Getting' Triggy With It

Date: 15 May 2013
Topic: Pythagorean Theorem and Trigonometric Ratios
Class: Grade 9
Ability Level: Mixed Ability
Teacher: Mr. Cyrus Alvarez

LESSON OBJECTIVES:

At the end of the lesson, the students should be able to:

1. Explain the characteristics of a right triangle and correctly identify its parts;
2. Explain the relationships between the different parts of the triangle;
3. Understand and correctly apply the Pythagorean Theorem and the three primary trigonometric ratios in solving right triangles.
4. Investigate applications of the Pythagorean Theorem and the three primary trigonometric ratios in real-life situations.

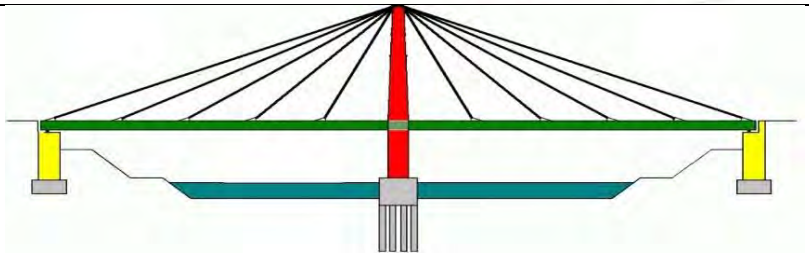
PRE-REQUISITES:

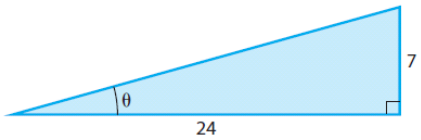
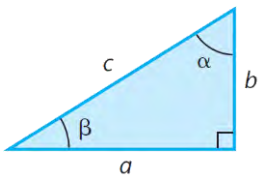
1. Students have mastery over the concepts of angles, measurements, ratio and proportion, exponents, radicals, and solutions to algebraic equations.
2. Students understand the basic properties of triangles, such as the sum of its interior angles, triangle inequality, similarity and congruence, etc.
3. Students should be familiar with the use of scientific calculators or similar technologies.

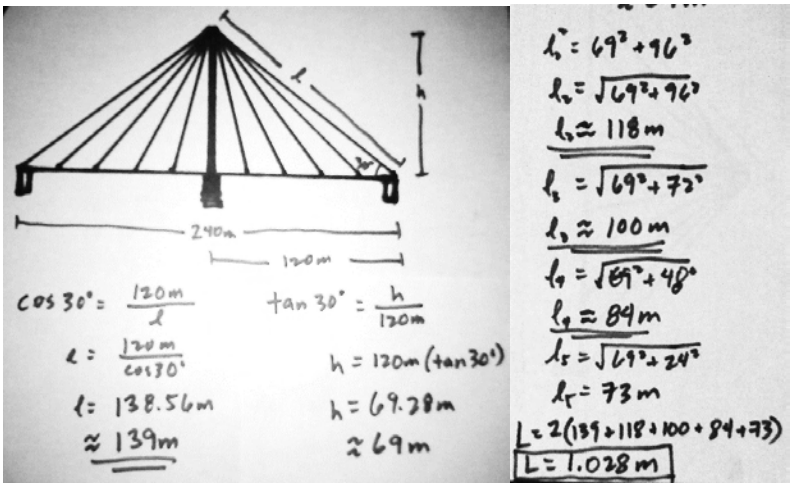
FOCUS:

Investigative activity
Cooperative Learning
Problem Solving

ACTIVITY	RESOURCES/MATERIALS
The teacher uses the following situation as an activation strategy: You are an engineer commissioned to design a fan-type cable-stayed foot bridge with one tower in the middle and 5 symmetric pairs of wire cables attached on top of the tower. The bridge is supposed to cover a length of 240 meters and the guy wires are supposed to be attached on the bridge at equal intervals. If the longest pair of wire cable has to be attached at a 30˚ angle from the ends of the bridge deck, how tall should the tower be? What is the total length of cable wires needed for the bridge?	 Source: http://www.qudamaa.com/vb/f95/most-famous-bridges-world-30774/

 The teacher explains that the tower and the cable wire create a right triangle with the bridge deck, thus the properties of right triangles may be used to solve the problems presented in the situation.	A cable-stayed bridge has one or more towers where support wire cables are attached from the bridge deck. A fan-type cable stayed bridge is one where the wire cables are connected to or passed over the top tower towards several points on the bridge deck.
The teacher asks about ideas regarding right triangles. Establish understanding that right triangles are triangles with one angle measuring 90°, and the two other angles are acute and complementary.	
The teacher builds the students' vocabulary with the following parts of a right triangle: • Hypotenuse – the longest side of a right triangle, which is opposite the right angle of a right triangle. • Legs – the two shorter sides of a right triangle. • Complementary Angles – angles whose measures add up to 90°. The two acute angles of a right triangle are complementary. • Adjacent Side – the leg of a right triangle that forms the angle with the hypotenuse. • Opposite Side – the leg of a right triangle that is not part of the angle, but is opposite the angle. •	
The teacher divides students into groups of 3-4 students and provides the Right Triangles Worksheetfor each group to • Establish understanding of the Pythagorean Theorem as the relationship of the length of the hypotenuse with the lengths of the legs ($a^2 + b^2 = c^2$, where a and b are the legs and c is the hypotenuse). • Establish understanding that in similar right triangles (i.e, right triangles whose angles have the same measures even if the sides do not have the same lengths), the ratio between two sides will always be the same. The teacher gives 10-15 minutes for the groups to work on the worksheet. The teacher moves around to check if: 1) the students are on track 2) all members are doing the said task	Worksheet 1 (attached) Materials: paper, scissors, protractor and ruler Technological requirement:Scientific calculator, Geogebra*, laptop*, digital camera*, and LCD projector* * if available

3) if technology is available, the teacher may take pictures of the students responses for easy presentation The teacher calls on the group to share their findings/answers to the worksheet questions. Possible answers to worksheet questions: 1.1. The hypotenuse decreases as well. 1.2. The ratios remain the same. 1.3. The measures of the acute angles of a right triangle depend on the ratios of its sides. 2.1. They are equal. 2.2. The sum of the squares of the legs is equal to the square of the hypotenuse. (Alternatively, if resources are available, the teacher may use Worksheet 1A, which utilizes Geogebra for the activity.)	
The teacher defines the three primary trigonometric ratios. • $\sin\theta = \frac{\text{Opposite Side}}{\text{Hypotenuse}}$ • $\cos\theta = \frac{\text{Adjacent Side}}{\text{Hypotenuse}}$ • $\tan\theta = \frac{\text{Opposite Side}}{\text{Adjacent Side}}$ • Provide mnemonic for easier recall: SOH-CAH-TOA	
The teacher defines the Pythagorean Theorem. • $\boldsymbol{a^2 + b^2 = c^2}$, where $\boldsymbol{a}$ and $\boldsymbol{b}$ are legs, and $\boldsymbol{c}$ is the hypotenuse	
The teacher provides sample problems: Given:  Find: (a) The hypotenuse (25) (b) sin θ $\left(\frac{7}{25}\right)$ (c) cos θ $\left(\frac{24}{25}\right)$ (d) tan θ $\left(\frac{7}{24}\right)$ Using the figure and the quantities given, find the other quantities: (a) β = 17.8°, *c* = 3.45 (α = 72.2°, *a* = 3.28, *b* = 1.05) (b) α = 23°, *a* = 54.0 (β = 67°, *b* = 127.2, *c* = 138.2) (c) β = 43°20', *a* =123 (α = 46°40', *b* = 116, *c* = 169) (d) α = 53.21°, *b* = 23.82 (β = 36°39', *a* = 32.02, *c* = 39.90)  	Source: Barnett, R. et al, 2008 Technological requirement: Scientific calculator

The teacher goes back to the student groups and instructs them to come up with solutions on the original problem posted at the beginning of the lesson. The Fan-Type Cable-Stayed Bridge Worksheet is distributed among the groups as guide. Solutions, complete with illustrations and calculations, are to be written on a piece of cartolina for presentation. The teacher moves around to check if: 1) the students are on track 2) all members are doing the said task 3) if technology is available, the teacher may take pictures of the students responses for easy presentation The teacher gives 10-15 minutes for the groups to work on the worksheet. The teacher calls on the group to share their solutions. Sample illustration and calculation: 	Worksheet 2 (attached) Materials: Ruler, protractor, white cartolina and markers Technological Requirement: Scientific calculator, laptop*, digital camera*, and LCD projector* * if available
The teacher summarizes lesson by playing the “Gettin’ Triggy With It” music video to emphasize the following points: 1. A right triangle is a triangle with one angle measuring 90° and the other two angles are acute and complementary. 2. The sides of a right triangle are related to each other via the Pythagorean Theorem. 3. The acute angles of a right triangle are dependent on the ratios of its sides. 4. These ratios are called trigonometric ratios. The three primary trigonometric ratios are sine, cosine and tangent.The sine of an angle is the ratio of the opposite side and the hypotenuse; the cosine of an angle is the	Getting’ Triggy With It music video http://www.youtube.com/watch?v=t2uPYYLH4Zo

ratio of the adjacent side and the hypotenuse; while the tangent of an angle is the ratio of the opposite side and the adjacent side. 5. In a right triangle, if at least two quantities are known, the other quantities may be derived using Pythagorean Theorem and/or the trigonometric ratios.	
The teacher gives the following homework to the students: 1. Journal/blog entry on their journals/CMS pages about what they have learned from the session and their insights on how this new knowledge can be applied in their lives. Students should also include answers to the following questions: a. What specific topics and/or skills helped you accomplish the activities? b. What made the tasks challenging? What are the difficulties that you encountered while doing the tasks? c. How can you improve on the difficulties you encountered? 2. Pair up students to work together on the Applications of Right Triangles Worksheet. 3. Divide students into groups of 5 and assign each group to come up with and film a unique music video that deals with the concepts discussed in this lesson.	Worksheet 3 (attached)

Evaluation/Assessment: Worksheet and class participation, both will be assessed using the Classwork/Participation Rubric and the Peer Evaluation Rubric (for group work) generated from iRubric – https://www.iRubric.com/

References:

Aufmann, Richard N., Barker, Vernon C. and Nation, Richard D. (2011). *College Algebra and Trigonometry, 11th ed.*
Barnett, Raymond A. et al (2008). *College Algebra with Trigonometry, 9th ed.*
Larson, Ron (2012). *Algebra and Trigonometry: Real Mathematics, Real People, 6th ed.*
McKeague, Charles P. and Turner, Mark D. (2008). *Trigonometry, 7th ed*.
Sullivan, Michael and Sullivan, Michael III (2009). *Algebra & Trigonometry, 6th ed.*

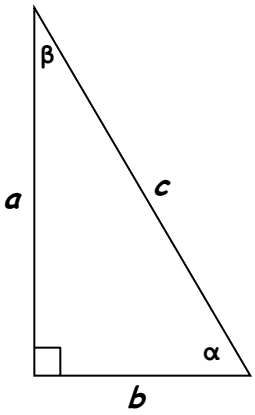
Worksheet 1: Right Triangles

Group No.: _____ **Date:** _____

Members: _____

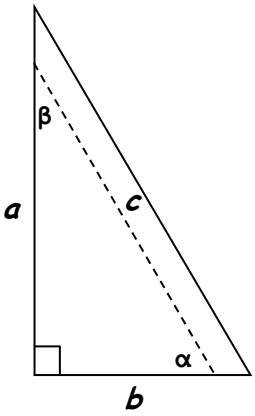
_____

Materials: paper, scissors, protractor, ruler, and scientific calculator



Procedure:

1. Cut a right triangle of any size from the piece of paper.
2. Identify the hypotenuse of the right triangle as side ***c***, and the legs as sides ***a*** and ***b***, respectively.
3. Measure the sides of the right triangle using the ruler and record your measurements on Table 1. Use centimeters as your unit of measurement.
4. Label the angle opposite leg ***a*** as **α** and the angle opposite leg ***b*** as **β.**



5. Measure the angles using the protractor and record your measurements on Table 1.
6. Cut the right triangle through a line parallel to one side. Make sure that angles **α** and **β** remains the same.
7. Measure the sides of the new triangle and record the measurements on Table 1.
8. Repeat (6) and (7).
9. Compute for the ratios as indicated in Table 1.
10. Answer the questions that follow.
11. Using the values in Table 1, compute for the values as indicated in Table 2.
12. Answer the questions that follow.

Table 1

	a	***b***	***c***	**α**	**β**	***a/c***	***b/c***	***a/b***
Triangle 1								
Triangle 2								
Triangle 3								

Questions based on Table 1:

1. When the lengths of the legs of the triangle are decreased, what happens to the length of the hypotenuse?

   _____
   _____

2. What do you notice about the ratio of the length of leg ***a*** to the hypotenuse ***c*** in the three triangles measured, given that the angles remain the same? How about ***b*** and ***c***? How about ***a*** and ***b***?

   _____
   _____

3. What conclusions can you draw regarding the ratios of the sides of a right triangle relative to its angles?

_____
_____

Table 2

	a^2	b^2	$a^2 + b^2$	c^2
Triangle 1				
Triangle 2				
Triangle 3				

Questions based on Table 2:

1. What do you notice about the sum of the squares of the legs of the right triangles in relation to the square of its hypotenuse?

_____
_____

2. What conclusion can you draw regarding the relationship between the lengths of the legs of a right triangle with the length of its hypotenuse?

_____
_____
_____

Worksheet 1A: Right Triangles

Group No.: _____ **Date**: _____

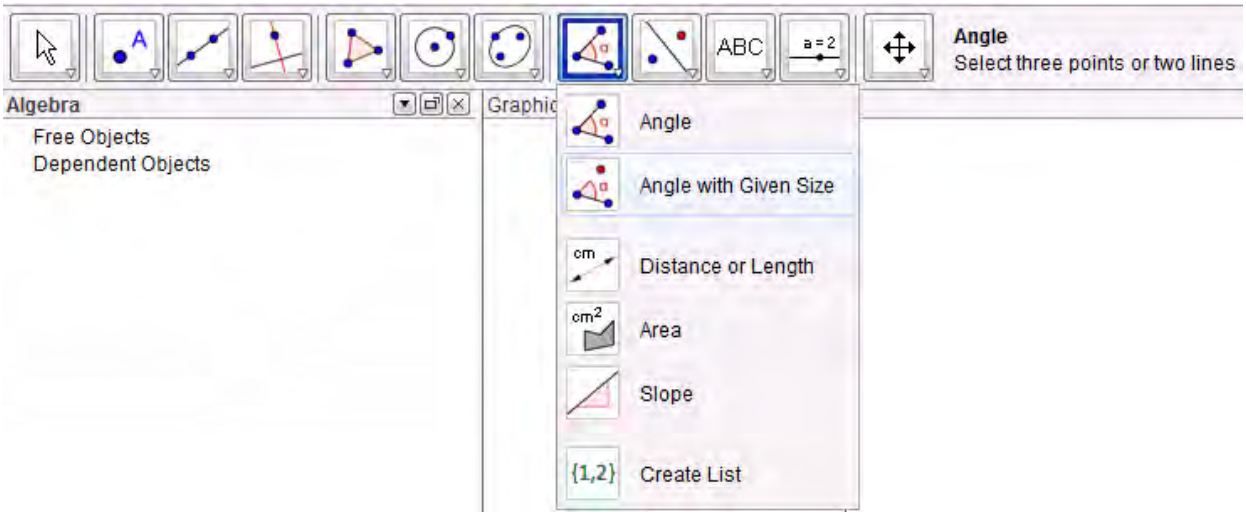
Members: _____

_____

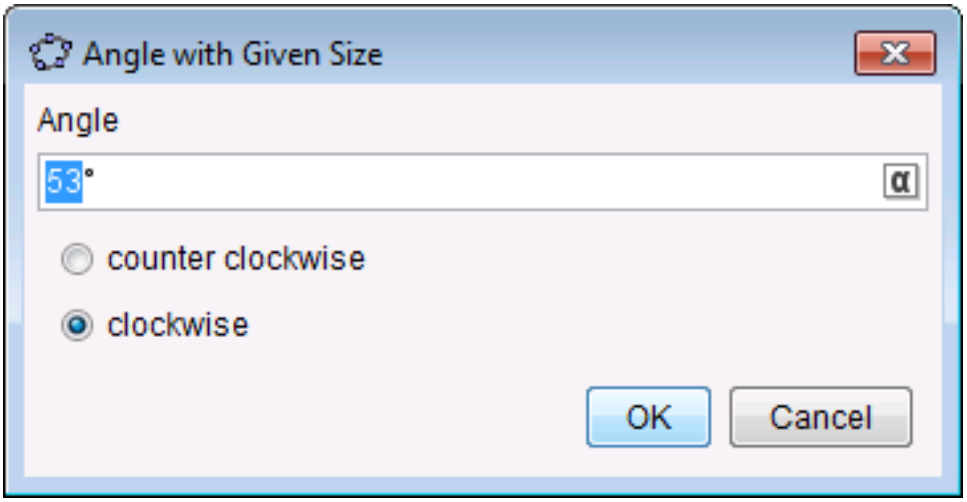
Technological Requirements:
Software: Geogebra (http://www.geogebra.org/)
Hardware: Computer, Scientific Calculator

Procedure:

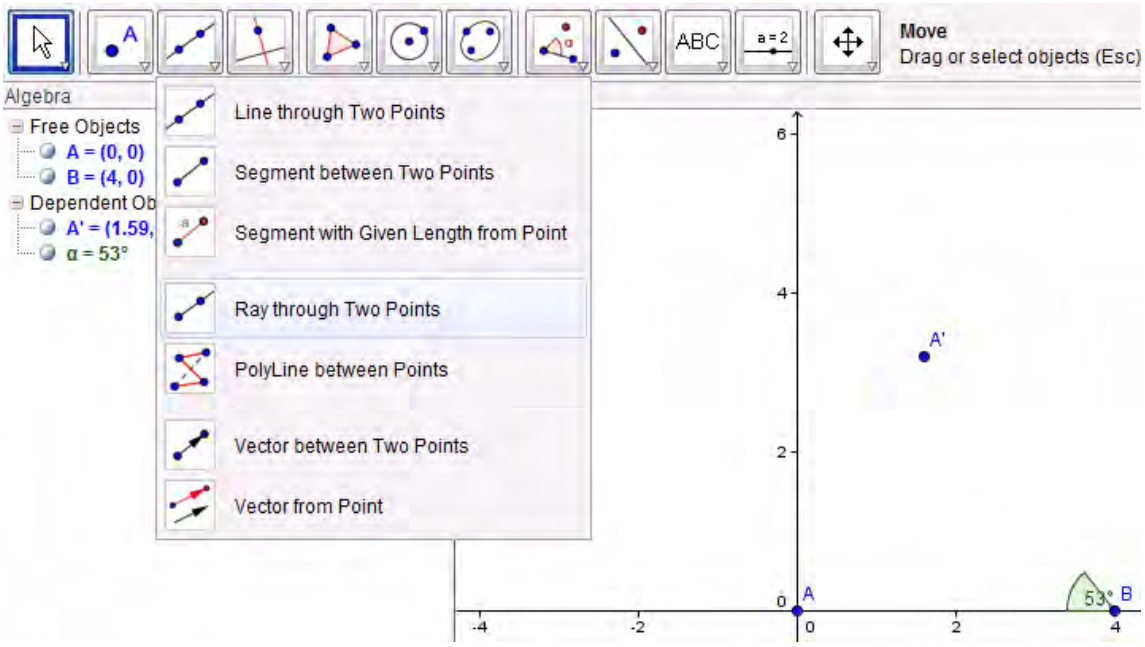
1. Click on Angle with Given Size. Click on the origin, then click on any point on the x-axis.



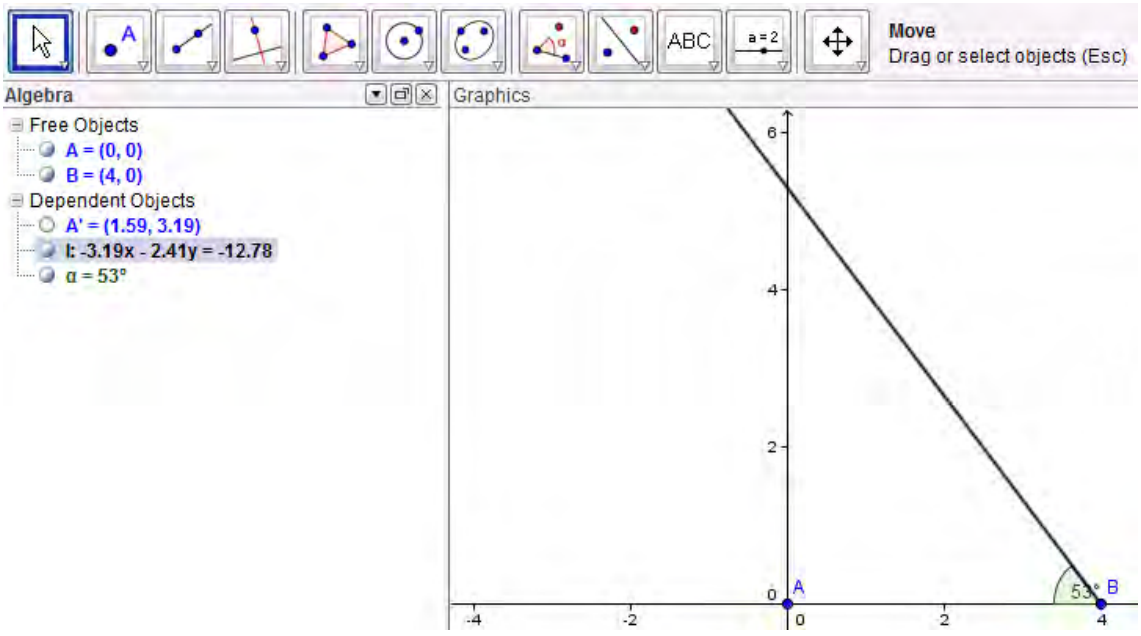
Provide any angle between 0° and 90°, and choose clockwise.



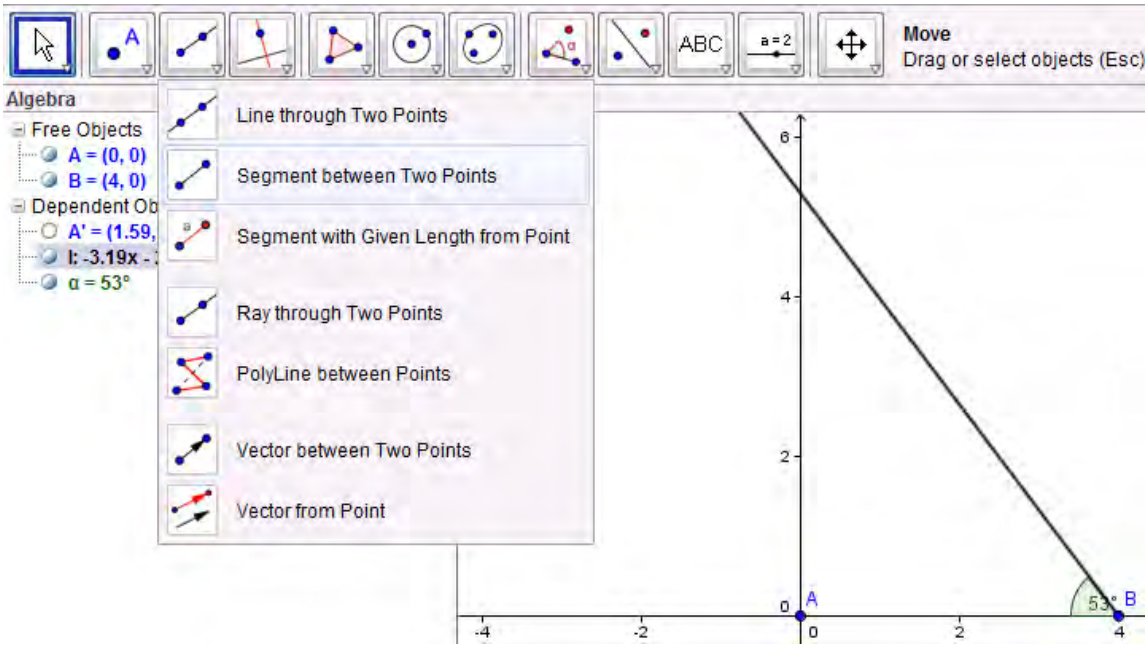
2. Define a ray going from point B to point A'.



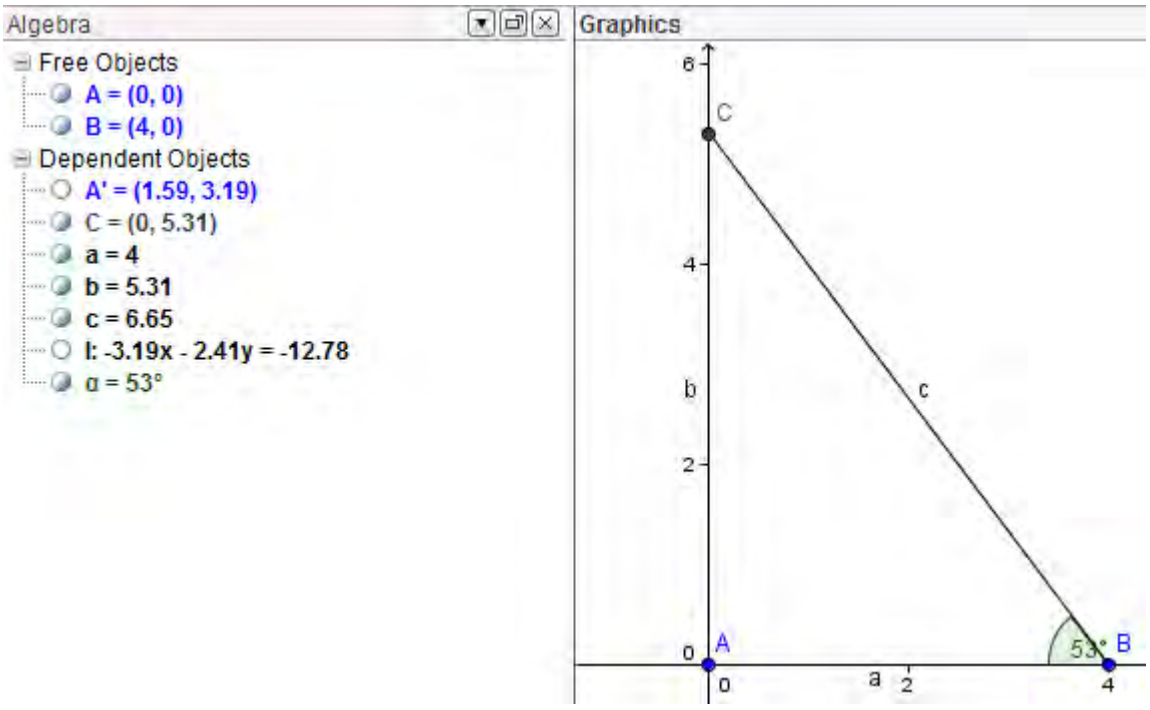
Label the ray as *l*. Hide the point A'.



3. Click on Segment between Two Points.



Click on Point A, then on Point B. Click again on Point A and then click on the intersection between the y-axis and the ray *l*. Finally, click on Point B and then click on the intersection between the y-axis and the ray *l*. Show the labels on these 3 line segments. Hide the ray *l*.



4. Record the values of **a**, **b** and **c** on the table below, as well as the measure of **α**. Then, compute for **β** = 90° – **α**.
5. Drag the Point B on any positive point on the x-axis. Record the values of **a**, **b** and **c** on the table. Do this again three times so that you will have 5 sets of data.
6. Compute for the ratios indicated on Table 1 (i.e., $\frac{a}{c}$, $\frac{b}{c}$, and $\frac{a}{b}$).
7. Compute for the quantities indicated on Table 2 as well.
8. Answer the questions that follow.

Table 1

	a	***b***	***c***	**α**	**β**	***a/c***	***b/c***	***a/b***
Triangle 1								
Triangle 2								
Triangle 3								
Triangle 4								
Triangle 5								

Questions based on Table 1:

1. When the lengths of the legs of the triangle are decreased, what happens to the length of the hypotenuse?

   _____

2. What do you notice about the ratio of the length of leg ***a*** to the hypotenuse ***c*** in the three triangles measured, given that the angles remain the same? How about ***b*** and ***c***? How about ***a*** and ***b***?

   _____

   _____

3. What conclusions can you draw regarding the measures of the acute angles of a right triangle in relation to its sides?

   _____

   _____

Table 2

	a^2	b^2	$a^2 + b^2$	c^2
Triangle 1				
Triangle 2				
Triangle 3				
Triangle 4				
Triangle 5				

Questions based on Table 2:

1. What do you notice about the sum of the squares of the legs of the right triangles in relation to the square of its hypotenuse? _____

   _____

2. What conclusion can you draw regarding the relationship between the lengths of the legs of a right triangle with the length of its hypotenuse? _____

   _____

   _____

Worksheet 2: Fan-Type Cable-Stayed Bridge

Group No.: _____ **Date:** _____

Members: _____

_____

Materials: Ruler, protractor, white cartolina, markers, and scientific calculator

Situation: You are a group of engineers hired to designa fan-type cable-stayed foot bridge. A cable-stayed bridge has one or more towers where support wire cables are attached from the bridge deck. A fan-type cable-stayed bridge is one where the wire cables are connected to or passed over the top tower towards several points on the bridge deck. You are expected to come up with a proposal for your bridge design. Afterwards, you will present your findings to the CEO (your teacher) of the Pythagoras Towers Company.

Problem:

The fan-type cable-stayed foot bridge that you're designing will have one tower in the middle and 5 symmetric pairs of wire cables attached on top of the tower.

The bridge is supposed to cover a length of 240 meters and the guy wires are supposed to be attached on the bridge at equal intervals. If the longest pair of wire cable has to be attached at a 30˚ angle from the ends of bridge deck, how tall should the tower be? What is the total length of cable wires needed for the bridge?

Requirements:

1. Create a proposal for the required lengths of guy wire needed to build the two towers given the specifications. Make sure that you have clear illustrations and calculations to substantiate your proposal.
2. Write your illustrations and complete solutions on the cartolina for presentation.
3. Answer the questions that follow.

Questions

1. Suppose that the contractor wanted to save some money and decided to reduce the number of support cables from 5 pairs to 4 pairs, which will still be all equally spaced throughout the bridge, how much cable wire will be saved?

2. If you can put the tower ANYWHERE on the bridge, where will you put it so that you can have the least amount of cable wires to use, considering that there should be a total of 10 cable wires attached uniformly on the bridge deck and the height of the tower remains the same?

Worksheet 3: Applications of Right Triangles

Names: _____ **Date:** _____

_____

Technological Requirements:
Hardware: Scientific Calculator

Procedure:

1. Write down two real-life situation problems that would involve the use of right triangles to solve. Make sure that your problem is original and not copied from anywhere.

PROBLEM No. 1

_____

_____

_____

_____

_____

_____

_____

_____

_____

PROBLEM No. 2

_____

_____

_____

_____

_____

_____

_____

_____

Names: _____ _____

2. Exchange questions with another pair (Page 1 of this worksheet). DO NOT show them your solutions. Write the names of the pair you exchanged questions with here: _____ _____.

   And write down your solutions to their questions here.

Names: _____ _____

3. Write down the solution for your problems here.

Names: _____ _____

4. Compare your solutions with the other pair's solutions.
5. Answer the following questions:
   a. Did both your groups get the same answers for your problems?

      _____

      a.1. If not, who got it right? _____

      How did someone get the wrong answer?

      _____

      _____

      _____

   b. Did you use the same steps to solve your problem?

      _____

      b.1. If not, which one is the easier or faster solution? Why?

      _____

      _____

      _____

   c. Are there any other possible solutions that may be easier or faster? If so, what is it?

      _____

      _____

      _____

      _____

   d. Are the other pair's problems too easy or too difficult? Why do you say so?

      _____

      _____

      _____

      _____

   e. What are your suggestions to improve the statement of their problem?

      _____

      _____

      _____

      _____