

LESSON
14-1

Rational and Irrational Numbers

Reteach

To write a fraction as a decimal, divide the numerator by the denominator.

A decimal may terminate.

$$\begin{array}{r} \frac{3}{4} = 4 \overline{) 3.00} \\ \underline{0.75} \\ 28 \\ \underline{20} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

A decimal may repeat.

$$\begin{array}{r} \frac{1}{3} = 3 \overline{) 1.00} \\ \underline{0.3} \\ 10 \\ \underline{9} \\ 10 \\ \underline{9} \\ 1 \end{array}$$

Complete to write each fraction as a decimal.

1. $\frac{15}{4} = 4 \overline{) 15.00}$

2. $\frac{5}{6} = 6 \overline{) 5.00}$

3. $\frac{11}{3} = 3 \overline{) 11.00}$

Every positive number has two square roots, one positive and one negative.

Since $5 \times 5 = 25$ and also $-5 \times -5 = 25$, both 5 and -5 are square roots of 25.

$$\sqrt{25} = 5 \text{ and } -\sqrt{25} = -5$$

Every positive number has one cube root.

Since $4 \times 4 \times 4 = 64$, 4 is the cube root of 64.

Find the two square roots for each number.

4. 81

5. 49

6. $\frac{25}{36}$

Find the cube root for each number.

7. 27

8. 125

9. 729

LESSON
14-2

Sets of Real Numbers

Reteach

Numbers can be organized into groups. Each number can be placed into one or more of the groups.

Real numbers include all rational and irrational numbers. All of the numbers that we use in everyday life are real numbers.

- If a real number can be written as a fraction, it is a **rational number**. If it cannot be written as a fraction, it is an **irrational number**.
- If a rational number is a whole number, or the opposite of a whole number, then it is an **integer**.
- If an integer is positive or 0, then it is a **whole number**.

You can use these facts to categorize any number.

A. What kind of number is 10?

Is it a real number? Yes.

Is it a rational number? Can it be written as a fraction? Yes: $\frac{10}{1}$

Is it an integer? Is it a whole number or the opposite of a whole number? Yes.

Is it a whole number? Yes.

So 10 is a real number, a rational number, an integer, and a whole number.

B. What kind of number is $\sqrt{\frac{9}{3}}$?

Is it a real number? Yes.

Is it a rational number? Can it be written as a fraction? No. $\frac{9}{3}$ simplifies

to 3. If you try to find the square root of 3, you will get a decimal answer that goes on forever but does not repeat: 1.7320508... This cannot be written as a fraction.

So $\sqrt{\frac{9}{3}}$ is a real, irrational number.

Answer each question to identify the categories the given number

$\sqrt{16}$

1. Is it a real number? _____
2. Is it a rational number? Can it be written as a fraction?

3. Is it an integer? Is it a whole number or the opposite of a whole number? _____
4. Is it a whole number? _____
5. List all of the categories $\sqrt{16}$ belongs to.

LESSON
14-3

Answers Ordering Real Numbers

Reteach

Compare and order real numbers from least to greatest.

Order $\sqrt{22}$, $\pi + 1$, and $4\frac{1}{2}$ from least to greatest.

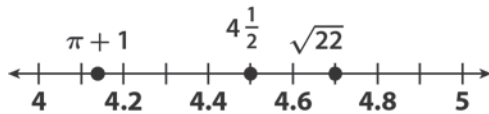
You can use a calculator to approximate irrational numbers.

$$\sqrt{22} \approx 4.69$$

You know that $\pi \approx 3.14$, so you can find the approximate value of $\pi + 1$.

$$\pi + 1 \approx 3.14 + 1 \approx 4.14$$

Plot $\sqrt{22}$, $\pi + 1$, and $4\frac{1}{2}$ on a number line.



On a number line, the values of numbers increase as you move from left to right. So, to order these numbers from least to greatest, list them from left to right.

$$\pi + 1, 4\frac{1}{2}, \text{ and } \sqrt{22}$$

Order each group of numbers from least to greatest.

1. $4, \pi, \sqrt{8}$

2. $5, \frac{17}{3}, \pi + 2$

3. $\sqrt{2}, 1.7, -2$

4. $2.5, \sqrt{5}, \frac{3}{2}$

5. $3.7, \sqrt{13}, \pi + 1$

6. $\frac{5}{4}, \pi - 2, \frac{\sqrt{5}}{2}$

LESSON
15-1

Integer Exponents

Reteach

A positive exponent tells you how many times to multiply the base as a factor. A negative exponent tells you how many times to divide by the base. Any number to the 0 power is equal to 1.

$$4^2 = 4 \cdot 4 = 16$$

$$4^5 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 1024$$

$$a^3 = a \cdot a \cdot a$$

$$4^{-2} = \frac{1}{4^2} = \frac{1}{4 \cdot 4} = \frac{1}{16}$$

$$4^{-5} = \frac{1}{4^5} = \frac{1}{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4} = \frac{1}{1024}$$

$$a^{-3} = \frac{1}{a^3} = \frac{1}{a \cdot a \cdot a}$$

When you work with integers, certain properties are always true. With integer exponents, there are also certain properties that are always true.

When the bases are the same and you multiply, you add exponents.

$$\begin{array}{l} 2^2 \cdot 2^4 = 2^{2+4} \\ \underbrace{2 \cdot 2} \cdot \underbrace{2 \cdot 2 \cdot 2 \cdot 2} = 2^6 \end{array}$$

$$a^m \cdot a^n = a^{m+n}$$

When the bases are the same and you divide, you subtract exponents.

$$\frac{2^5}{2^3} = 2^{5-3}$$

$$\frac{2 \cdot 2 \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2}}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2}} = 2^2$$

$$\frac{a^m}{a^n} = a^{m-n}$$

When you raise a power to a power, you multiply.

$$(2^3)^2 = 2^{3 \cdot 2}$$

$$(2 \cdot 2 \cdot 2)^2$$

$$(2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2) = 2^6$$

$$(a^m)^n = a^{m \cdot n}$$

Tell whether you will add, subtract, or multiply the exponents. Then simplify by finding the value of the expression.

1. $\frac{3^6}{3^3} \rightarrow$ _____

2. $8^2 \cdot 8^{-3} \rightarrow$ _____

3. $(3^2)^3 \rightarrow$ _____

4. $5^3 \cdot 5^1 \rightarrow$ _____

5. $\frac{4^2}{4^4} \rightarrow$ _____

6. $(6^2)^2 \rightarrow$ _____

LESSON
15-2

Scientific Notation with Positive Powers of 10


Reteach

You can change a number from standard notation to scientific notation in 3 steps.

1. Place the decimal point between the first and second digits on the left to make a number between 1 and 10.
2. Count from the decimal point to the right of the last digit on the right.
3. Use the number of places counted in Step 2 as the power of ten.

Example

Write 125,000 in scientific notation.


<p>1.25 the</p> <p>1.25. 125,000 </p> <p>1.25 × 10⁵</p>	<p>1) The first and second digits to the left are 1 and 2, so place the decimal point between the two digits to make the number</p> <p>2) The last digit in 125,000 is 5 places to the right.</p> <p>3) The power of 10 is 5.</p>
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You can change a number from scientific notation to standard notation in 3 steps.

1. Find the power of 10.
2. Count that number of places to the right.
3. Add zeros as needed.

Example

Write 5.96×10^4 in standard notation.

<p>10⁴ 5.9600 </p> <p>59,600</p>	<p>1) The power of 10 is 4.</p> <p>2) Move the decimal point 4 places to the right.</p> <p>3) Add two zeros.</p>
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Complete to write each number in scientific notation.

1. 34,600

The number between 1 and 10: _____

The power of 10: _____

The number in scientific notation: _____

2. 1,050,200

The number between 1 and 10: _____

The power of 10: _____

The number in scientific notation: _____

Write each number in standard notation.

3. 1.057×10^3

4. 3×10^8

5. 5.24×10^5

LESSON
15-3

Scientific Notation with Negative Powers of 10

Reteach

You can convert a number from standard form to scientific notation in 3 steps.

4. Starting from the left, find the first non-zero digit. To the right of this digit is the new location of your decimal point.
5. Count the number of places you moved the decimal point. This number will be used in the exponent in the power of ten.
6. Since the original decimal value was less than 1, your power of ten must be negative. Place a negative sign in front of the exponent.

Example

Write 0.00496 in standard notation.

- | | |
|-----------------------|--|
| 4.96 | 1) The first non-zero digit is 4, so move the decimal point to the right of the 4. |
| 4.96×10^3 | 2) The decimal point moved 3 places, so the whole number in the power of ten is 3. |
| 4.96×10^{-3} | 3) Since 0.00496 is less than 1, the power of ten must be negative. |

You can convert a number from scientific notation to standard form in 3 steps.

1. Find the power of ten.
2. If the exponent is negative, you must move the decimal point to the left. Move it the number of places indicated by the whole number in the exponent.
3. Insert a leading zero before the decimal point.

Example

Write 1.23×10^{-5} in standard notation.

- | | |
|-----------|---|
| 10^{-5} | 1) Find the power of ten. |
| .0000123 | 2) The exponent is -5 , so move the decimal point 5 places to the left. |
| 0.0000123 | 3) Insert a leading zero before the decimal point. |

Write each number in scientific notation.

1. 0.0279

2. 0.00007100

3. 0.0000005060

Write each number in standard notation.

4. 2.350×10^{-4}

5. 6.5×10^{-3}

6. 7.07×10^{-5}

LESSON
16-1

Representing Proportional Relationships

Reteach

A **proportional relationship** is a relationship between two sets of quantities in which the ratio of one quantity to the other quantity is constant. If you divide any number in one group by the corresponding number in the other group, you will always get the same quotient.

Example: Martin mixes a cleaning spray that is 1 part vinegar to 5 parts water.

Proportional relationships can be shown in tables, graphs, or equations.

Martin's Cleaning Spray

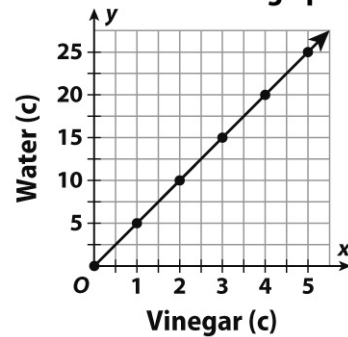
Water (c)	5	10	15	20	25
Vinegar (c)	1	2	3	4	5

Notice that if you divide the amount of water by the amount of vinegar, the quotient is always 5.

Graph

On the graph, you can see that for every 1 unit you move to the right on the x-axis, you move up 5 units on the y-axis.

Martin's Cleaning Spray



Equation

Let y represent the number of cups of water.

Let x represent the cups of vinegar.

$$y = 5x$$

Use the table below for Exercises 1–3.

Distance driven (mi)	100	200		400		600
Gas used (gal)	5		15			30

1. There is a proportional relationship between the distance a car drives and the amount of gas used. Complete the table.

2. Find each ratio. $\frac{\text{miles}}{\text{gallons}} \rightarrow \frac{100}{5} = \frac{200}{15} = \frac{400}{\quad} = \frac{600}{30}$

Each ratio is equal to _____.

3. a. Let x represent gallons of gas used. Let y represent _____.

b. The equation that describes the relationship is _____.

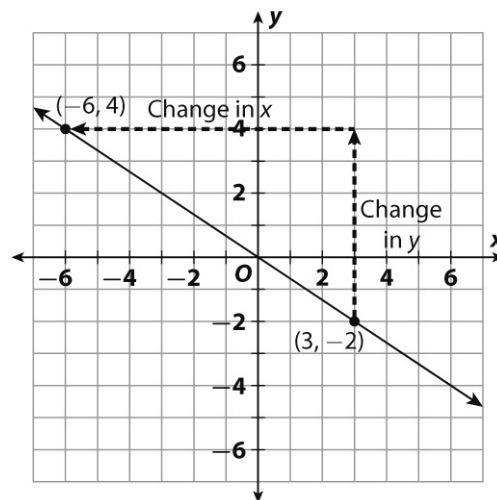
LESSON
16-2

Rate of Change and Slope

Reteach

Look at the relationships between the table, the graph, and the slope.

First value (x)	Second value (y)
-6	4
-3	2
0	0
3	-2



To find the slope, choose two points, using the table or graph. For example, choose $(-6, 4)$ and $(3, -2)$.

Change in y: $4 - (-2) = 6$

Change in x: $-6 - 3 = -9$

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{6}{-9} = -\frac{2}{3}$$

Use the example above to complete Exercises 1 and 2.

- The slope is negative. In the table, as the values of x decrease, the values of y _____.
- The slope is negative. In the graph, as you move from left to right, the line of the graph is going _____ (up or down).

Solve.

- Suppose the slope of a line is positive. Describe what happens to the value of x as the value of y increases.

- Suppose the slope of a line is positive. Describe what happens to the graph of the line as you move from left to right.
- Two points on a line are $(3, 8)$ and $(-3, 2)$. What is the slope of the line?

LESSON
17-1

Representing Linear Nonproportional Relationships

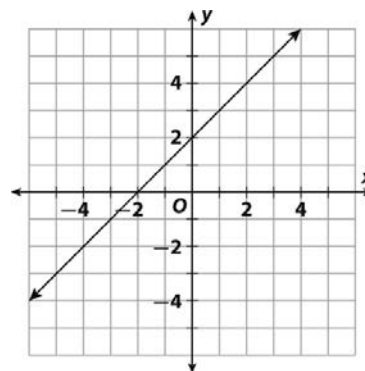
Reteach

A relationship will be proportional if the ratios in a table of values of the relationship are constant. The graph of a proportional relationship will be a straight line through the origin. If either of these is not true, the relationship is nonproportional.

To graph the solutions of an equation, make a table of values. Choose values that will give integer solutions.

A. Graph the solutions of $y = x + 2$.

x	-2	-1	0	1	2
y	0	1	2	3	4



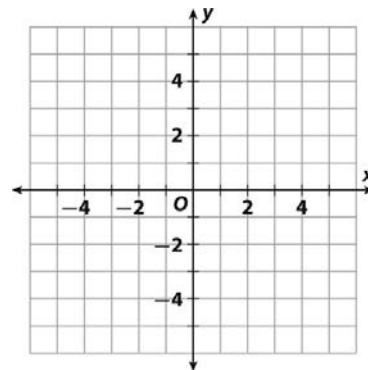
B. Tell whether the relationship is proportional. Explain.

The graph is a straight line, but it does **not** go through the origin, so the relationship is not proportional.

Make a table and graph the solutions of each equation.

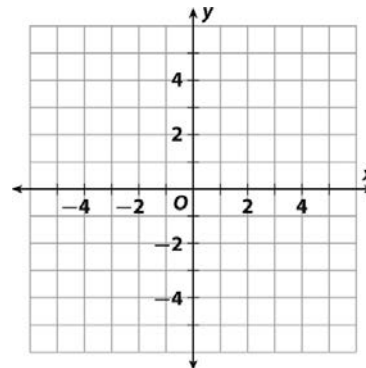
1. $y = 3x + 1$

x	-2	-1	0	1	2
y					



2. $y = -x - 2$

x	-2	-1	0	1	2
y					



LESSON
17-2

Determining Slope and y-intercept

Reteach

The **slope** of a line is a measure of its tilt, or slant.

The slope of a straight line is a constant ratio, the “rise over run,” or the **vertical change** over the **horizontal change**.

You can find the slope of a line by comparing any two of its points.

The vertical change is the difference between the two y-values, and the horizontal change is the difference between the two x-values.

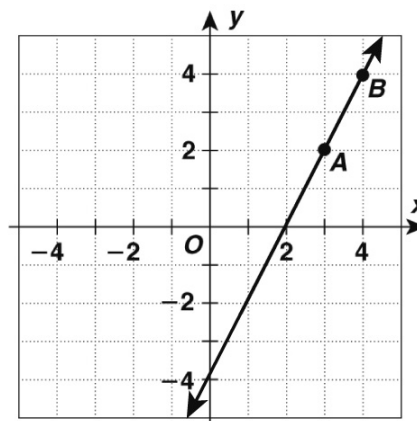
The **y-intercept** is the point where the line crosses the y-axis.

- A.** Find the slope of the line shown.
point A: (3, 2) point B: (4, 4)

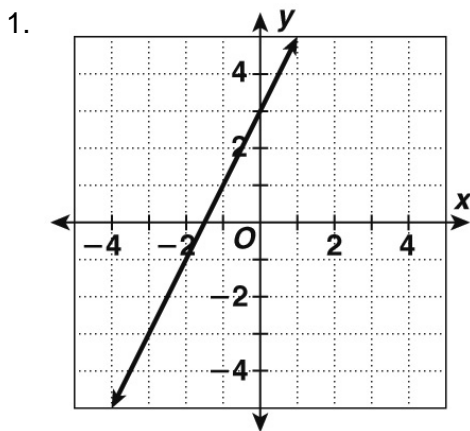
$$\begin{aligned} \text{slope} &= \frac{4 - 2}{4 - 3} \\ &= \frac{2}{1}, \text{ or } 2 \end{aligned}$$

So, the slope of the line is 2.

- B.** Find the y-intercept of the line shown.
The line crosses the y-axis at (0, -4).
So, the y-intercept is -4.

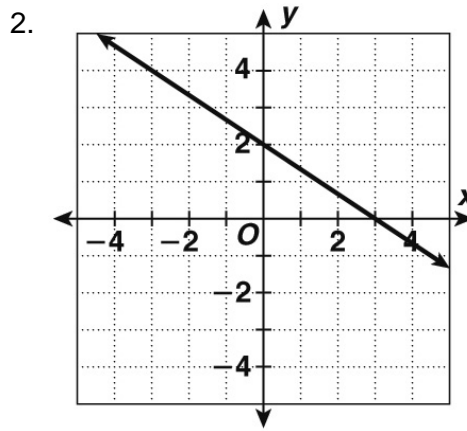


Find the slope and y-intercept of the line in each graph.



slope $m =$ _____

y-intercept $b =$ _____



slope $m =$ _____

y-intercept $b =$ _____

LESSON
18-1

Equations with Variables on Both Sides

Reteach

Solve the equation $5x = 2x + 12$.

$$\begin{array}{r} 5x = 2x + 12 \\ -2x \quad -2x \\ \hline 3x = 12 \\ \frac{3x}{3} = \frac{12}{3} \\ x = 4 \end{array}$$

To collect on left side, subtract $2x$ from both sides of the equation.

Divide by 3.

Check: Substitute into the original equation.

$$\begin{aligned} 5x &= 2x + 12 \\ 5(4) &\stackrel{?}{=} 2(4) + 12 \\ 20 &\stackrel{?}{=} 8 + 12 \\ 20 &= 20 \end{aligned}$$

Solve the equation $-6z + 28 = 9z - 2$

$$\begin{array}{r} -6z + 28 = 9z - 2 \\ +6z \quad +6z \\ \hline 28 = 15z - 2 \\ +2 \quad +2 \\ \hline 30 = 15z \\ \frac{30}{15} = \frac{15z}{15} \\ 2 = z \end{array}$$

To collect on right side, add $6z$ to both sides of the equation.

Add 2 to both sides of the equation.

Divide by 15.

Check: Substitute into the original equation.

$$\begin{aligned} -6z + 28 &= 9z - 2 \\ -6(2) + 28 &\stackrel{?}{=} 9(2) - 2 \\ -12 + 28 &\stackrel{?}{=} 18 - 2 \\ 16 &= 16 \end{aligned}$$

Complete to solve and check each equation.

1. $9m + 2 = 3m - 10$

$$\begin{array}{r} 9m + 2 = 3m - 10 \\ -[\quad] \quad -[\quad] \\ \hline 6m + 2 = -10 \\ -[\quad] \quad -[\quad] \\ \hline 6m = [\quad] \\ \frac{6m}{[\quad]} = \frac{-12}{[\quad]} \\ m = [\quad] \end{array}$$

To collect on left side, subtract _____ from both sides.

Subtract _____ from both sides.

Divide by _____.

Check: Substitute into the original equation.

$$\begin{aligned} 9m + 2 &= 3m - 10 \\ 9(\underline{\quad}) + 2 &\stackrel{?}{=} 3(\underline{\quad}) - 10 \\ \underline{\quad} + 2 &\stackrel{?}{=} \underline{\quad} - 10 \\ \underline{\quad} &= \underline{\quad} \end{aligned}$$

2. $-7d - 22 = 4d$

$$\begin{array}{r} -7d - 22 = 4d \\ +[\quad] \quad +[\quad] \\ \hline -22 = 11d \\ \frac{-22}{[\quad]} = \frac{11d}{[\quad]} \\ [\quad] = d \end{array}$$

To collect on right side, add _____ to both sides.

Divide by _____.

Check: Substitute into the original equation.

$$\begin{aligned} -7d - 22 &= 4d \\ -7(\underline{\quad}) - 22 &\stackrel{?}{=} 4(\underline{\quad}) \\ \underline{\quad} - 22 &\stackrel{?}{=} \underline{\quad} \\ \underline{\quad} &= \underline{\quad} \end{aligned}$$

LESSON
18-2

Equations with Rational Numbers

Reteach

To solve an equation with a variable on both sides that involves fractions, first get rid of the fractions.

$$\text{Solve } \frac{3}{4}m + 2 = \frac{2}{3}m + 5.$$

Multiply both sides of the equation by 12, the LCM of 4 and 3.

Check: Substitute into the original equation.

$$12\left(\frac{3}{4}m + 2\right) = 12\left(\frac{2}{3}m + 5\right)$$

Multiply each term by 12.

$$\frac{3}{4}m + 2 = \frac{2}{3}m + 5$$

$$12\left(\frac{3}{4}m\right) + 12(2) = 12\left(\frac{2}{3}m\right) + 12(5)$$

Simplify.

$$\frac{3}{4}(36) + 2 \stackrel{?}{=} \frac{2}{3}(36) + 5$$

Subtract $8m$ from both sides.

$$27 + 2 \stackrel{?}{=} 24 + 5$$

$$9m + 24 = 8m + 60$$

Simplify.

$$29 = 29$$

$$\begin{array}{r} -8m \quad -8m \\ \hline m + 24 = 60 \end{array}$$

Subtract 24 from both sides.

$$\begin{array}{r} -24 \quad -24 \\ \hline m = 36 \end{array}$$

Simplify.

Complete to solve and check your answer.

1. $\frac{1}{4}x + 2 = \frac{2}{5}x - 1$

$$[\quad] \left(\frac{1}{4}x + 2 \right) = [\quad] \left(\frac{2}{5}x - 1 \right)$$

Multiply both sides of the equation by _____ the LCM of 4 and 5.

Check: Substitute into the original equation.

$$\frac{1}{4}x + 2 = \frac{2}{5}x - 1$$

$$[\quad] \left(\frac{1}{4}x \right) + [\quad] (2)$$

Multiply each term by ____.

$$(\quad) + 2 \stackrel{?}{=} \frac{2}{5}(\quad) - 1 \frac{1}{4}$$

$$\begin{array}{r} [\quad]x + [\quad] = [\quad]x - [\quad] \\ \hline -5x \quad -5x \end{array}$$

Simplify.

$$\quad + 2 \stackrel{?}{=} \quad - 1$$

Subtract ____.

$$\begin{array}{r} 40 = 3x - 20 \\ \hline +20 \quad +20 \end{array}$$

Simplify.

$$\quad = \quad$$

Add ____.

$$[\quad] = 3x$$

Simplify.

$$\frac{60}{[\quad]} = \frac{3x}{[\quad]}$$

Divide both sides by _____

$$[\quad] = x$$

Simplify.

Answers

<p>LESSON 14-1</p> <ol style="list-style-type: none"> 3.75 $0.8\bar{3}$ $3.\bar{6}$ 9, -9 7, -7 $\frac{5}{6}, -\frac{5}{6}$ 3 5 9 	<p>LESSON 14-2</p> <ol style="list-style-type: none"> Yes Yes; $\sqrt{16} = 4$, which can be written as $\frac{4}{1}$. Yes Is it a whole number? Yes. real, rational, integer, whole 	<p>LESSON 14-3</p> <ol style="list-style-type: none"> $\sqrt{8}, \pi, 4$ $5, \pi + 2, \frac{17}{3}$ -2, $\sqrt{2}, 1.7$ $\frac{3}{2}, \sqrt{5}, 2.5$ $\sqrt{13}, 3.7, \pi + 1$ $\frac{\sqrt{5}}{2}, \pi - 2, \frac{5}{4}$
<p>LESSON 15-1</p> <ol style="list-style-type: none"> subtract; 27 add; $\frac{1}{8}$ multiply; 729 add; 625 subtract; $\frac{1}{16}$ multiply; 1296 	<p>LESSON 15-2</p> <ol style="list-style-type: none"> 3.46 4 3.46×10^4 1.0502 6 1.0502×10^6 1057 300,000,000 524,000 	<p>LESSON 15-3</p> <ol style="list-style-type: none"> 2.79×10^{-2} 7.1×10^{-5} 5.06×10^{-7} 0.000235 0.0065 0.0000707

LESSON
16-1

1.

Distance Driven (mi)	100	200	300
Gas Used (gal)	5	10	15

2. a. $10; 300; 20; \frac{500}{25}$

b. 20

3 a. number of miles driven

b. $y = 20x$

LESSON
16-2

1. increase

2. down

3. When the slope is positive, as the value of y increases, the value of x increases.

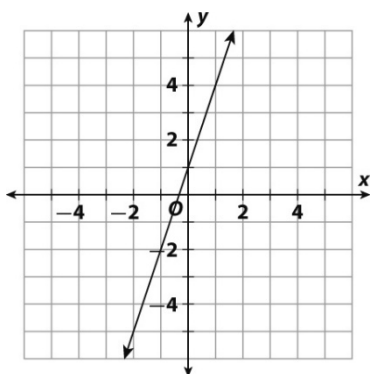
4. When the slope is positive, as you move from left to right, the line goes up.

5. slope = 1

LESSON
17-1

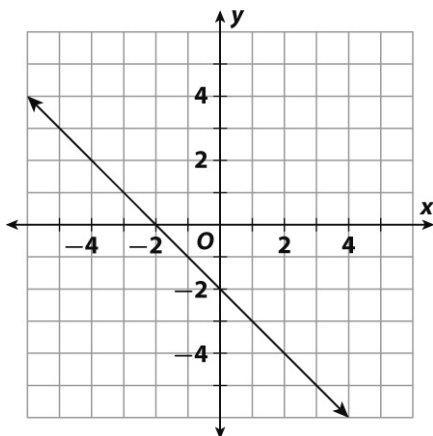
1.

x	-2	-1	0	1	2
y	-5	-2	1	4	7



2.

x	-2	-1	0	1	2
y	0	-1	-2	-3	-4



LESSON
17-2

1.

slope = 2

y-intercept = 3

2.

slope = $-\frac{2}{3}$;

y-intercept = 2

LESSON
18-1

$$\begin{array}{r}
 1. \quad 9m + 2 = 3m - 10 \\
 \underline{-[3m] \quad -[3m]} \\
 6m + 2 = -10 \\
 \underline{-[2] \quad -[2]} \\
 6m = [-12] \\
 \frac{6m}{[6]} = \frac{-12}{[6]} \\
 m = [-2]
 \end{array}$$

To collect on left side, subtract 3m from both sides.
Subtract 2 from both sides.
Divide by 6.

Check: Substitute into the original equation.

$$\begin{array}{l}
 9m + 2 = 3m - 10 \\
 9(-2) + 2 \stackrel{?}{=} 3(-2) - 10 \\
 -18 + 2 \stackrel{?}{=} -6 - 10 \\
 \underline{-16 = -16}
 \end{array}$$

$$\begin{array}{r}
 2. \quad -7d - 22 = 4d \\
 \underline{+[7d] \quad +[7d]} \\
 -22 = 11d \\
 \frac{-22}{[11]} = \frac{11d}{[11]} \\
 [-2] = d
 \end{array}$$

To collect on right side, add 7d to both sides.
Divide by 11.

Check: Substitute into the original equation.

$$\begin{array}{l}
 -7d - 22 = 4d \\
 -7(-2) - 22 \stackrel{?}{=} 4(-2) \\
 \underline{14 - 22 \stackrel{?}{=} -8} \\
 -8 = -8
 \end{array}$$

LESSON
18-2

$$\begin{array}{l}
 1. \\
 [20] \left(\frac{1}{4}x + 2 \right) = [20] \left(\frac{2}{5}x - 1 \right) \\
 [20] \left(\frac{1}{4}x \right) + [20](2) = [20] \left(\frac{2}{5}x \right) - [20](1) \\
 [5]x + [40] = [8]x - [20] \\
 \underline{-5x \quad -5x} \\
 40 = 3x - 20 \\
 \underline{+20 \quad +20} \\
 [60] = 3x \\
 \frac{60}{[3]} = \frac{3x}{[3]} \\
 [20] = x
 \end{array}$$

Multiply both sides of the equation by 20 the LCM of 4 and 5.

Multiply each term by 20.

Simplify.

Subtract 5x.

Simplify.

Add 20.

Simplify.

Divide both sides by 3.

Simplify.

Check: Substitute into the original equation.

$$\begin{array}{l}
 \frac{1}{4}x + 2 = \frac{2}{5}x - 1 \\
 \frac{1}{4}(\underline{20}) + 2 \stackrel{?}{=} \frac{2}{5}(\underline{20}) - 1 \\
 \underline{5 + 2 \stackrel{?}{=} 8 - 1} \\
 \underline{7 = 7}
 \end{array}$$