LESSON 8 Arithmetic Sequences and Boiling Water

LEARNING OBJECTIVES

- Today I am: deciding if data about boiling water at different elevation is linear or not.
- So that I can: write arithmetic sequences.
- I'll know I have it when I can: write the general term of any arithmetic sequence if I'm given the starting term and the common difference.

Opening Exploration

Source: Illustrative Mathematics

1. Below is a table showing the approximate boiling point of water at different elevations:

Elevation (meters above sea level)	Boiling Point (degrees Celsius)
0	100
500	98.2
1000	96.5
1500	94.7
2000	93.1
2500	91.3

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A. Based on the table, if we were to model the relationship between the elevation and the boiling point of water, would a linear function be appropriate? Explain why or why not.

B. Below are some additional values for the boiling point of water at higher elevations:

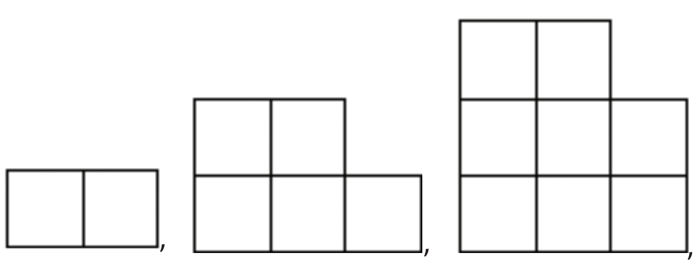
Elevation (meters above sea level)	Boiling Point (degrees Celsius)
5000	83.2
6000	80.3
7000	77.2
8000	74.3
9000	71.5

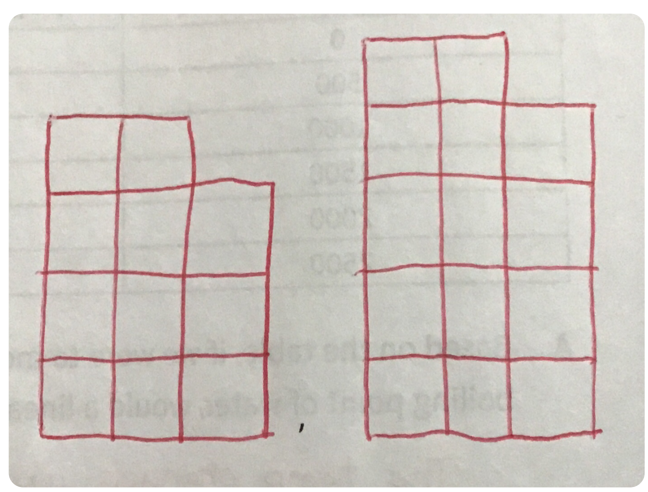
Based on the table, if we were to model the relationship between the elevation and the boiling point of water, would a linear function be appropriate? Explain why or why not.

C. When the information from both tables is combined, would a linear function be appropriate to model this data? What kind of function would you use to model the data? Why?

The data from the Opening Exploration doesn't have **exactly** the same difference of temperature for each 1000 meter rise in elevation. But it is **close** to being a consistent change of −3 degrees Celsius for each 1000 meter rise in elevation. Real world data is often "messy" and the models you use to describe the data are based on patterns that display a very clear rule. Let's look at other sequence patterns that show a linear relationship.

2. Draw the next two terms of the following sequence:





2 5 8 11 14

3. How could you describe this pattern so that someone else could draw the sixth term? The 10th term?

Add 3 blocks to the base.

4. We can also look at this pattern with numbers. How do the figures relate to the sequence 2, 5, 8, …?

1 2 3 4 5

2, 5, 8, 11, 14, ...

5. Find the next three terms of the sequence: 2, 5, 8, . . .

2, 5, 8, 11, 14, 17, . . .

6. Do the values you got in Exercise 5 correspond to the figures you drew in Exercise 2?

We can continue to add 3 to each term to get the next term, but to answer questions like, "How many squares would be in the 100th term?" is difficult with this type of description. In Lesson 1, we used some explicit formulas and found that explicit formulas make questions about the 100th term much easier to answer. In this lesson, you'll learn to write explicit formulas for *arithmetic* sequences.

7. We'll continue with the same sequence 2, 5, 8, . . . So 2 is term 1 ($n = 1$), 5 is term 2 ($n = 2$), 8 is term 3 ($n = 3$) and so forth. Fill in the rest of the table below.

n	Term	Work	Pattern
1	2	$2 = 2$	$2 + 3(0) = 2$
2	5	$2 + 3 = 5$	$2 + 3(1) = 5$
~~4~~ 3	8	$2 + 3 + 3 = 8$	$2+3(2)$
~~5~~ 4	11	$2+3+3+3=11$	$2+3(3)$
~~6~~ 5	14	$2+3+3+3+3=14$	$2+3(4)$
~~7~~ 6	17	$2+3+3+3+3+3=17$	$2+3(5)$
n			$\underline{2} + 3(\underline{n-1}) = f(n)$

$2+3(n-1)$

$\underline{2}+3n\underline{-3}$

3n − 1

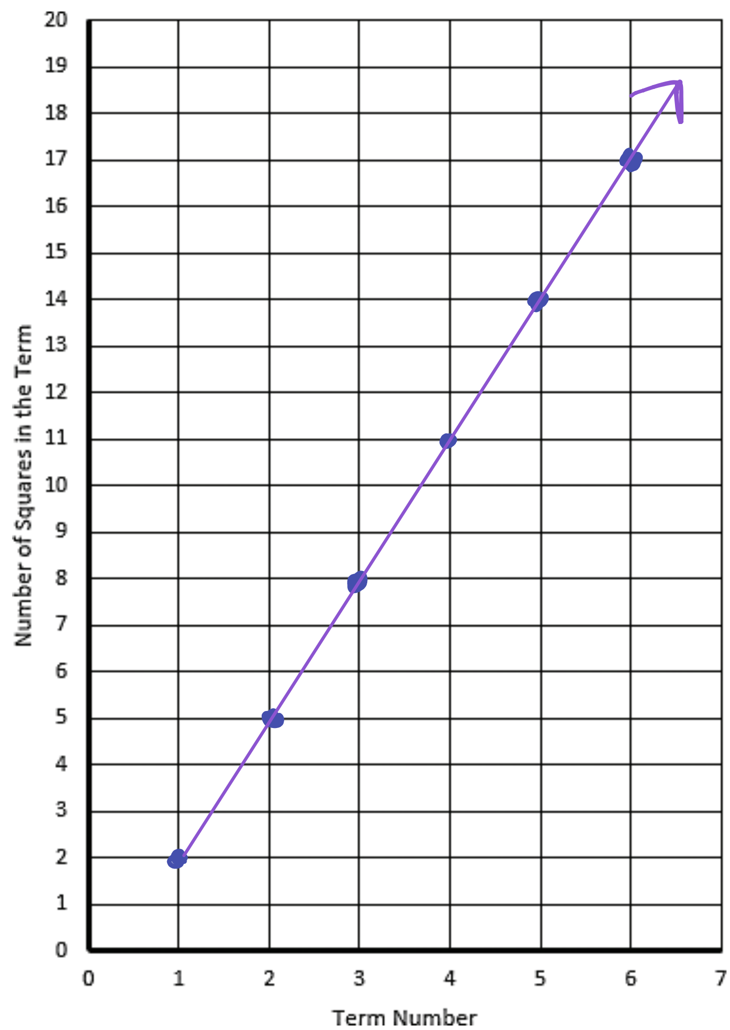
8. A. Graph the data in the chart.

B. What pattern(s) do you notice in the graph?

(1, 2)
(2, 5)
(3, 8)
(4, 11)

linear pattern

3n−1



In an *arithmetic sequence*, each term after the first term differs from the preceding term by a constant amount. The difference between consecutive terms is called the *common difference* of the sequence.

9. The sequence 2, 5, 8, . . . is an arithmetic sequence. What is the common difference of this sequence?

common difference (d) = 3

10. How can you easily find the common difference for an arithmetic sequence?

2, 5, 8, 11, 14, 17, ...

Subtract: (term after) − (term before)

11. If the points in Exercise 8 were connected, what would be the slope of the line formed? How does this relate to the common difference?

slope = common difference.

12. A. What would the "0th" term be? Is there a figure that could be drawn of the 0th term? Explain.

$0^{th} \rightarrow -1$

$f(0) \rightarrow -1$

n	term
0	? -1
1	2
2	5
3	8

B. How can you determine the 15th term? What would it be?

$f(n) = 3n - 1$

$a_n = 3n - 1$

$f(15) = 3(15) - 1 = 44$

$a_1 \rightarrow 1^{st}$ term

$a_2 \rightarrow 2^{nd}$ term

Although we could find the 15th term by adding 3 over and over, it would be easy to make a mistake somewhere in the calculations. And finding the 101st term would be tedious and time-consuming! Instead of continuing to add on 3 each time, we'll write an explicit formula for this sequence.

add / subtract same number

The n^{th} term or the ***general term*** of an arithmetic sequence is given by the ***explicit*** formula

1st term

$$\boxed{f(n) = f(1) + d(n - 1)} \text{ or } a_n = a_1 + d \cdot (n - 1)$$

common diff

where $f(1)$ or a_1 is the first term of the sequence and d is the common difference

$f(1)$ $d = 3$

13. A. Write the explicit formula for the sequence 2, 5, 8, 11, 14, ...

$f(n) = 2 + 3(n - 1)$

$= 2 + 3n - 3$

B. Compare this formula to the pattern in the table in Exercise 7 for the n^{th} term.

$\boxed{f(n) = 3n - 1}$ ← explicit formula

$y = mx + b$

C. Find the 15th term and the 101st term for this sequence.

$a_{101} = 3(101) - 1$

$= 303 - 1$

$a_{101} = 302$

$f(n) = f(1) + d(n-1)$

14. Find the common difference for the arithmetic sequences and then write the explicit formula for each one.

A. 142, 138, 134, 130, 126, . . .

$f(1) = 142$

$d = -4$

$f(n) = 142 - 4(n-1)$
$= 142 - 4n + 4$
$f(n) = -4n + 146$

B. −5, −2, 1, 4, 7, . . .

$a_1 = -5$

$d = 3$

$a_n = -5 + 3(n-1)$
$= -5 + 3n - 3$
$a_n = 3n - 8$

15. Write the first six terms of arithmetic sequence where $f(1) = -8$, $d = 5$.

1st term

$\underline{-8}$, $\underline{-3}$, $\underline{2}$, $\underline{7}$, $\underline{12}$, $\underline{17}$

+5

$f(n) = -8 + 5(n-1)$
$f(n) = -8 + 5n - 5$
$f(n) = 5n - 13$

Boiling Water Revisited

16. Let's look at the boiling water again.

A. Graph the data.

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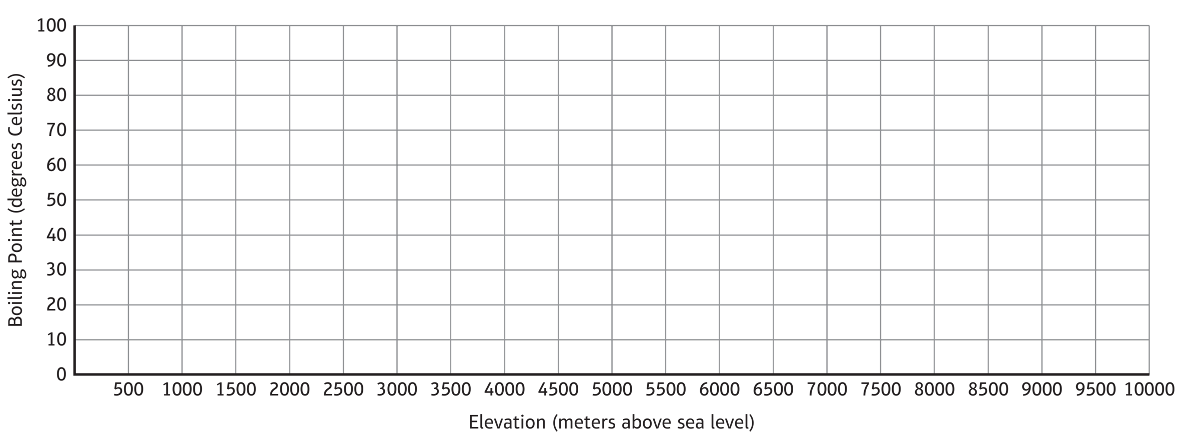
Elevation (meters above sea level)	0	500	1000	1500	2000	2500	5000	6000	7000	8000	9000
Boiling Point (degrees Celsius)	100	98.2	96.5	94.7	93.1	91.3	83.2	80.3	77.2	74.3	71.5



B. Does your graph agree with your predictions in Exercise 1?

C. Write an equation of a linear function that would model this data best. Then draw the graph of your function on the grid.

D. Use your model to predict the boiling temperature of water at 4000 feet, 7500 feet and 10,000 feet.

E. What are the problems with this model? For what values do you think this model doesn't make sense?

Lesson Summary

Arithmetic Sequences

The n^{th} term (the ***general term*** or ***explicit formula***) of an arithmetic sequence with the first term $f(1)$ or a_1 and common difference d is

$$f(n) = f(1) + d(n - 1) \longleftarrow \text{Function notation}$$

or

$$a_n = a_1 + d \cdot (n - 1) \longleftarrow \text{“}a\text{” sub notation}$$

NAME: ______________________ PERIOD: __________ DATE: __________

Homework Problem Set

1. Write a formula for the n^{th} term of the arithmetic sequence 1, 5, 9, 13, ... Then use the formula to find $f(20)$.

2. Find the $f(8)$ of the arithmetic sequence when $f(1) = 4$ and whose common difference is -7.

3. Daniel gets a job with a starting salary of \$70,000 per year with an annual raise of \$3,000. What will Daniel's salary be in the 10^{th} year? Write an explicit formula and then solve.

4. Save-A-Lot Theater ticket prices were originally \$1 each. Prices have risen by 50 cents each year since. What is the price of a ticket 8 years later? Write an explicit formula and then solve.

5. The graph shows how the cost of a snowboarding trip depends on the number of boarders.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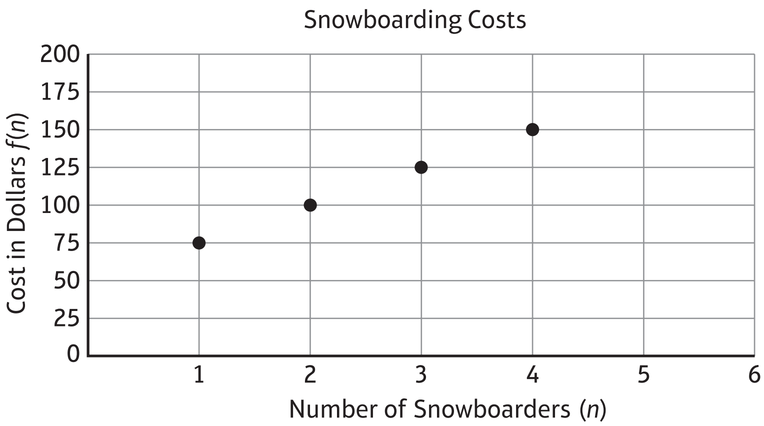
A. Fill in the chart of the data.



n	$f(n)$
1	
2	
3	
4	

B. Write the explicit rule.

C. Draw a line connecting the data points. What is the y-intercept? What is the slope? Write an equation of the line in slope-intercept form.

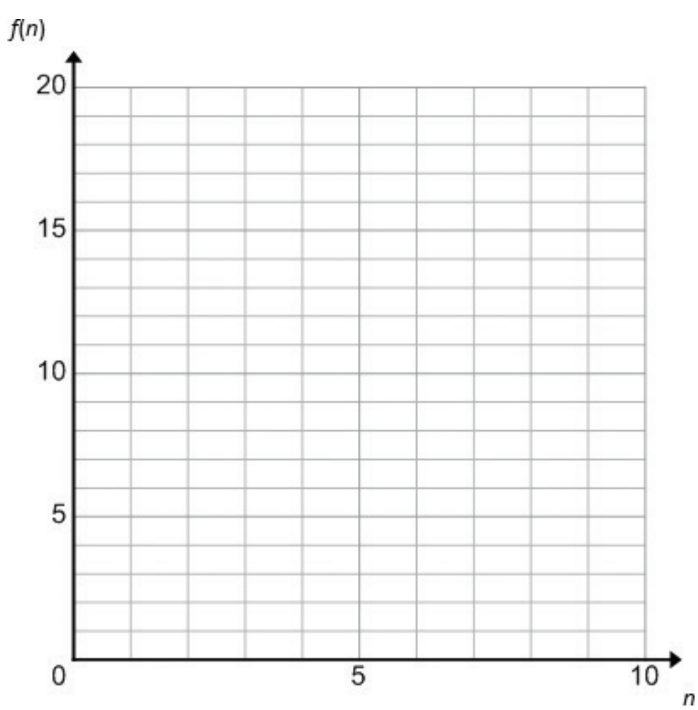
D. What do you notice about the answers in Parts B and C? Explain (use m and d in your response).

6. Consider the sequence that follows a plus 3 pattern: 4, 7, 10, 13, 16, ...

   A. Write a formula for the sequence using the $f(n)$ notation.

   B. Does the formula $f(n) = 3(n - 1) + 4$ generate the same sequence? Why might some people prefer this formula?

   C. Graph the terms of the sequence as ordered pairs $(n, f(n))$ on the coordinate plane. What do you notice about the graph?

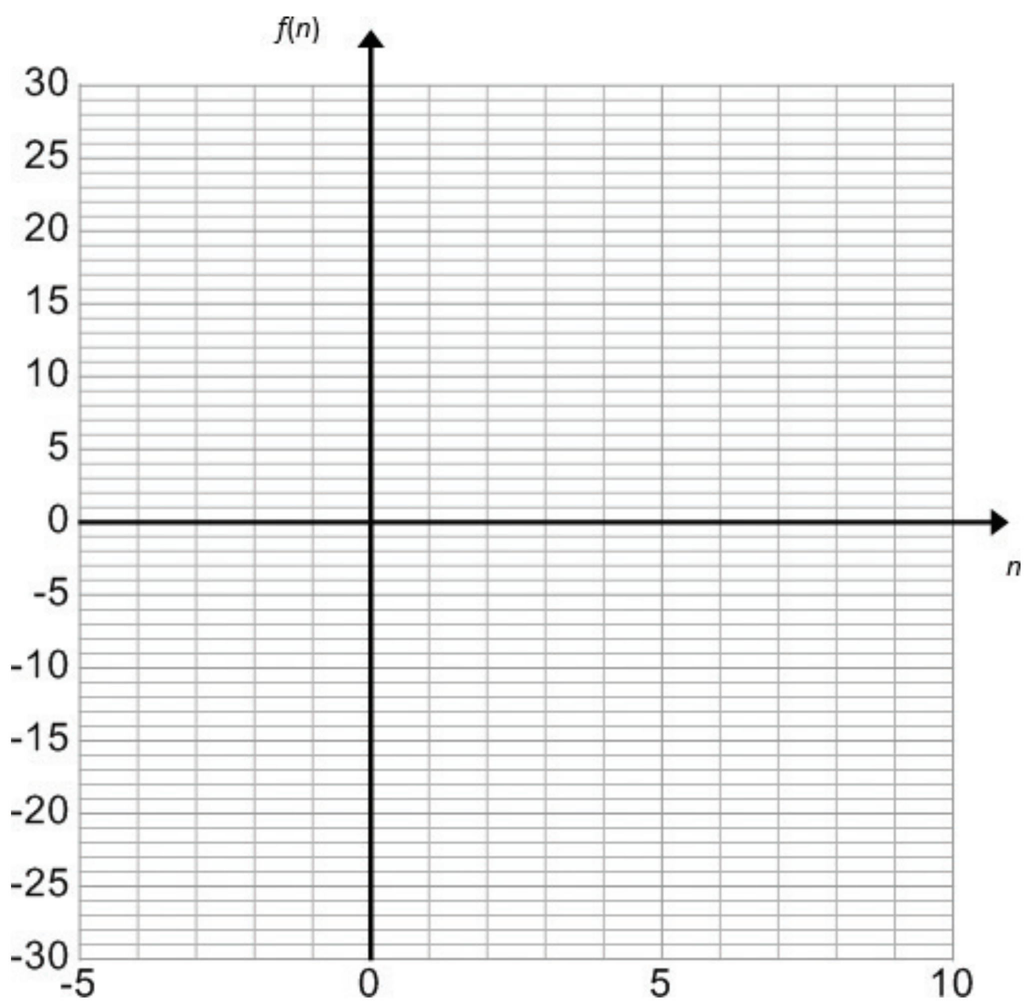


7. Consider a sequence that follows a minus 5 pattern: 30, 25, 20, 15, ...

   A. Write a formula for the n^{th} term of the sequence. Be sure to specify what value of n your formula starts with.

   B. Using the formula, find the 20^{th} term of the sequence.

   C. Graph the terms of the sequence as ordered pairs (n, $f(n)$) on the coordinate plane.



Challenge Problems 8–16

Find an explicit form $f(n)$ for each of the following arithmetic sequences (assume a is some real number and x is some real number).

8. $-34, -22, -10, 2, \dots$

9. $\frac{1}{5}, \frac{1}{10}, 0, -\frac{1}{10}, \dots$

10. $x + 4, x + 8, x + 12, x + 16, \dots$

11. $a, 2a + 1, 3a + 2, 4a + 3, \dots$

12. Consider the arithmetic sequence 13, 24, 35, ….

    A. Find an explicit form for the sequence in terms of n.

    B. Find the 40^{th} term.

    C. If the n^{th} term is 299, find the value of n.

13. If $-2, a, b, c, 14$ forms an arithmetic sequence, find the values of a, b, and c.

14. $3 + x, 9 + 3x, 13 + 4x, \ldots$ is an arithmetic sequence for some real number x.

    A. Find the value of x.

    B. Find the 10^{th} term of the sequence.

15. Find an explicit form $f(n)$ of the arithmetic sequence where the 2^{nd} term is 25 and the sum of the 3^{rd} term and 4^{th} term is 86.

16. In the right triangle figure below, the lengths of the sides a cm, b cm, and c cm of the right triangle form a finite arithmetic sequence. If the perimeter of the triangle is 18 cm, find the values of a, b, and c.