $x = -4$ & 2

Relative Minimum: $x = -1$

d) Over the interval, $-1 < x < 2$, is $f(x)$ increasing or decreasing? How can you tell? Graph goes up

e) Over which interval(s), is $f(x)$ decreasing?

$-4 < x < -1 \quad \text{or} \quad x > 2$

f) What is the interval(s) for which $f(x) > 0$? What is a quick way of seeing this visually?

$-6 < x < 4$ if it is above the x-axis then it is > 0.

g) Determine if f is an odd or even polynomial. Then determine if the sign of the leading coefficient is positive or negative.

2. Use the graph below to answer the given questions:

a) Determine the x-intercept(s) of the graph.

x-intercepts: $-4, -1, \& \ 2$

b) Determine the y-intercept(s) of graph.

y-intercepts: -2

c) Describe the end behavior of the function.

As $x \to -\infty, y \to -\infty$ and

As $x \to +\infty, y \to +\infty$

d) Approximate the x-coordinates of the relative maximums and relative minimums.

Relative Maximum: $x \approx -2.75$ Relative Minimum: $x \approx .75$

e) Determine the interval(s) where the function is increasing.

$x < -2.75 \quad \text{and} \quad x > .75$

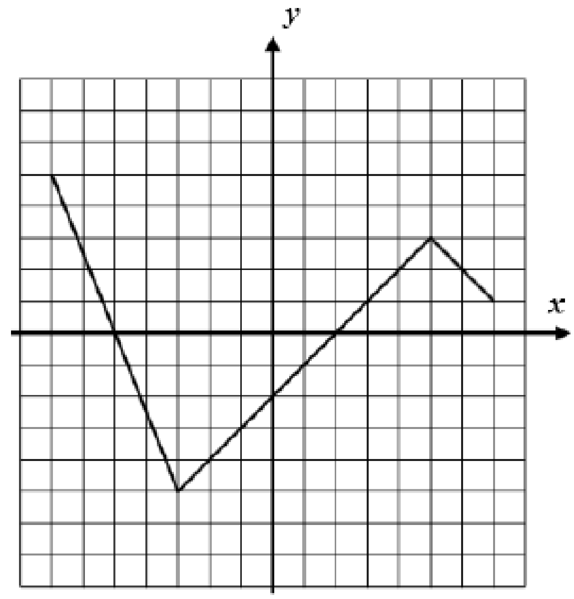
f) What is the interval(s) for which $f(x) < 0$? What is a quick way of seeing this visually?

$-\infty < x < -4 \ \text{and} \ -1 < x < 2$ if it is below the x-axis then it is < 0.

g) Determine if the function is an odd or even polynomial. Then determine if the sign of the leading coefficient is positive or negative.

Homework 8.1: Key Features of Polynomial Graphs

1. The piecewise linear function $f(x)$ is shown to the right. Answer the following questions based on its graph.



(a) Evaluate each of the following based on the graph:

(i) $f(4) =$ (ii) $f(-3) =$

(b) State the zeros of $f(x)$.

(c) Over which of the following intervals is $f(x)$ always increasing?

(1) $-7 < x < -3$

(2) $-3 < x < 5$

(3) $-5 < x < 5$

(4) $-5 < x < 3$

(d) State the coordinates of the relative maximum and the relative minimum of this function.

Relative Maximum: ____________ Relative Minimum: ____________

(e) Over which of the following intervals is $f(x) < 0$?

(1) $-7 < x < -3$ (2) $2 \le x \le 7$ (3) $-5 < x < 2$ (4) $-5 \le x \le 2$

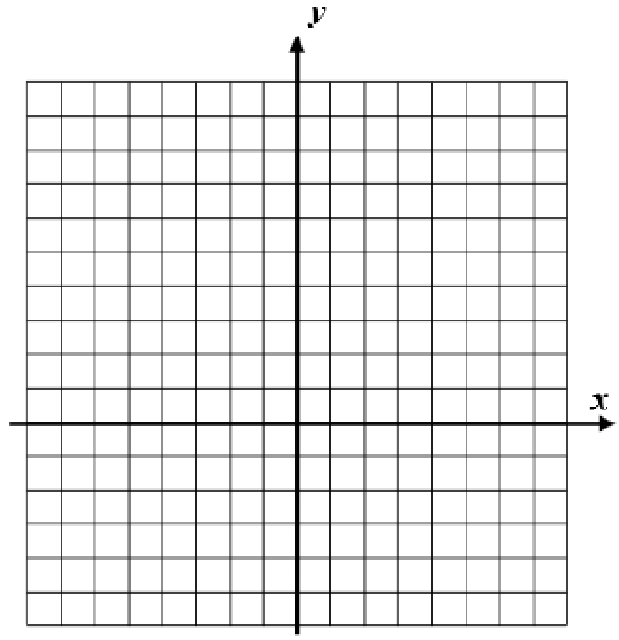
2. A continuous function has a domain of $-7 \le x \le 10$ and has selected values shown in the table below. The function has exactly two zeroes and a relative maximum at $(-4, 12)$ and a relative minimum at $(5, -6)$

x	-7	-4	-1	0	2	5	7	10
$f(x)$	8	12	0	-2	-5	-6	0	4

(a) State the intervals on which $f(x)$ is decreasing.

(b) State the interval over which $f(x) < 0$.

3. For the function $g(x) = 9 - (x+1)^2$ do the following.



(a) Sketch the graph of g on the axes provided.

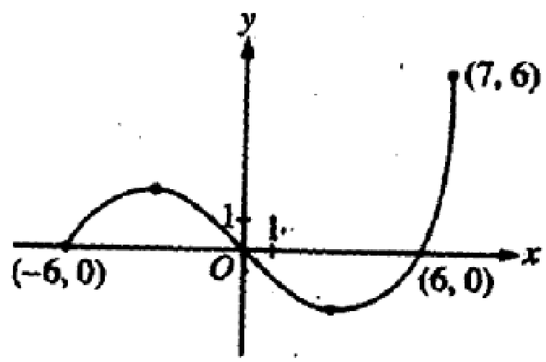
(b) State the zeros of g.

(c) Over what interval is $g(x)$ decreasing?

(d) Over what interval is $g(x) \geq 0$?

(e) State the range of g.

4. Based on the graph of the function f below, what are the values of x for which $f(x)$ is negative?



(a) $-6 < x < 0$

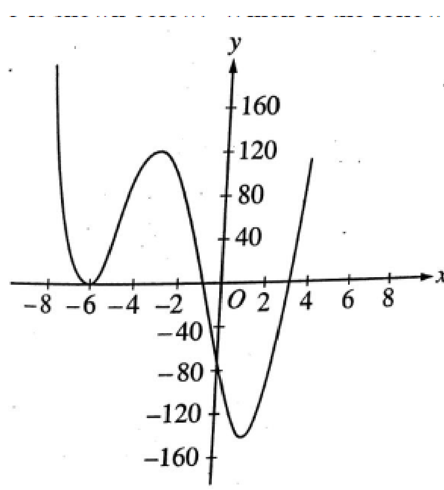
(b) $0 < x < 6$

(c) $6 < x < 7$

(d) $-6 < x < 6$

(e) $-6 < x < 0$ and $6 < x < 7$

5. The graph of $y = x^4 + 10x^3 + 10x^2 - 96x + c$ is shown below. Which of the following could be the value of c?



(a) $3{,}240$

(b) $1{,}080$

(c) 72

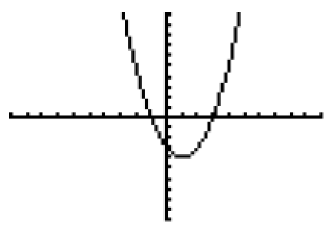
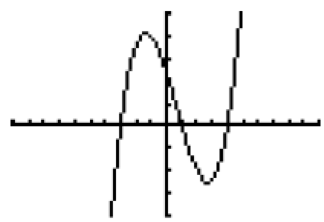
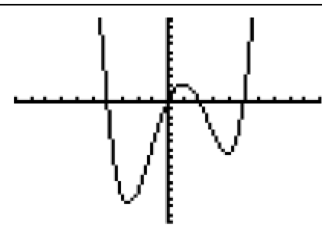
(d) -72

(e) $-3{,}240$

Lesson 8. 2: Sketching a Polynomial in Factored Form

Learning Goal: How do we sketch a polynomial in factored form by using its characteristics?

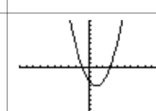
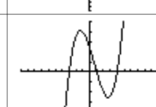
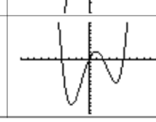
Discovery: For any particular polynomial, can we determine how many relative maxima or minima there are? Consider the following polynomial functions in factored form and their graphs.

Polynomial	Graph	Degree of Polynomial	# of x-intercepts	# of relative max/min
$f(x) = (x + 1)(x - 3)$				
$g(x) = (x + 3)(x - 1)(x - 4)$				
$h(x) = (x)(x + 4)(x - 2)(x - 5)$				

What observations can we make from this information?

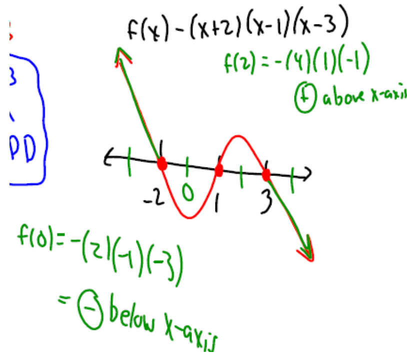
Looks like the degree of a polynomial equals the number of x-intercepts.

Looks like the number of max/min is one less than the degree of the polynomial.

Polynomial	Graph	Degree of Polynomial	# of x-intercepts	# of relative max/min
$x \cdot x$ $f(x) = (x + 1)(x - 3)$ $\boxed{x^2}$		2	2	1
$g(x) = (x + 3)(x - 1)(x - 4)$ x^3		3	3	2
$h(x) = (x)(x + 4)(x - 2)(x - 5)$ x^4		4	4	3

Practice: Identify the following about the function below: location of the zeros, degree of the polynomial, sign of the leading coefficient and end behavior:

$$f(x) = -(x+2)(x-1)(x-3)$$



$x = -2, 1, 3$

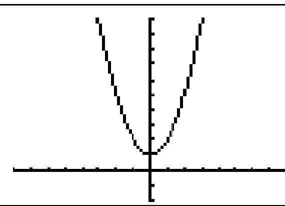
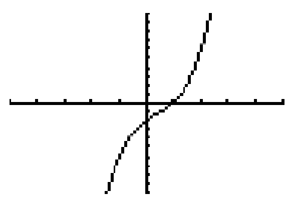
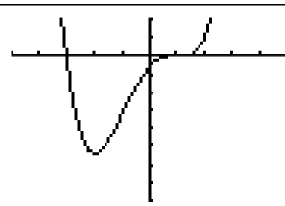
Degree = 3

Leading coefficient = $-x^3$

So it is negative, Odd

end behavior: $x \to -\infty, y \to \infty$ and $x \to \infty, y \to \infty$

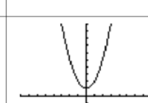
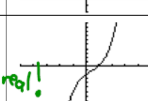
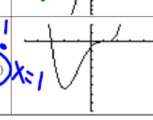
Discovery: Is this true for every polynomial? Consider the examples below.

Polynomial	Graph	Degree of Polynomial	# of x-intercepts	# of relative max/min
$r(x) = x^2 + 1$				
$s(x) = (x^2 + 2)(x - 1)$				
$t(x) = (x + 3)(x - 1)(x - 1)(x - 1)$				

What observations can we make from this information?

The number of x-intercepts for a graph can be UP TO the degree of the polynomial.

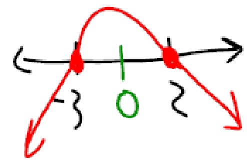
The number of max/min for a graph can be UP TO one less than the degree of the polynomial.

Polynomial	Graph	Degree of Polynomial	# of x-intercepts	# of relative max/min
$x^2+1=0$, $x^2=-1$, $x^2=\pm i$ (imaginary) $r(x) = x^2 + 1$		2	0	1
$x^2+2=0$, $x^2=-2$, $x=\pm i\sqrt{2}$ $s(x) = (x^2 + 2)(x - 1)$ $\boxed{x=1}$ real!		3	1	0
same zero! $t(x) = (x + 3)(x - 1)(x - 1)(x - 1)$ $x=1$		4	2	1

Practice: Identify the following about the function below: location of the zeros, degree of the polynomial, sign of the leading coefficient, and end behavior:

$$f(x) = -(x^2 + 9)(x - 2)(x + 3)$$

x intercepts$=-3,2$
$x=\pm 3i$ (not an $x-$intercept)



Degree $= 4$

Leading coefficient $= -x^4$

So it is negative, even

end behavior: $x \to \infty, y \to -\infty$ and $x \to -\infty, y \to -\infty$

$f(x) = -(x^2+9)(x-2)(x+3)$

$f(0) = -(9)(-2)(3)$

$= (+)$ above x-axis!

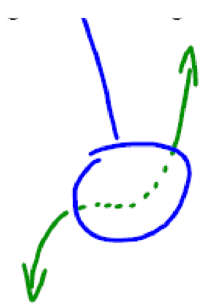
SUMMARY

- ✓ By looking at the factored form of a polynomial, we can identify important characteristics of the graph such as x-intercepts and degree of the function, which in turn allow us to develop a sketch of the graph.
- ✓ A polynomial function of degree n may have up to n x-intercepts.
- ✓ A polynomial function of degree n may have up to $n - 1$ relative maxima and minima.

Model Problem: Consider the function $f(x) = (x - 4)(x - 8)(x - 1)$.

a. Find the x-intercepts for the graph of f.

$x = 1, 4, 8$

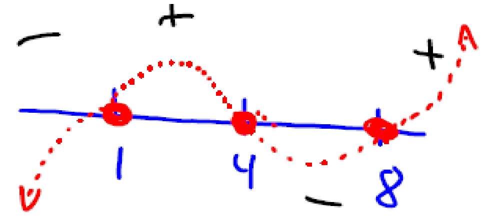


b. What is the degree of the polynomial and the sign of the leading coefficient? Leading coefficient $= +x^3$, positive odd

c. What is the end behavior of the function f?

end behavior: $x \to -\infty, y \to -\infty$ and $x \to \infty, y \to \infty$

d. To sketch a graph of f, we need to consider whether the function is positive or negative on the intervals $1 < x < 4$ and $4 < x < 8$ to determine if the graph is above or below the x-axis between x-intercepts. How can we determine this? Test a point in between the x-intercepts.



e. For $1 < x < 4$, is the graph above or below the x-axis?

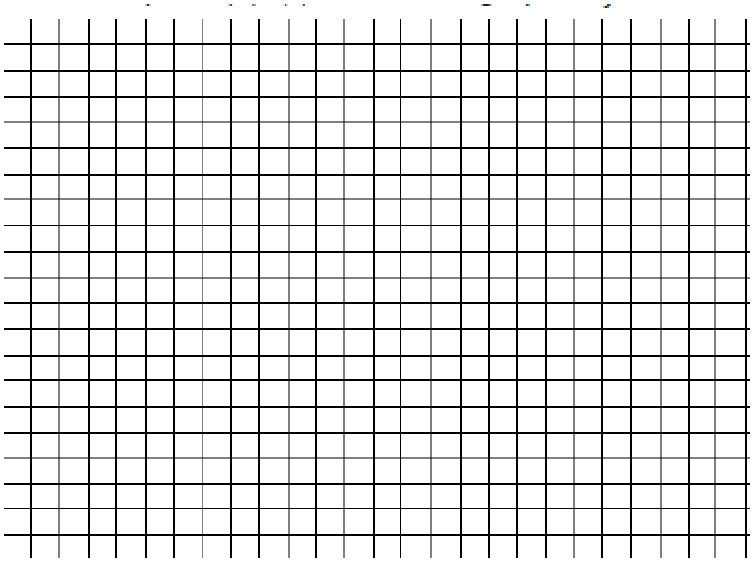
Pick $x = 3$ $\quad (x-4)(x-8)(x-1)$

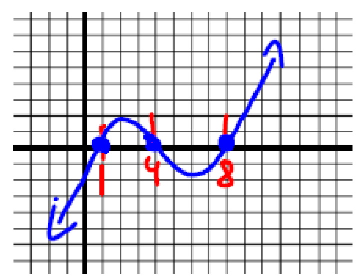
$(3-4)(3-8)(3-1) = (-1)(-5)(2) = +10$ so above

f. For $4 < x < 8$, is the graph above or below the x-axis?

Pick $x = 5$ $\quad (5-4)(5-8)(5-1) = (1)(-3)(4) = -12$ so below

g. Use the information generated in parts (a) – (f) to sketch a graph of f.

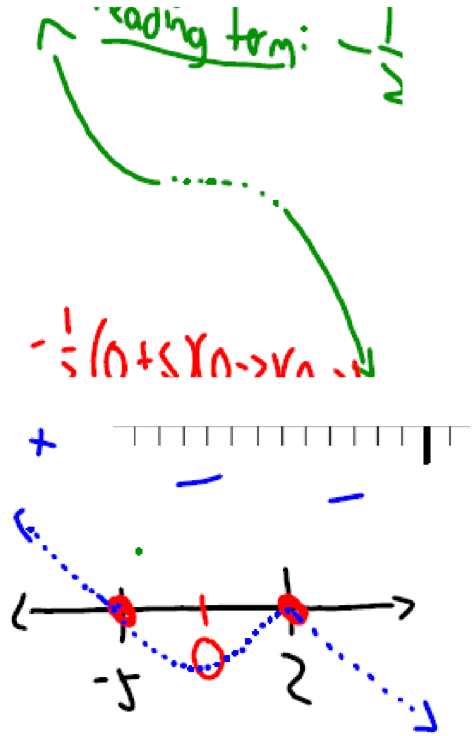




<u>**SUMMARY FOR GRAPHING POLYNOMIAL FUNCTIONS**</u>

1. **Zeros** – Factor the polynomial to find all its real zeros; these are the x-intercepts of the graph.
2. **Test Points** – Test a point between the x-intercepts to determine whether the graph of the polynomial lies above or below the x-axis on the intervals determined by the zeros.
3. **End Behavior** – Determine the end behavior of the polynomial by looking at the degree of the polynomial and the sign of the leading coefficient.
4. **Graph** – Plot the intercepts and other points you found when testing. Sketch a smooth curve that passes through these points and exhibits the required end behavior.

a. Sketch a graph of the function $f(x) = -\frac{1}{2}(x+5)(x-2)(x-2)$ by finding the zeros and determining the sign of the values of the function between zeros.



Zeros: $x = -5$ and $x = 2$

Leading term: $-\frac{1}{2}x^3$

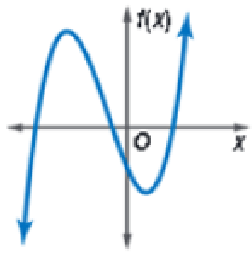
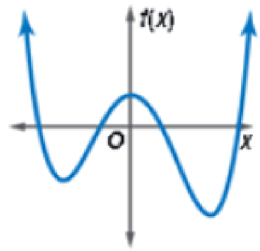
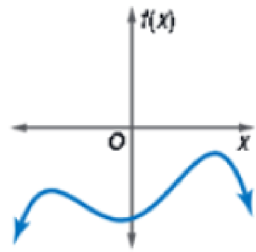
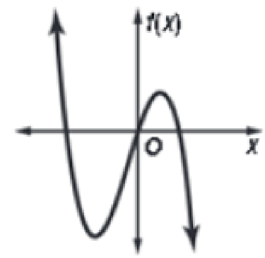
Pick $x = 0$

$-\frac{1}{2}(0+5)(0-2)(0-2) = -\frac{1}{2}(5)(-2)(-2) = -10$ So below

b. Sketch the graph of the function $g(x) = x(x^2+3)(x-4)$ by finding the zeros and determining the sign of the values of the function between zeros.

Homework 8. 2: Sketching a Polynomial in Factored Form

1. For each graph, determine whether it represents an odd or even-degree polynomial and determine the sign of the leading coefficient (positive or negative).

2. Describe the end behavior of the graph of the functions given below.

a. $f(x) = -5x^4 + 7x^3 - 6x^2 + 9x + 2$

b. $h(x) = 7x^7 + 12x^5 - 6x^3 - 2x - 18$

c. $g(x) = -2x^4 + 12x^8 + 17 + 15x^2$

d. $f(x) = 11 - 18x^2 - 5x^5 - 12x^4 - 2x$

e. $p(x) = (x-2)(x-3)(x-4)$

f. $p(x) = -4x(x-3)(x+5)(x-1)$

3. Sketch a graph of the function $f(x) = -(x+2)(x-4)(x-4)$ by finding the zeros and determining the sign of the values of the function between zeros.

4. Sketch a graph of the function $g(x) = (x^2+1)(x-1)(x+3)$ by finding the zeros and determining the sign of the values of the function between zeros.

5. A function f has zeros at $-1, 3, \text{and } 5$. We know that $f(-2)$ and $f(2)$ are negative, while $f(4)$ and $f(6)$ are positive. Sketch a graph of f.

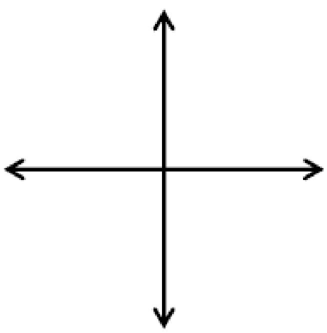
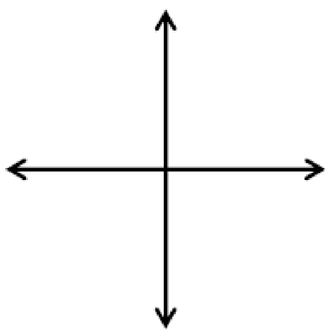
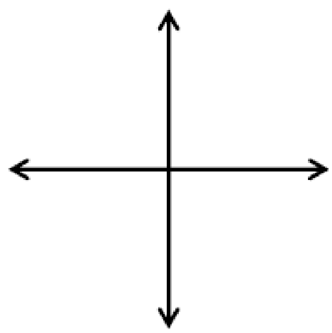
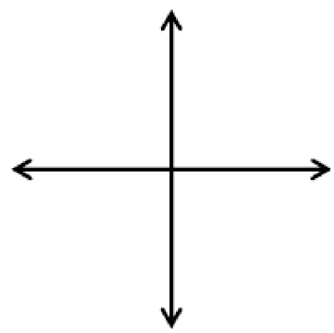
Lesson 8.3: Odd and Even Functions

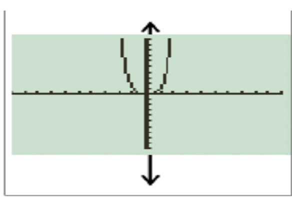
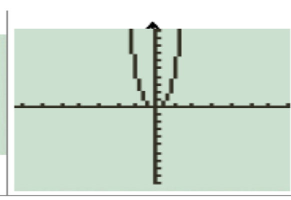
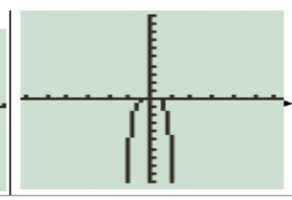
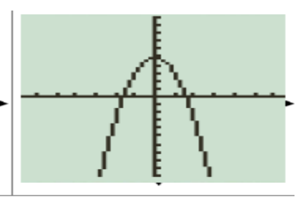
Learning Goals:

1) What is an odd function and how do we determine them graphically and algebraically?
2) What is an even function and how do we determine them graphically and algebraically?

Investigating Even Functions: Use your graphing calculator to graph each of the following functions. Draw a rough sketch of the graph under its equation.

$f(x) = x^4$ $\quad$ $f(x) = 2x^4 + 2x^2$ $\quad$ $f(x) = -3x^6 - 2x^4$ $\quad$ $f(x) = -2x^2 + 5$

All of the above functions are called **EVEN functions**

a. What type of symmetry does each graph have? Reflections over the y-axis!

b. What is special about the exponents of each term in the functions? All exponents are even (including the exponent of 0)

c. What happens when you evaluate $f(-x)$ for each of the functions?

$f(-x) = (-x)^4 = x^4$

$f(-x) = 2(-x)^4 + 2(-x)^2 = 2x^4 + 2x^2$

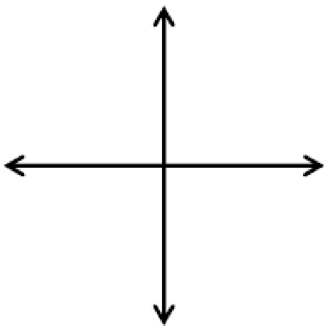
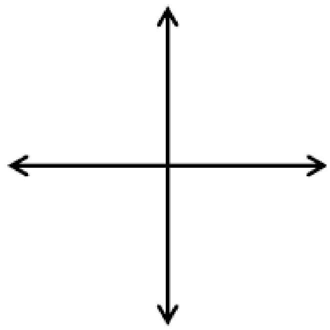
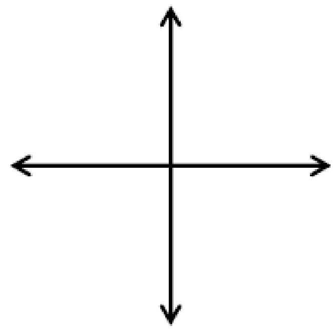
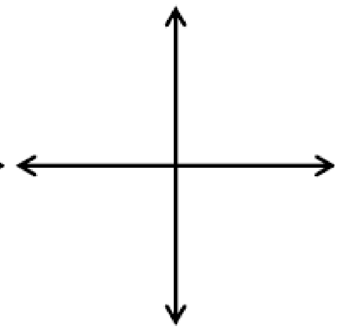
$f(-x) = -3(-x)^6 - 2(-x)^4 = -3x^6 - 2x^4$

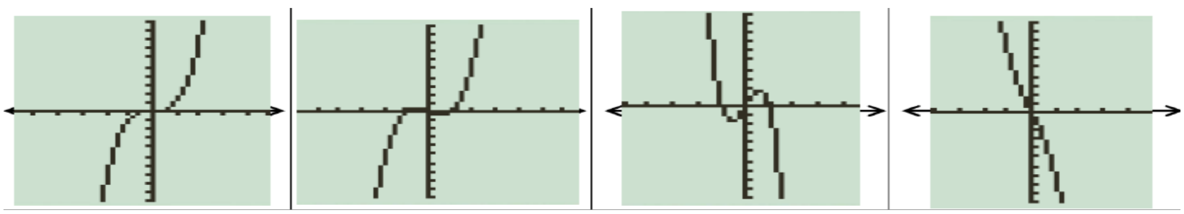
$f(-x) = -2(-x)^2 + 5 = -2x^2 + 5$

Even function if $f(-x) = f(x)$

Investigating Odd Functions: Use your graphing calculator to graph each of the following functions. Draw a rough sketch of the graph under its equation.

$f(x) = x^3$ $\quad$ $f(x) = x^3 - x$ $\quad$ $f(x) = -x^5 - 3x^3 + 4x$ $\quad$ $f(x) = -2x^3 - 5x$



All of the above functions are called **ODD functions**

a. What type of symmetry does each graph have? Reflections over the origin $(0,0)$. To test for it, turn the paper upside down.

b. What is special about the exponents of each term in the functions? All exponents are odd!

c. What happens when you evaluate $f(-x)$ for each of the functions?

$f(-x) = (-x)^3 = -x^3$

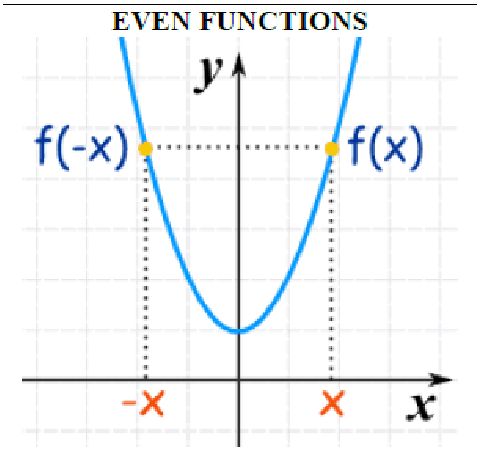
$f(-x) = (-x)^3 - (-x) = -x^3 + x$

$f(-x) = -(-x)^5 - 3(-x)^3 + 4(-x) = x^5 + 3x^3 - 4x$

$f(-x) = -2(-x)^3 - 5(-x) = 2x^3 + 5x$

Odd function if $f(-x) = -f(x)$

Summary of Even and Odd Functions



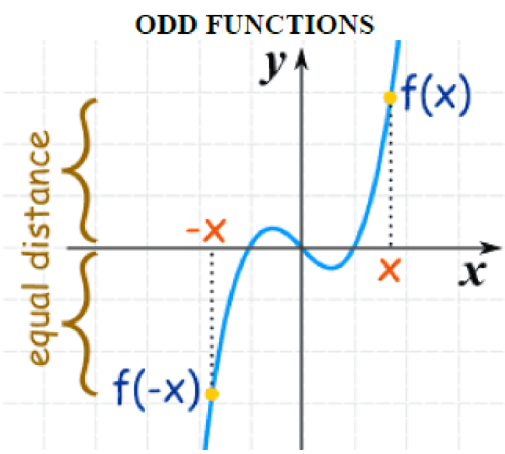
An **EVEN** function has the following properties:

I. Its graph is symmetric about the <u>y-axis.</u>

II. The exponents of all the terms in its equation are <u>even.</u>

III. $f(-x) = f(x)$

TRICK: Can you fold the graph in half along the y-axis and it aligns perfectly? Then it is **even**.



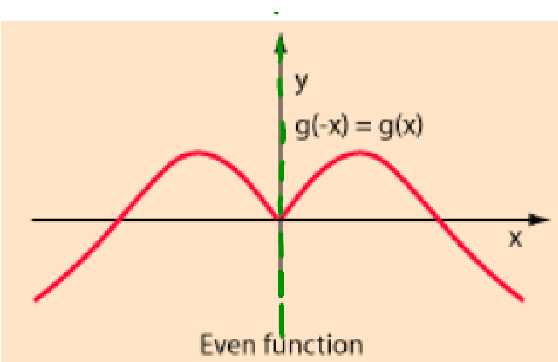
An **ODD** function has the following properties:

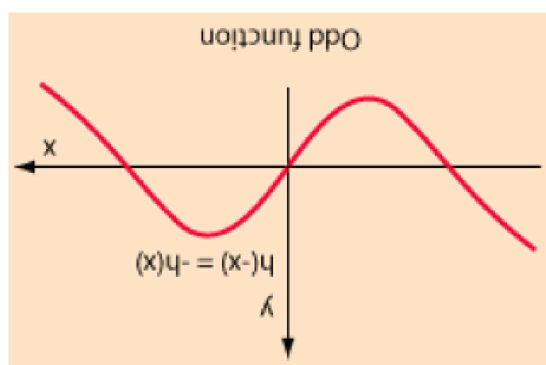
I. Its graph is symmetric about the <u>origin.</u>

II. The exponents of all the terms in its equation are <u>odd.</u>

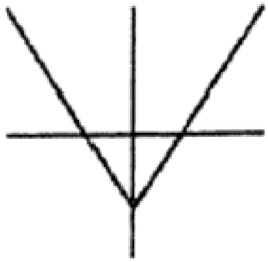
III. $f(-x) = -f(x)$

TRICK: Can you rotate the graph upside down and it still looks the same? Then it is **odd**.

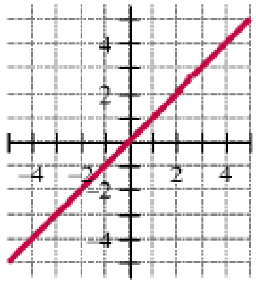




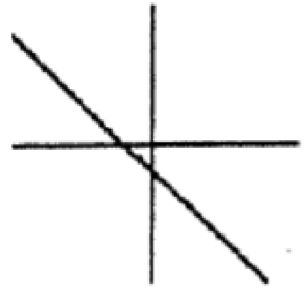
1. Determine whether each of the following functions are even, odd, or neither.



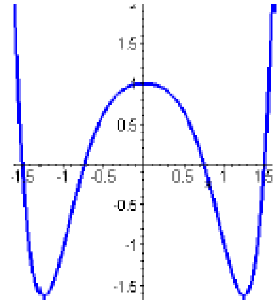
Even $r_{y-\text{axis}}$



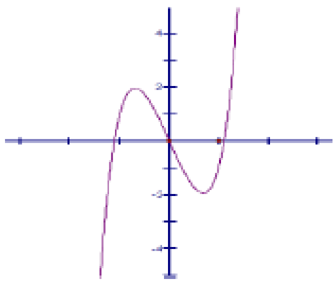
Odd r_{origin}



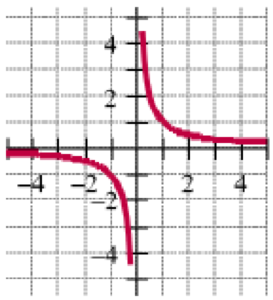
Neither



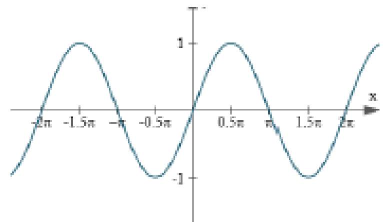
Even $r_{y-\text{axis}}$



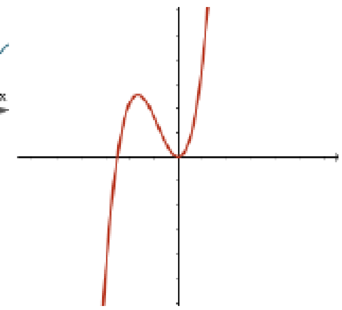
Odd r_{origin}



Odd r_{origin}



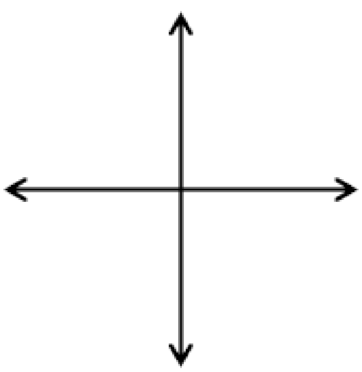
Odd r_{origin}

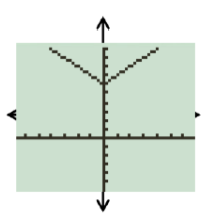


Neither

2. Graphically, determine whether each function is odd, even, or neither.

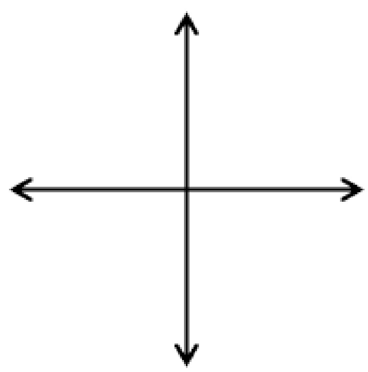
a) $f(x) = |x| + 6 = x + 6$

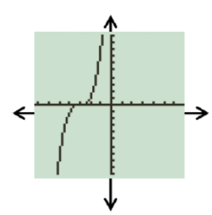




Even $|-x| + 6 = x + 6$ (same equation)

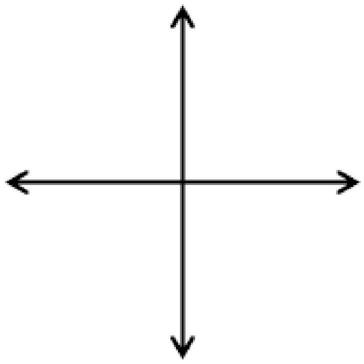
b. $h(x) = (x+3)^3 = (-x+3)^3$

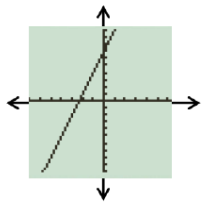




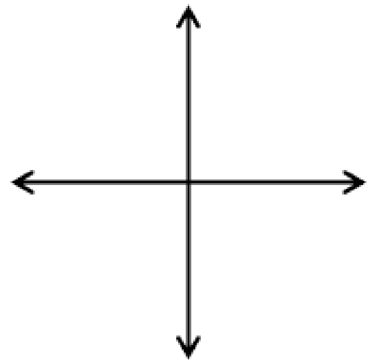
Neither

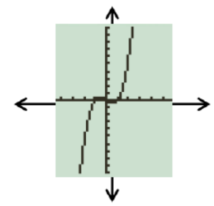
c) $f(x) = 3x + 7$





d. $f(x) = x^3 - x$





Neither $3(-x) + 7 = -3x + 7$

Odd $(-x)^3 - (-x) = -x^3 + x = -(x^3 - x)$

3. Algebraically, determine whether each function is odd, even, or neither.

$f(-x) = f(x)$ is even $\qquad f(-x) = -f(x)$ is odd

a) $f(x) = 3x^4 - 5x^2 + 17$

$f(-x) = 3(-x)^4 - 5(-x)^2 + 17$

$f(-x) = 3x^4 - 5x^2 + 17$

Same = EVEN

b) $f(x) = |x| = x$

$f(-x) = |-x|$

$f(x) = x$

Same = EVEN

c) $f(x) = 12x^7 + 6x^3 - 2x$

$f(-x) = 12(-x)^7 + 6(-x)^3 - 2(-x)$

$f(-x) = -12x^7 - 6x^3 + 2x$

Opposites = ODD

d) $f(x) = \frac{x^2-5}{2x^3+x}$

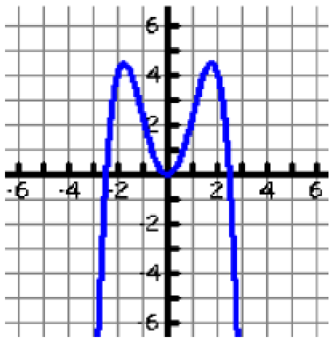
$f(-x) = \frac{(-x)^2-5}{2(-x)^3+(-x)}$

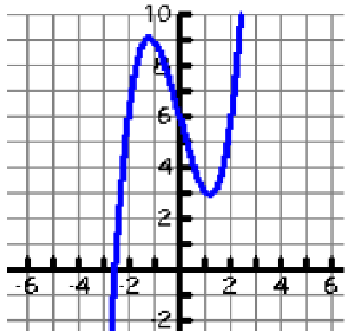
$f(-x) = \frac{x^2-5}{-2x^2-x} = \frac{x^2-5}{-(2x^2+x)}$

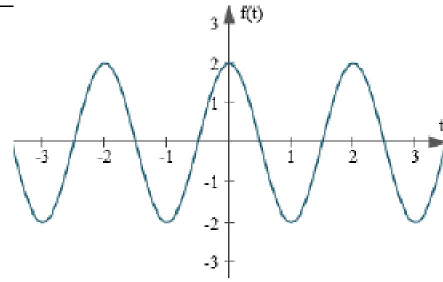
Opposites = ODD

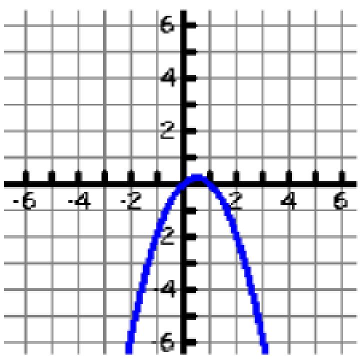
Homework 8.3: Odd and Even Functions

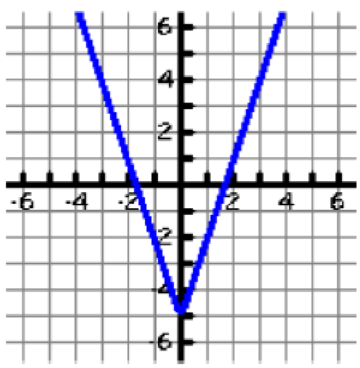
1. Determine whether each of the following functions are odd, even, or neither. Justify your answer.

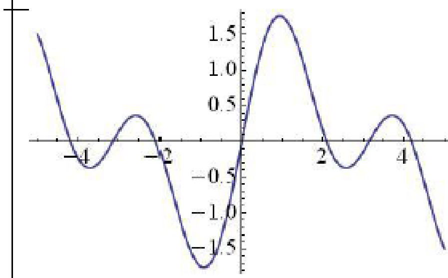








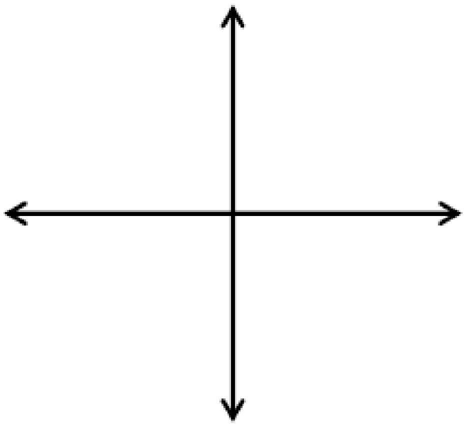




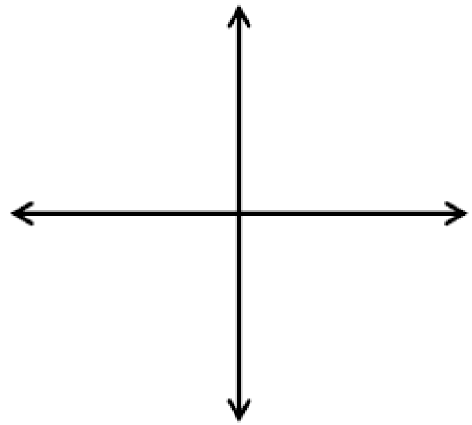
2. If $f(x)$ is an even function and $f(3) = 5$ then what is the value of $2f(-3)$?

3. Graphically, determine whether each function is odd, even, or neither. Justify your answer.

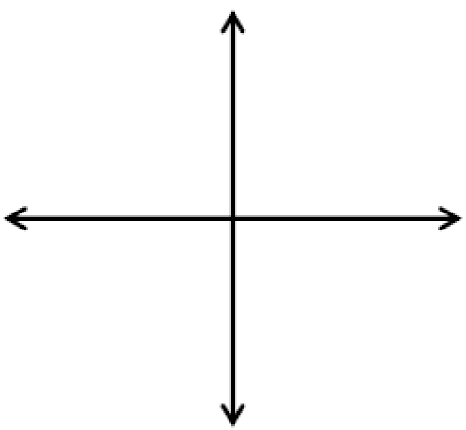
a. $f(x) = |x - 2|$



b. $h(x) = -(x^2 - 1)(x^2 - 4)$



c. $f(x) = x + 2x^3 - x^5$



4. Algebraically, determine whether each function is odd, even, or neither.

a. $f(x) = -3x^2 + 7$

b. $f(x) = 3x^3 - 4x$

c. $f(x) = 2x^2 - 4x$

d. $f(x) = \frac{x^5 - x}{x^3}$

5. Use the graph below to answer the given questions:

a. Determine the x-intercept(s) of the graph.

b. Determine the y-intercept(s) of the graph.

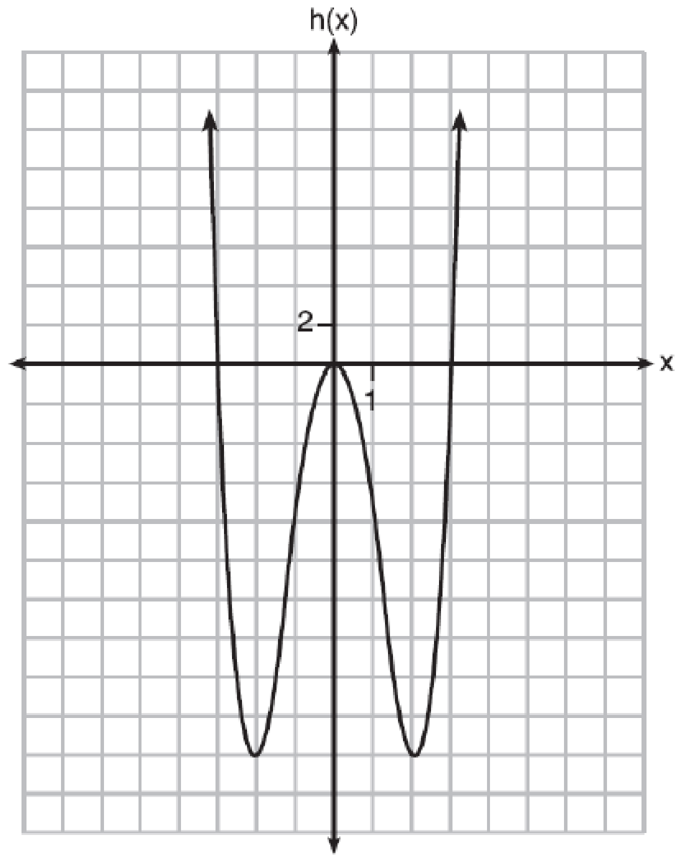
c. Describe the end behavior of the function.

d. Determine the x-coordinates of the relative maximums and relative minimums.

e. Determine the interval(s) where the function is increasing.

f. What is the interval(s) for which $f(x) < 0$?

g. Determine if the function is an odd or even polynomial. Then determine if the sign of the leading coefficient is positive or negative.



Lesson 8.4: Solving Polynomial Equations of Higher Degree

Learning Goal: How can we solve polynomial equations of degree greater than two?

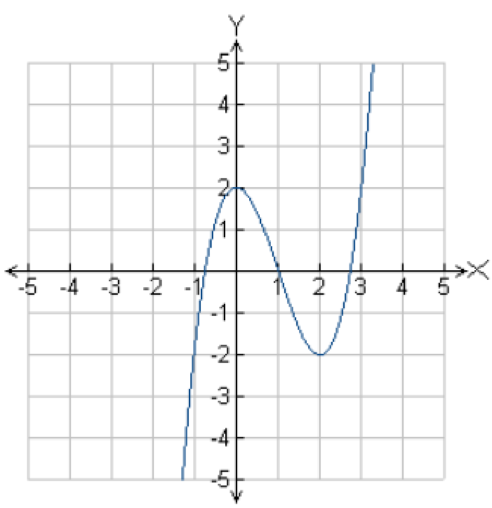
Do now:

1. What are the questions to ask yourself when factoring?
   - Is there a GCF?
   - How many terms are there?
   - See if you can factor more?

2. What are the methods we used to solve quadratic equations?
   - Quadratic Formula, Complete the Square, Square-Root Method, Factoring

Finding the roots of a higher degree polynomial equation by inspecting the graph.

1. The function $f(x) = x^3 - 3x^2 + 2$ is graphed below. Use the graph to approximate the solutions to the equation $x^3 - 3x^2 + 2 = 0$



Look for x-intercepts! $f(x) = 0$ $\qquad$ $x = -.8, 1, 2.8$

Match each polynomial equation with the graph of its related polynomial function.

2. $x^2 + 5x + 4 = 0$

3. $x^3 - 2x^2 - x + 2 = 0$

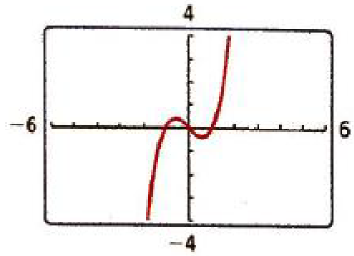
4. $x^3 + x^2 - 2x = 0$

5. $x^3 - x = 0$

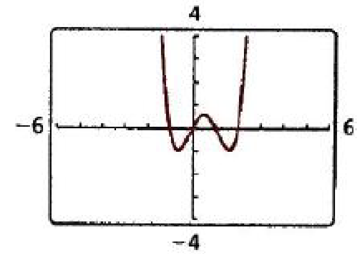
6. $x^4 - 5x^2 + 4 = 0$

7. $x^4 - 2x^3 - x^2 + 2x = 0$

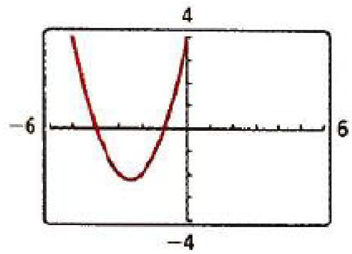
A.



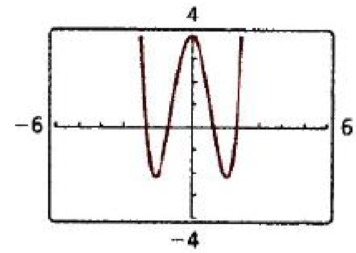
B.



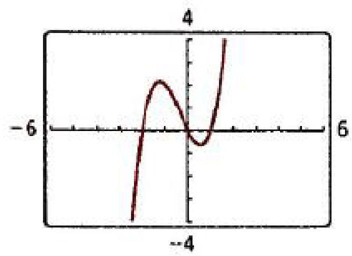
C.



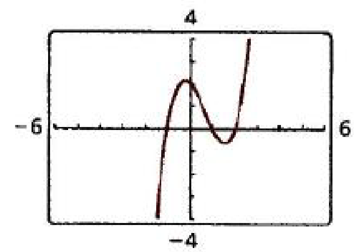
D.



E.



F.



Use the strategies learned in solving quadratic equations to solve the following equations. Express any irrational solutions in simplest radical form.

8. $h^5 - 10h^3 + 21h = 0$

$h(h^4 - 10h^2 + 21) = 0$ GCF $= h$

$h(h^2 - 7)(h^2 - 3) = 0$ Sum/Product

$h = 0,\ \ h^2 = 7,\ \ h^2 = 3$

$h = 0, \pm\sqrt{7}, \pm\sqrt{3}$ Degree is 5 because there are 5 solutions!

9. Find all roots of the given function: $f(x) = 16x^4 - 1$

$0 = 16x^4 - 1$

$0 = (4x^2 - 1)(4x^2 + 1)$

$0 = (2x - 1)(2x + 1)(4x^2 + 1)$

$2x + 1 = 0 \quad 2x + 1 = 0 \quad 4x^2 + 1 = 0$

$4x^2 = -1$

$x^2 = -\frac{1}{4}$

$x = \pm\sqrt{-\frac{1}{4}}$

$x = \pm\frac{1}{2}, \pm\frac{i}{2}$ Degree is 4 because there are 4 solutions!

10. $x^4 = 13x^2 - 36$ Solve for 0

$x^4 - 13x^2 + 36 = 0$ Sum/Product

$(x^2 - 4)(x^2 - 9) = 0$

$(x - 2)(x + 2)(x - 3)(x + 3) = 0$

$x = \pm 2, \pm 3$ Degree is 4 because there are 4 solutions!

11. $6k^3 - 15k = k^2$ Solve for 0

$6k^3 - k^2 - 15k = 0$ GCF $= k$

$k(6k^2 - k - 15) = 0$ AC Method $6(-15) = \underline{90}$

$k(6k^2 \underline{+9k - 10k} - 15) = 0$ $\underline{9, -10}$

$k[3k(2k + 3) - 5(2k + 3)] = 0$

$k(3k - 5)(2k + 3) = 0$

$k = 0, 3k - 5 = 0, 2k + 3 = 0$

$k = 0, \frac{5}{3}, -\frac{3}{2}$ Degree is 3 because there are 3 solutions!

12. Find the zeros of the function: $f(x) = x^3 - 125$

$0 = x^3 - 125$ Difference of Cubes (SOAP) $a = \sqrt[3]{x} = x$ $b = \sqrt[3]{125} = 5$

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$= (x-5)(x^2+5x+25)$

$x-5=0$ $\quad$ $x^2+5x+25=0$ $\quad$ Quadratic Formula or Complete the Square

$x = 5, \frac{-5\pm 5i\sqrt{3}}{2}$

13. Consider the polynomial function $Q(x) = x^5 - 3x^4 + 4x^3 - 12x^2 - 5x + 15$

Solve the function when $Q(x) = 0$

$x^5 - 3x^4 + 4x^3 - 12x^2 - 5x + 15 = 0$ $\quad$ Grouping

$x^4(x-3) + 4x^2(x-3) - 5(x-3) = 0$

$(x^4 + 4x^2 - 5)(x-3) = 0$ $\quad$ Sum/Product

$(x^2-1)(x^2+5)(x-3) = 0$ $\quad$ DOPs

$(x-1)(x+1)(x^2+5)(x-3) = 0$

$x = \pm 1, \pm i\sqrt{5}, 3$

14. Find the zeros for the function $p(x) = (x^2-8)(x^5-4x^3)$ in simplest form.

$x^2 - 8 = 0$ $\quad$ $x^5 - 4x^3 = 0$

$x^2 = 8$ $\quad$ $x^3(x^2-4) = 0$

$x = \pm 2\sqrt{2}, 0, \pm 2$

15. How many complex roots does the following polynomial function have?

$p(x) = (x^2-5)(x^2+4)(x^2+10)(2x+6)$ $\quad$ complex # $= a + bi$ form

Total of 4!

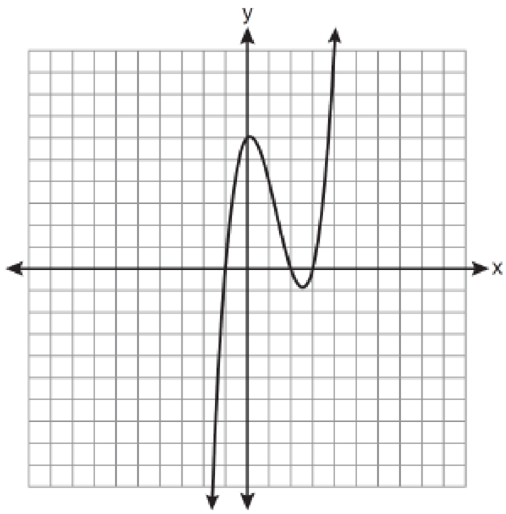
16. Find all solutions to the function: $f(x) = x(x^2-7)(x^2-3)$

$x = 0$ $\quad$ $x^2 - 7 = 0$ $\quad$ $x^2 - 3 = 0$

$x = \pm\sqrt{7}$ $\quad$ $x = \pm\sqrt{3}$

Homework 8.4: Solving Polynomial Equations of Higher Degree

1. The graph of $y = f(x)$ is shown below. What is the product of the roots of the equation $f(x) = 0$? (1) -36 (2) -6 (3) 6 (4) 4



Use the strategies learned in solving quadratic equations to solve the following equations. Express any irrational solutions in simplest radical form.

2. Find the zeros of the function: $f(x) = 3x^4 - 6x^2 + 3$

3. Find the zeros of the function: $f(x) = x^3 + x^2 - 6x$

4. $2a^3 - 3a^2 - 18a + 27 = 0$

5. $y^3 - 48y = 2y^2$

6. $y^4 - 14y^2 = -45$

7. $5m^3 - 7m^2 = 6m$

8. Find the zeros of the function: $f(x) = x^3 + 8$

9. $z^5 - 18z^3 = -32z$

Lesson 8.5: Synthetic Division

Learning Goals:

1) How do we perform long division on polynomials?
2) How do we perform synthetic division on polynomials?
3) When can we use synthetic division when asked to divide polynomials?

Do Now: Divide the following using long division.

Divide: $x^3 + 4x^2 - 3x - 5$ by $x + 3$ Is there a missing term?

$$
\begin{array}{rl}
 & x^2 + x - 6 \\
x+3 & \overline{|x^3 + 4x^2 - 3x - 5} \\
 & \underline{-(x^3 + 3x^2)} \downarrow \quad \downarrow \\
 & x^2 - 3x \quad \downarrow \\
 & \underline{-(x^2 + 3x)} \quad \downarrow \\
 & -6x - 5 \\
 & \underline{-(-6x - 18)} \\
 & 13
\end{array}
$$

$$x^2 + x - 6 + \frac{13}{x+3}$$

What is the degree of the following polynomials? Highest exponent!

1) $5x^2 - 7x + 5$ 2) $4x + 6x^5 + 8x^2 - 4$ 3) $8x^7 - 4x^2 + 9x^{10}$

Now we will take the "do now" problem and divide by a method called synthetic division: Divisor must be degree 1

Step 1: Arrange the coefficients in descending order. (Remember to include placeholders for any missing variables)

Step 2: Write the constant of the divisor $x - r$ (in this case -3)

Step 3: Bring down the first coefficient.

Step 4: Multiply the first coefficient by r (in this case -3). Place that product under the 2nd coefficient.

Step 5: Add the column. Then multiply that sum by r.

Step 6: Repeat step 5 for all coefficients.

Step 7: The final sum represents the remainder. The other numbers are the coefficients of the quotient polynomial which has degree one less than the dividend.

Synthetic division can only be used when: The divisor is degree one!

The divisor is a factor of the dividend when: The remainder is 0

Divide: $x^3 + 4x^2 - 3x - 5$ by $x + 3$ using synthetic division.

$$\begin{array}{r|rrrr} -3 & 1 & 4 & -3 & -5 \\ \oplus & \downarrow & -3 & -3 & 18 \\ \hline & 1 & 1 & -6 & \underline{13} \rightarrow \text{is the remainder} \end{array}$$

$x^2 + x - 6 + \frac{13}{x+3}$

Practice: Divide each of the following using synthetic division.

1. Divide $x^3 - x^2 + 2$ by $x + 1$ $\qquad$ Degree is 3, missing x and $x + 1 = -1$

$$\begin{array}{r|rrrr} -1 & 1 & -1 & 0 & 2 \\ \oplus & \downarrow & -1 & 2 & -2 \\ \hline & 1 & -2 & 2 & \underline{0} \rightarrow \text{no remainder} \end{array}$$

$x^2 - 2x + 2$

2. Divide and find the factors of $(2x^3 - 3x^2 + x) \div (x - 1)$

Degree is 3, missing no $x's$ and $x - 1 = 1$

$$\begin{array}{r|rrrr} 1 & 2 & -3 & 1 & 0 \\ \oplus & \downarrow & 2 & -1 & 0 \\ \hline & 2 & -1 & 0 & \underline{0} \end{array} \rightarrow \text{no remainder}$$

$2x^2 - x$ GCF $= x$

$x(2x - 1)(x - 1)$

Divide each of the following using synthetic division. Then state whether the binomial is a factor of the polynomial.

3. $(2x^4 + 4x^3 - x^2 + 9) \div (x + 1)$

Degree is 4, missing x and $x + 1 = -1$

$$\begin{array}{r|rrrrr} -1 & 2 & 4 & -1 & 0 & 9 \\ \oplus & \downarrow & -2 & -2 & 3 & -3 \\ \hline & 2 & 2 & -3 & 3 & \underline{6} \end{array} \rightarrow \text{is the remainder}$$

$2x^3 + 2x^2 - 3x + 3 + \frac{6}{x+1}$, so $x + 1$ is not a factor because the remainder is not 0

4. $(2x^3 - 3x^2 - 10x + 3) \div (x - 3)$ Degree is 3, missing no x and $x - 3 = 3$

$$\begin{array}{r|rrrr} 3 & 2 & -3 & -10 & 3 \\ \oplus & \downarrow & 6 & 9 & -3 \\ \hline & 2 & 3 & -1 & \underline{0} \end{array} \rightarrow \text{no remainder}$$

$2x^2 + 3x - 1$ would need to do quadratic or complete square

$(2x^2 + 3x - 1)(x - 3)$

5. Use synthetic division to find all the factors of $x^3 + 6x^2 - 9x - 54$, is one of the factors $x - 3$? $\text{Degree is 3, missing no } x \text{ and } x - 3 = 3$

$$\begin{array}{r|rrrr} 3 & 1 & 6 & -9 & -54 \\ \oplus & \downarrow & 3 & 27 & 54 \\ \hline & 1 & 9 & 18 & \underline{0} \end{array} \rightarrow \text{no remainder}$$

$(x^2 + 9x + 18)(x - 3)$

$(x + 3)(x + 6)(x - 3)$

6. Suppose we know that the polynomial equation $4x^3 - 12x^2 + 3x + 5 = 0$ has three real solutions that one of the factors of $4x^3 - 12x^2 + 3x + 5$ is $(x - 1)$. How can we find all three solutions to the given equation?

Synthetic or Long Division

$$\begin{array}{r|rrrr} 1 & 4 & -12 & 3 & 5 \\ \oplus & \downarrow & 4 & -8 & -5 \\ \hline & 4 & -8 & -5 & \underline{0} \end{array} \rightarrow \text{no remainder}$$

$(4x^2 - 8x - 5)(x - 1)$

$(4x^2 + 2x - 10x - 5)(x - 1)$

$2x(2x + 1) - 5(2x + 1)(x - 1)$

$(2x - 5)(2x + 1)(x - 1)$

$x = \frac{5}{2}, -\frac{1}{2}, 1$

7. Consider the polynomial $P(x) = x^3 + kx^2 + x + 6$.

a. Find the value of k so that $x + 1$ is a factor of P.

$\text{Degree is 3, missing no } x \text{ and } x + 1 = -1$

$$\begin{array}{r|rrrr} -1 & 1 & k & 1 & 6 \\ \oplus & \downarrow & -1 & 5 & -6 \\ \hline \end{array}$$

will need to work backwards for a bit

$1 \qquad k-1 \qquad 6 \qquad \underline{0} \rightarrow \text{no remainder}$

$-1(k-1) = 5$

$-k+1 = 5$

$-k = 4$

$k = -4$

b. Find the other two factors of P for the value of k found in part (a).

$x^2 - 5x + 6$

$(x-2)(x-3)$

Homework 8.5: Synthetic Division

Divide each of the following using synthetic division.

1. $(x^2 + 20x + 91) \div (x + 7)$

2. $(3x^4 - 2x^3 + 5x^2 - 4x - 2) \div (x + 1)$

3. $(x^4 + x^3 - 1) \div (x - 2)$

4. $(x^4 - 8x^2 + 16) \div (x + 2)$

5. Determine all of the factors of the expression $3x^3 + 7x^2 - 18x + 8$ is one of the factors $x - 1$?

6. If $p(x) = 2x^3 + cx^2 - 5x - 6$ and $x + 2$ is a factor of $p(x)$, find the value of c.

   (1) -5 (2) -2 (3) 3 (4) 2

7. Which binomial is a factor of the polynomial $x^3 + 3x^2 - 2x - 8$?

   (1) $x - 1$ (2) $x + 1$ (3) $x - 2$ (4) $x + 2$

Lesson 8.6: Factor and Remainder Theorems

Learning Goals:

1) What is the Fundamental Theorem of Algebra?
2) What is the Factor Theorem and why do we need it?
3) What is the Remainder Theorem and why do we need it?

Fundamental Theorem of Algebra:

- Every polynomial equation with degree greater than zero has at least one root in the set of complex numbers.
- Every polynomial $p(x)$ of degree $n,\ n > 0$. Can be written as a product of a constant $k,\ k \neq 0$, and n linear factors: $p(x) = k(x - r_1)(x - r_2) \ldots (x - r_n)$
- A polynomial equation of degree n has exactly n complex roots, namely $r_1, r_2, r_3, \ldots r_n$

$x = 3$ $\qquad$ $x = -1$

$(x - 3)$ $\qquad$ $(x + 1)$

Consider the polynomial function $f(a) = 2a^2 + 3a - 8$. Since 2 is a factor of 8, it is possible that $a - 2$ is a factor of $2a^2 + 3a - 8$.

$a = 2$

$(a - 2) = 0$

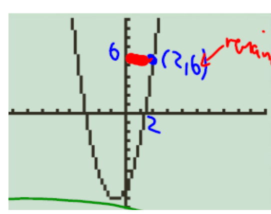
$$
\begin{array}{r}
2a + 7 \\
a - 2\ \overline{)\,2a^2 + 3a - 8} \\
-\underline{(2a^2 - 4a)}\ \downarrow \\
7a - 8 \\
-\underline{(7a - 14)} \\
6 \rightarrow \text{remainder}
\end{array}
$$

$2a + 7 + \frac{6}{a-2}$

substitution: $f(2) = 2(2)^2 + 3(2) - 8 = 6$

Graphically: $y = 2x^2 + 3x - 8$

$f(2) = 6$



For all of these reasons $(a - 2)$ is NOT a factor!

The Remainder Theorem:
If a polynomial $P(x)$ is divided by $x-r$, the remainder is a constant $P(r)$ and $P(x)=(x-r)\cdot Q(x)+P(r)$ where $Q(x)$ is a polynomial with degree one less than the degree of $P(x)$. $(2a^2+3a-8)\div(a-2)=2a+7+\frac{6}{a-2}$ $(2a^2+3a-8)=(2a+7)(a-2)+6$

Divide the following using synthetic division:

2. Divide x^3+4x^2-3x-5 by $x+3$. Determine if $x+3$ is a factor of the polynomial. $\text{Degree is 3, missing no } x \text{ and } x+3=-3$

$$\begin{array}{r|rrrr} -3 & 1 & 4 & -3 & -5 \\ \oplus & \downarrow & -3 & -3 & 18 \\ \hline & 1 & 1 & -6 & \underline{13} \rightarrow \text{remainder} \end{array}$$

$f(-3)=13$

$x^2+x-6+\frac{13}{x+3}$, Therefore $(x+3)$ is NOT a factor because the remainder is not 0.

3. Divide x^3-x^2+2 by $x+1$. Determine if $x+1$ is a factor of the polynomial.

$\text{Degree is 3, missing an } x \text{ and } x+1=-1$

$$\begin{array}{r|rrrr} -1 & 1 & -1 & 0 & 2 \\ \oplus & \downarrow & -1 & 2 & -2 \\ \hline & 1 & -2 & 2 & \underline{0} \rightarrow \text{no remainder} \end{array}$$

$f(-1)=0$

x^2-2x+2 , Therefore $(x+1)$ is a factor because the remainder is 0.

The Factor Theorem:
The binomial $x-r$ is a factor of the polynomial $P(x)$ if and only if $P(r)=0$. Remainder is 0…It crosses the x-axis!

Use the remainder theorem for each division. State whether the binomial is a factor of the polynomial. Justify your answer.

4. $(2x^4 + 4x^3 - x^2 + 9) \div (x + 1)$

$(x + 1) = 0$ $\quad$ $2(-1)^4 + 4(-1)^3 - (-1)^2 + 9)$

$x = -1$ $\quad$ $2 - 4 - 1 + 9 = 6$

$x + 1$ is NOT a factor because the remainder is not equal to 0.

5. $(2x^3 - 3x^2 - 10x + 3)(x - 3)$

$(x - 3) = 0$ $\quad$ $2(3)^3 - 3(3)^2 - 10(3) + 3$

$x = 3$ $\quad$ $2(27) - 3(9) - 30 + 3$

$54 - 27 - 27 = 0$

$x - 3$ is a factor because the remainder is equal to 0.

6. $(n^4 - n^3 - 10n^2 + 4n + 24) \div (n + 2)$ Degree is 4, missing no n and $n + 2 = -2$

$$
\begin{array}{r|rrrrr}
-2 & 1 & -1 & -10 & 4 & 24 \\
\oplus & \downarrow & -2 & 6 & 8 & -24 \\
\hline
 & 1 & -3 & -4 & 12 & \underline{0} \rightarrow \text{ no remainder}
\end{array}
$$

$n + 2$ is a factor because the remainder is equal to 0.

7. Find the value of k so that $\frac{x^3 - kx^2 + 2}{x - 1}$ has a remainder of 8.

$\underbrace{(x^3 - kx^2 + 2)} \div \underbrace{(x - 1)}$

$f(1) = 8$ $\quad$ $x = 1$

$8 = (1)^3 - k(1)^2 + 2$

$8 = 1 - k + 2$

$5 = -k \quad$ so $k = -5$

8. What is the value of k such that $x - 7$ is a factor of $f(x) = 2x^3 - 13x^2 - kx + 105$? Justify your answer.

$x - 7 = 0$

$x = 7$

$f(7) = 0$

$2(7)^3 - 13(7)^2 - k(7) + 105 = 0$

$686 - 637 - 7k + 105 = 0$

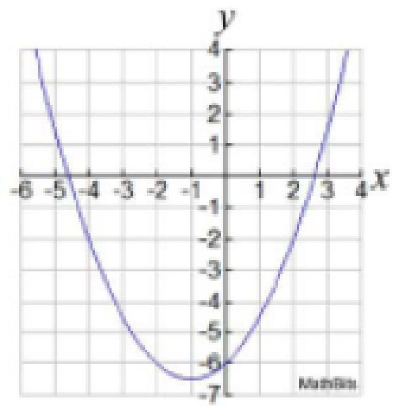
$154 - 7k = 0$

$7k = 154$

$k = 22$

9. The graph of the quadratic function $f(x)$ is shown below. What is the remainder when $f(x)$ is divided by $(x + 2)$?

(1) -2 (2) 2 (3) -4 (4) -6

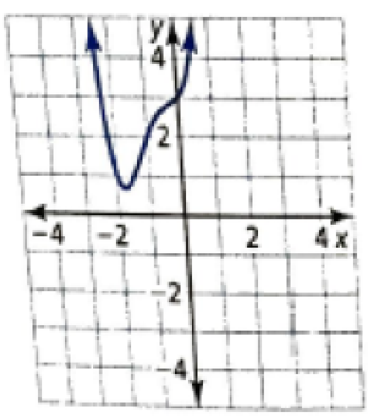


$x + 2 = 0$

$x = -2$

$f(-2) = -6$

10. The graph of the function $f(x) = x^4 + 3x^3 + 2x^2 + x + 3$ is shown. Can you use the factor theorem to factor $f(x)$? Explain. No because it never crosses the x-axis!



Homework 8.6: Factor and Remainder Theorems

Divide using Synthetic Division:

1. $(3x^2 + 4x - 12) \div (x + 5)$

2. $(x^4 - 3x^2 + 12) \div (x + 1)$

Use the Remainder Theorem to find the remainder for each division. State whether the binomial is a factor of the polynomial.

3. $(2x^4 + 4x^3 - x^2 + 9) \div (x + 1)$

4. $(2x^3 - 3x^2 - 10x + 3) \div (x - 3)$

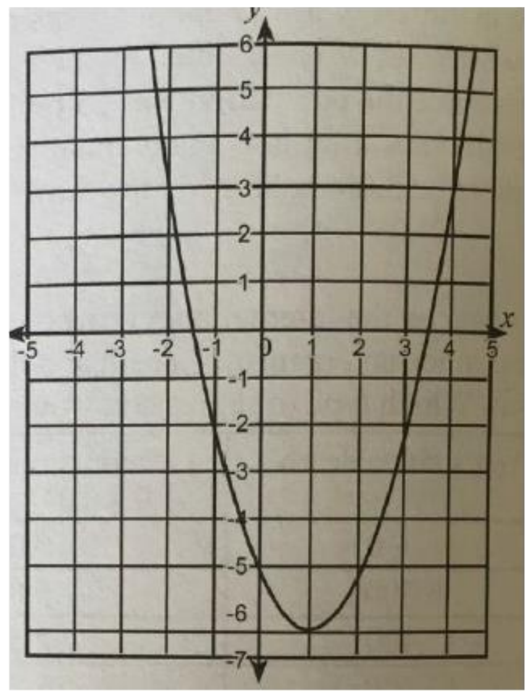
5. $(y^3 + y^2 - 10) \div (y + 3)$

6. $(n^4 - n^3 - 10n^2 + 4n + 24) \div (n + 2)$

7. Use synthetic division to find all the factors of $x^3 + 6x^2 - 9x - 54$ is one of the factors is $x - 3$?

8. Find the value of k so that $\frac{kx^2+x-k}{x+2}$ has a remainder of 16.

9. The graph of the quadratic function, $f(x)$, is shown to the right. What is the remainder when $f(x)$ is divided by $x-2$? (1) -5 (2) 3 (3) -3 (4) 5



10. If $p(x) = 2x^3 + cx^2 - 5x - 6$ and $x+2$ is a factor of $p(x)$, find the value of c.

(1) -5 (2) -2 (3) 3 (4) 2

Lesson 8.7: Solving Polynomial Equations Given a Root

Learning Goals:

1) How can we solve polynomial equations given a root?
2) How do we sketch the graph of a polynomial given its equation?

Do Now:

1) What are the solutions to $x^4 - 3x^2 - 4 = 0$? Sum/Product

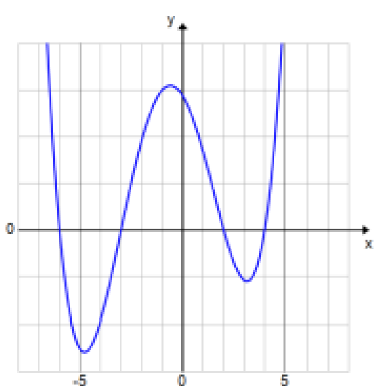
$(x^2 - 4)(x^2 + 1) = 0$

$(x - 2)(x + 2)(x^2 + 1) = 0$

$x = \pm 2, \pm i$

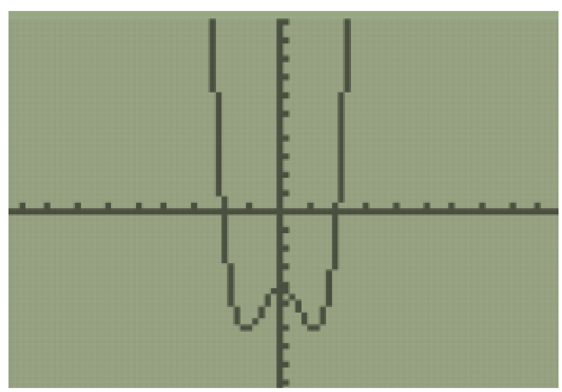
2) Find the real zeros of the polynomials whose graphs are given.

(a)



$x = -6, -3, 2, 4$

(b)



$x = \pm 2$

(c) Write the function in (a) in factored form using the zeros found.

$f(x) = (x + 6)(x + 3)(x - 2)(x - 4)$

3) If $P(-2) = 0$, what is one factor of $P(x)$?

$x = -2$ so the factor would be $(x + 2)$

Solving a Polynomial

1) Determine one of the roots
   - Either it's given or you must use the graph to find an x-intercept
2) Use synthetic or long division to divide the polynomial by this root.
3) Solve the remaining polynomial using an appropriate method.
   - Factoring, square root method, completing the square, quadratic formula

****If asked to write the function as a product of its factors, you must use the roots to write the equation in the form $\boldsymbol{f(x)=(x-r_1)(x-r_2)(x-r_3)}$ …where $\boldsymbol{r_1, r_2, r_3, \ldots}$ are the roots**

Example 1: Consider the polynomial function $P(x) = x^3 - 8x^2 - 29x + 180$

a) Verify that $\underline{P(9) =}\ 0$. Since $P(9) = 0$, what must one of the factors of P be?

$x = 9$

$P(9) = (9)^3 - 8(9)^2 - 29(9) + 180 = 0$

Since $P(9) = 0$, x must $= 0$; therefore $(x - 9)$ is a factor!

b) Find the remaining two factors of P. Degree is 3, missing no x and $x - 9 = 9$

Use synthetic division!

$$
\begin{array}{r|rrrr}
9 & 1 & -8 & -29 & 180 \\
\oplus & \downarrow & 9 & 9 & -180 \\
\hline
 & 1 & 1 & -20 & \underline{0} \rightarrow \text{no remainder}
\end{array}
$$

$x^2 + x - 20$ Sum/Product

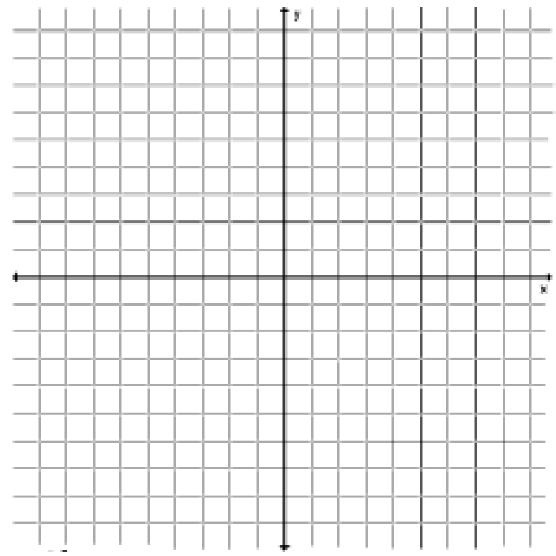
$\underline{(x-4)(x+5)}(x-9)$

c) State the zeros of P.

$P(x) = (x - 4)(x + 5)(x - 9)$

$x = 4, -5, 9$

d) Sketch the graph of P



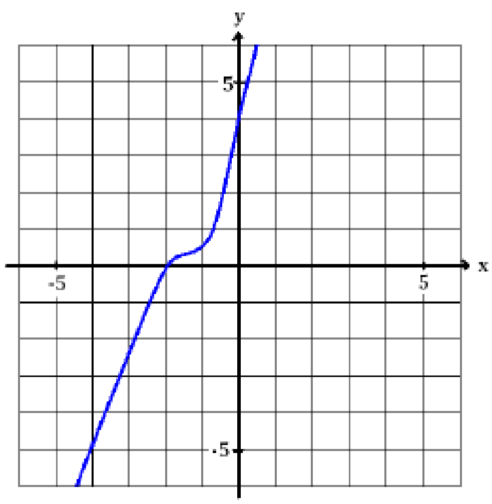
plot zeros

end behaviors (from leading coefficient)

test points

graph

Example 2: The graph of the polynomial function $f(x) = x^3 + 4x^2 + 6x + 4$ is shown below.



a) Based on the appearance of the graph, what does the real solution to the equation $x^3 + 4x^2 + 6x + 4 = 0$ appear to be? Jiju does not trust the accuracy of the graph. Prove to her algebraically that your answer is in fact a zero of

$y = f(x)$.

or synthetic division would work too!

$f(-2) = (-2)^3 + 4(-2)^2 + 6(-2) + 4$

$x = -2$

$= -8 + 4(4) - 12 + 4$

$f(-2) = 0$

$= -8 + 16 - 12 + 4 = 0$ no remainder

b) Find the two complex number zeros of $y = f(x)$.

Use synthetic division to find the other zero!

$(x^3 + 4x^2 + 6x + 4) \div (x + 2)$

$$
\begin{array}{r|rrrr}
-2 & 1 & 4 & 6 & 4 \\
\oplus & \downarrow & -2 & -4 & -4 \\
\hline
 & 1 & 2 & 2 & \underline{0} \rightarrow \text{ no remainder}
\end{array}
$$

$x^2 + 2x + 2 = 0$ Complete the Square!

$x^2 + 2x + 1 = -2 + 1$

$(x + 1)^2 = -1$

$x + 1 = \pm i$

$x = -1 \pm i$

c) Write f as a product of three linear factors.

$x = -2, -1 \pm i$

$f(x) = (x + 2)(x + 1 - i)(x + 1 + i)$

Example 3: Consider the quartic function $y = x^4 - x^3 - 27x^2 + 25x + 50$.

a) Show that $x = 2$ is a solution to the equation.

$$
\begin{array}{r|rrrrr}
2 & 1 & -1 & -27 & 25 & 50 \\
\oplus & \downarrow & 2 & 2 & -50 & -50 \\
\hline
 & 1 & 1 & -25 & -25 & \underline{0} \rightarrow \text{ no remainder}
\end{array}
$$

So $x = 2$ is a solution because the remainder is zero!

b) Find all zeros of the equation.

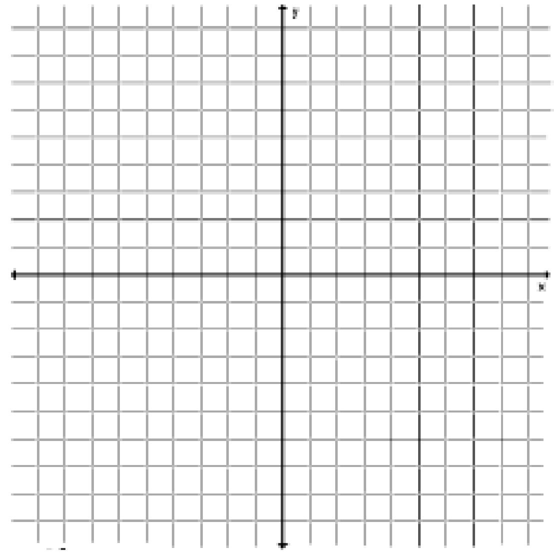
$x^3 + x^2 - 25x - 25 = 0$ Grouping!

$x^2(x + 1) - 25(x + 1) = 0$

$(x^2 - 25)(x + 1) = 0$

$(x + 5)(x - 5)(x + 1) = 0$ $x = \pm 5, -1, 2$

c) Sketch the graph of the polynomial.



plot zeros

end behaviors (from leading coefficient)

test points

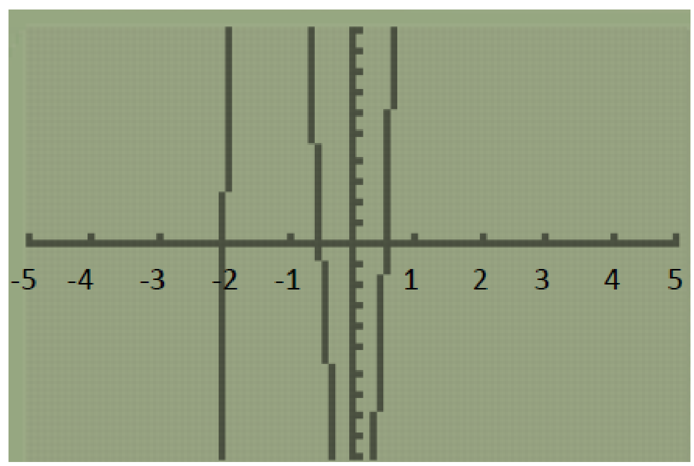
graph

Example 4: Consider the polynomial function

$$f(x) = 8x^5 + 16x^4 + 30x^3 + 60x^2 - 8x - 16$$

a) Use the graph to state one solution to the above equation. Explain.

-2, because it is where the graph crosses the x-axis.



b) Find all the zeros of $f(x)$.

$\underline{8x^5 + 16x^4} + \underline{30x^3 + 60x^2 - 8x - 16} = 0$

Grouping

$8x^4(x+2) + 30x^2(x+2) - 8(x+2) = 0$

$(8x^4 + 30x^2 - 8)(x+2) = 0$

AC Method

$(8x^4 - 2x^2 + 32x^2 - 8)(x+2) = 0$

$(2x^2(4x^2 - 1) + 8(4x^2 - 1))(x+2) = 0$

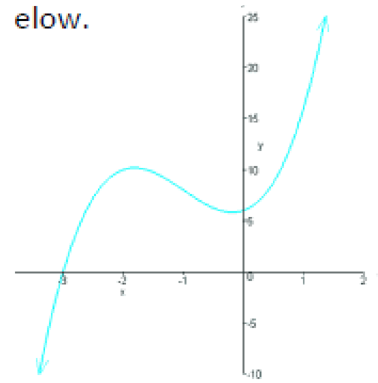
$(2x^2 + 8)(4x^2 - 1)(x+2) = 0$

$x = \pm 2i, \pm\frac{1}{2}, -2$

c) Write f as a product of its factors. $(2x^2 + 8)(4x^2 - 1)(x+2) = 0$

Homework 8.7: Solving Polynomial Equations Given a Root

1. The graph of the polynomial function $f(x) = x^3 + x^2 - 4x + 6$ is shown below.



a. What does the real solution appear to be? Algebraically, show that it is indeed a solution to the equation.

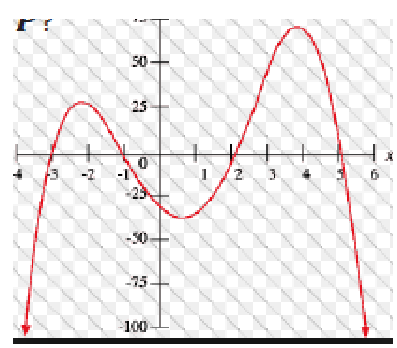
b. Find all other roots of the polynomial.

2. Consider the polynomial $P(x) = x^4 + 3x^3 - 28x^2 - 36x + 144$.

a. Is 3 a zero of the polynomial P?

b. Is $x - 2$ one of the factors of P?

c. The graph of P is shown to the right. What are the zeros of P?



d. Write the equation of P in factored form.

3. Consider the polynomial function $P(x) = 8x^4 + 12x^3 - 2x^2 - 9x - 3$.

a. Verify that $x = -1$ is a solution to the equation.

b. Find the remaining zeros of P.

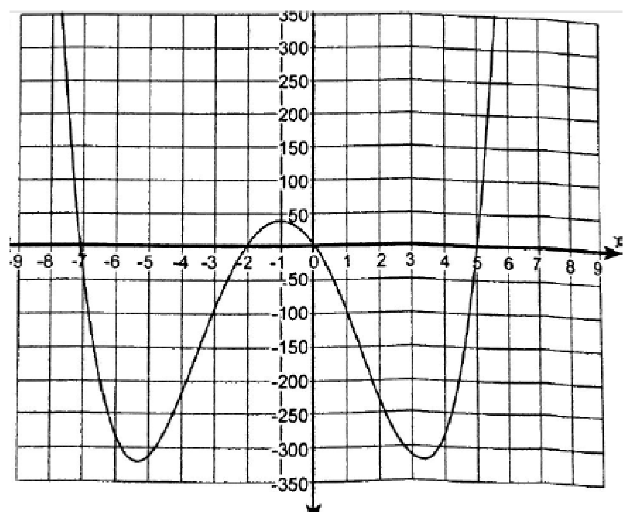
4. A quartic function, $p(x)$, is graphed to the right. Which of the following is the correct factorization of $p(x)$?

(1) $(x-7)(x-2)(x+5)$

(2) $(x+7)(x+2)(x-5)$

(3) $x(x-7)(x-2)(x+5)$

(4) $x(x+7)(x+2)(x-5)$



5. Sketch a graph of the function $f(x) = x^4 - 4x^3 + 2x^2 + 4x - 3$ by determining the sign of the values of the function between the zeros $-1, 1, \text{and } 3$

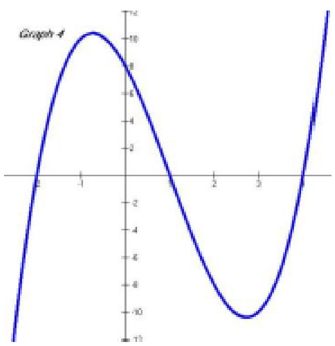
Lesson 8.8: Writing the Equations of a Polynomial

Learning Goals:

1) How can we write the equation of a polynomial function given its roots and multiplicity?
2) How can we write the equation of a polynomial function given its roots in standard form?

Warm-Up: Based on yesterday's lesson, use the graph to find the roots of the polynomial and write each polynomial as the product of its linear factors.

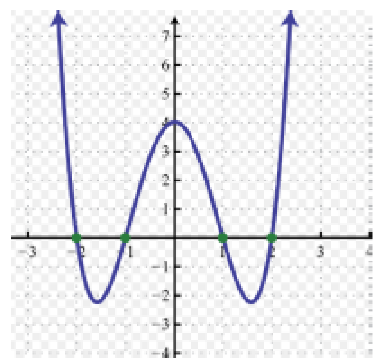
a.



$x = 1, -2, 4$

$(x-1)(x+2)(x-4)$

b.



$x = -2, -1, 1, 2$

$(x+2)(x+1)(x-1)(x-2)$

The **real zeros** of a polynomial function may be found by factoring (where possible) or by finding where the graph touches the x-axis. The number of times a zero occurs is called its **multiplicity**. In an equation, the **multiplicity** of the zero is represented by the **exponent** of each factor.

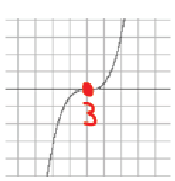
k is ODD

The graph crosses the

x-axis at $(c, 0)$

$(x-3)^1$

$(x-3)^3$



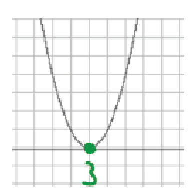
k is EVEN

The graph is tangent to the

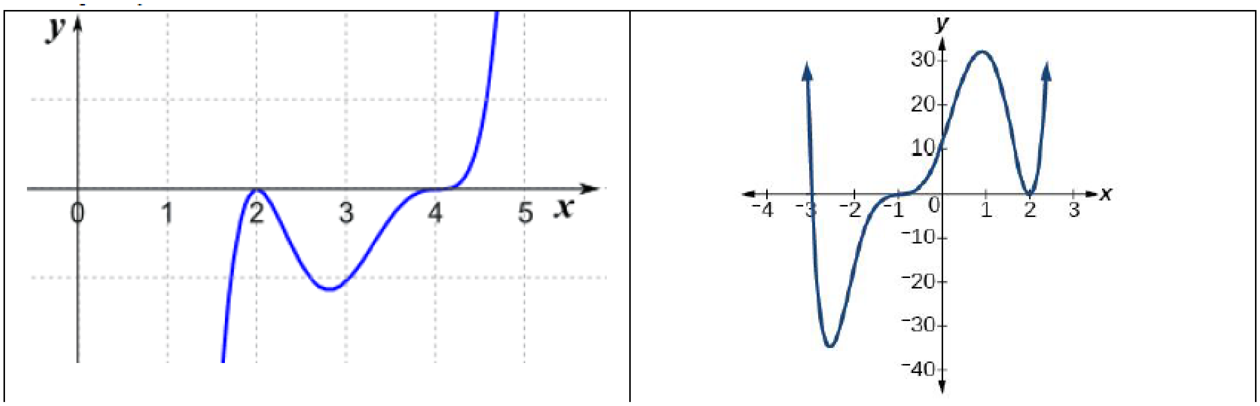
x-axis at $c, 0)$

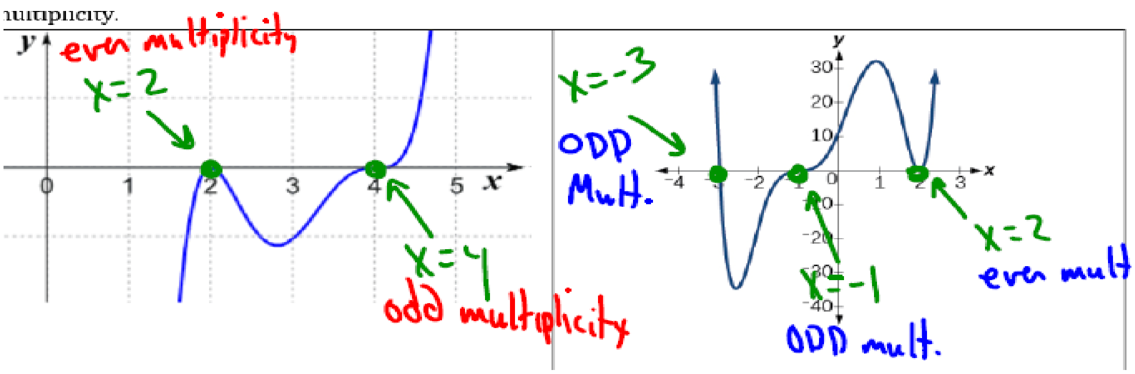
$(x-3)^2$

$(x-3)^4$



Directions: For each of the following graphs, state the zeros and if the zero has an odd or even multiplicity.





For each of the following, state the zeros and the multiplicity of each zero.

1) $f(x) = (x+2)^2(x-1)^3$

Leading Coefficient: x^5

$x = -2 \rightarrow$ multiplicity is 2 (tangent)

$x = 1 \rightarrow$ multiplicity is 3 (cross)

2) $f(x) = x^3(x+2)^4(x-3)^5$

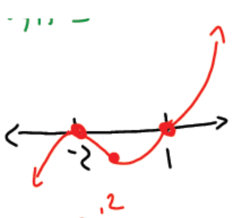
Leading Coefficient: x^{12}

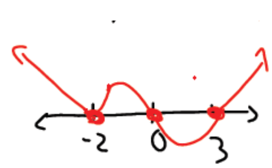
$x = 0 \rightarrow$ multiplicity is 3 (cross)

$x = -2 \rightarrow$ multiplicity is 4 (tangent)

$x = 3 \rightarrow$ multiplicity is 5 (cross)

Check the values out on the graph to see if it is above or below the x-axis!





Writing the Equation of a Polynomial is Standard Form

- The equation of a polynomial function can be expressed in many forms. The two most common forms are **factored form** and **standard form**.
- **Pay close attention to the directions** to see what form the polynomial must be expressed as. If the question does not state what form to use, just stop at the factored form! (less work)

3. Write a polynomial function in factored form, with zeros $-1, 2, \text{and } 5$.

$$x = -1 \quad x = 2 \quad x = 5$$

$$P(x) = (x+1)(x-2)(x-5)$$

4) Write a polynomial function, in standard form of least degree, with zeros $-1, 2, \text{and } 5$. Get factored form then multiply!

$$P(x) = (x+1)(x-2)(x-5)$$

$$= x^2 - x - 2)(x-5)$$

$$= x^3 - 5x^2 - x^2 + 5x - 2x + 10$$

$$= x^3 - 6x^2 + 3x + 10$$

5) Write a polynomial function in standard form with least degree whose roots are $4, 3i, \text{and } -3i$. Use conjugates!

$$x = 4 \quad x = 3i \quad x = -3i$$

$$P(x) = (x-4)(x-3i)(x+3i)$$

$$P(x) = (x-4)(x^2+9)$$

$$P(x) = x^3 + 9x - 4x^2 - 36$$

$$P(x) = x^3 - 4x^2 + 9x - 36$$

6) Write a polynomial function in factored form whose zeros are $2, 4-i, \text{and } 4+i$

$$x = 2 \quad x = 4-i \quad x = 4+i$$

$$P(x) = (x-2)(x-4+i)(x-4-i)$$

7) Write a polynomial function in factored form that has the following zeros and multiplicities. What is the degree of your polynomial. Add multiplicity to get degree = 20

Zero	Multiplicity
2	3
-4	1
6	6
-8	10

$f(x) = (x - 2)^2(x + 4)(x - 6)^6(x + 8)^{10}$

Another (will vary): $f(x) = -20(x - 2)^2(x + 4)(x - 6)^6(x + 8)^{10}$

For each of the following, write a polynomial function in standard form with least degree whose roots are given. To be in Standard Form it must be multiplied out and be in order from highest to lowest degree!

8) -2 and 1

$f(x) = (x + 2)(x - 1)$

$f(x) = x^2 - x + 2x - 2$

$f(x) = x^2 + x - 2$

9) $-\frac{2}{3}$ and 5

$f(x) = (3x + 2)(x - 5)$

$f(x) = 3x^2 - 15x + 2x - 10$

$f(x) = 3x^2 - 13x - 10$

10) 2 and $-\frac{3}{2}$ where the root 2 has a multiplicity of 2

$f(x) = (x - 2)^2(2x + 3)$

$f(x) = (x - 2)(x - 2)(2x + 3)$

$f(x) = (x^2 - 4x + 4)(2x + 3)$

$f(x) = 2x^3 + 3x^2 - 8x^2 - 12x + 8x + 12$

$f(x) = 2x^3 - 5x^2 - 4x + 12$

What if one of the zeros is irrational or complex? Always include conjugate

$x^2 - 7 = 0$

$x^2 = 7$

$x = \pm\sqrt{7}$

$x^2 + 16 = 0$

$x^2 = -16$

$x = \pm 4i$

11) $-2, 3, -4i$

$f(x) = (\underbrace{(x+2)(x-3)})(\underbrace{(x+4i)(x-4i)})$

$f(x) = (x^2 - x - 6)(x^2 - 16i^2)$ $\quad i^2 = -1$

$f(x) = (x^2 - x - 6)(x^2 + 16)$

$f(x) = x^4 + 16x^2 - x^3 - 16x - 6x^2 - 96$

$f(x) = x^4 - x^3 + 10x^2 - 16x - 96$

12) $-1, 3i$

$f(x) = (x-1)(\underbrace{(x-3i)(x+3i)})$

$f(x) = (x-1)(x^2 - 9i^2)$ $\quad i^2 = -1$

$f(x) = (x-1)(x^2 + 9)$

$f(x) = x^3 + 9x - x^2 - 9$

$f(x) = x^3 - x^2 + 9x - 9$

13) $\sqrt{2}, -\sqrt{2}, 3, 2i$

$f(x) = (\underbrace{(x-\sqrt{2})(x+\sqrt{2})})(x-3)(\underbrace{(x-2i)(x+2i)})$

$f(x) = (x^2 - 2)(x-3)(x^2 - 4i^2)$ $\quad i^2 = -1$

$f(x) = \underbrace{(x^2 - 2)(x-3)}(x^2 + 4)$

$f(x) = (x^3 - 3x^2 - 2x + 6)(x^2 + 4)$

$f(x) = x^5 + 4x^3 - 3x^4 - 12x^2 - 2x^3 - 8x + 6x^2 + 24$

$f(x) = x^5 - 3x^4 + 2x^3 - 6x^2 - 8x + 24$

Homework 8.8: Writing the Equations of a Polynomial

1. Find the zeros of the following polynomial functions, with their multiplicities.

a) $f(x) = (x+1)(x-1)(x^2+1)$

b) $g(x) = (x-4)^3(x-2)^8$

c) $h(x) = x(2x-3)^5$

d) $k(x) = (3x+4)^{100}(x-17)^4$

2. Find a polynomial function of least degree in standard form that has zeros at $1, 3, \text{and } 5i$

3. Find a polynomial function that has a zero at 2 of multiplicity 5 and a zero at -4 of multiplicity 3.

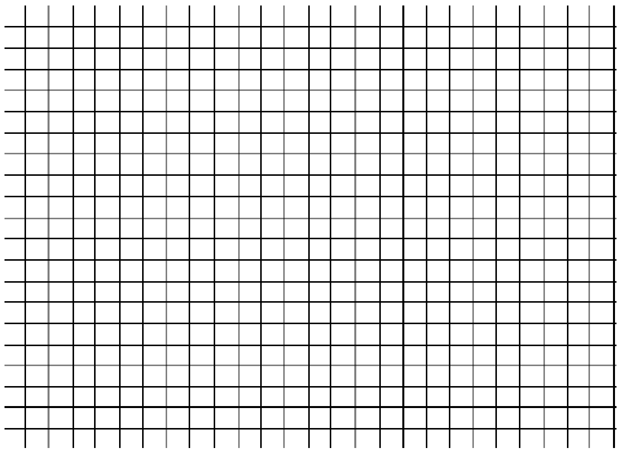
4. Find a polynomial function that has the following zeros and multiplicities. What is the degree of your polynomial?

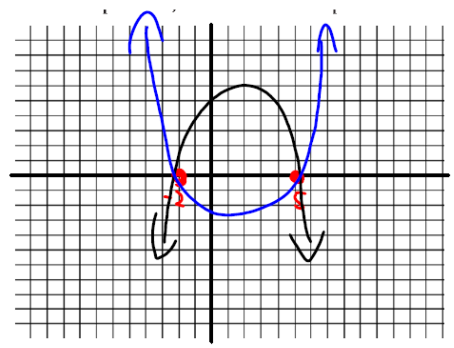
x	Multiplicity
-3	1
-1	1
1	2
2	1

Lesson 8.9: Writing an Equation to Model a Polynomial Function

Learning Goal: How do we write an equation to model a polynomial function?

a) On the axes provided, draw two different parabolas that have roots -2 and 5.





b) Look at the graphs you drew. What would change in the equation to represent the two different functions?

Black: $-(x+2)(x-5)$

Blue: $(x+2)(x-5)$

c) Find two different polynomial functions that have zeros at $1, 3,$ and 5 of multiplicity 1.

$f(x) = (x-1)(x-3)(x-5)$ will vary for other one, I used:

$f(x) = -(x-1)(x-3)(x-5)$

d) Find two different polynomial functions that have a zero at 2 of multiplicity 5 and a zero at -4 of multiplicity 3.

$f(x) = (x-2)^5(x+4)^3$ will vary for other one, I used:

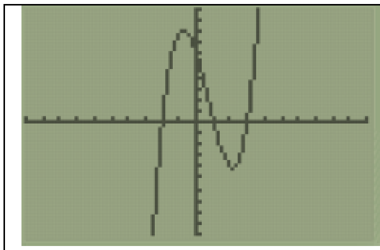
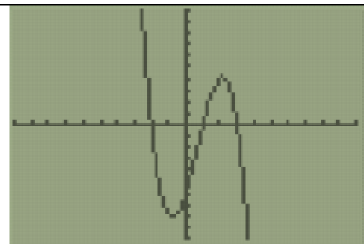
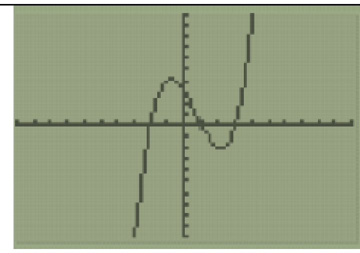
$f(x) = -2(x-2)^5(x+4)^3$

Recall the Fundamental Theorem of Algebra

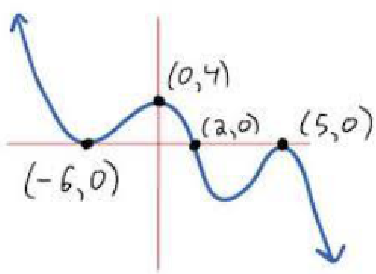
Every polynomial $p(x)$ of degree $n, n > 0$, can be written as a product of a **constant** $\boldsymbol{c}, \boldsymbol{c} \neq \mathbf{0}$, and n linear factors: $p(x) = \boldsymbol{c}(x - r_1)(x - r_2) \ldots (x - r_n)$ where $r_{1,}, r_2, r_3, \ldots r_n$ are the zeros of the polynomial.

When no $\boldsymbol{c}$ is written we assume that $\boldsymbol{c} = \mathbf{1}$.

Each function will have its own unique constant value. You must use another point on the graph to determine the value of $\boldsymbol{c}$ for each function.

		
$f(x) = (x+2)(x-1)(x-3)$	$f(x) = -(x+2)(x-1)(x-3)$	$f(x) = \frac{1}{2}(x+2)(x-1)(x-3)$

Model Problem: The graph of polynomial f is given below Write a formula for f in factored form using c for the constant factor.



$$f(x) = c(x+6)(x-2)(x-5)$$

If f passes through the point $(0, 4)$, find the constant factor c.

$$4 = c(0+6)(0-2)(0-5)$$

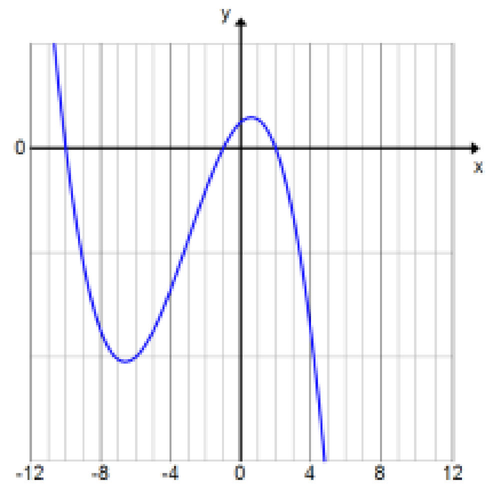
$$4 = c(6)(-2)(-5)$$

$$4 = 60c$$

$$c = -\frac{1}{15}$$

$$f(x) = -\frac{1}{15}(x+6)(x-2)(x-5)$$

Example 1: The graph to the right is of a third-degree polynomial function f.



a) State the zeros of f. $x = -10, -1, 2$

b) Write the formula for f in factored form using $\boldsymbol{c}$ for the constant factor.

$f(x) = c(x + 10)(x + 1)(x - 2)$

c) Use the fact that $f(-4) = -54$ to find the constant factor $\boldsymbol{c}$.

This is a point on the graph at $x = -4$ and $y = -54$!

$f(x) = c(x + 10)(x + 1)(x - 2)$

$-54 = c(-4 + 10)(-4 + 1)(-4 - 2)$

$-54 = c(6)(-3)(-6)$

$-54 = 108c$

$-\frac{54}{108} = -\frac{1}{2} = c$

d) Verify your equation by using the fact that $f(1) = 11$.

Again, this is a point on the graph at $x = 1$ and $y = 11$! But also use the value of c!

$f(x) = c(x + 10)(x + 1)(x - 2)$

$11 = -\frac{1}{2}(1 + 10)(1 + 1)(1 - 2)$

$11 = -\frac{1}{2}(11)(2)(-1)$

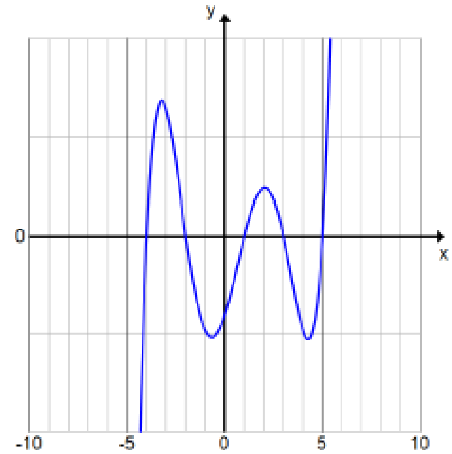
$11 = -\frac{1}{2}(-22)$

$11 = 11\surd$

Example 2: Consider the graph of a degree 5 polynomial shown below, with x-intercepts, $-4, -2, 1, 3,$ and 5

a) Write a formula for a possible polynomial function that the graph represents using $\boldsymbol{c}$ as the constant factor.

$$f(x) = c(x+4)(x+2)(x-1)(x-3)(x-5)$$



b) Suppose the y-intercept is -4. Find the value of $\boldsymbol{c}$ so that the graph of P has y-intercept -4. This would mean it crosses at $(0,-4)$ so $x = 0$ and $y = -4$!

$$-4 = c(0+4)(0+2)(0-1)(0-3)(0-5)$$

$$-4 = c(4)(2)(-1)(-3)(-5)$$

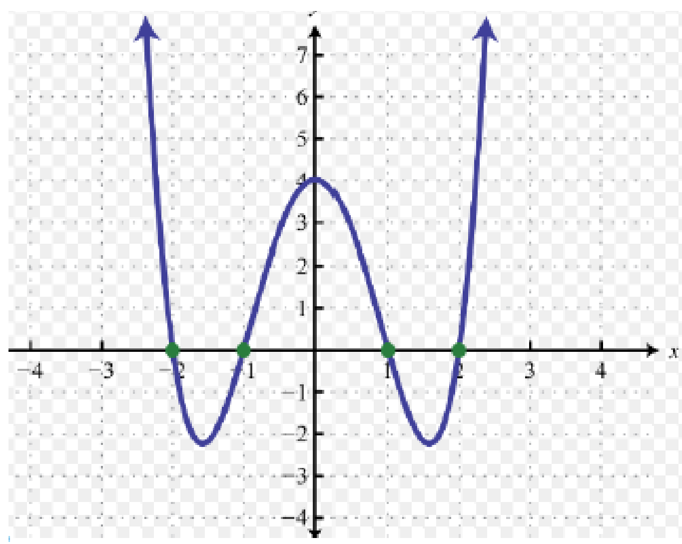
$$-4 = -120c$$

$$\frac{-4}{-120} = \frac{1}{30} = c$$

$$f(x) = \frac{1}{30}(x+4)(x+2)(x-1)(x-3)(x-5)$$

Example 3: The graph below is of a fourth-degree polynomial function f.

a) State the zeros of f. $x = -2, -1, 1, 2$



b) Write a formula for f in factored form using $\boldsymbol{c}$ for the constant factor.

$$f(x) = c(x+2)(x+1)(x-1)(x-2)$$

c) Use the y-intercept to find the constant factor $\boldsymbol{c}$.

Look at the graph and see that the y-intercept is $(0,4)$ so $x = 0$ and $y = 4$!

$$f(x) = c(x+2)(x+1)(x-1)(x-2)$$

$$4 = c(0+2)(0+1)(0-1)(0-2)$$

$$4 = c(2)(1)(-1)(-2)$$

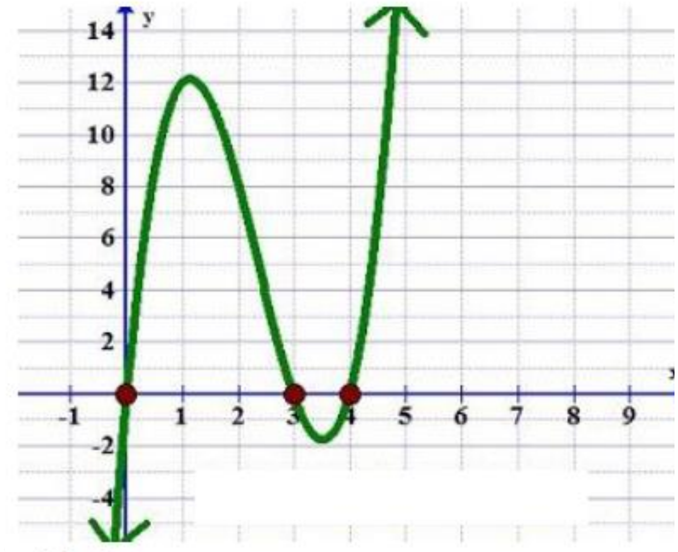
$$4 = 4c$$

$$1 = c$$

$$f(x) = 1(x+2)(x+1)(x-1)(x-2) \text{ or } f(x) = (x+2)(x+1)(x-1)(x-2)$$

Example 4:

a) Write a formula for a possible polynomial function that the graph represents using $\boldsymbol{c}$ as the constant factor.



The zeros are $x = 0, 3, 4$

$f(x) = cx(x-3)(x-4)$

b) Use the fact that $f(1) = 12$ to find the constant factor $\boldsymbol{c}$.

This is a point on the graph at

$x = 1$ and $y = 12$!

$f(x) = cx(x-3)(x-4)$

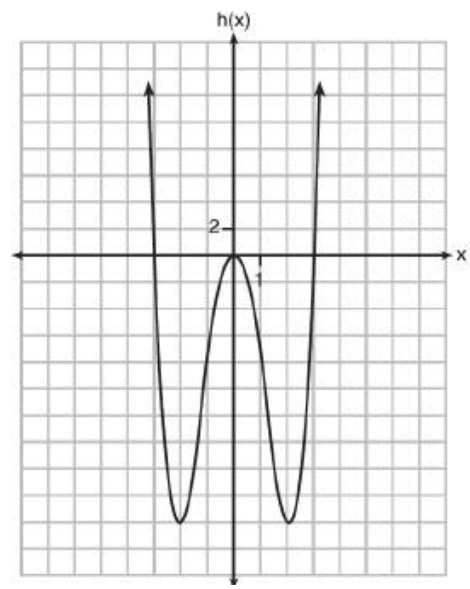
$12 = c(1)(1-3)(1-4)$

$12 = c(-2)(-3)$

$12 = 6c$

$2 = c$

Example 5: The graph below is of a fourth-degree polynomial function f with x-intercepts $-3, 0,$ and 3. The root of 0 has a multiplicity of two.



Write a formula for f in factored form to model this function. (hint: Be sure to find $\boldsymbol{c}$ using a point on the graph) Use $(2, -10)$

$f(x) = c(x+3)(x-3)(x-0)^2$

$-10 = c(2+3)(2-3)(2-0)^2$

$-10 = c(5)(-1)(2)^2$

$-10 = c(5)(-1)(4)$

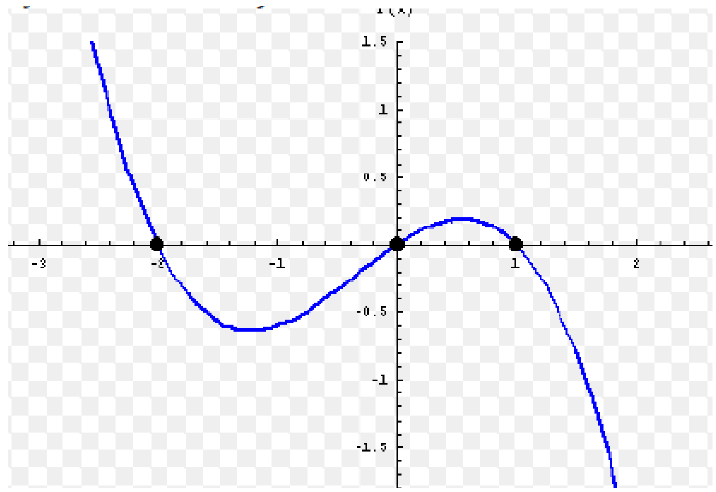
$-10 = -20c$

$\frac{-10}{-20} = \frac{1}{2} = c$ $\qquad$ $f(x) = \frac{1}{2}(x+3)(x-3)(x-0)^2$

Homework 8.9: Writing an Equation to Model a Polynomial Function

1. The graph to the right is of a third-degree polynomial function f.

a. State the zeros of f.



b. Write a formula for f in factored form using $\boldsymbol{c}$ for the constant factor.

c. Use the fact that $f(-7) = -56$ to find the constant factor $\boldsymbol{c}$.

2. Write a polynomial equation in standard form with least degree that has roots $2, -3,$ and $3i$.

3. Suppose a polynomial of degree 4 has roots $-3, 1, 4, \text{and } 8$.

a) Write a possible function that could represent this polynomial using $\boldsymbol{c}$ as the constant factor.

b) Suppose the y-intercept is 6. Find the value of $\boldsymbol{c}$ so that the graph of P has y-intercept 6.

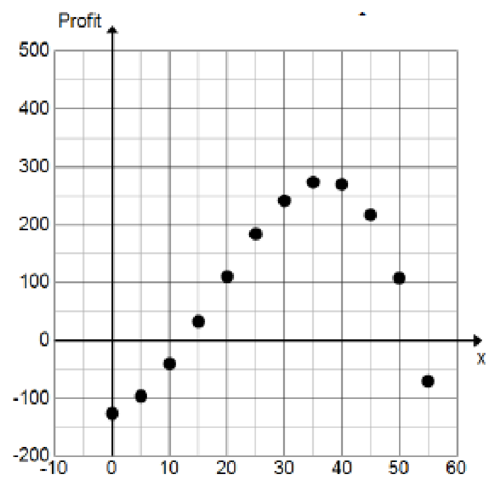
4. A polynomial function P has zeros of $2, 2, -3, -3, \text{and } 4$. Find a possible formula for P, and state its degree. Why is the degree of the polynomial not 3?

Lesson 8.10: Modeling with Polynomial Functions

Learning Goal: How do we use polynomial functions to model real-life situations?

Warm-Up: Use the graph to answer the given questions as best you can.

For a fundraiser, members of the math club decide to make and sell "Pythagoras may have been Fermat's first problem but not his last!" t-shirts. They are trying to decide how many t-shirts to make and sell at a fixed price. They surveyed the level of interest of students around school and made a scatterplot of the number of t-shirts sold (x) versus profit shown below.



a) Identify the y-intercept. Interpret its meaning within the context of this problem.

y-intercept is -125. This represents the money they spent to start the business (make the t-shirts).

b) What is the smallest number of t-shirts they can sell and still make a profit? 15 t-shirts

c) How many t-shirts should they sell in order to maximize the profit? About 35 t-shirts

d) What is the maximum profit? About $275

e) What would cause the profit to start decreasing?

Maybe they are running out of t-shirts

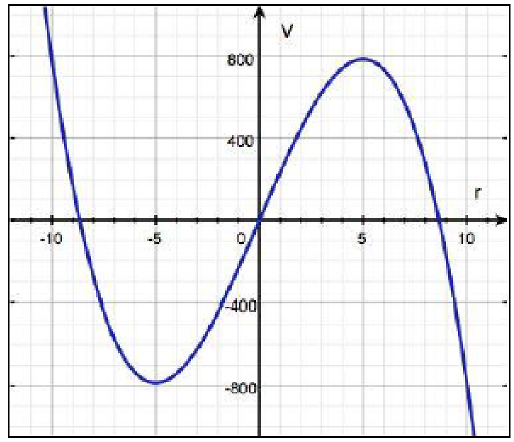
They made too many t-shirts and the demand was not high enough!

Maybe the shirts cost too much (increased price)

Steps to Writing a Polynomial Function to Model a Given Situation

- Find the x-intercepts.
- Write the equation in factored form using the x-intercepts and a constant factor c.
- Plug a point from the graph into the equation to find the value of c.
- Write the equation with c plugged back in to the factored form.

Example 1: Jeannie wishes to construct a cylinder closed at both ends. The figure below shows the graph of a cubic polynomial function used to model the volume of the cylinder as a function of the radius if the cylinder is constructed using $150\pi \text{ cm}^2$ of material. Use the graph to answer the questions below. Estimate values to the nearest half unit on the horizontal axis and to the nearest 50 units on the vertical axis.



a) What is the most volume that Jeannie's cylinder can enclose?

Look for the maximum on the graph and it is $(5, 800)$ so $V = 800$

b) What radius yields the maximum volume?

$r = 5$

c) What are the zeros of the function V?

$x = -9, 0, 9$

d) Find an equation to represent this function using the zeros. Find the value of c so that this formula fits the graph. [hint: remember to plug in a value from the graph to find c] Use the point $(5{,}800)$

$V = c(r+9)(r-0)(r+9)$ or $V = c\,r(r+9)(r-9)$

$800 = c\,(5)(5+9)(5-9)$

$800 = c\,(5)(14)(-4)$

$800 = -280c$

$\frac{800}{-280} = -\frac{20}{7} = c$

$V = -\frac{20}{7}\,r(r+9)(r-9)$

e) Use the graph to estimate the volume of the cylinder with $r = 2$ cm.

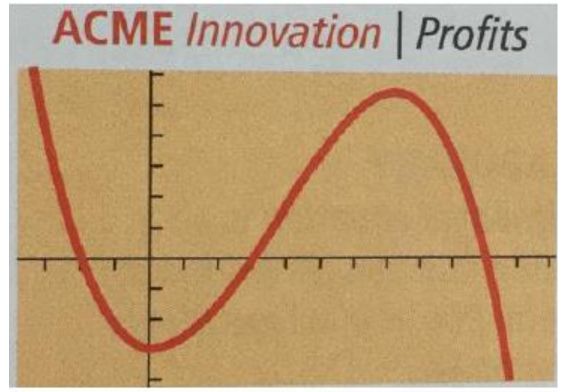
Looks like $V = 450$

f) Use your formula for V to find the volume of the cylinder when $r = 2$ cm. How close is the value from the formula to the value on the graph?

$V = -\frac{20}{7}(2)(2+9)(2-9)$

$V = -\frac{20}{7}(2)(11)(-7) = 440$, so the estimate is pretty close!

Example 2: Acme Innovations makes and sells lamps. Their profit, P, in hundreds of dollars earned, is a function of the number of lamps sold x, in thousands. From historical data, they know that their company's profit is modeled by the function shown below.



a) State the zeros of the function. $x = -2, 3, 10$

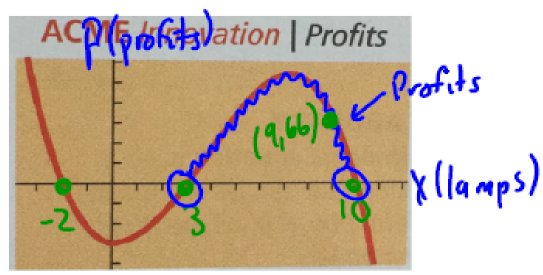
b) State the interval where Acme Innovations will produce a profit in lamp sales. $3 < x < 10$

c) When the company sells 9 thousand lamps, their profit will be 66 hundred dollars. Using this information and the zeros, write a polynomial function to represent the profit from their lamp sales.

Point on the graph! Use a constant factor c.

Use the $(9, 66)$

$P(x) = c(x+2)(x-3)(x-10)$



$66 = c(9+2)(9-3)(9-10)$

$66 = c(11)(6)(-1)$

$66 = -66c$

$c = -1$

$P(x) = -(x+2)(x-3)(x-10)$

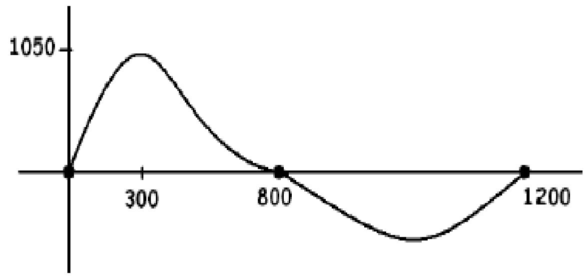
Example 3: The population of tortoises on an island is modeled by the function $P(x) = -x^3 + 6x^2 + 12x + 325$ where x is the number of years since 2015. Algebraically estimate the population in 2023.

$$x = 2023 - 2015 = 8$$

$$P(8) = 8^3 + 6(8)^2 + 12(8) + 325$$

$$p(8) = 293$$

Example 4: Geographers sit at a café discussing their field work site, which is a hill and a neighboring riverbed. The hill is approximately $1{,}050$ feet high, 800 feet wide, with a peak about 300 feet east of the western base of the hill. The rive is about 400 feet wide. They know the rive is shallow, no more than about 20 feet deep. They make the following crude sketch on a napkin, playing the profile of the hill and riverbed on a coordinate system with the horizontal axis representing ground level.



a) Write a cubic polynomial function H that could represent the curve shown. Be sure that your formula satisfies $H(300) = 1050$.

$$x = 0 \quad x = 800 \quad x = 1200 \quad \text{a point on the graph is } (300, 1050)$$

$$H(x) = c(x)(x - 800)(x - 1200)$$

$$1050 = c(300)(300 - 800)(300 - 1200)$$

$$1050 = c(300)(-500)(-900)$$

$$1050 = 135000000c$$

$$c = \frac{1050}{135000000}$$

b) Using this equation, determine the depth of the riverbed at $1{,}000$ feet.

$x = 1000$ $$H(x) = \frac{1050}{135000000}(x)(x - 800)(x - 1200)$$

$$H(1000) = \frac{1050}{135000000}(1000)(1000 - 800)(1000 - 1200)$$

$$H(1000) = -311.\overline{1}$$

c) Based on the context of the question, why is the equation you found above not a suitable model for this hill and riverbed? Explain. The riverbed is only 20 feet deep, not 311!

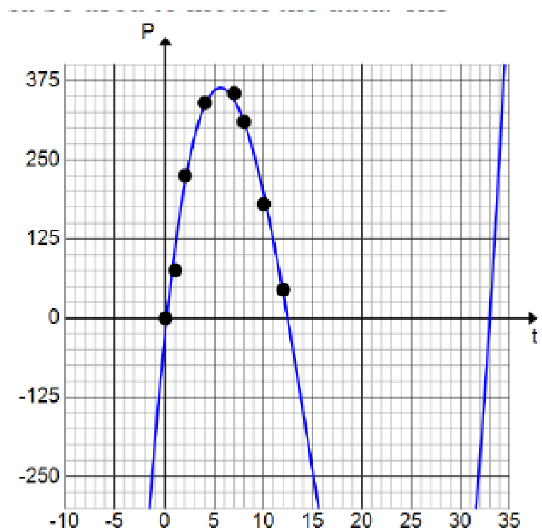
Example 5: The owners of Dizzy Lizzy's, an amusement park, are studying the wait time at their most popular roller coaster. The table below shows the number of people standing in line for the roller coaster t hours after Dizzy Lizzy's opens.

t (hours)	0	1	2	4	7	8	10	12
P (people in line)	0	75	225	345	355	310	180	45

Jaylon made a scatterplot and decided that a cubic function should be used to model the data. His scatter plot and curve are shown below.

a) Estimate the t-intercepts (x-intercepts) of the function used to model this data.

$x = 0, 12.5, 33$



b) Use the t-intercepts to write a formula for the function of the number of people in line, f, after t hours.

$f(x) = c\, x(x - 12.5)(x - 33)$

c) Use the relative maximum $f(6) = 350$ to find the leading coefficient of f.

$x = 6$ and $y = 350$

$350 = c\,(6)(6 - 12.5)(6 - 33)$

$350 = c\,(6)(-6.5)(-27)$

$350 = 1053c$

$\frac{350}{1053} = c$

d) Use your function f to calculate the number of people in line 10 hours after the park opens.

$f(x) = \frac{350}{1053}\,(10)(10 - 12.5)(10 - 33)$

$f(x) = \frac{350}{1053}\,(10)(-2.5)(-23)$

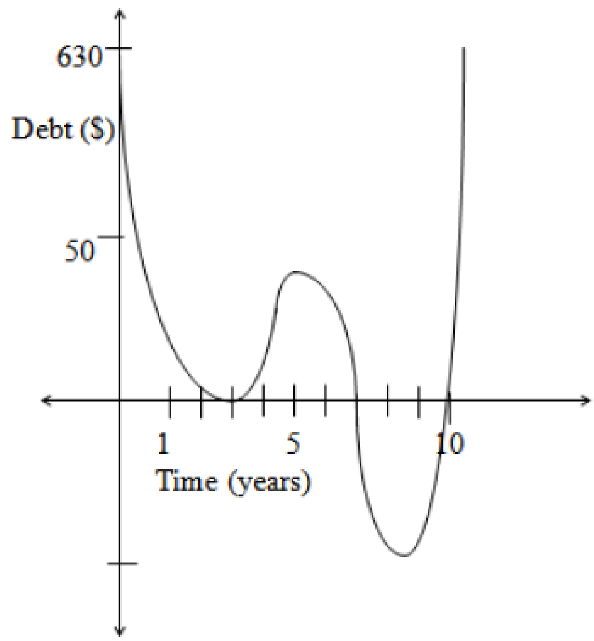
$f(x) = \frac{350}{1053}\,(575)$

$f(x) = \frac{201250}{1053} \approx 191.12 \approx 191$

Example 6: Bailey borrowed $\$630$ from her parents to use towards buying a used car. It is common for her to borrow money, but she is good about paying her parents back. Bailey's borrowing habit can be modeled by the graph below.

He didn't do

Write a quartic polynomial function D that could represent the curve shown (here, t represents the time, in years, along the horizontal axis, and $D(x)$ is the debt, in dollars.) Be sure that your formula satisfies $D(5) = 45$. [HINT: the graph appears to be tangent to the x-axis at one of the zeros]



x-intercepts at $3, 7, 10$?

$$D(x) = c(x-a)^2(x-b)(x-d)$$

$$D(x) = c(x-3)^2(x-7)(x-10)$$

Interpret what the relative minimum at $x = 8.5$ means, in terms of the context of the problem.

She hadn't paid her parents in awhile.

Example 7: Lauren wants to determine the length and height of her DVD stand. The function $f(x) = x^3 + 14x^2 + 57x + 72$ represents the volume of the DVD stand, where the width is $x + 3$ units. What are the possible dimensions for the length and height of the DVD stand?

$$V = l * w * h$$

$$x^3 + 14x^2 + 57x + 72 = l(x+3)h$$

Synthetic division

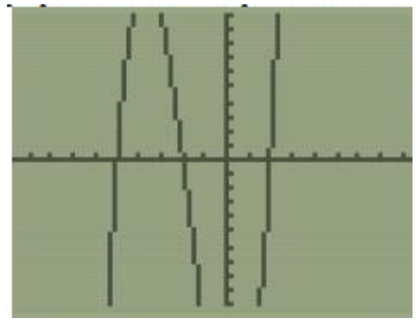
$$\begin{array}{r|rrrr} -3 & 1 & 14 & 57 & 72 \\ \oplus & \downarrow & -3 & -33 & -72 \\ \hline & 1 & 11 & 24 & \underline{0} \rightarrow \text{no remainder} \end{array}$$

$$(x^2 + 11x + 24)(x+3)$$

$$\underline{(x+3)(x+8)}(x+3)$$

Homework 8.10: Modeling with Polynomial Functions

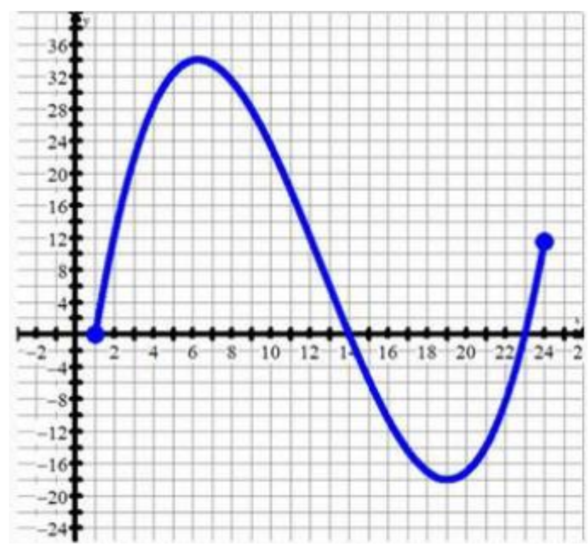
1. A population of parasites in an experiment can be modeled by the graph given below where t is the time in days, and $f(t)$ is the population of the parasites at any given time t.



Write a polynomial function that models this curve. Be sure that your equation satisfies $f(7) = 540$.

2. John wanted to see his business profits for the last two years. He created the following curve to show his profit, $P(t)$, where $t = 1$ is January 2015, $t = 2$ is February 2015 … and $t = 24$ is December 2016.

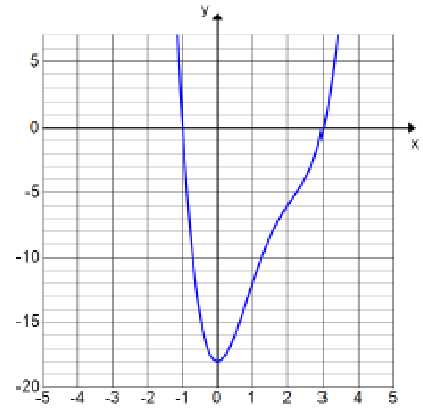
a) Write a polynomial function that models this curve. Be sure that your equation satisfies $P(6) = 34$.



b) Interpret what the relative minimum at $x = 19$ means, in terms of the context of the problem.

3. Consider the polynomial function $P(x) = x^4 - 6x^3 + 11x^2 - 18$.

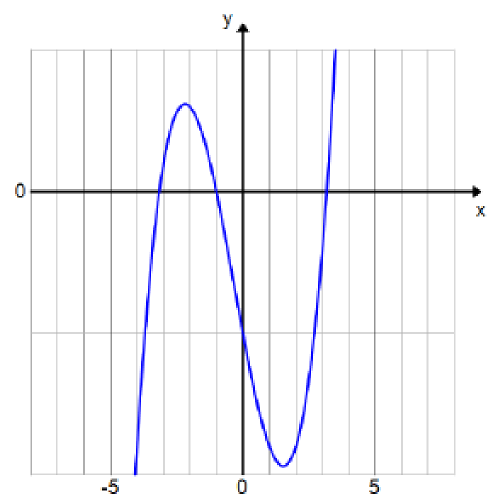
a) Use the graph to find the real zeros of P.



b) Confirm that the zeros are correct by evaluating the function P at those values.

c) Express P in terms of two linear factors and one quadratic factor.

4. The graph shown has x-intercepts at $\sqrt{10}, -1,$ and $-\sqrt{10}$. Could this be the graph of $P(x) = x^3 + x^2 - 10x - 10$? Explain how you know.



5. Find the zeros of the following polynomial function, with its multiplicities.

$$k(x) = (5x + 8)^{10}(x - 14)^4(x + 3)^2$$