8.4 Midsegments of Triangles

Essential Question: How are the segments that join the midpoints of a triangle's sides related to the triangle's sides?

Objectives

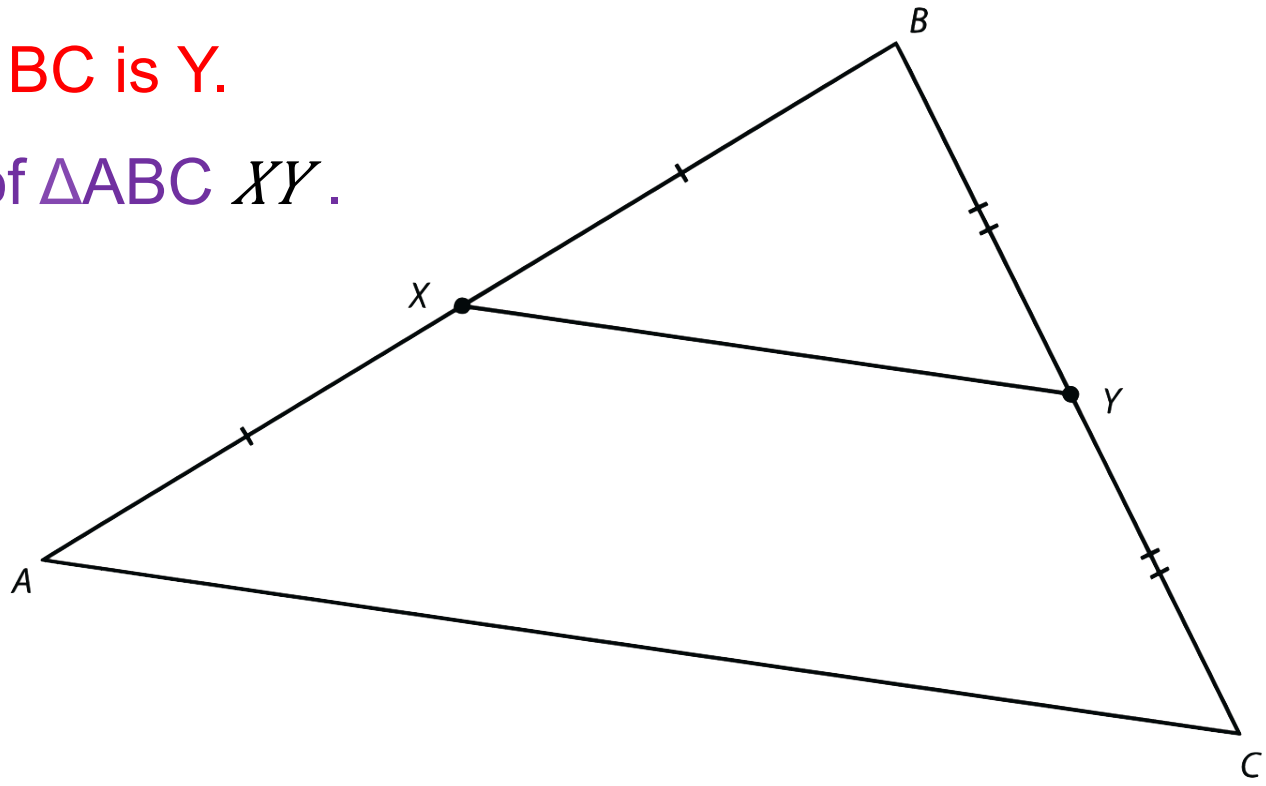
Apply properties of midsegments

Vocabulary

Midsegment of a triangle

Key Concepts

- The midpoint is the point on a line segment that divides the segment into two equal parts.
- A midsegment of a triangle is a line segment that joins the midpoints of two sides of a triangle.
- In the diagram below, the midpoint of AB is X.
- The midpoint of BC is Y.
- A midsegment of ΔABC XY.



Triangle Midsegment Theorem

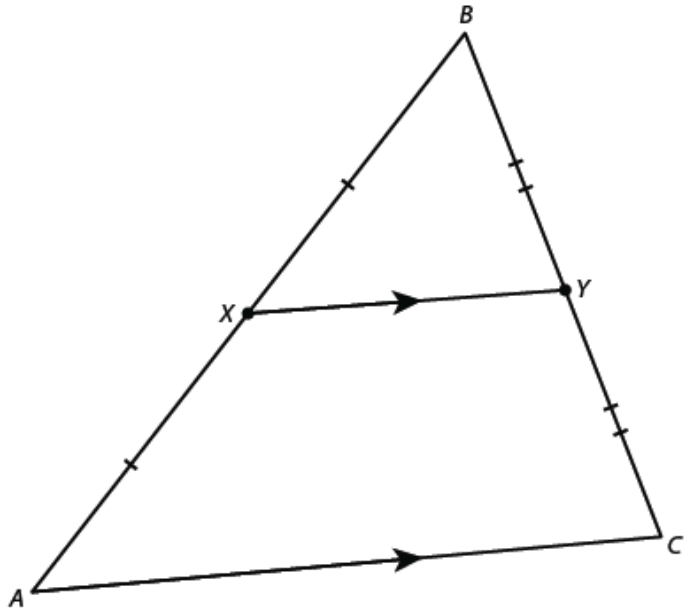
Theorem

Triangle Midsegment Theorem

A midsegment of a triangle is parallel to the third side and is half as long.

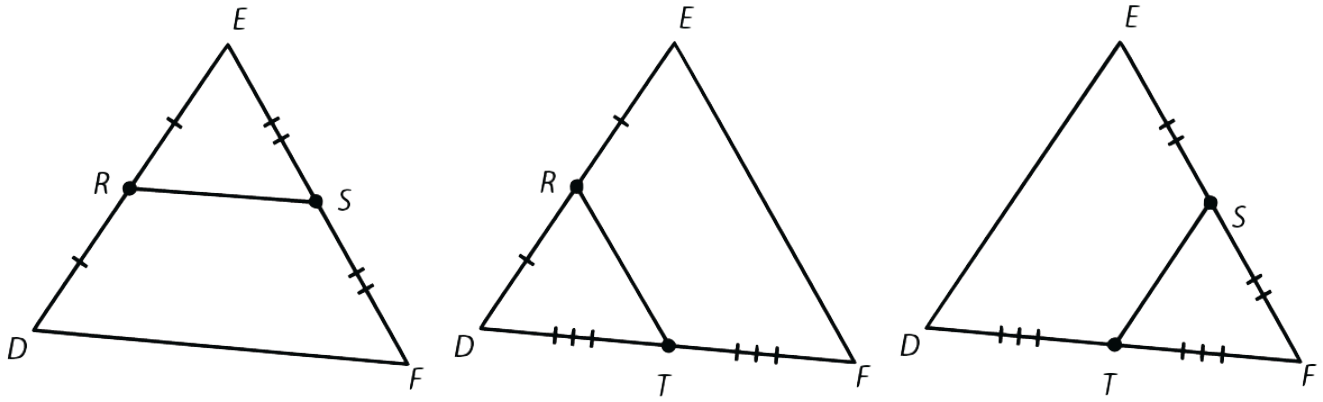
$\overline{AC} \parallel \overline{XY}$

$XY = \frac{1}{2}AC$

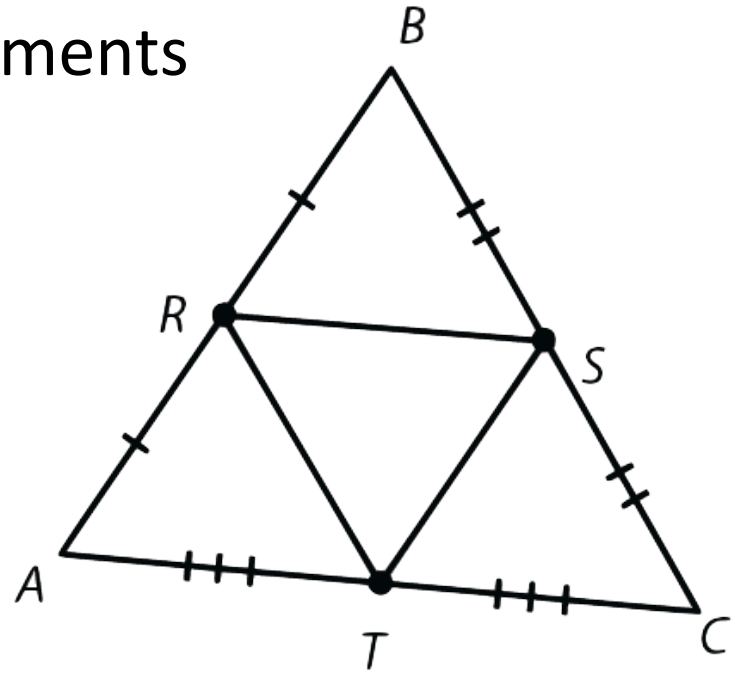


Key Concepts, *continued*

- Every triangle has three midsegments.



- When all three of the midsegments of a triangle are connected, a **midsegment triangle** is created.
- In the diagram right, $\Delta ABC \sim \Delta TSR$.



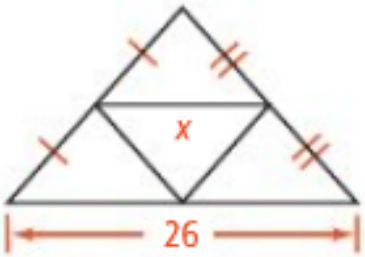
Common Errors/Misconceptions

- assuming a segment that is parallel to the third side of a triangle is a midsegment
- incorrectly writing and solving equations to determine lengths
- incorrectly calculating slope
- incorrectly applying the Triangle Midsegment Theorem to solve problems
- misidentifying or leaving out theorems, postulates, or definitions when writing proofs

QUICK PRACTICE

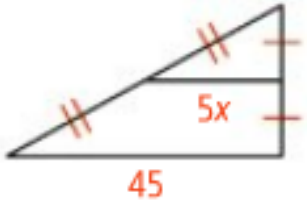
Algebra Find the value of x.

1



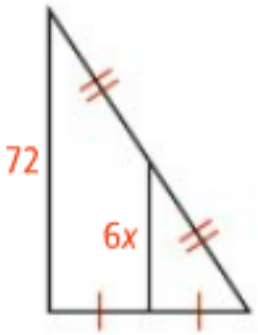
$x=13$

2



$x=4.5$

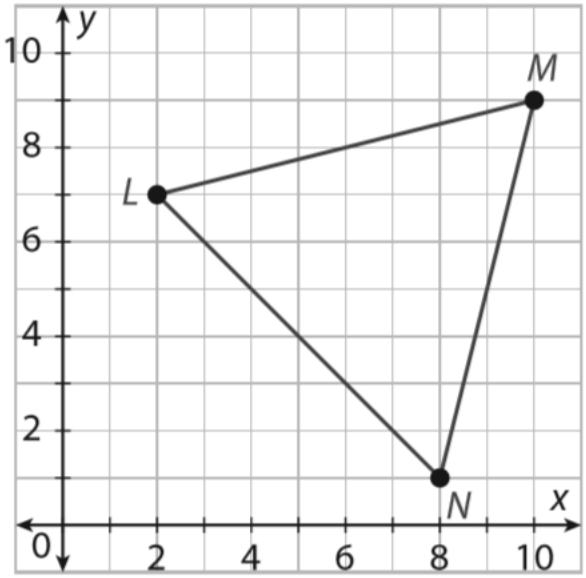
3.



$x=6$

Example 1 **Show that the given midsegment of the triangle is parallel to the third side of the triangle and is half as long as the third side.**

Ⓑ The vertices of $\triangle LMN$ are $L(2, 7)$, $M(10, 9)$, and $N(8, 1)$. P is the midpoint of $\overline{LM}$, and Q is the midpoint of $\overline{MN}$.



Show that $\overline{PQ} \parallel \overline{LN}$ and $PQ = \frac{1}{2}LN$. Sketch $\overline{PQ}$.

Step 1 The midpoint of $\overline{LM} = \left(\frac{2 + \square}{2}, \frac{7 + \square}{2}\right) = (\square, \square)$.
Graph and label this point P.

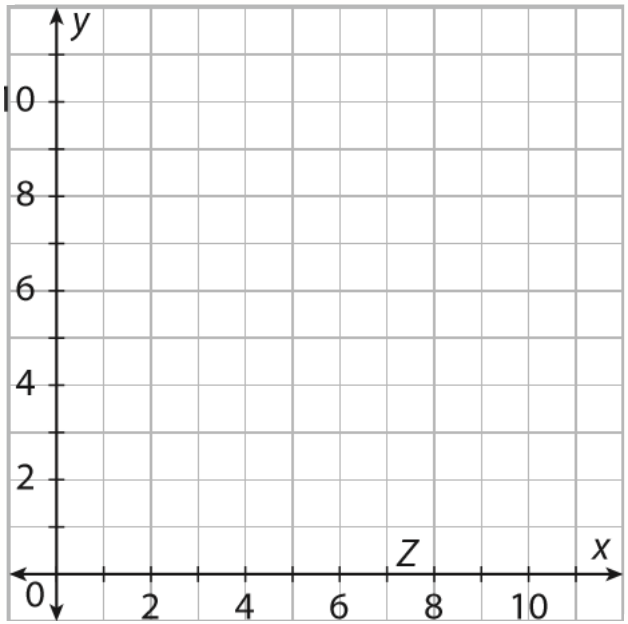
The midpoint of $\overline{NM} = \left(\frac{\square + \square}{2}, \frac{\square + \square}{2}\right)$
$= (\square, \square)$. Graph and label this point Q. Use a straightedge to draw $\overline{PQ}$.

Step 2 Slope of $\overline{PQ} = \frac{5 - 8}{9 - \square} = \square$ Slope of $\overline{LN} = \frac{\square - \square}{\square - \square} = \square$

Since the slopes are the same, $\overline{PQ}$ and $\overline{LN}$ are _____.

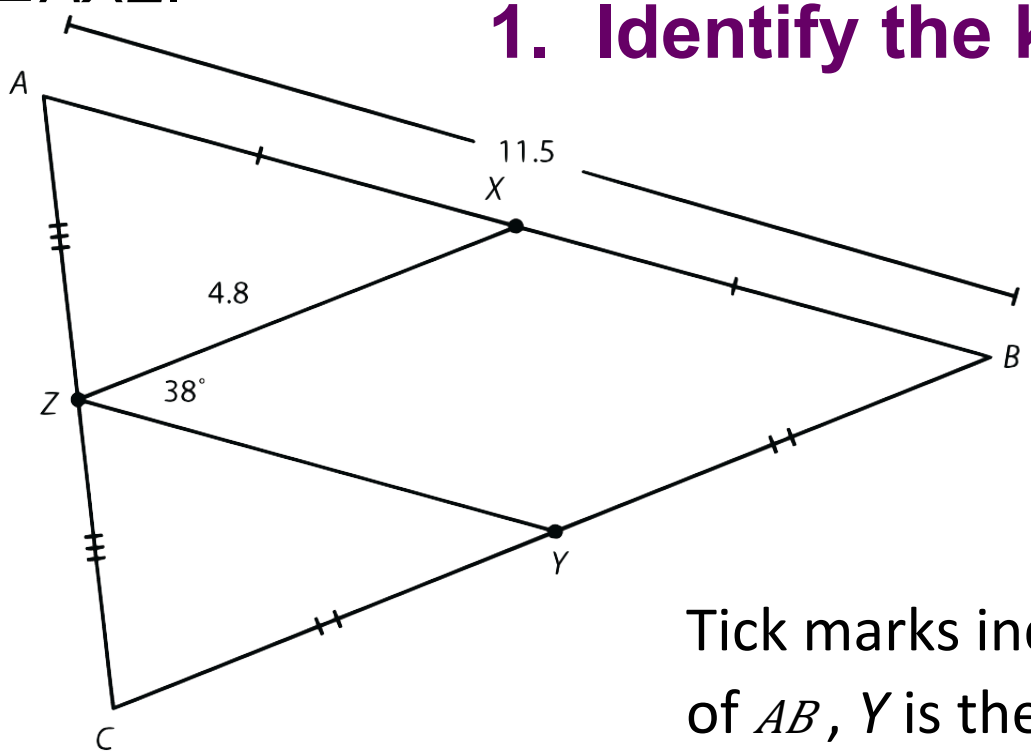
Your Turn

4. The vertices of $\triangle XYZ$ are $X(3, 7)$, $Y(9, 11)$, and $Z(7, 1)$. U is the midpoint of $\overline{XY}$, and W is the midpoint of $\overline{XZ}$. Show that $\overline{UW} \parallel \overline{YZ}$ and $UW = \frac{1}{2}YZ$. Sketch $\triangle XYZ$ and $\overline{UW}$.



EXAMPLE **Using the Triangle Midsegment**

Find the lengths of *BC* and *YZ* and the measure of ∠*AXZ*.



1. Identify the known information.

Tick marks indicate that *X* is the midpoint of $\overline{AB}$, *Y* is the midpoint of BC, and *Z* is the midpoint of AC.

$\overline{XZ}$ and $\overline{YZ}$ are midsegments of ΔABC

EXAMPLE **Using the Triangle Midsegment (cont.)**

2. Calculate the length of *BC*.

$\overline{XZ}$ is the midsegment that is parallel to BC .

The length of $\overline{XZ}$ is $1/2$ the length of BC .

$$XZ = \frac{1}{2}BC$$ Triangle Midsegment Theorem

$$4.8 = \frac{1}{2}BC$$ Substitute 4.8 for *XZ*.

$$BC = 9.6$$ Solve for *BC*.

EXAMPLE **Using the Triangle Midsegment (cont.)**

3. Calculate the measure of *YZ*.

$\overline{YZ}$ is the midsegment parallel to $\overline{AB}$.

The length of $\overline{YZ}$ is $1/2$ the Length of $\overline{AB}$.

$$YZ = \frac{1}{2}AB$$ Triangle Midsegment Theorem

$$YZ = \frac{1}{2}(11.5)$$ Substitute 11.5 for *AB*.

$$YZ = 5.75$$ Solve for *YZ*.

EXAMPLE **Using the Triangle Midsegment (cont.)**

4. Calculate the measure of $\angle AXZ$.

$\overline{YZ} \parallel \overline{AB}$ — Triangle Midsegment Theorem

$m\angle AXZ = m\angle XZY$ — Alternate Interior Angles Theorem

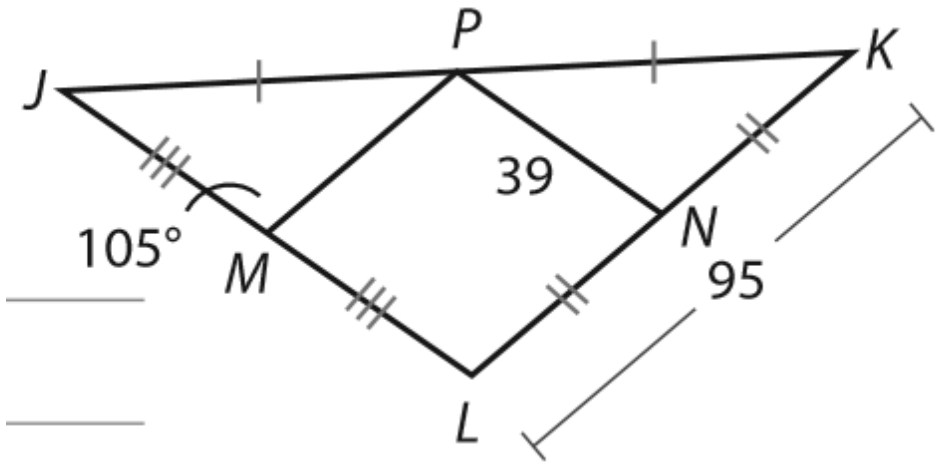
$m\angle AXZ = 38$

5. State the answers.

BC is 9.6 units long.
YZ is 5.75 units long.
$m\angle AXZ$ is 38°.

Your Turn

6. Find JL, PM, and m $\angle MLK$.



PRACTICE QUESTIONS

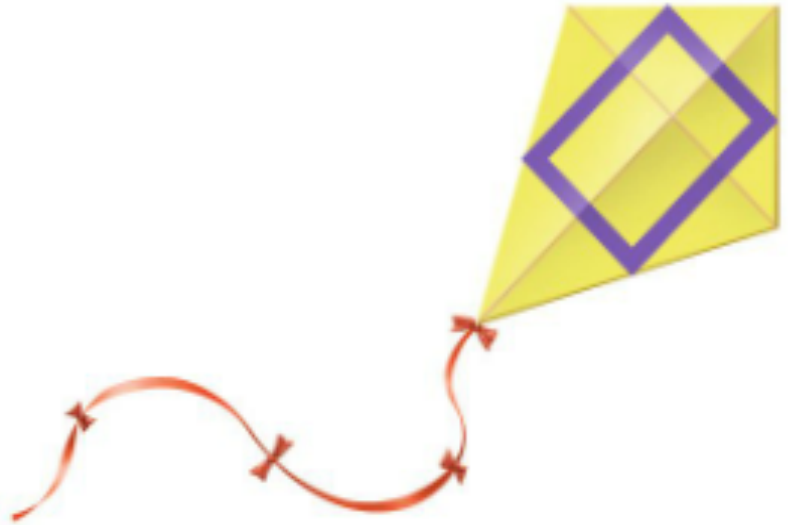
Kite Design You design a kite to look like the one at the right. Its diagonals measure 64 cm and 90 cm. You plan to use ribbon, represented by the purple rectangle, to connect the midpoints of its sides. How much ribbon do you need?

(A) 77 cm

(C) 154 cm

(B) 122 cm

(D) 308 cm



PRACTICE QUESTIONS

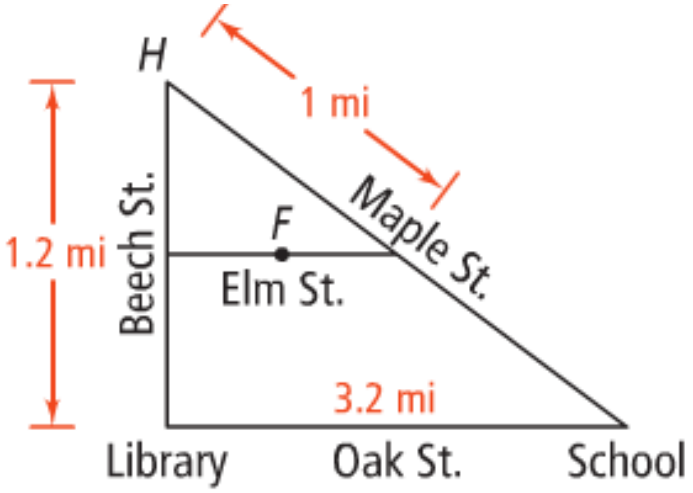
Your home is at point *H*. Your friend lives at point *F*, the midpoint of Elm Street. Elm Street intersects Beech Street and Maple Street at their midpoints.

1. Your friend walks to school by going east on Elm and then turning right on Maple. How far in miles does she walk?

$0.8+1=1.8$ *mi*

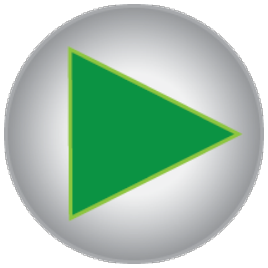
2. You walk your dog along this route: Walk from home to Elm along Maple. Walk west on Elm to Beech, south on Beech to the library, and east on Oak to school. Then walk back home along Maple. How far in miles do you walk?

$1+1.6+0.6+3.2+2=8.4$ *mi*

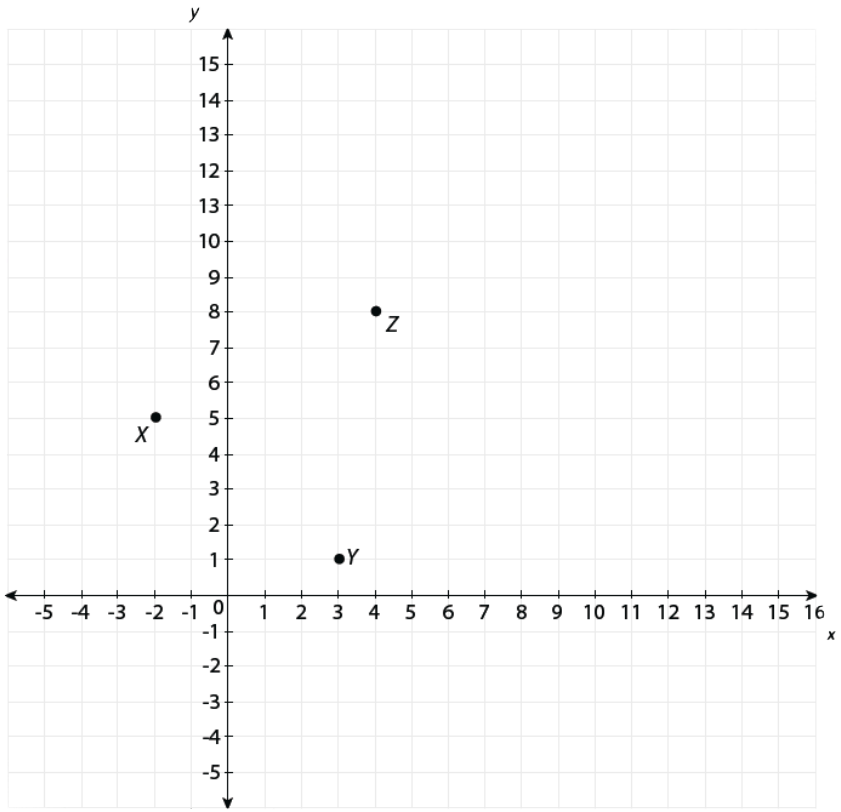


CHALLENGE!!

The midpoints of a triangle are X (–2, 5), Y (3, 1), and Z (4, 8). Find the coordinates of the vertices of the triangle.

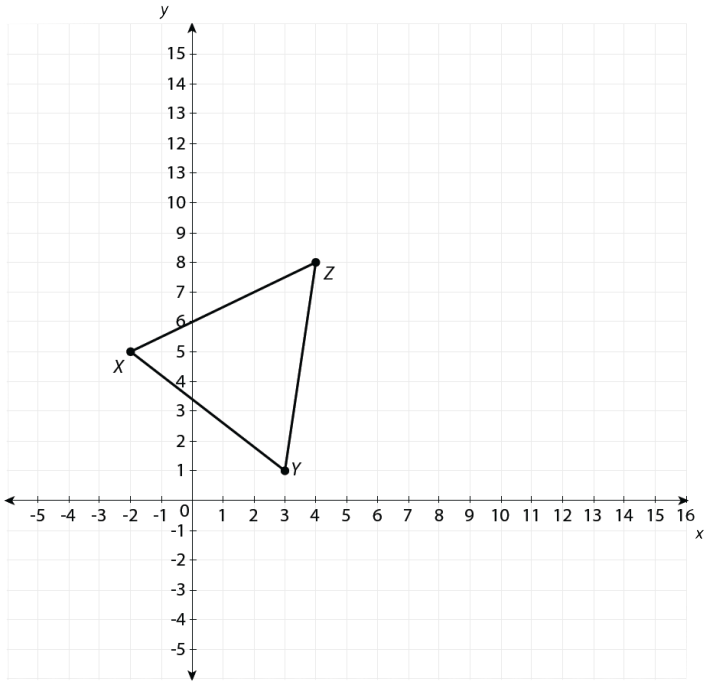


1. Plot the midpoints on a coordinate plane.



Guided Practice: Example 3, *continued*

2. Connect the midpoints to form the midsegments $\overline{XY}$, $\overline{YZ}$, and $\overline{XZ}$.



3. Calculate the slope of each midsegment.

Calculate the slope of $\overline{XY}$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope formula

$$m = \frac{(1) - (5)}{(3) - (-2)}$$ Substitute (–2, 5) and (3, 1) for (x_1, y_1) and (x_2, y_2).

$$m = -\frac{4}{5}$$ Simplify.

The slope of $\overline{XY}$ is $-\frac{4}{5}$.

Calculate the slope of $\overline{YZ}$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope formula

$$m = \frac{(8) - (1)}{(4) - (3)}$$ Substitute (3, 1) and (4, 8) for (x_1, y_1) and (x_2, y_2).

$$m = \frac{7}{1} = 7$$ Simplify.

The slope of $\overline{YZ}$ is 7.

Calculate the slope of $\overline{XZ}$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ Slope formula

$$m = \frac{(8) - (5)}{(4) - (-2)}$$ Substitute (–2, 5) and (4, 8) for (x_1, y_1) and (x_2, y_2).

$$m = \frac{3}{6} = \frac{1}{2}$$ Simplify.

The slope of $\overline{XZ}$ is $\frac{1}{2}$.

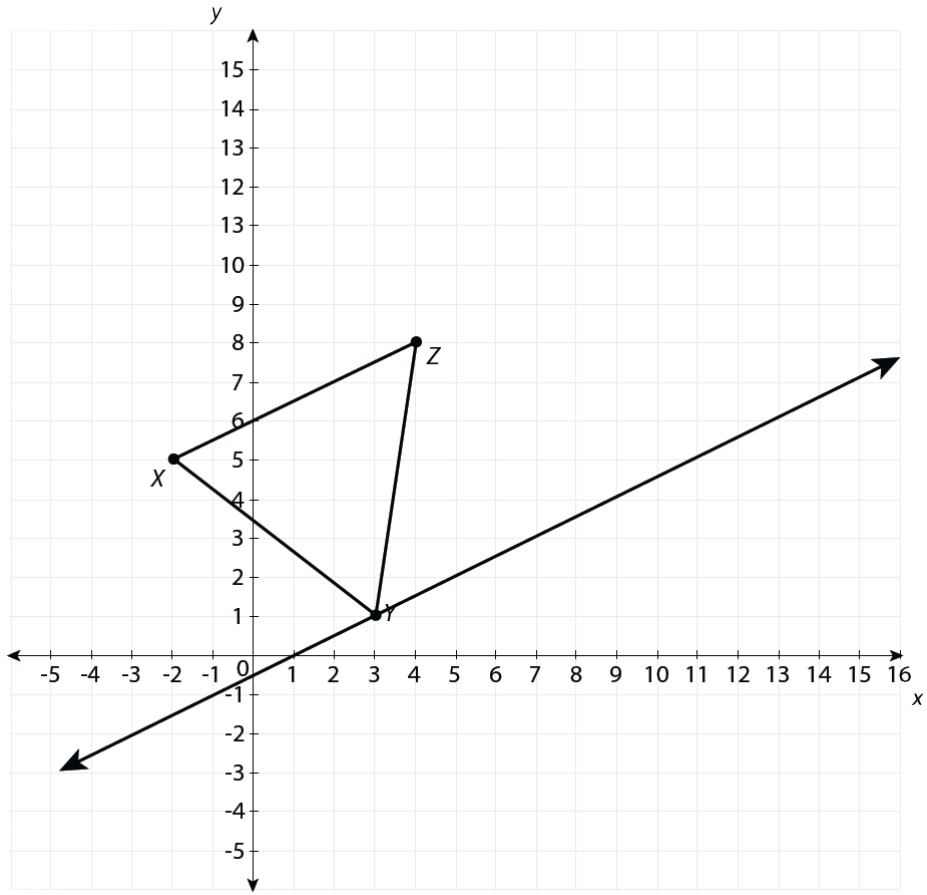
4. Draw the lines that contain the midpoints.

The endpoints of each midsegment are the midpoints of the larger triangle.

Each midsegment is also parallel to the opposite side.

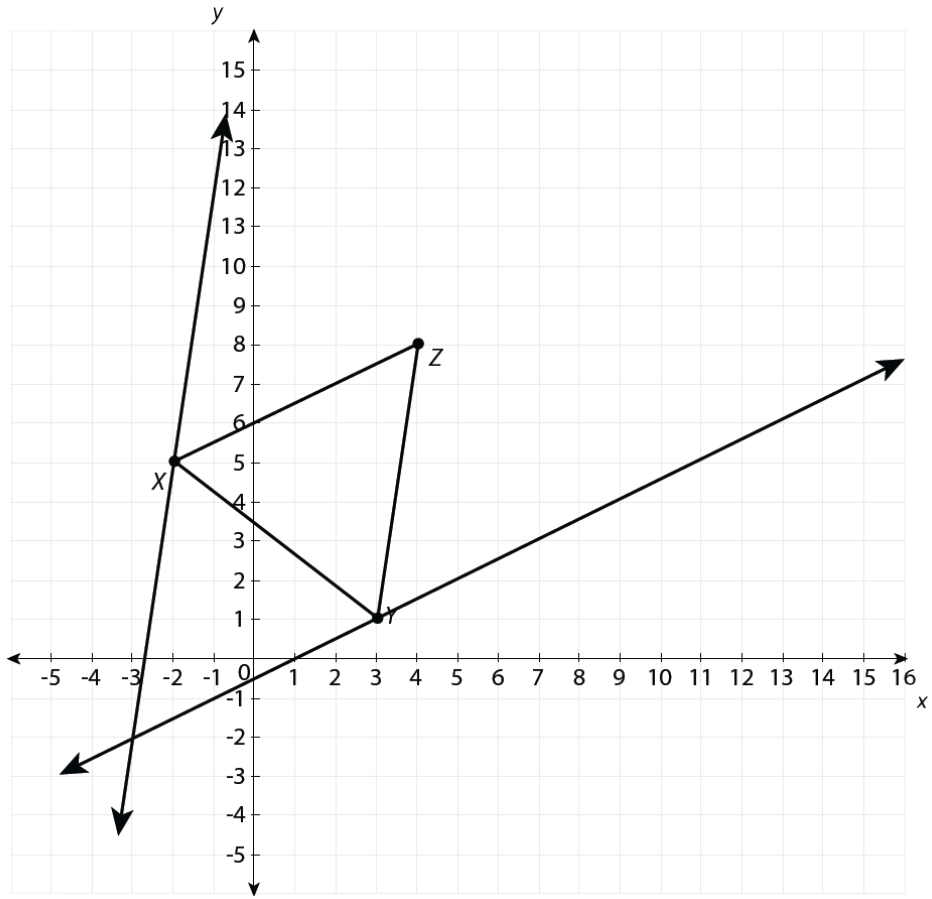
The slope of $\overline{XZ}$ is $\frac{1}{2}$.

From point *Y*, draw a line that has a slope of $\frac{1}{2}$.



The slope of $\overline{YZ}$ is 7.

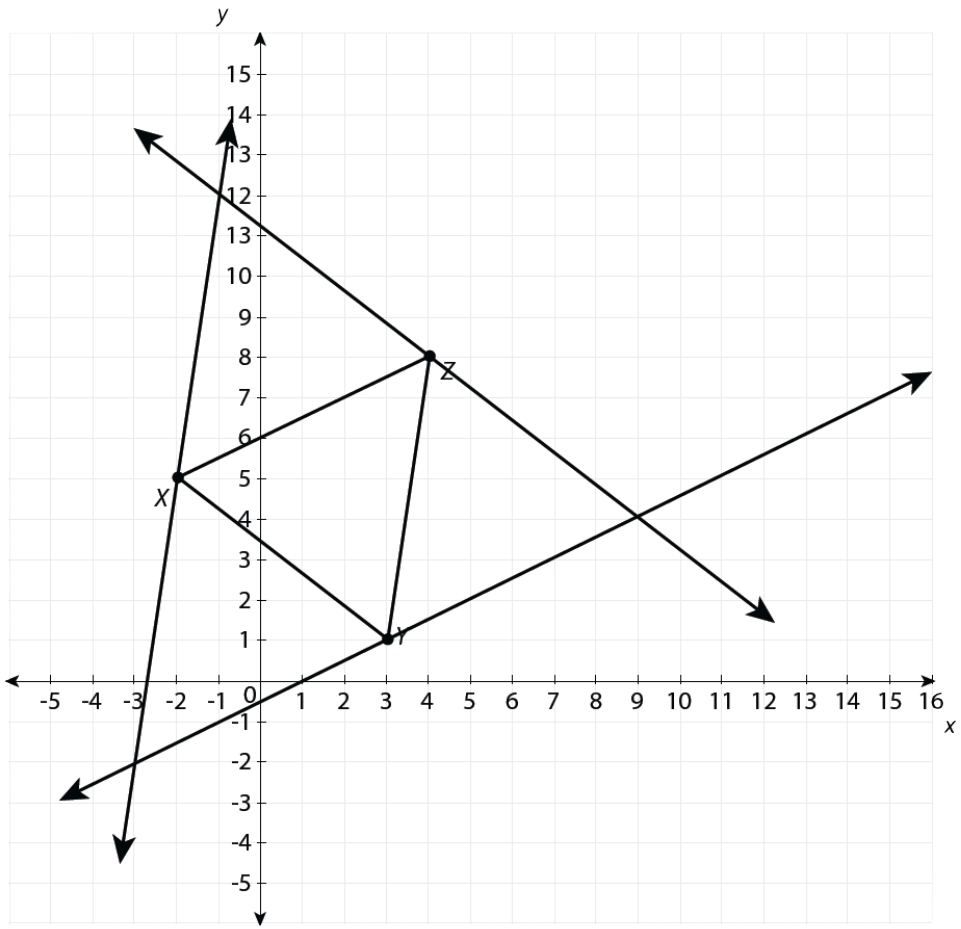
From point *X*, draw a line that has a slope of 7.



The slope of $\overline{XY}$ is $-\frac{4}{5}$.

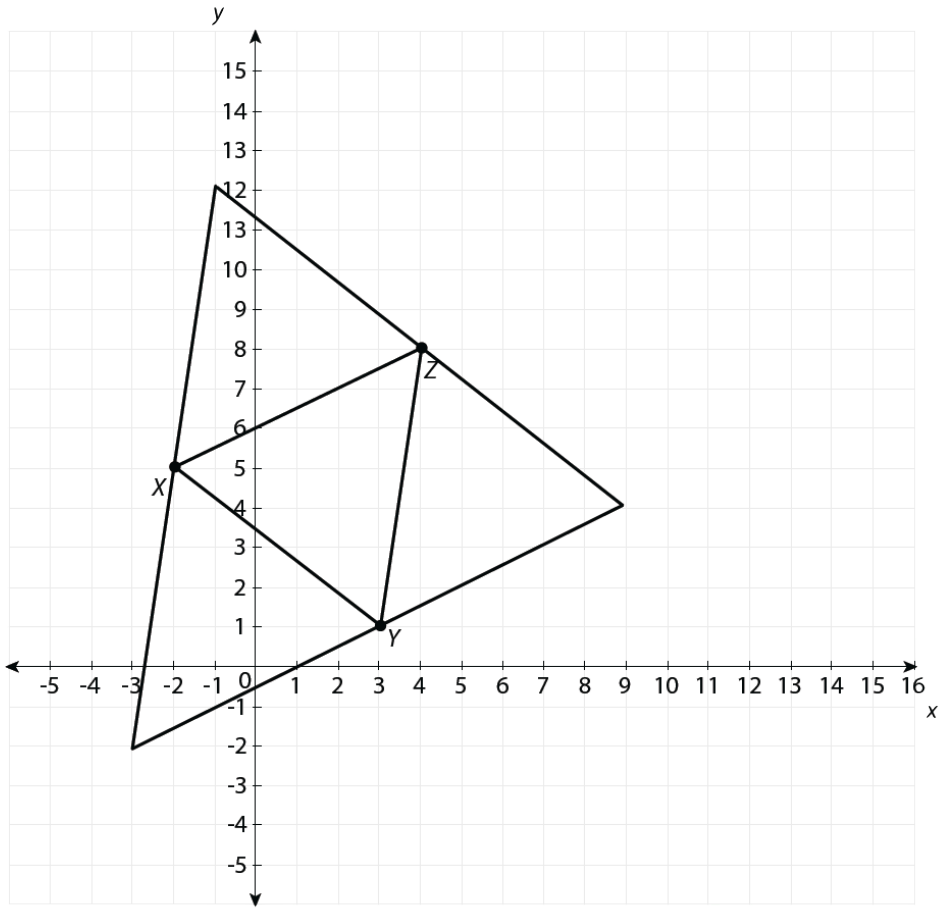
From point Z, draw a line that has a slope of $-\frac{4}{5}$.

The intersections of the lines form the vertices of the triangle.



5. Determine the vertices of the triangle.

The vertices of the triangle are (–3, –2), (9, 4), and (–1, 12), as shown on the following slide.



8.4 Classwork
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- **GO ONLINE and complete 8.4 HW**
- **Alternative:**
  Honors: 2, 4-6, 8, 12-13, 17, 20, 22, 24
- **Regular**: **2, 4, 6, 8, 11, 20, 22**

Reminders:

❑ …