Lightweight Techniques for Private Heavy Hitters

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Technion

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Mozilla wants to know...
which URLs most often crash the browser?

 stanford.edu/images/logo.png

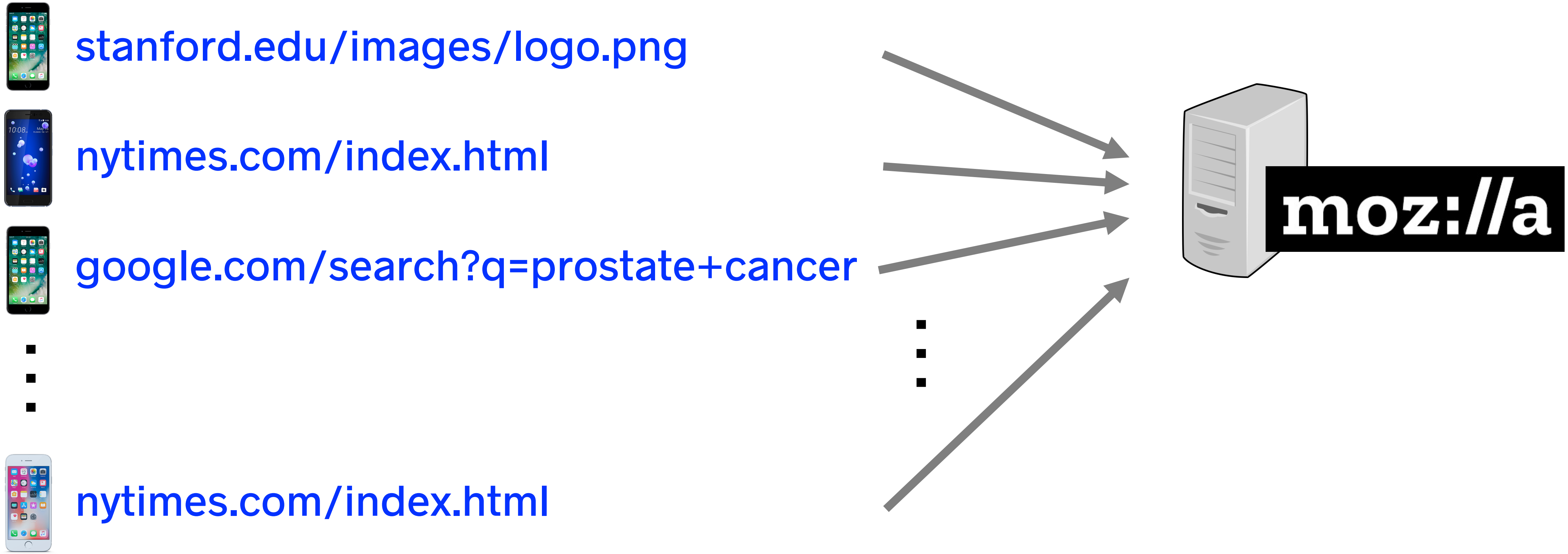
 nytimes.com/index.html

 google.com/search?q=prostate+cancer

⋮

 nytimes.com/index.html

Today: Non-private data collection



We show: Mozilla can learn the most-often reported URLs, without learning which client reported which URL

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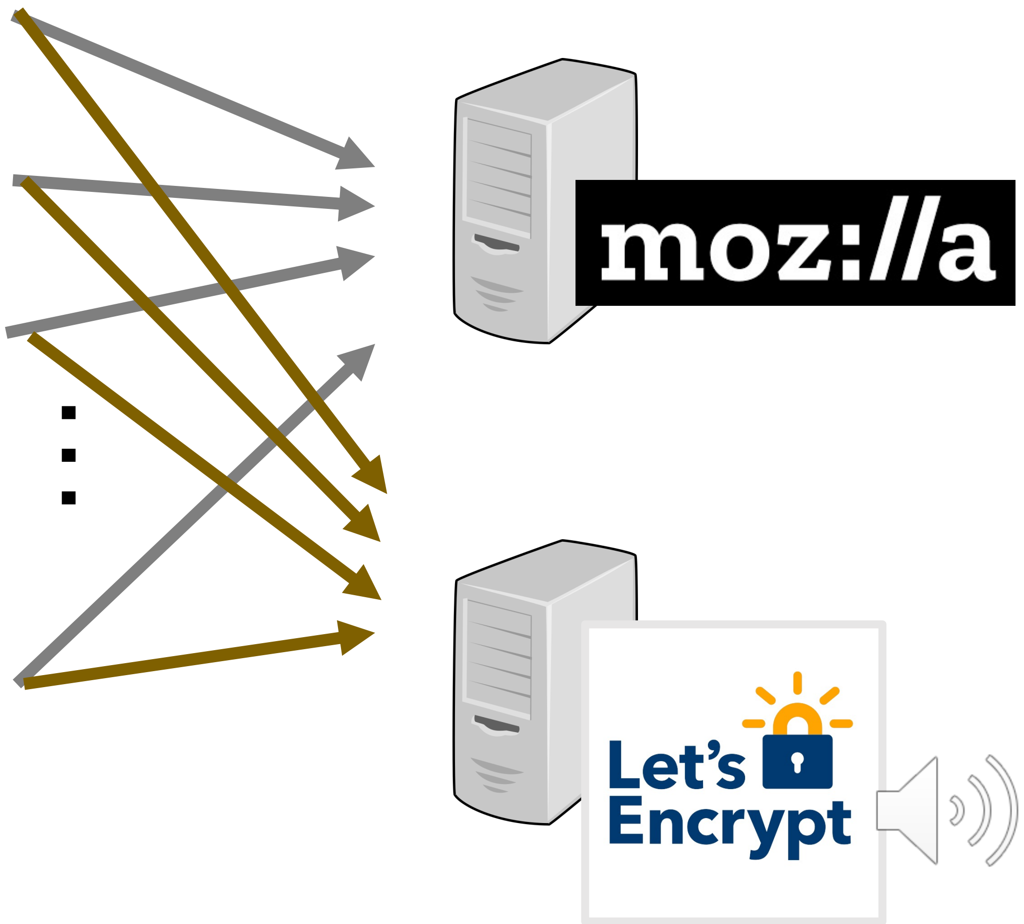

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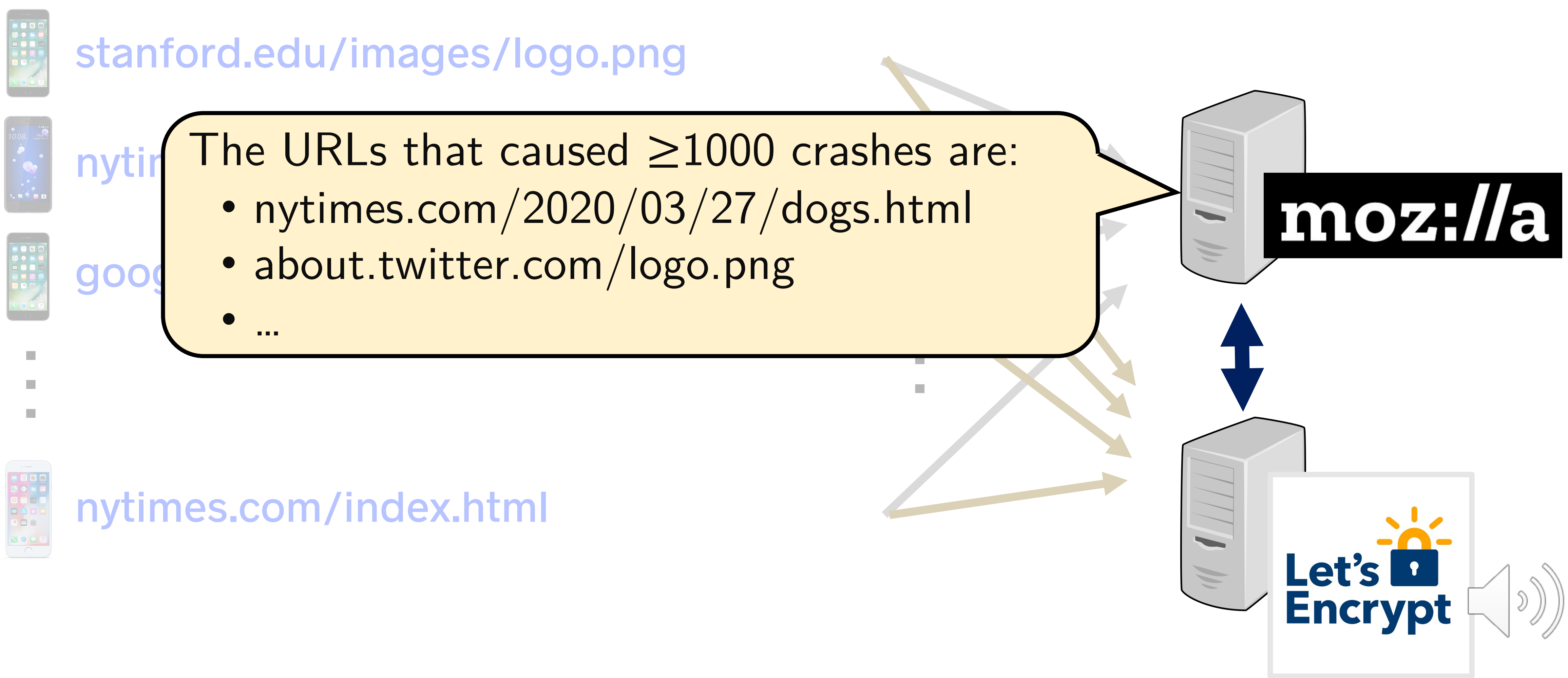

Private heavy-hitters problem

In setting of local differential privacy: [Bassily, Smith 2015] [Qin, Yan, Yu, Khalil, Xiao, Ren 2016] [Bassily, Nissim, Stemmer, Guha 2017] [Bun, Nelson, Stemmer 2019] ...

Millions of clients

Each client holds an n-bit string

(e.g., $n \approx 256$)

Two data-collection servers

Should learn the set of strings that $\geq t$ clients hold

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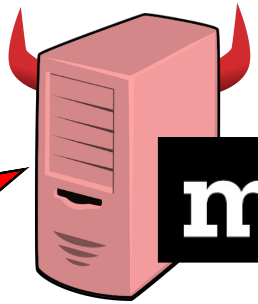
Privacy* against one malicious server, colluding with malicious clients



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Correctness against malicious clients.

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Minimal communication and computation costs

string

(e.g., $n \approx 256$)

Two

Shou

that $\geq t$ clients hold

Support 100s of submissions per second

(Using 100-1000x less bandwidth than general-purpose MPC)

Applications

- Which URLs crash Firefox most often?
- Which phone apps consume the most battery life?
- Which passwords are most popular?
- Which programs consume the most CPU?
- Where do users of my app spent their time?
- …

This talk

- The private heavy-hitters problem
- New tools
  - Incremental distributed point functions
  - Malicious-secure sketching
- Evaluation

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  - –Incremental distributed point functions
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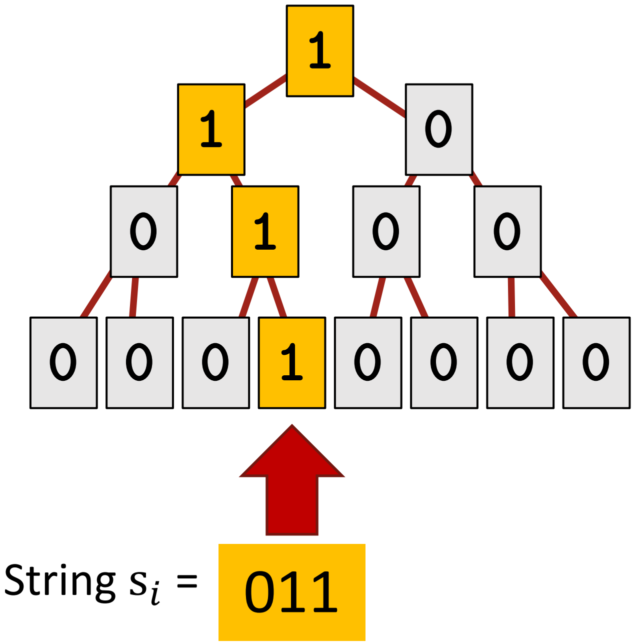
Private heavy hitters: A warm-up scheme

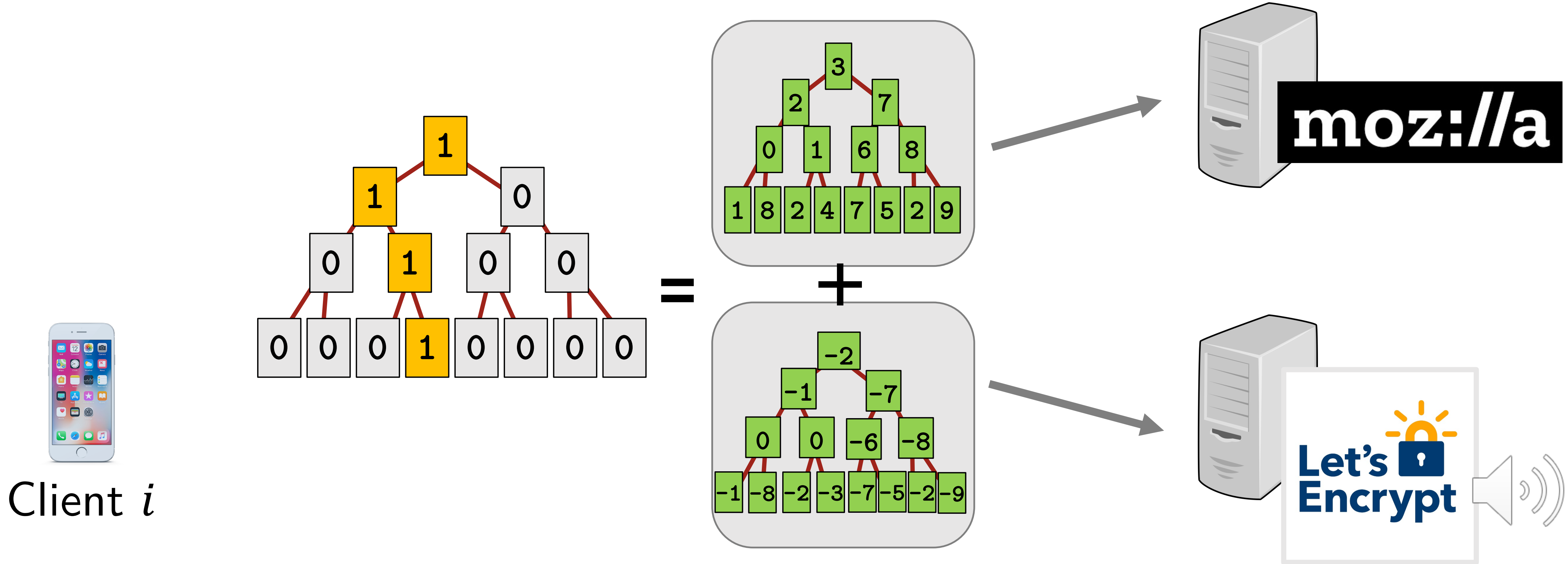
1. Client i with string s_i prepares a binary tree, with 1s on the path to the s_i-th leaf of the tree.

Client i



Private heavy hitters: A warm-up scheme

2. Each client secret-shares the labels on the tree's nodes and sends one share to each of the servers.



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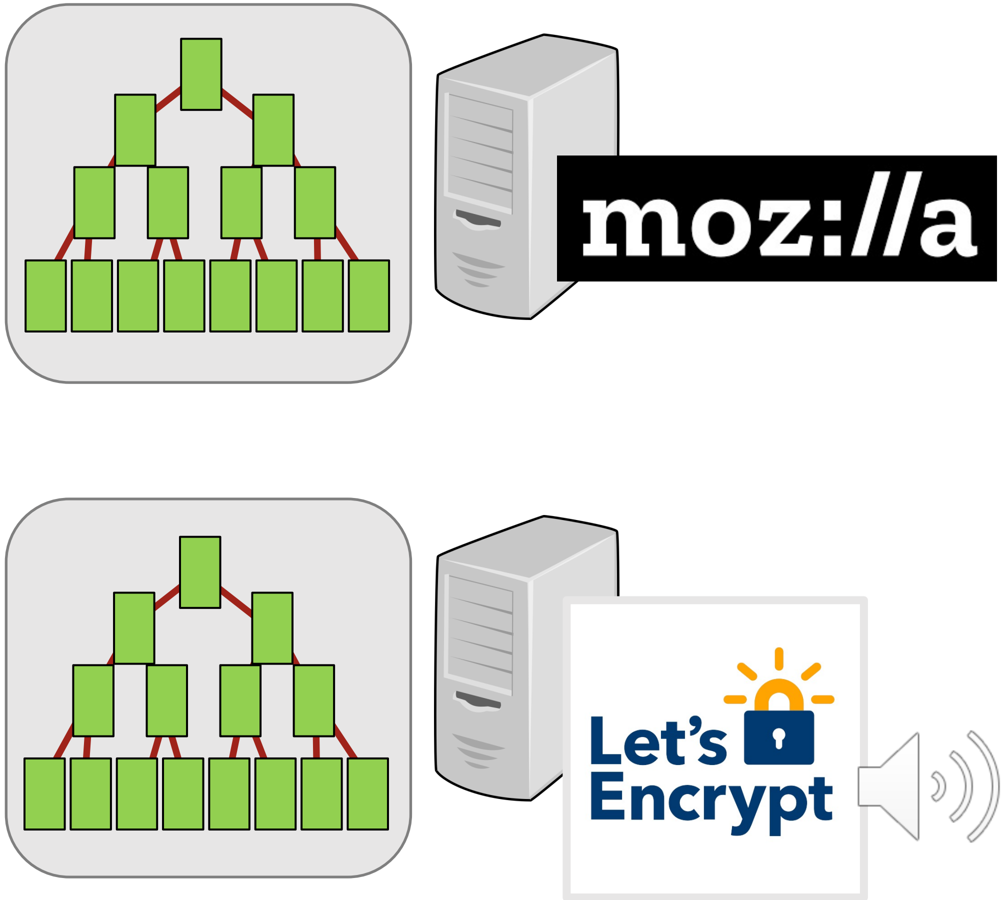
   → **Single message from client to servers**



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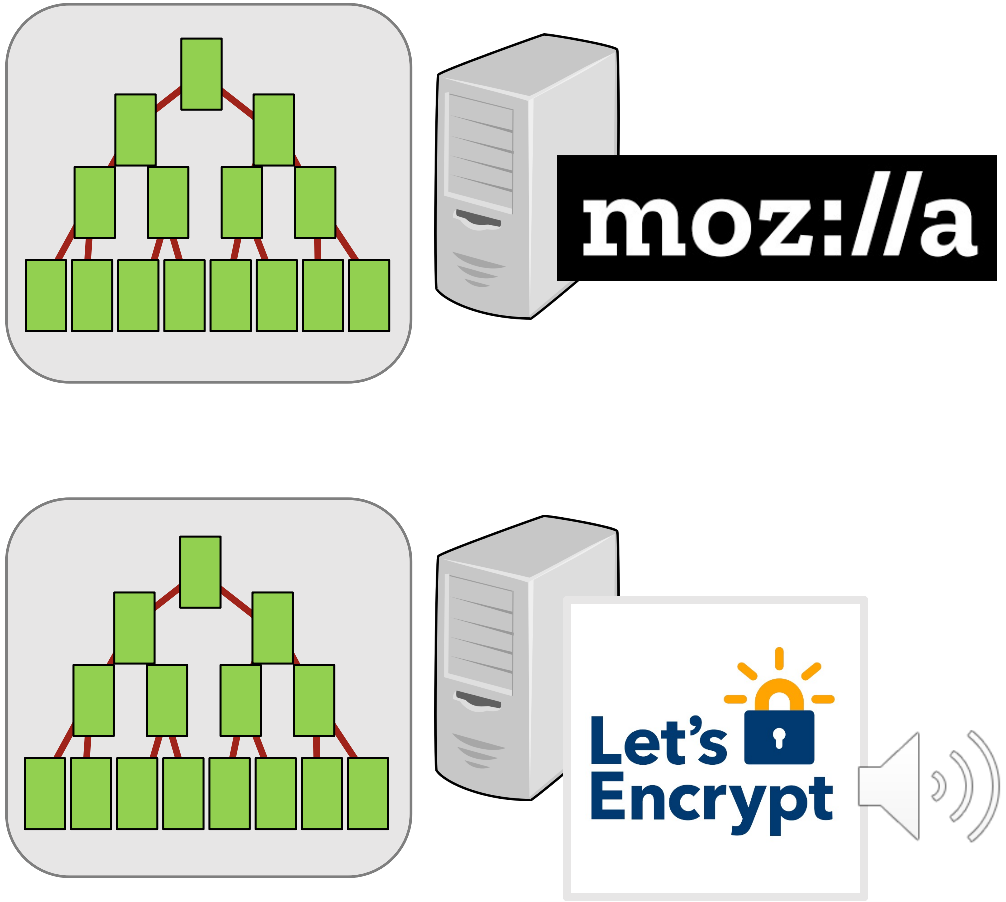

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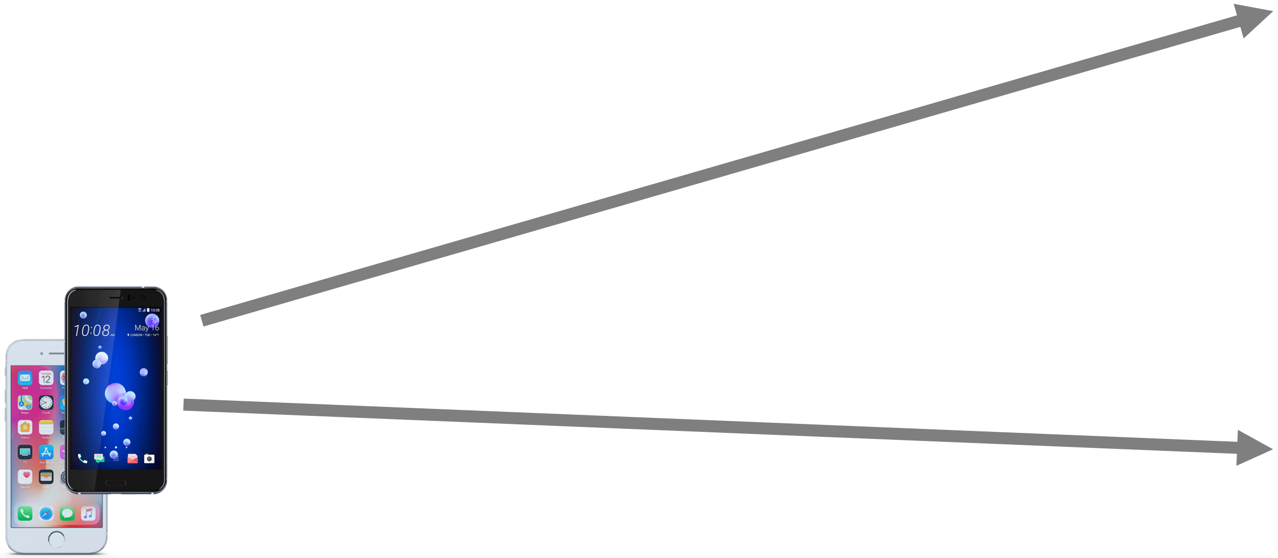

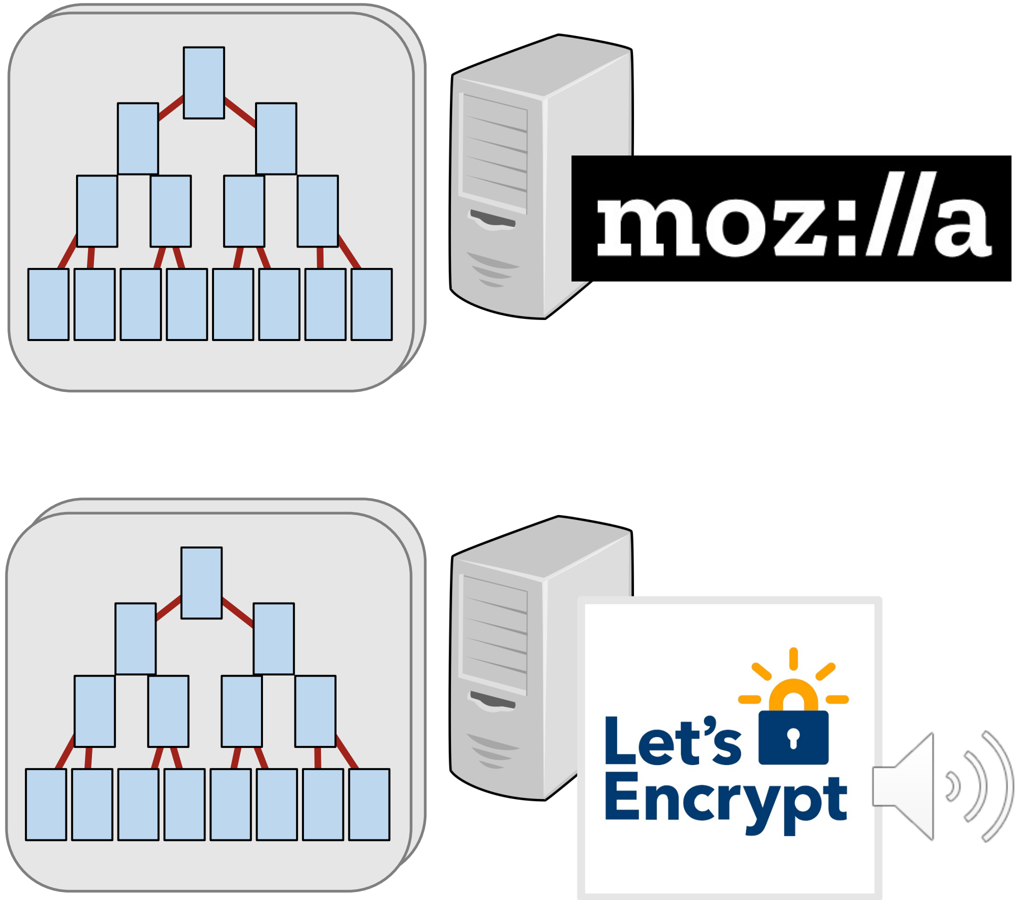

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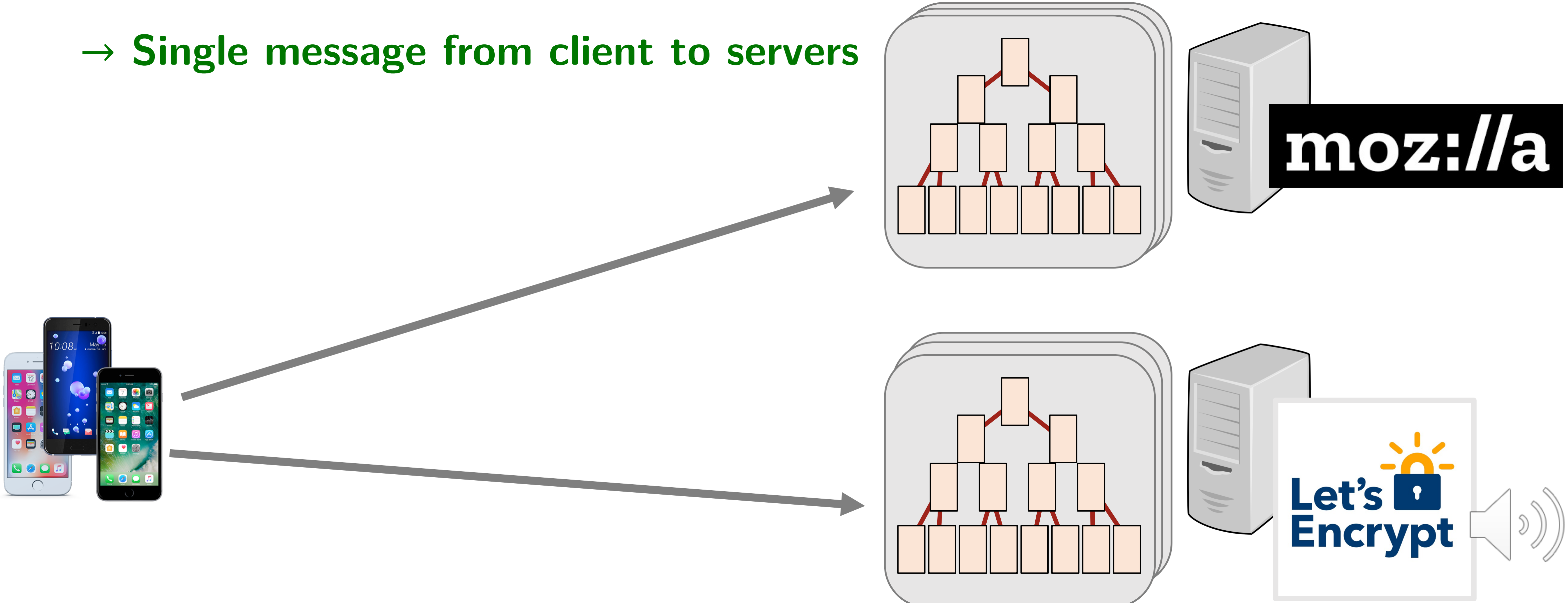

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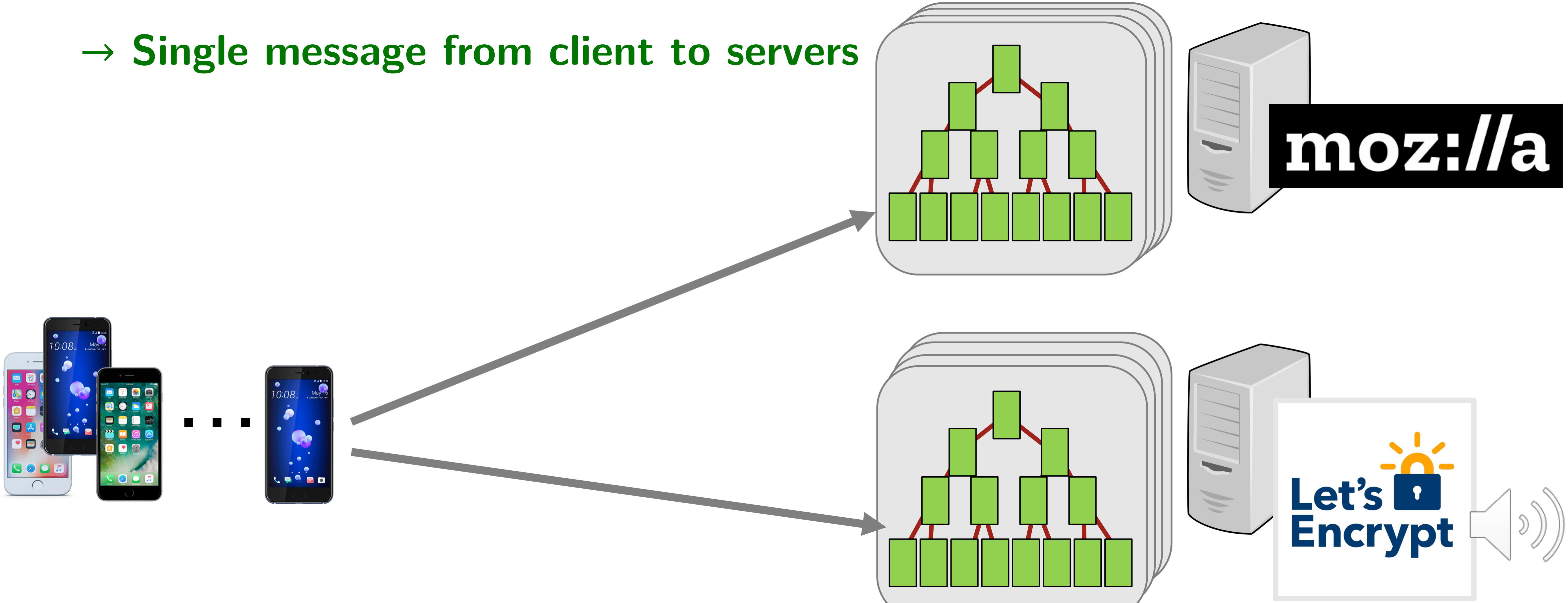

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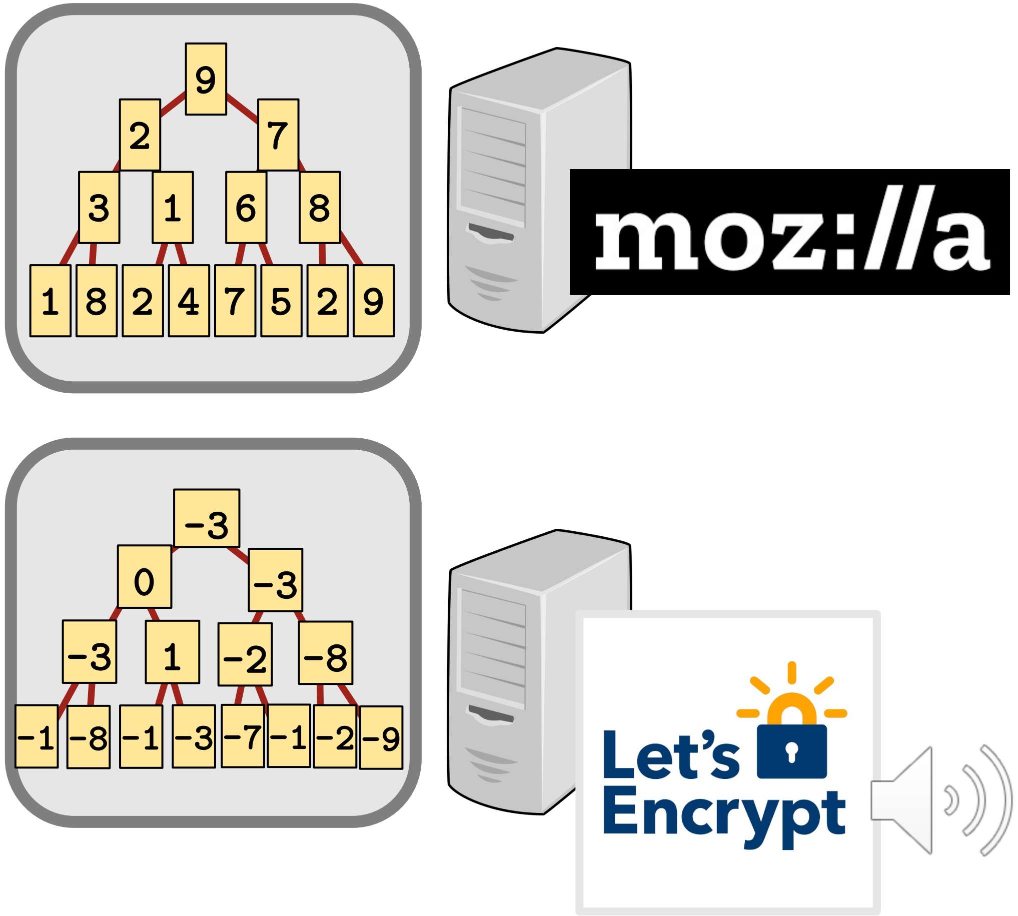
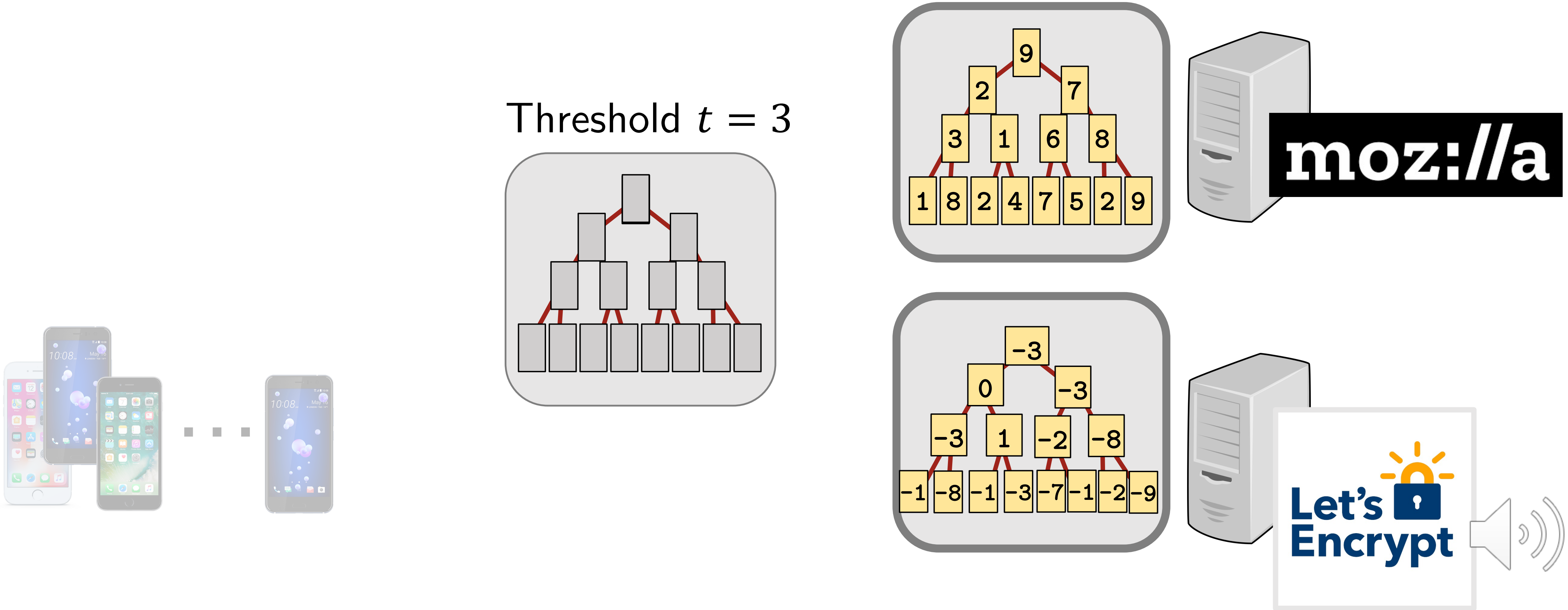
3. Servers sum up shares from each client to get "aggregate" shares.



Private heavy hitters: A warm-up scheme

4. Servers publish shares to perform BFS search for heavy hitters.



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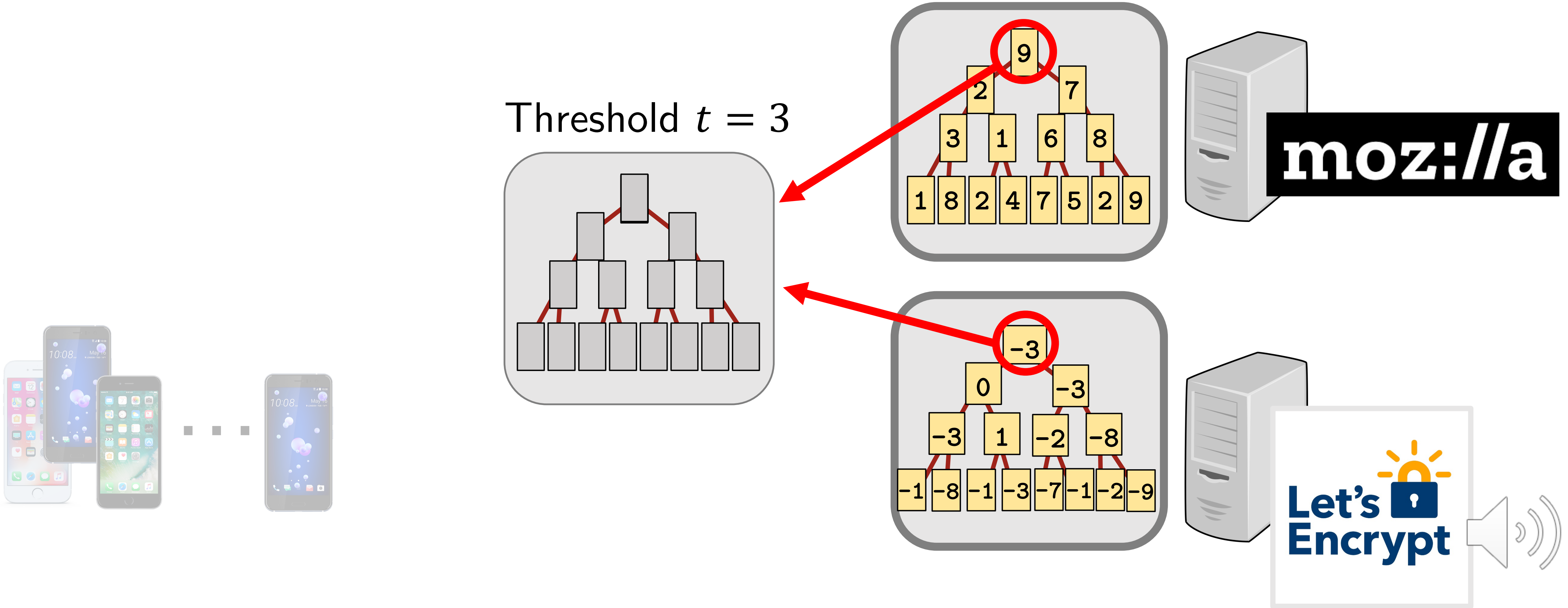

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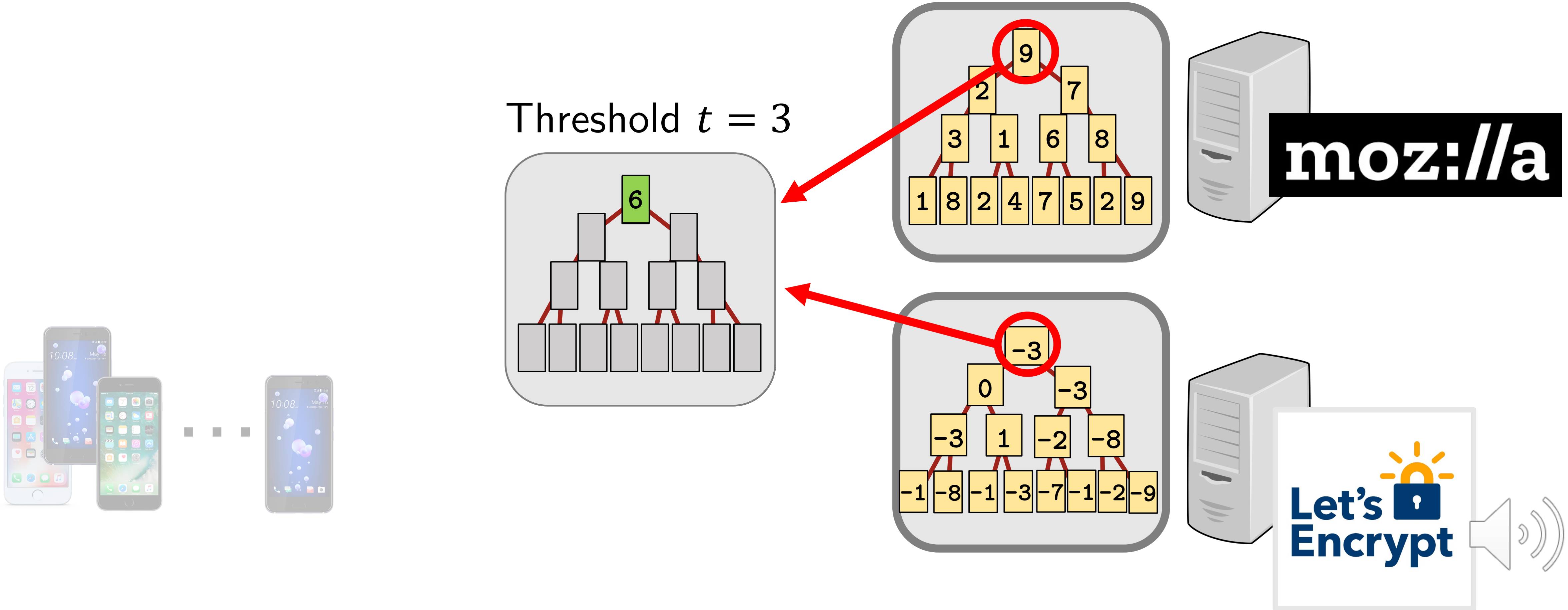

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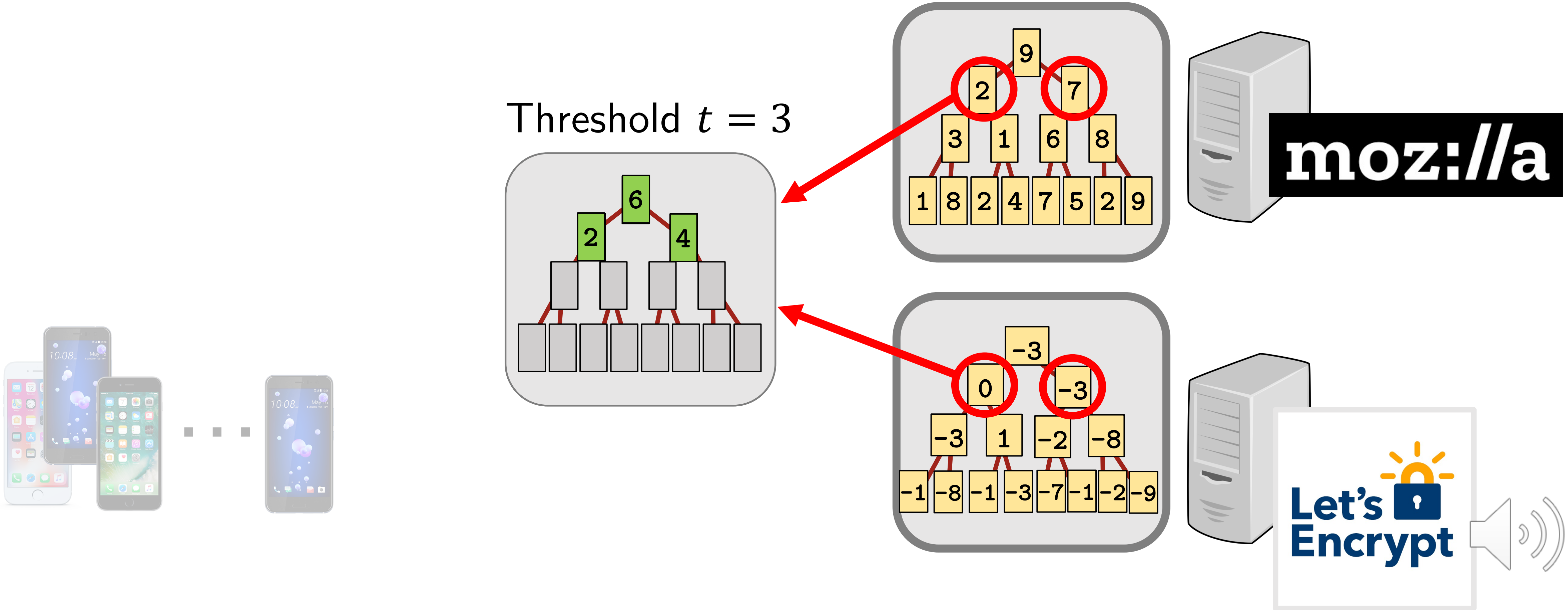

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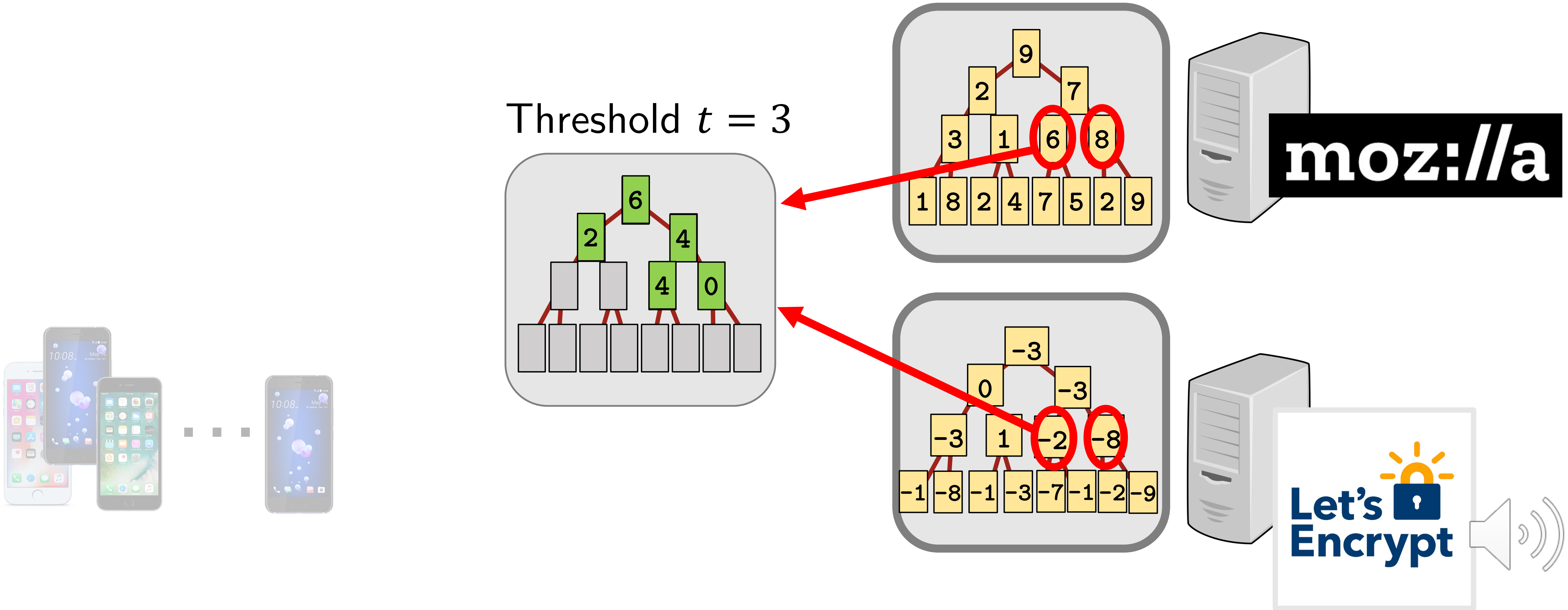

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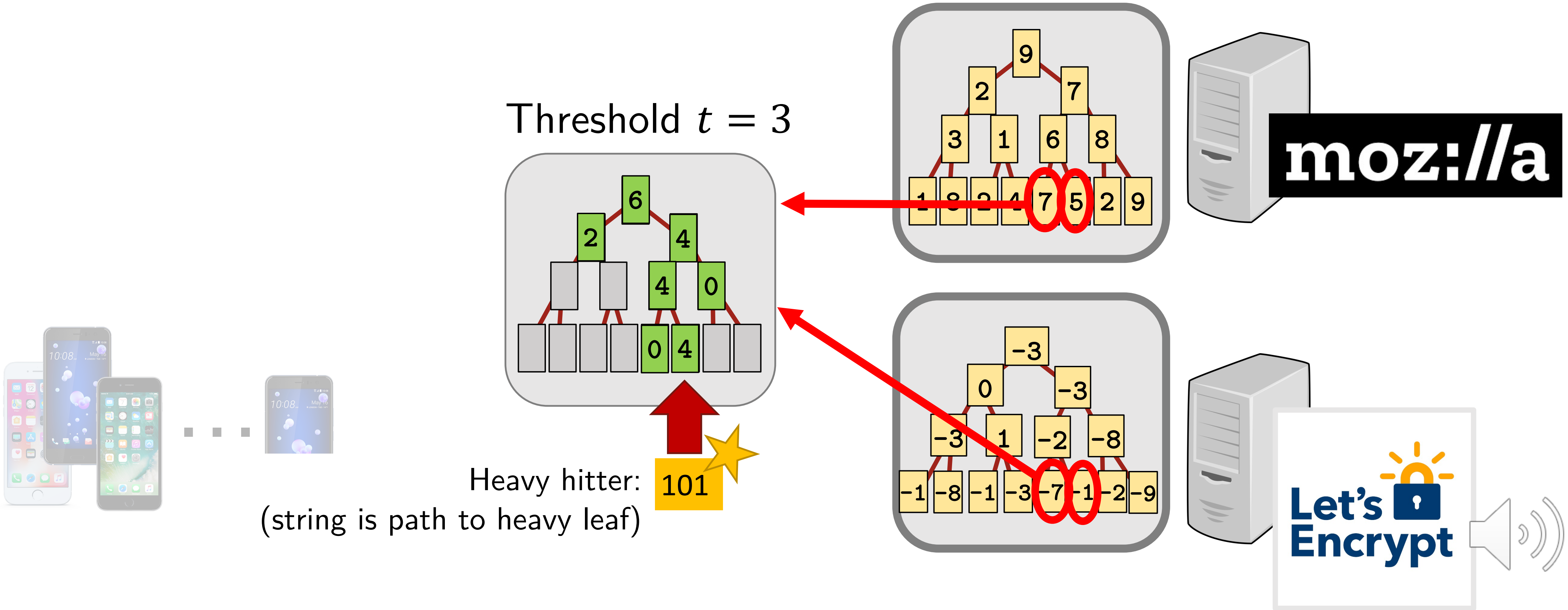

Warm-up scheme: Properties

Correctness
Servers find exactly the set of heavy hitters (no error).

Privacy
If one server is honest, servers learn only the set of heavy “prefixes”

Are we done?

Technical challenges

1. Each tree is exponentially large ⇒ Client cannot materialize it

Idea: Incremental distributed point functions.
→ Succinct secret sharing of a tree with one non-zero path
→ Communication $O(\lambda n)$ instead of $O(\lambda n^2)$ [with normal DPF]

2. Client can send malformed secret shares ⇒ Data corruption

Idea: Malicious-secure sketching.
→ Servers can test whether a secret-shared vector is non-zero in a single coordinate.
→ No interaction with client, $O(\lambda)$ comm b/w servers.

+ Extractable distributed point functions (see paper)

Technical challenges

e.g., when strings are URLs, locations, passwords

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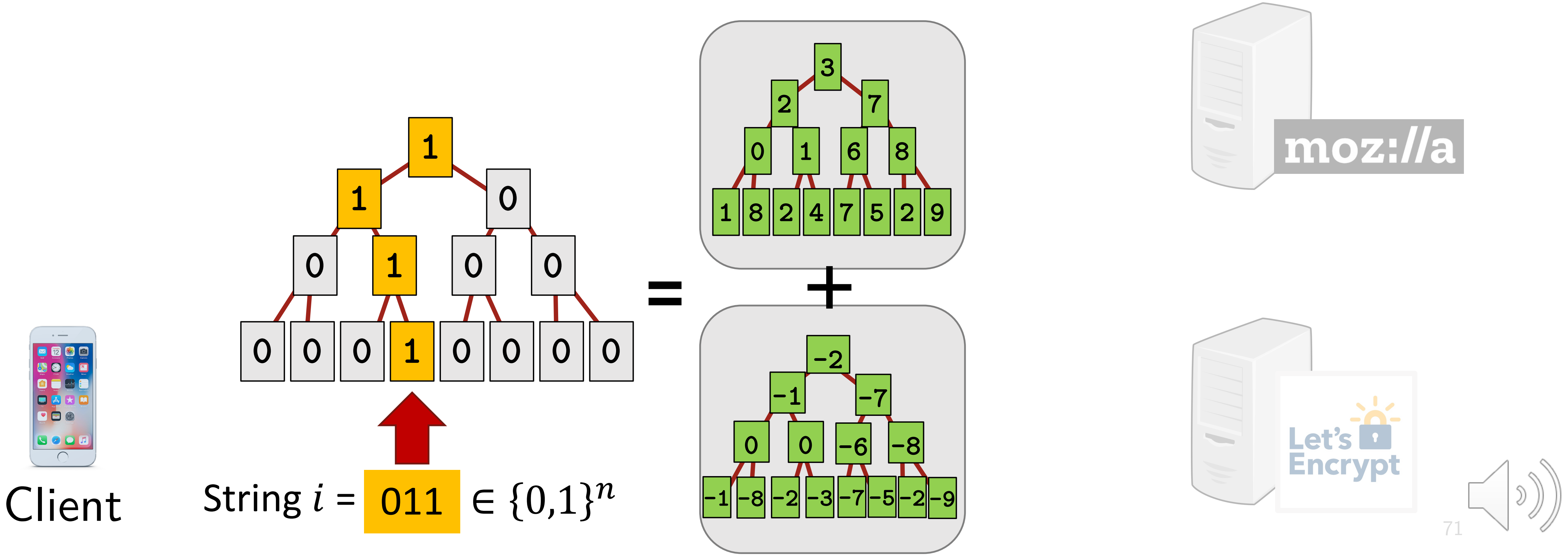
Contribution 1:

Incremental distributed point functions (DPFs)

Each client secret-shares the labels on a tree with one non-zero path and sends one share to each server. **Communication $\approx 2^n$ ☹**



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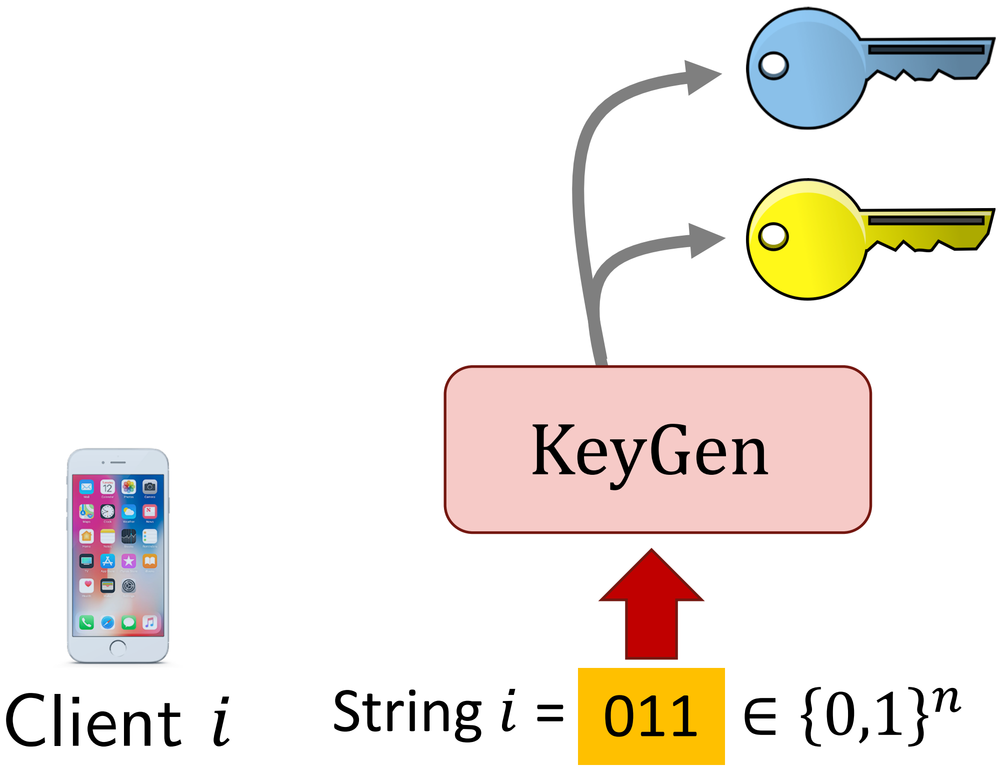
Incremental distributed point functions (DPFs)

With incremental DPFs, client only sends each server a short key



For a tree of depth n,
and security parameter $\lambda \approx 128$,
the keys have size $O(\lambda n)$. ☺

Using standard DPFs would
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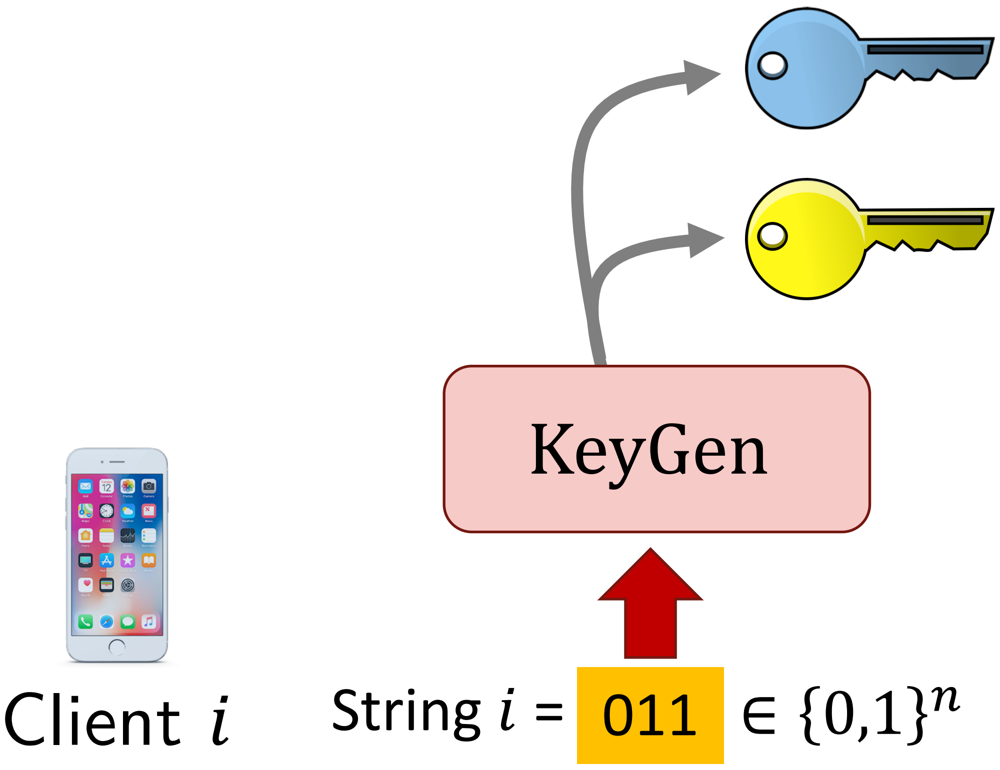

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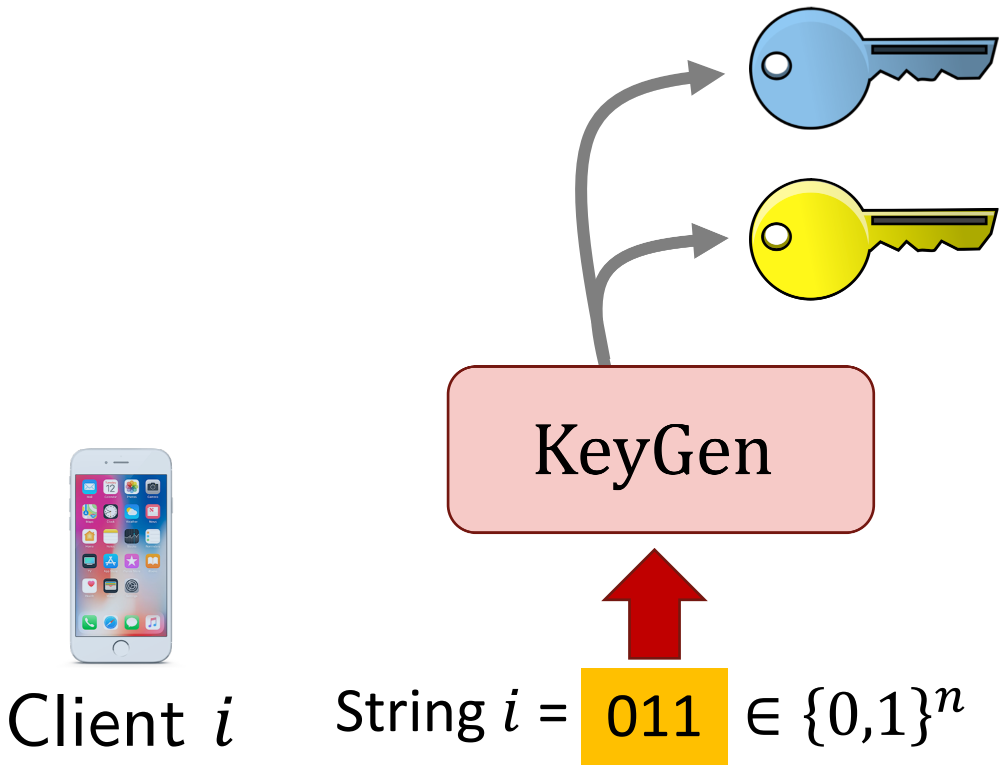

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The servers expand the key into shares of a tree, one node at a time

Evaluating the keys on a path takes $O(n)$ AES ops.

Standard DPFs would require $O(n^2)$ AES ops.

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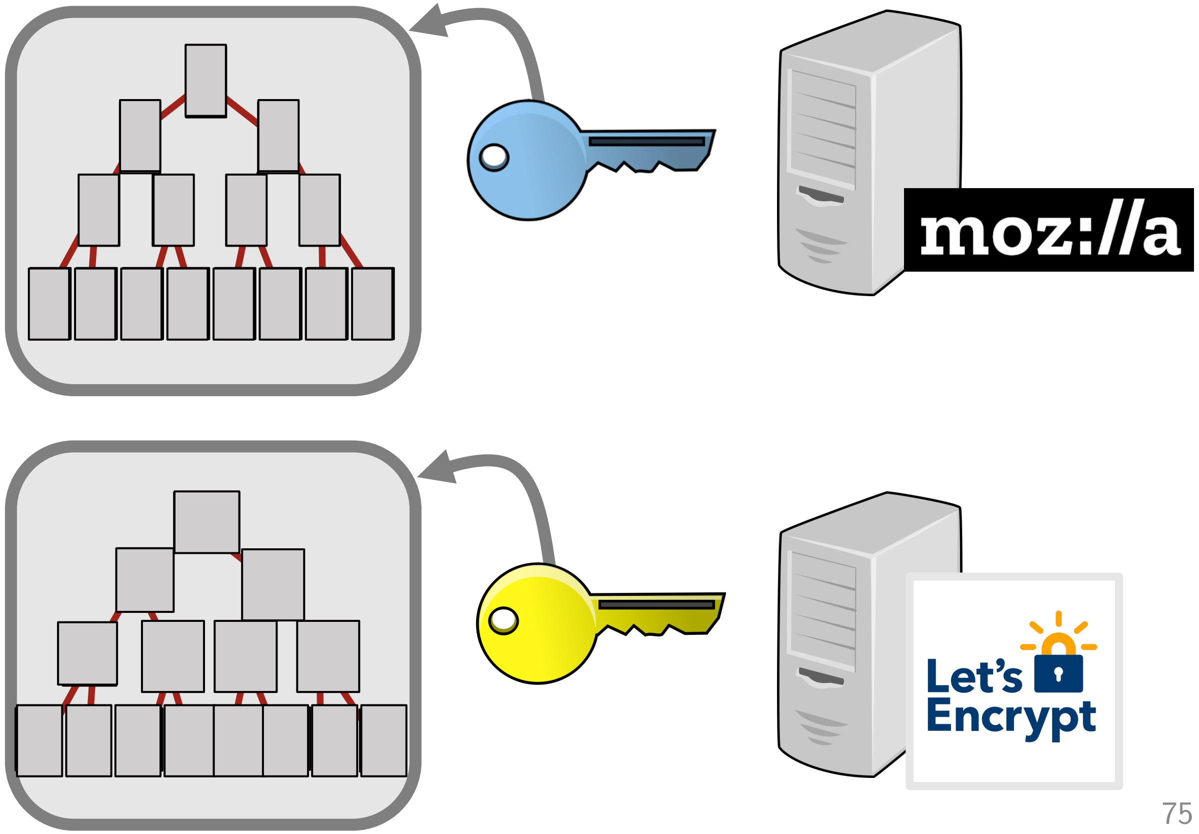

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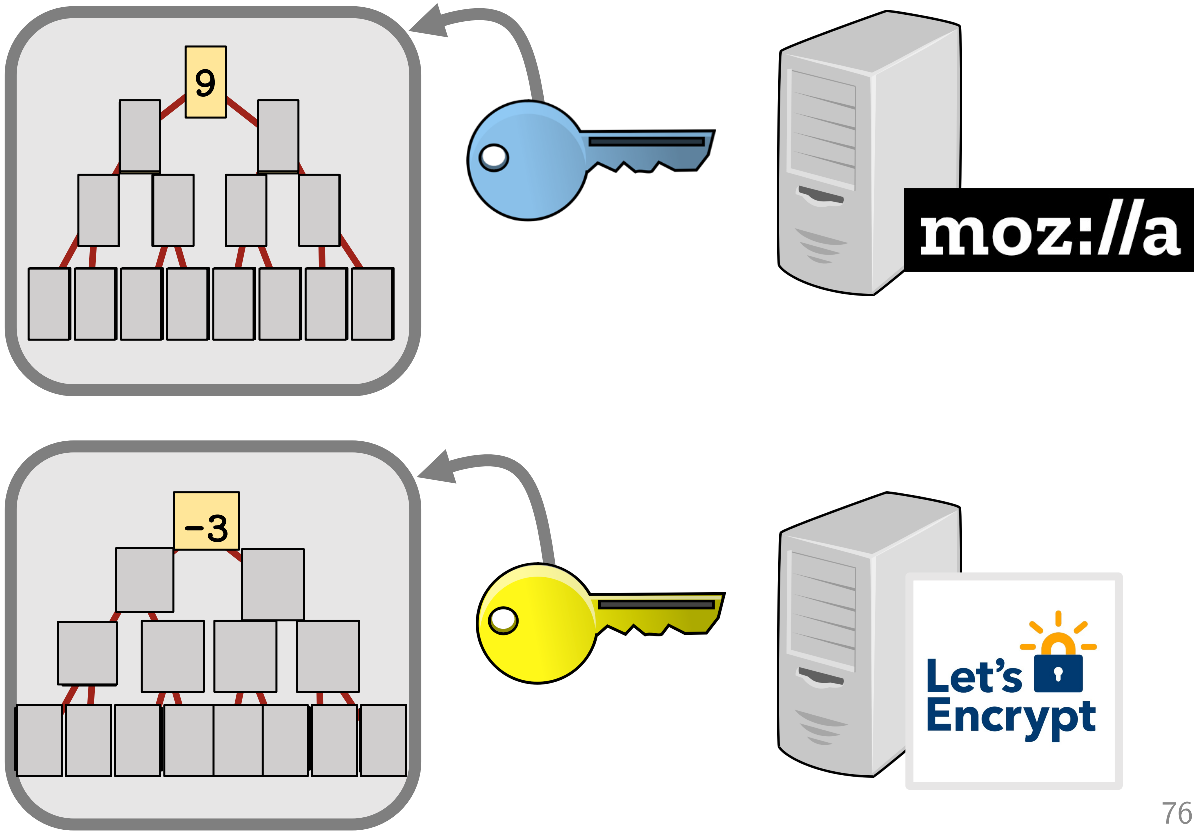

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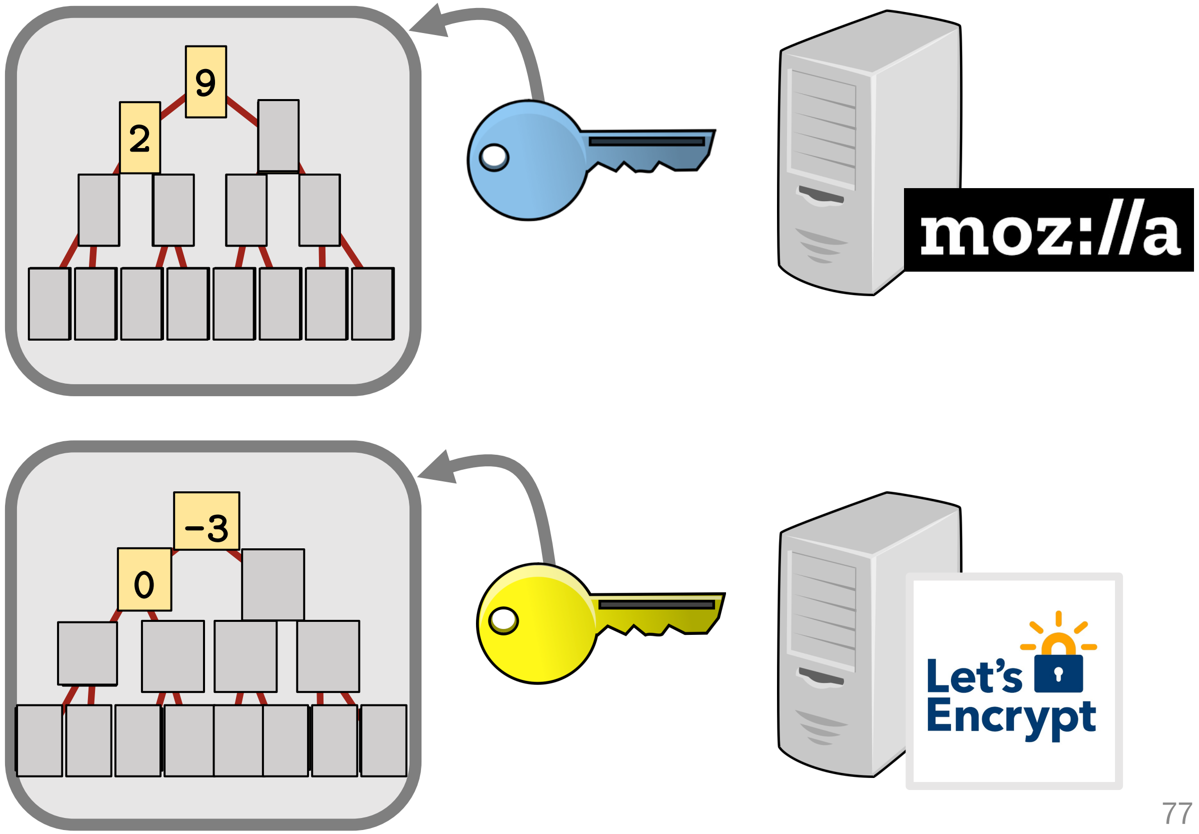

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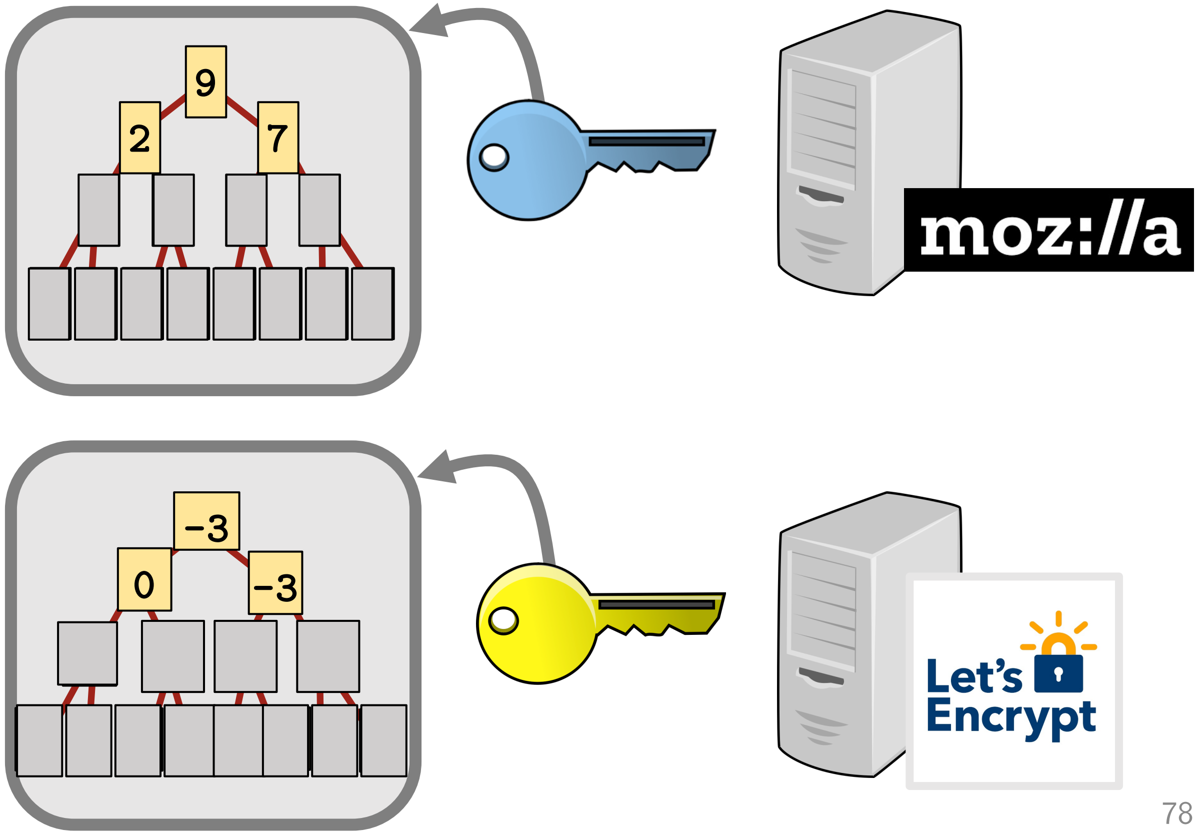

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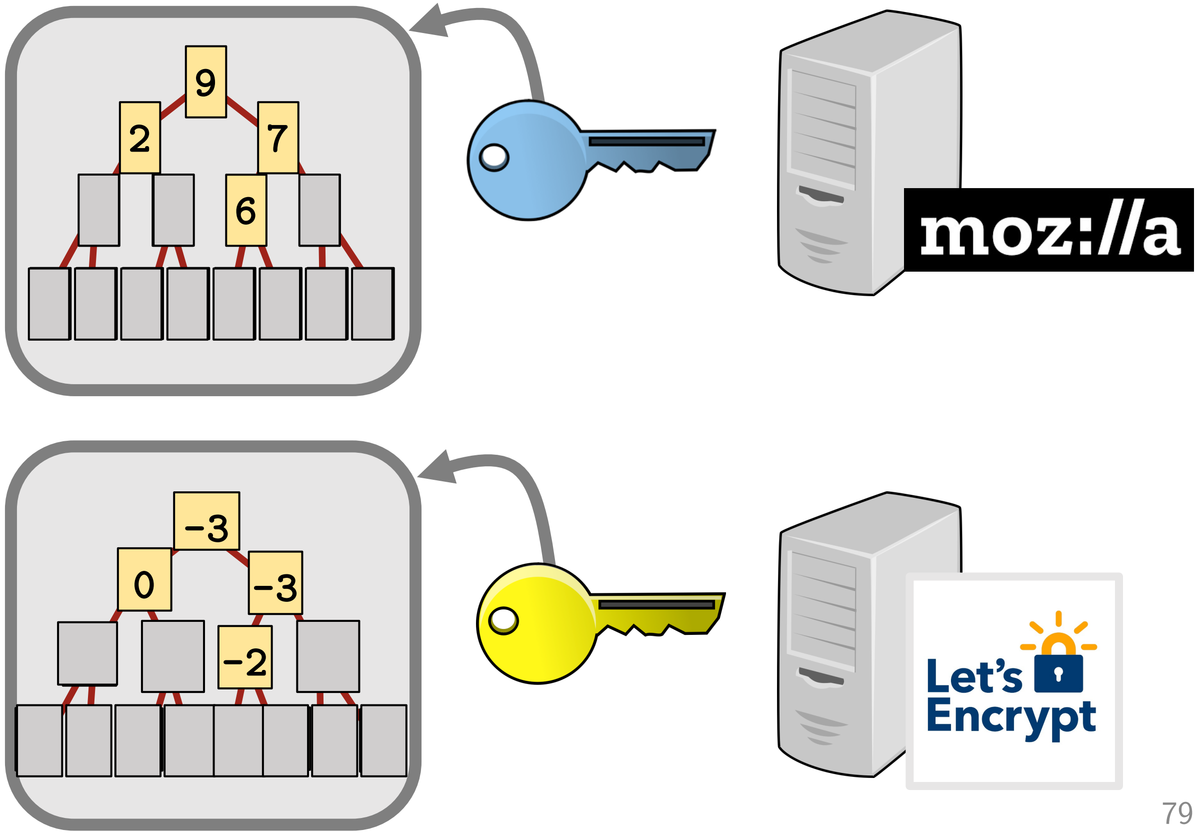

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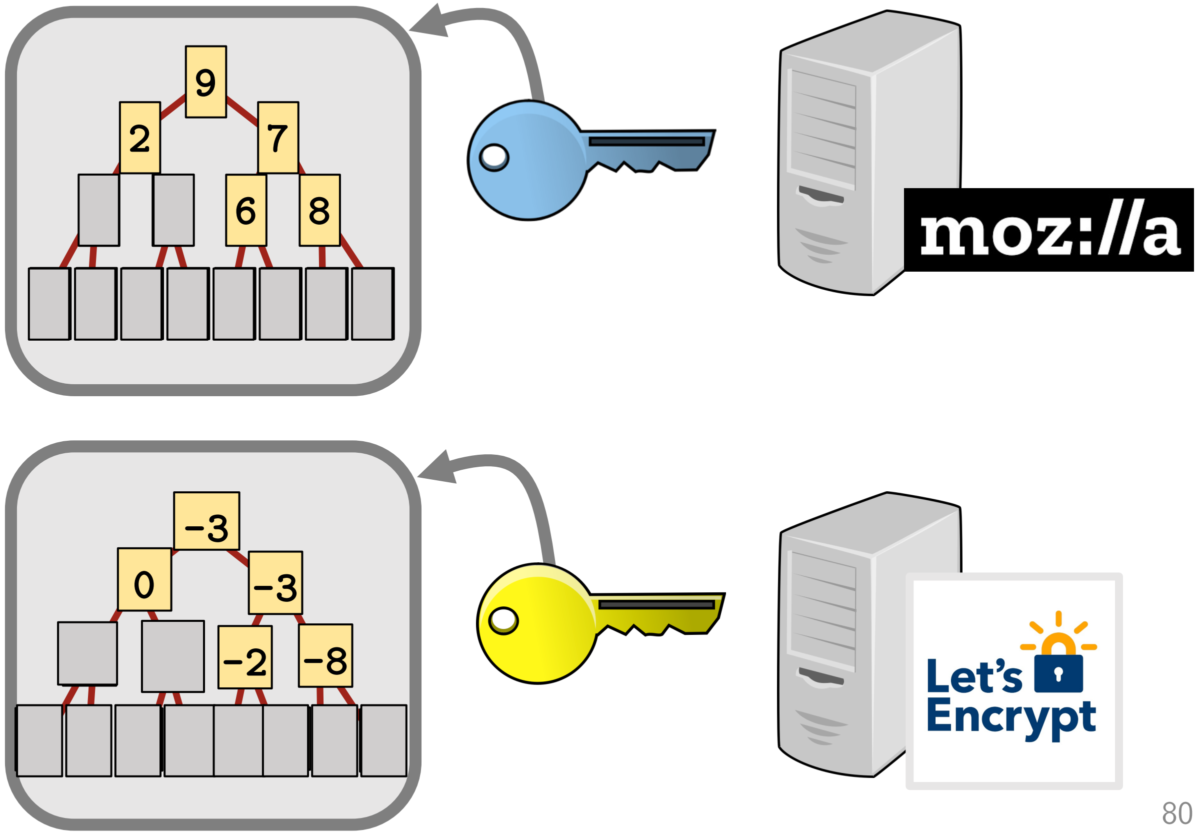

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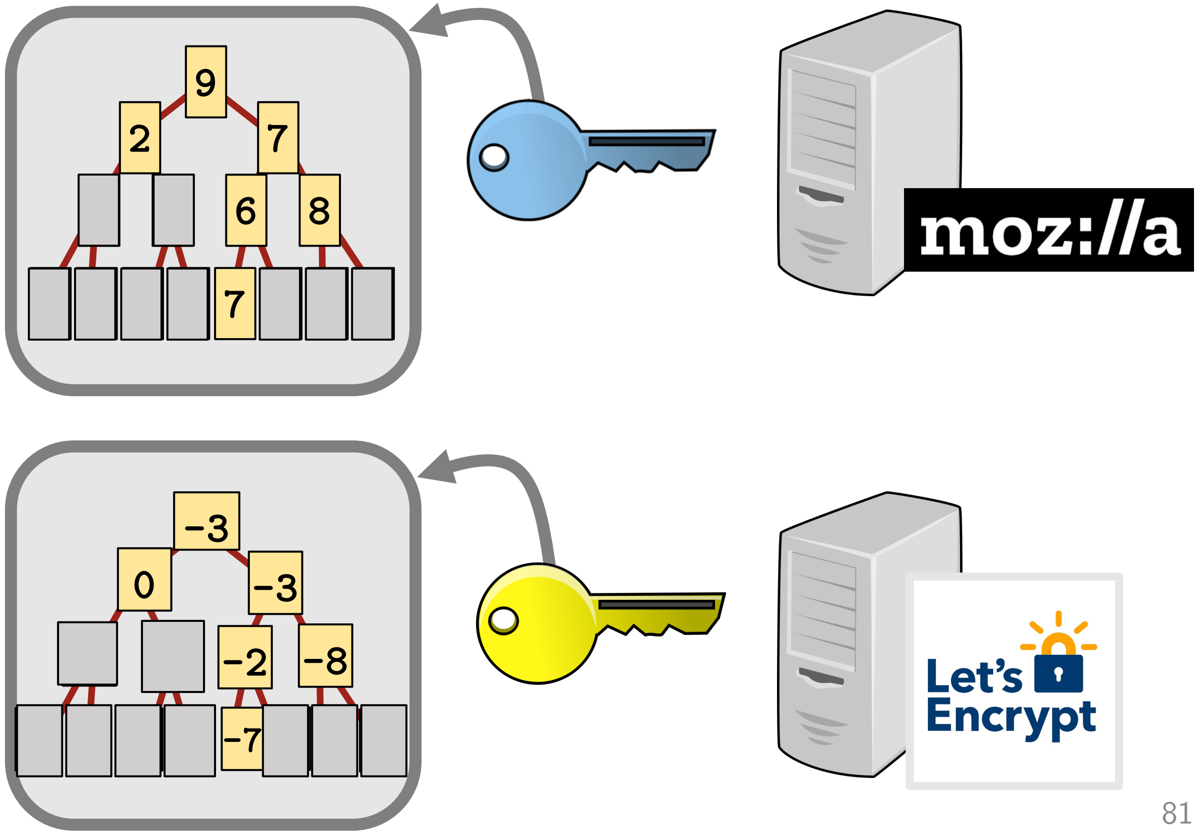

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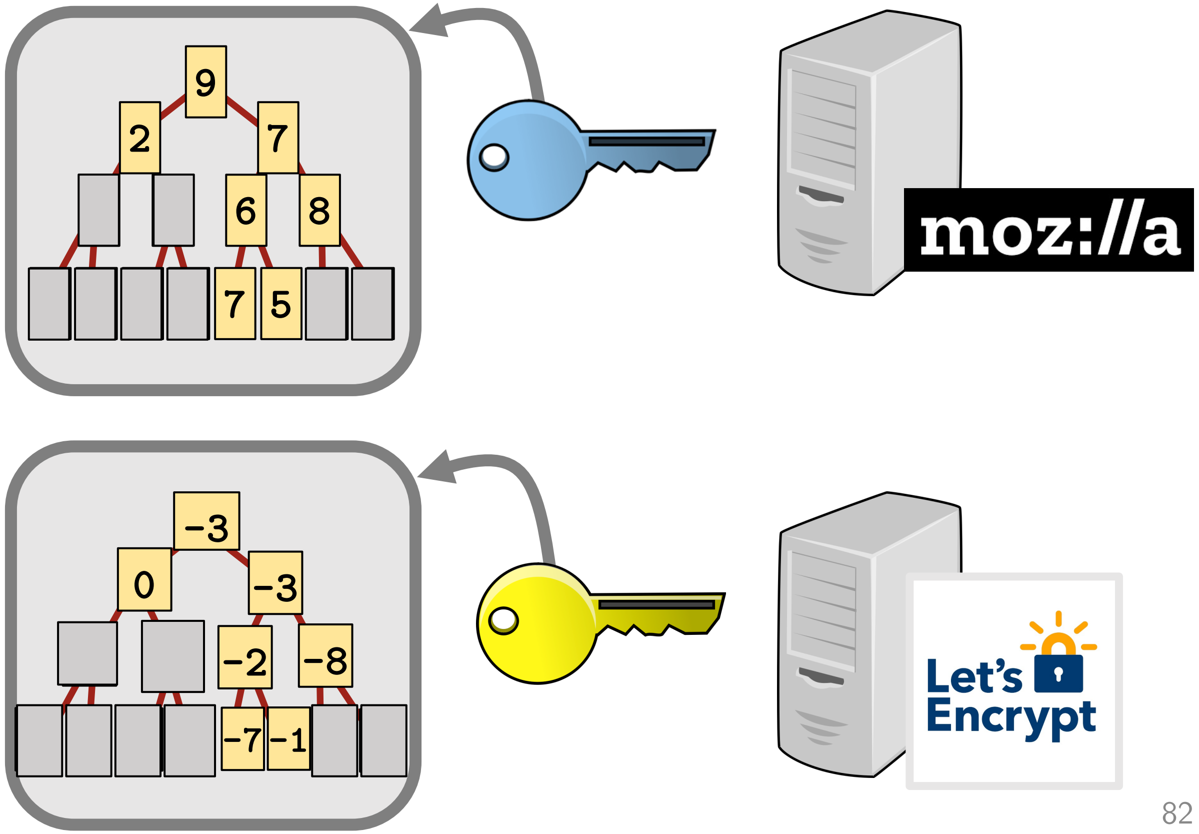

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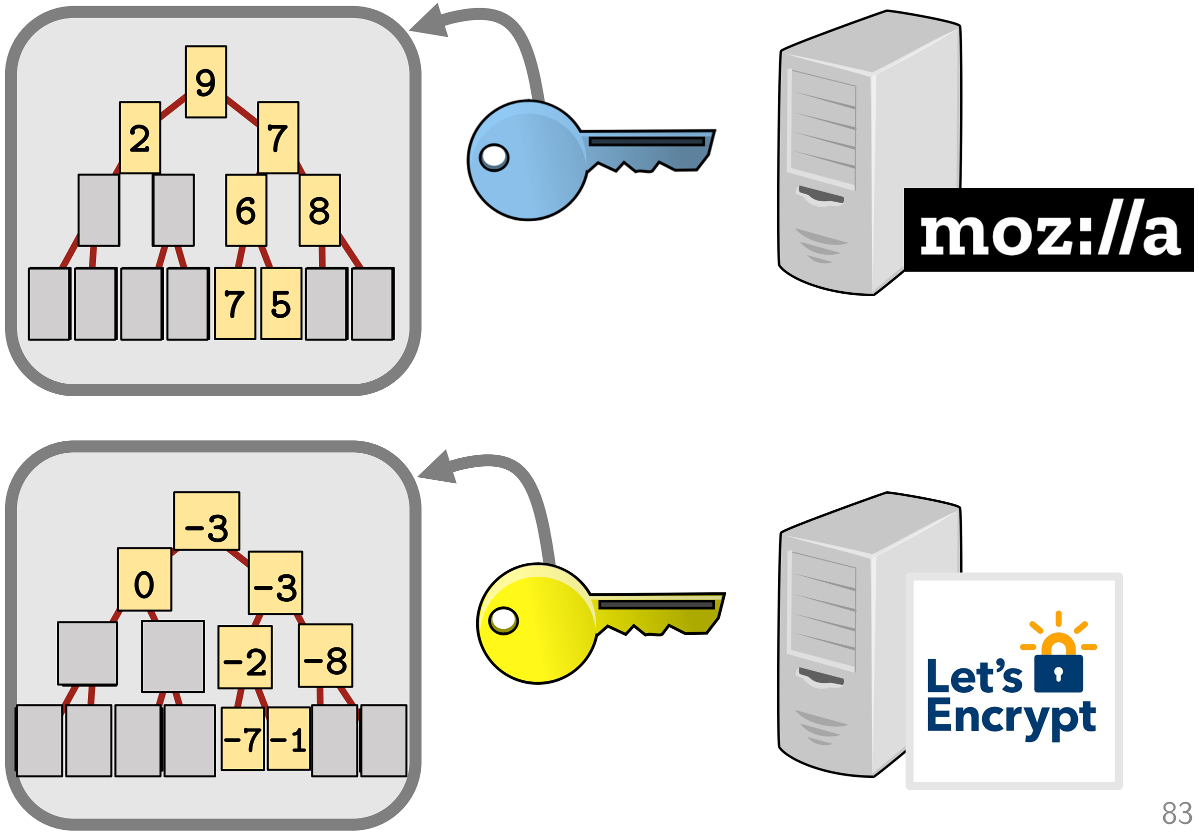

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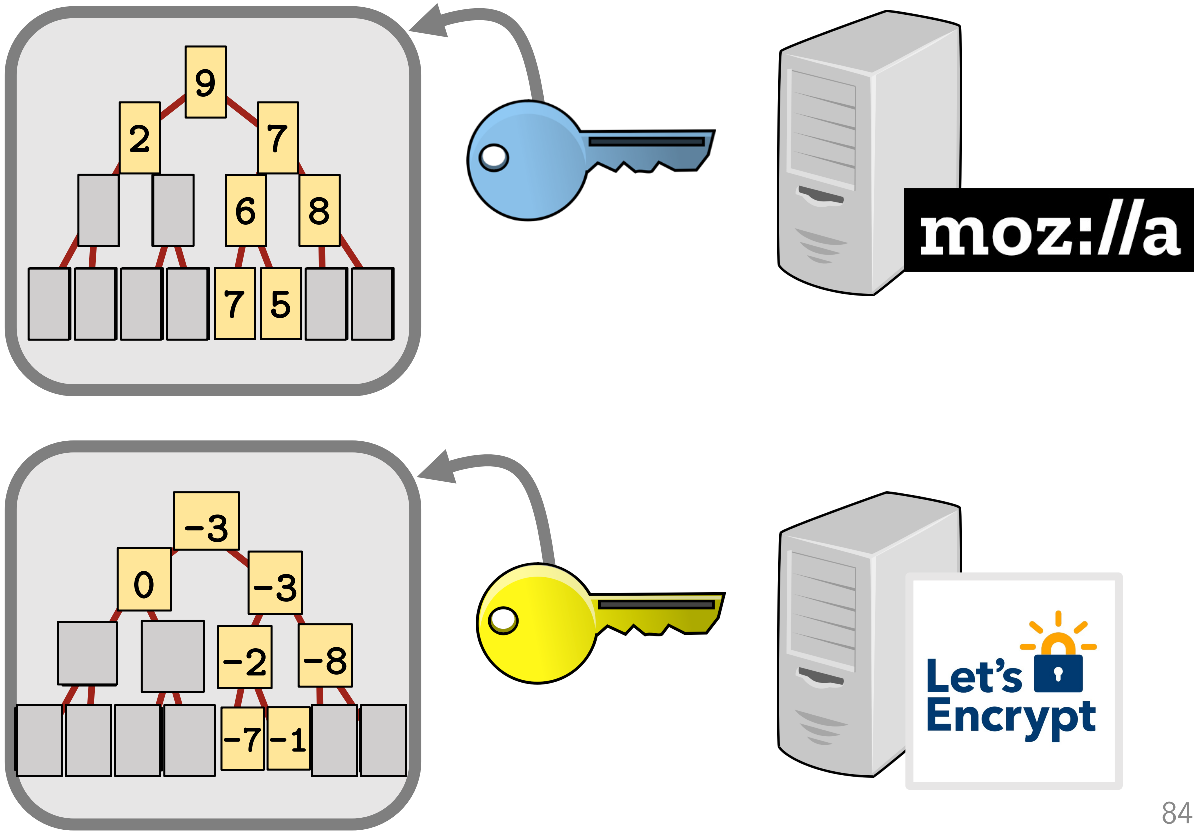

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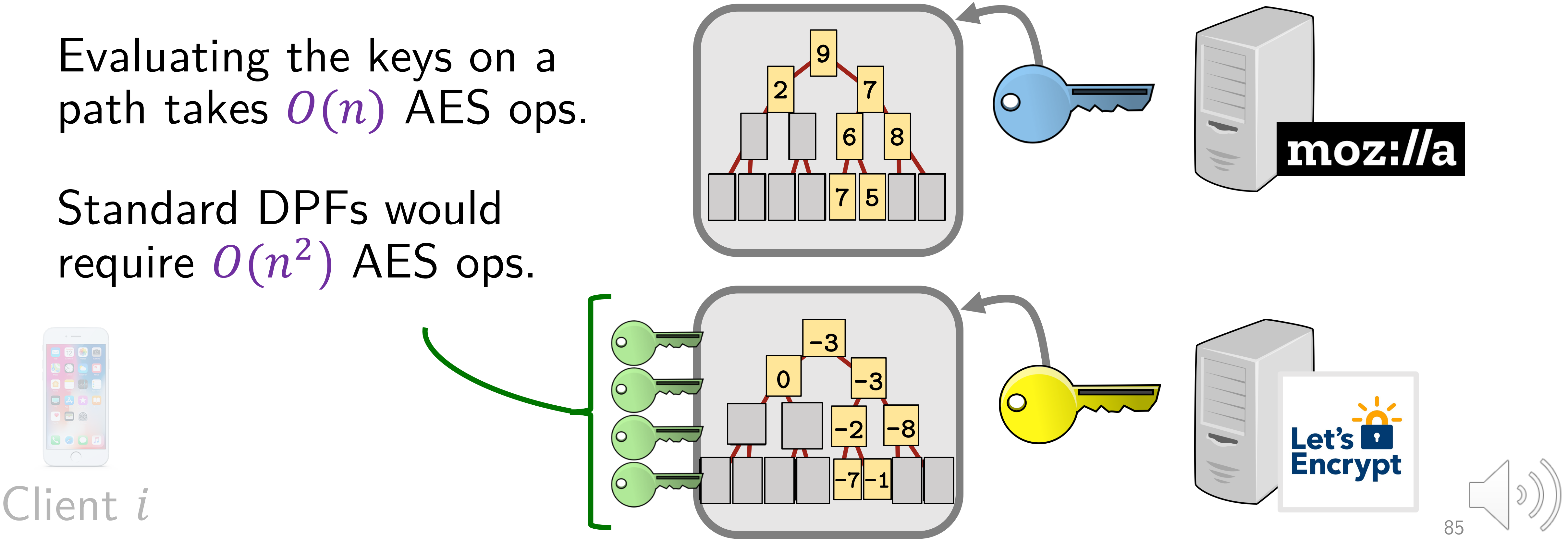

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Incremental distributed point functions (DPFs)

Construction

- Our incremental DPF builds on the DPF of BGI16
- Just requires symmetric-key operations (PRG/AES)
- The BGI16 DPF already uses a tree structure internally
  - Our construction just exposes this structure explicitly

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Malicious-secure sketching

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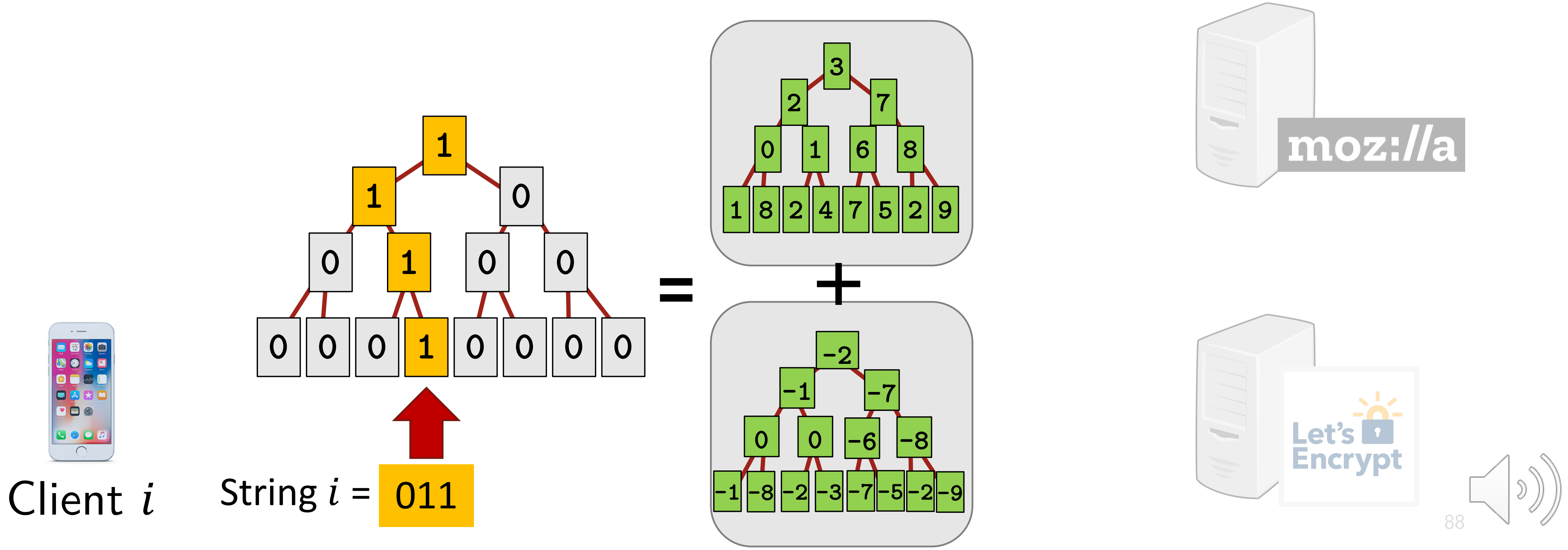
Servers cannot detect this!



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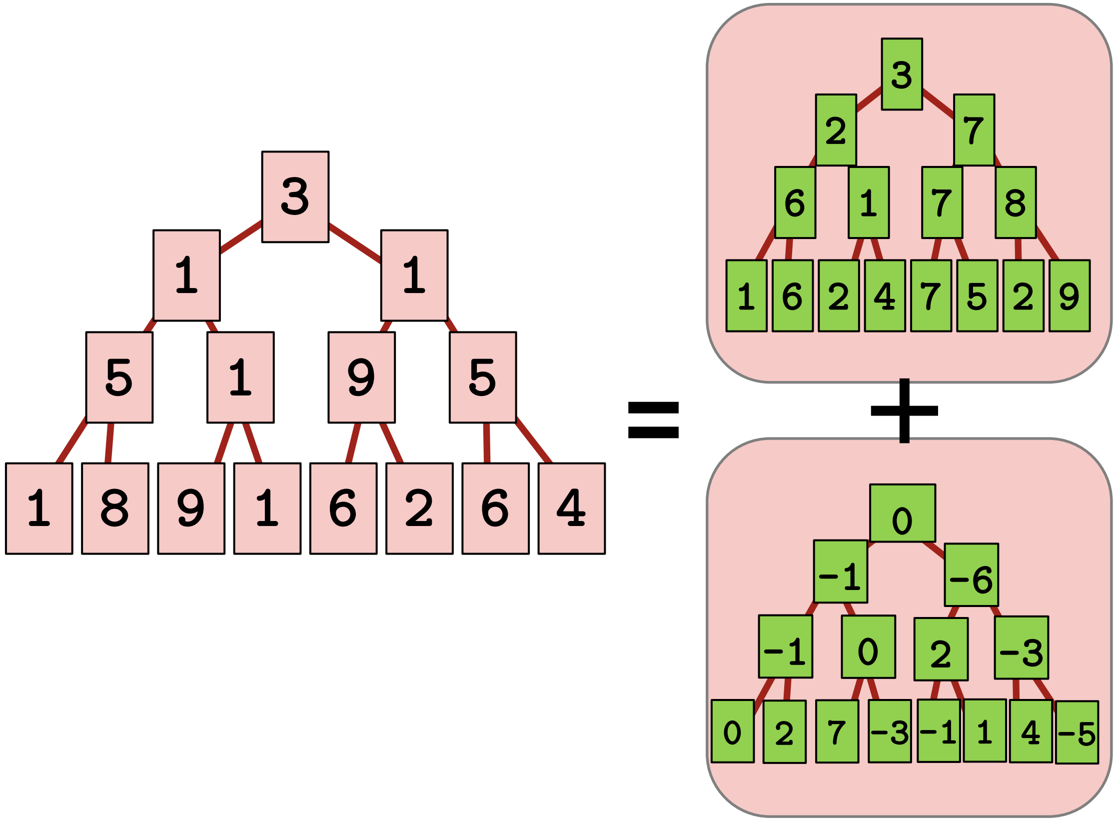

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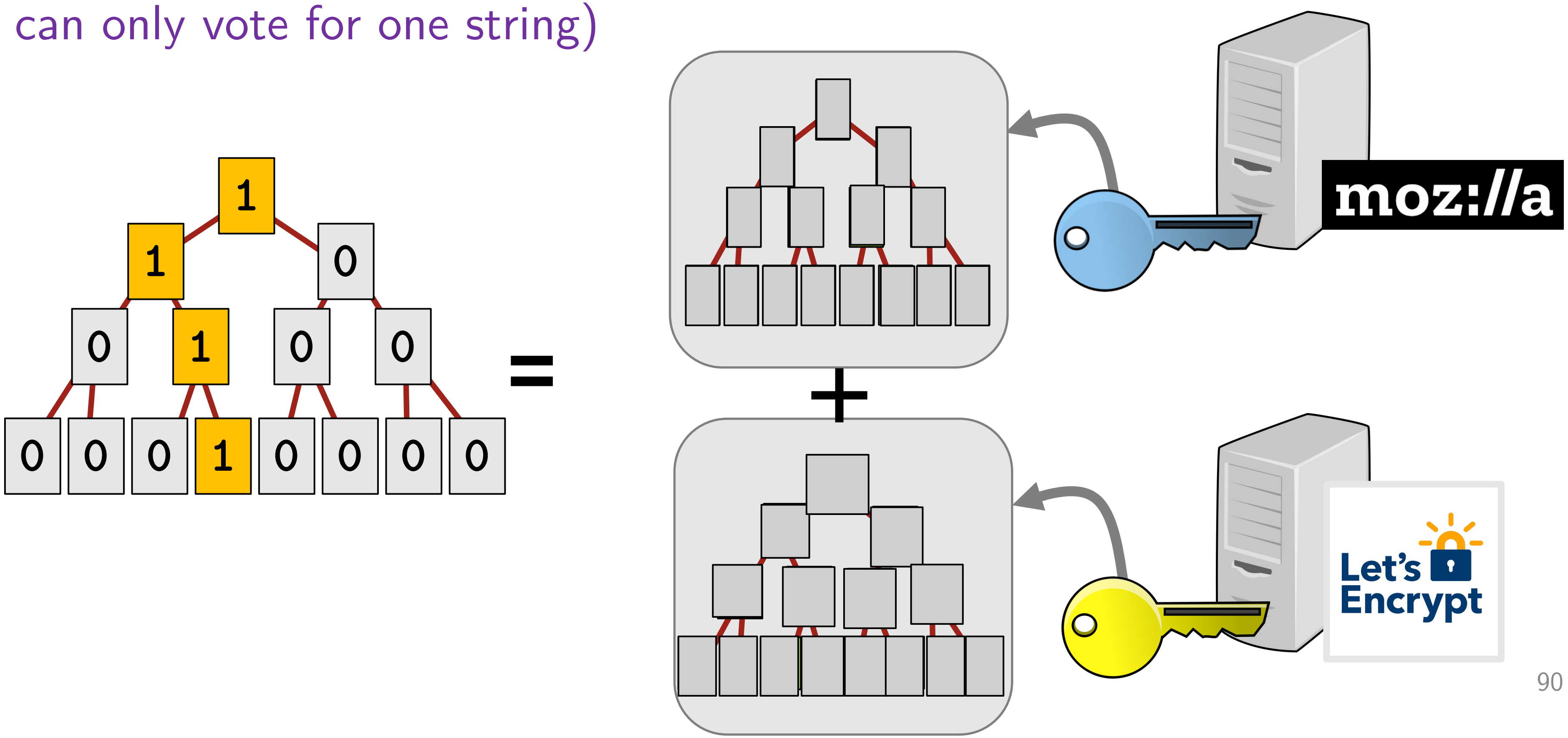

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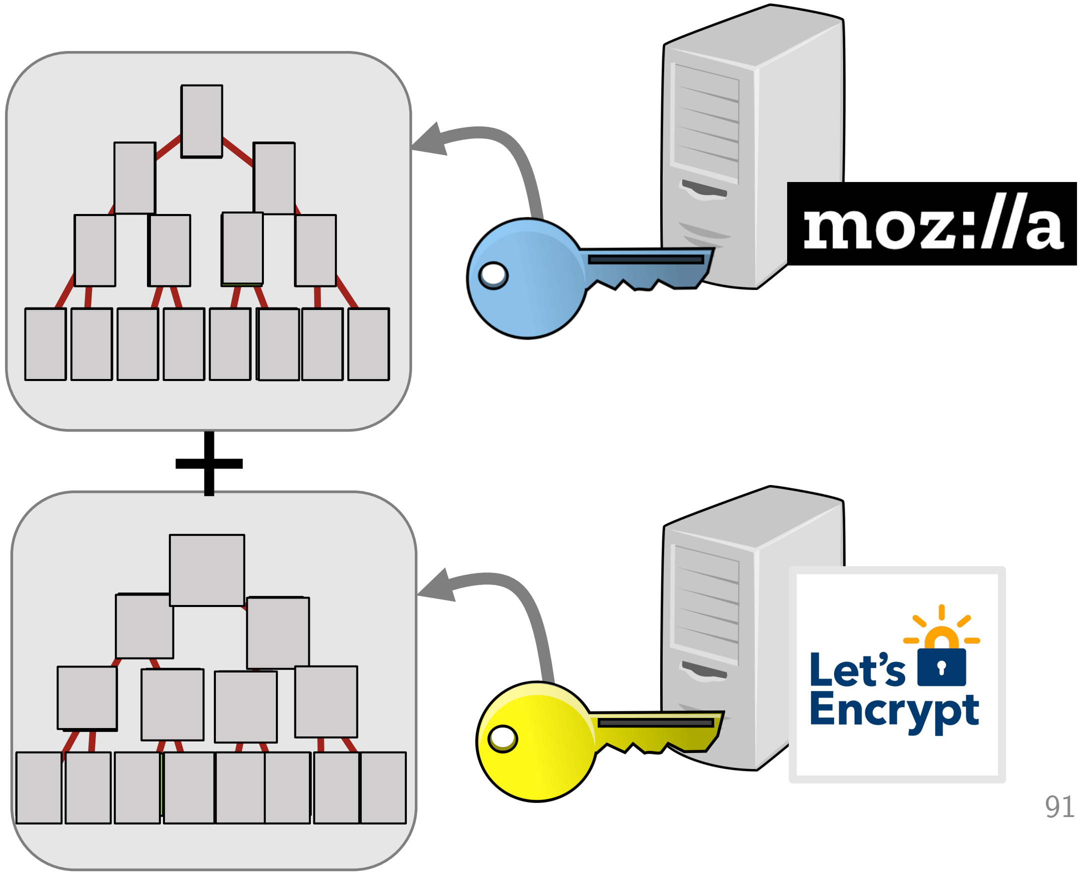

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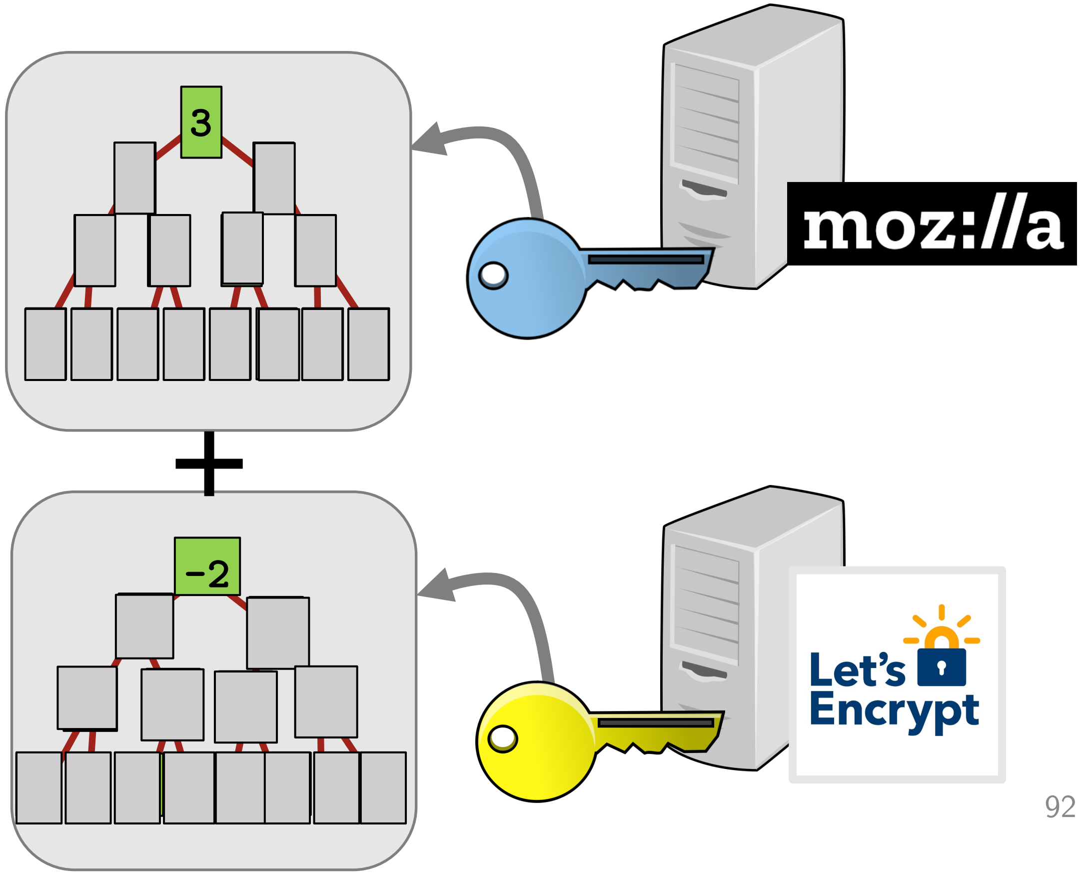

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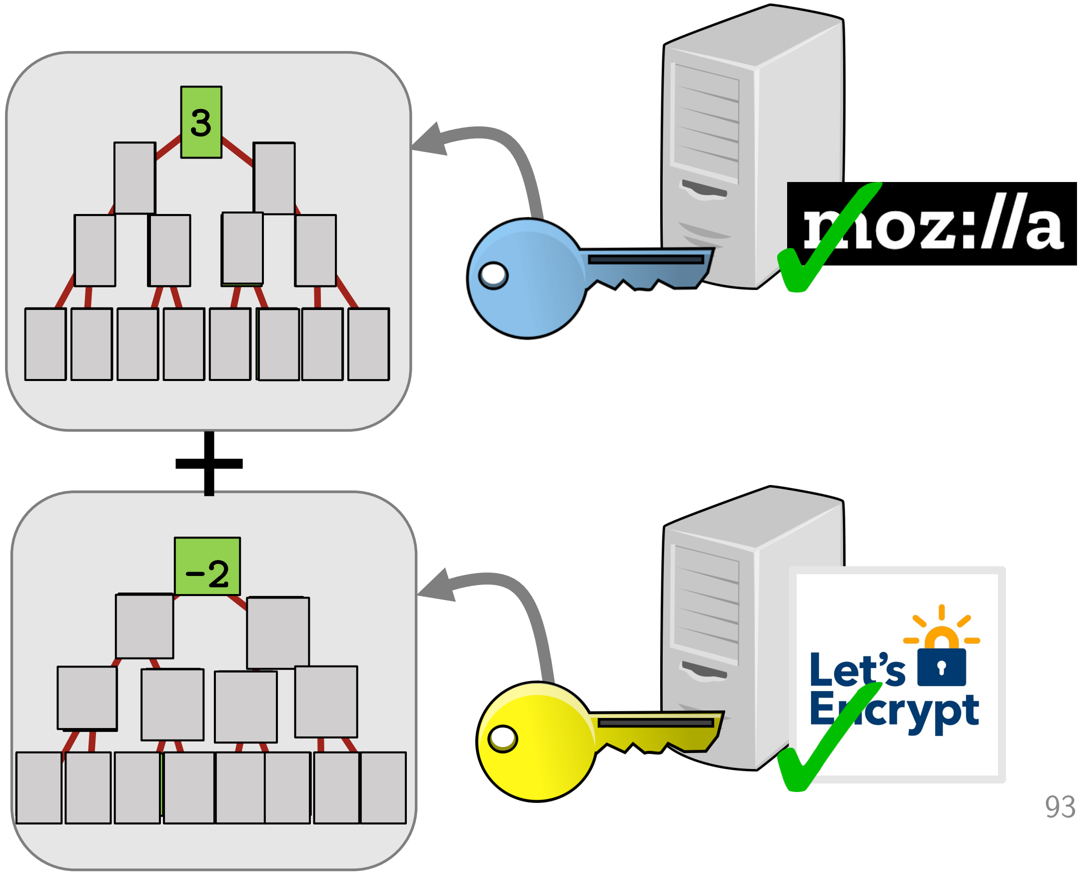

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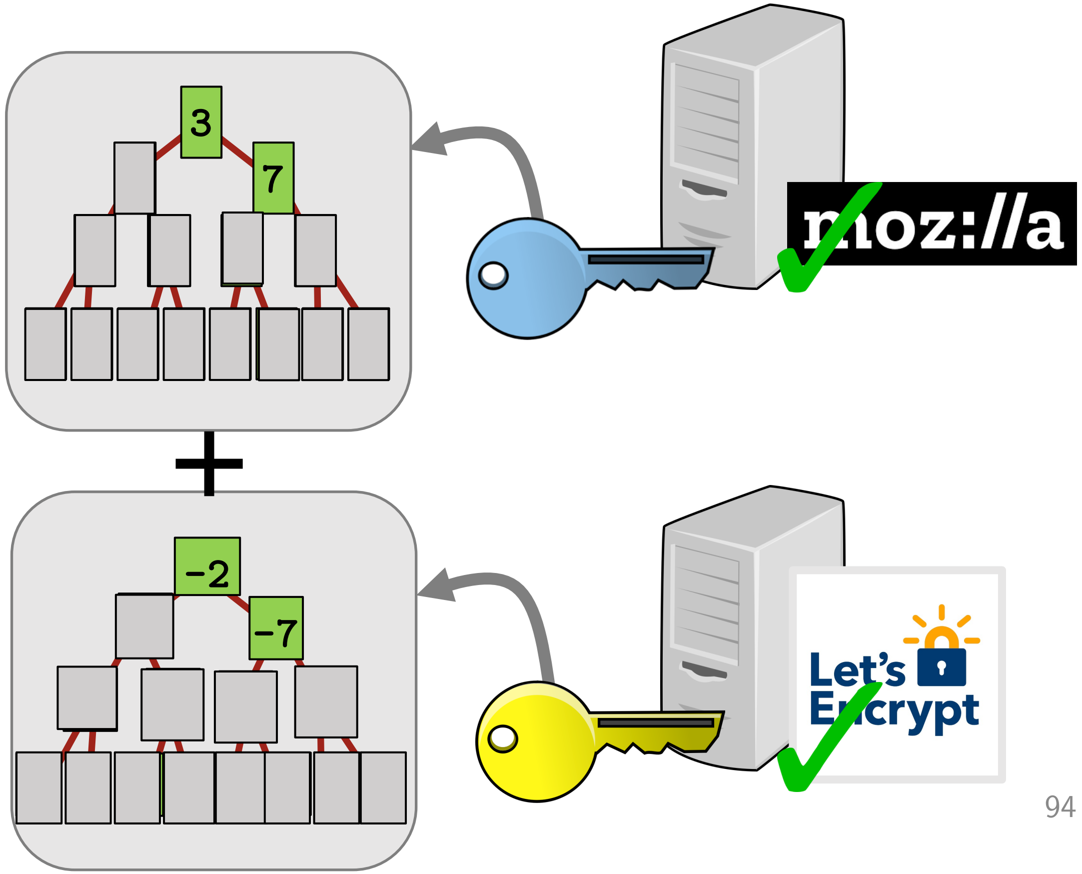

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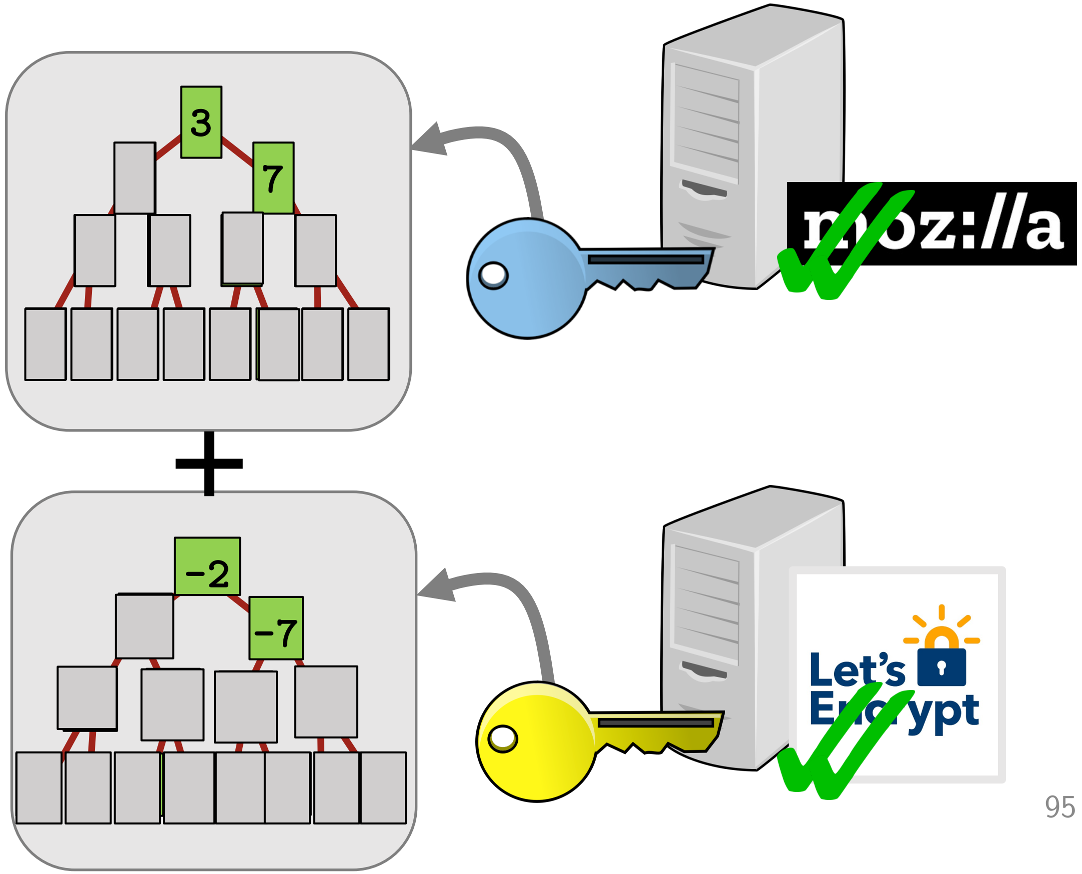
Idea: The servers run a protocol on each client’s key at each layer to check that this invariant holds.

→ Protocol checks that servers hold a secret sharing of a vector of Hamming weight one.



Contribution 2:

Malicious-secure sketching

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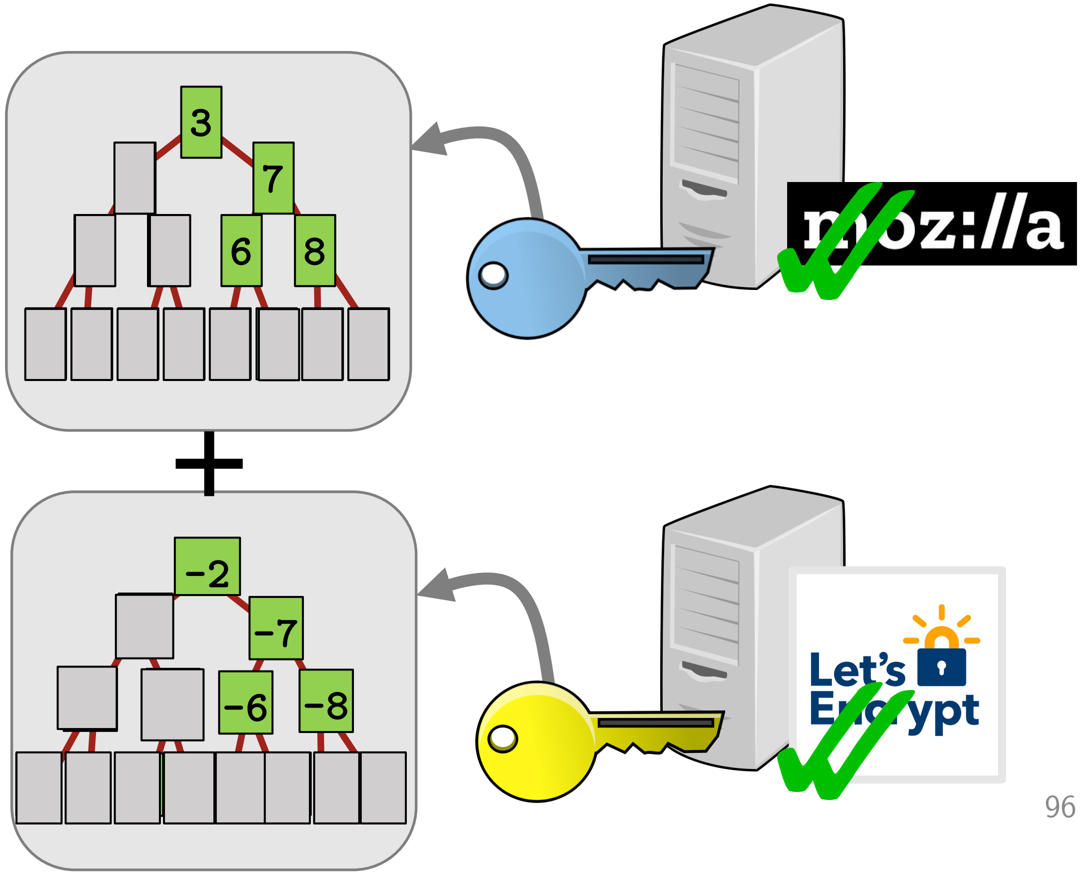

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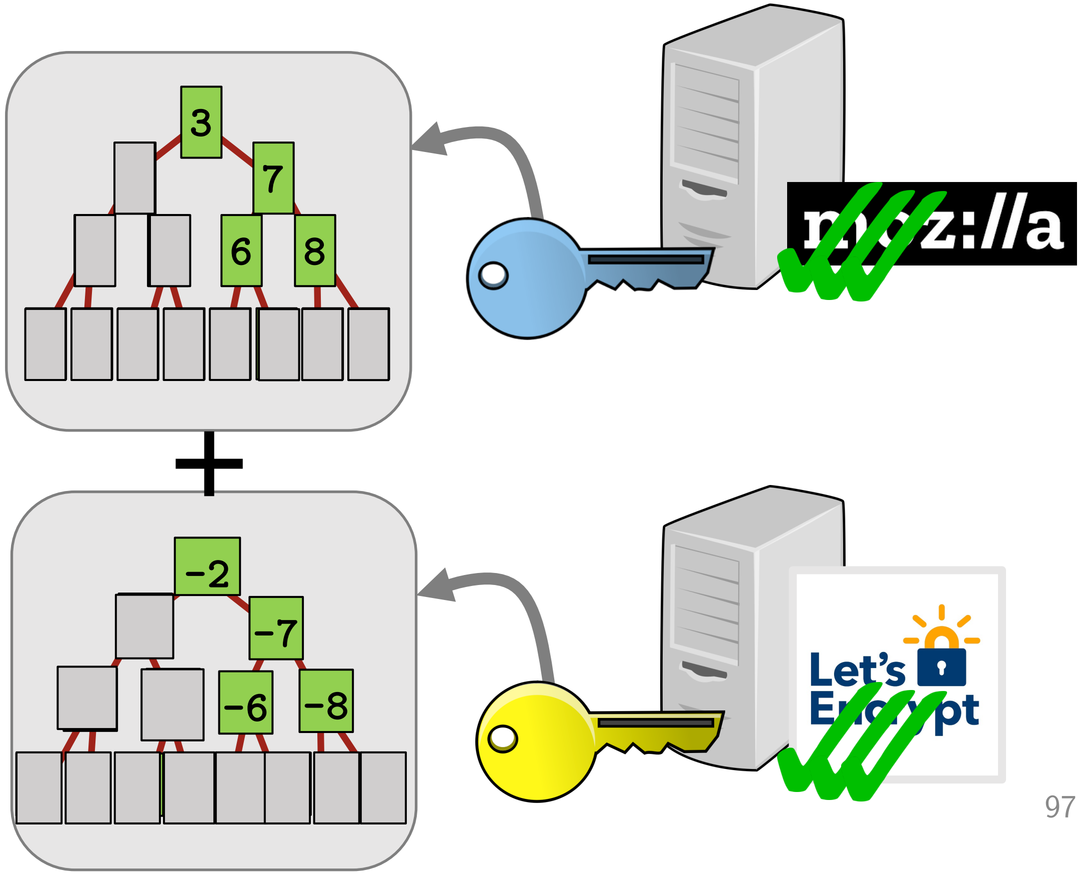

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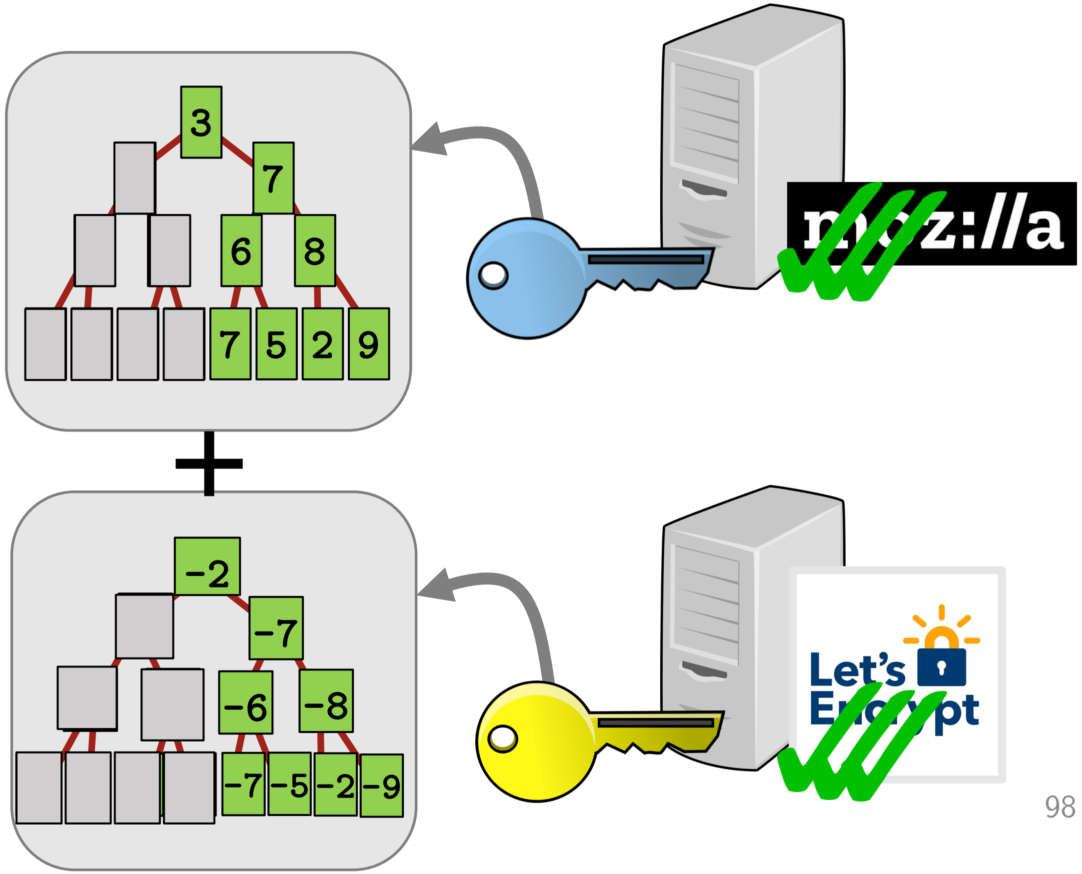

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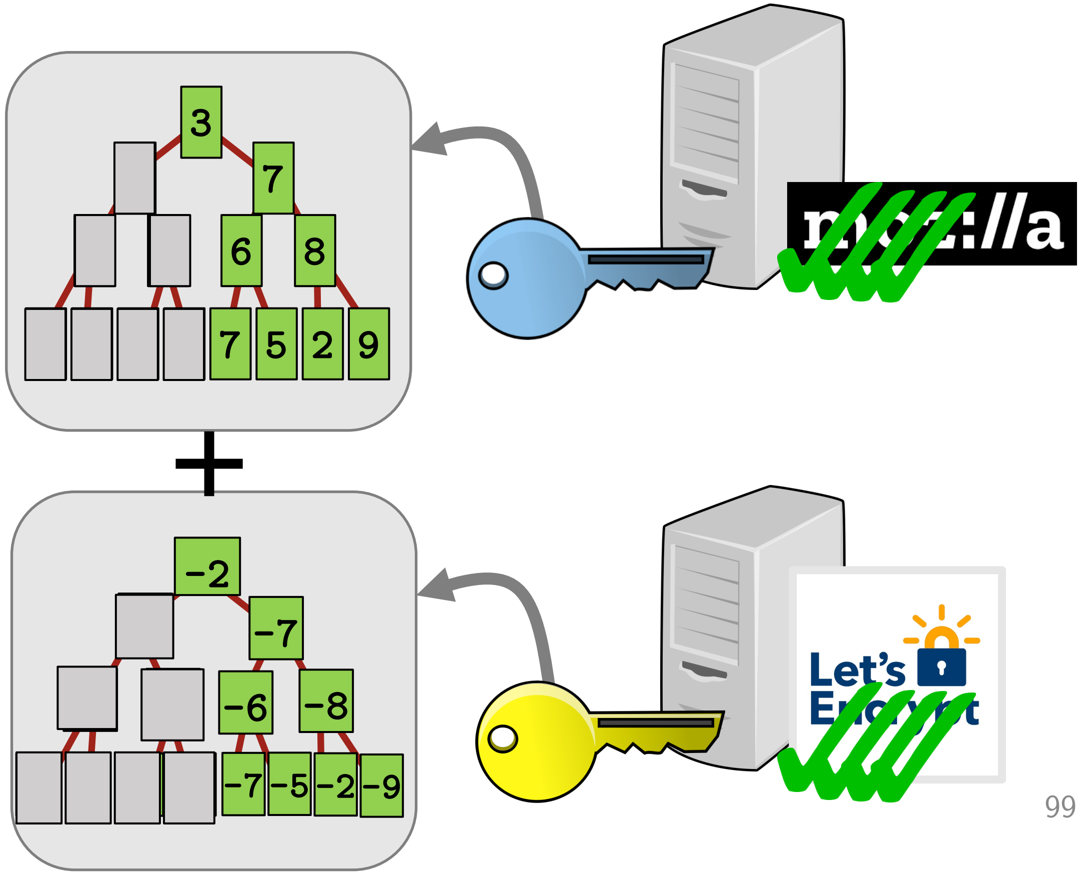

Contribution 2:

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Prior work allows testing whether shared vector has Hamming weight 1

– with security against semi-honest servers [BGI16]

– when servers can interact with the client [BBCGI19,ECZB19]

– with additional non-colluding servers [CBM15,APY20]

Our technique has none of these limitations.

Idea:

- Convert semi-honest-secure scheme [BGI16] into malicious-secure one.
- To do so, we use "algebraic manipulation detection" codes [CDFP08]

Technical challenges

✓ 1. **Each tree is exponentially large ⇒ Client cannot materialize it**

Idea: Incremental distributed point functions.
→ Succinct secret sharing of a tree with one non-zero path
→ Communication $O(\lambda n)$ instead of $O(\lambda n^2)$ [with normal DPF]

✓ 2. **Client can send malformed secret shares ⇒ Data corruption**

Idea: Malicious-secure sketching.
→ Servers can test whether a secret-shared vector is non-zero in a single coordinate.
→ No interaction with client, $O(\lambda)$ comm b/w servers.

+ Extractable distributed point functions (see paper)

This talk

- The private heavy-hitters problem
- New tools
  - Incremental distributed point functions
  - Malicious-secure sketching
- Evaluation

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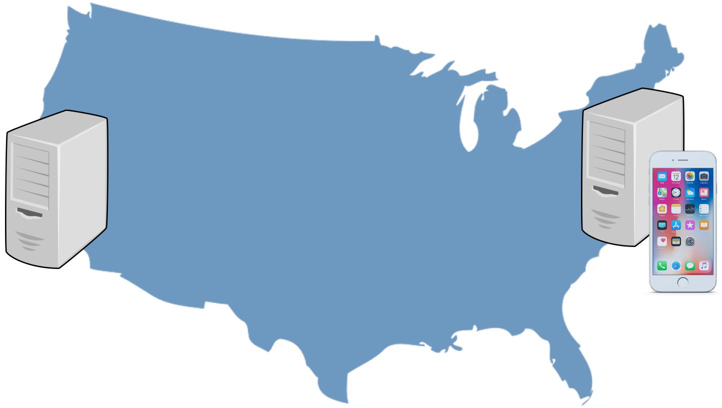
Implementation



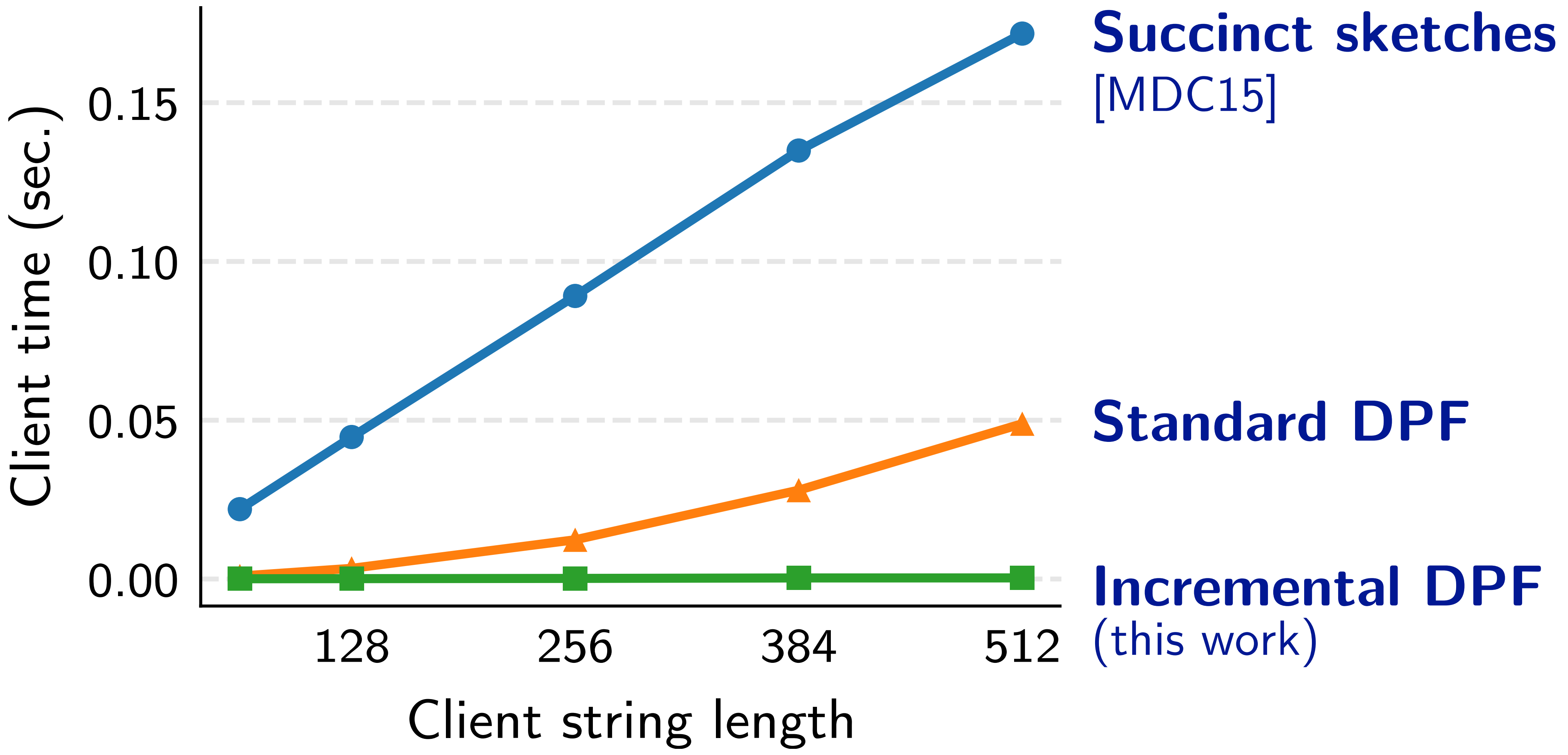
Roughly 3,500 lines of Rust

- Our open-source implementation: `github.com/henrycg/heavyhitters`
- Google's C++ implementation of incremental DPF: `github.com/google/distributed_point_functions`

Experimental setup

- Servers on opposite sides of U.S.
  - Amazon EC2 us-east-1 (VA) and us-west-1 (CA)
- Simulated clients in us-east-1
- Each server is one c4.8xlarge (36 vCPU, 60 GiB RAM)

Incremental DPFs save computation



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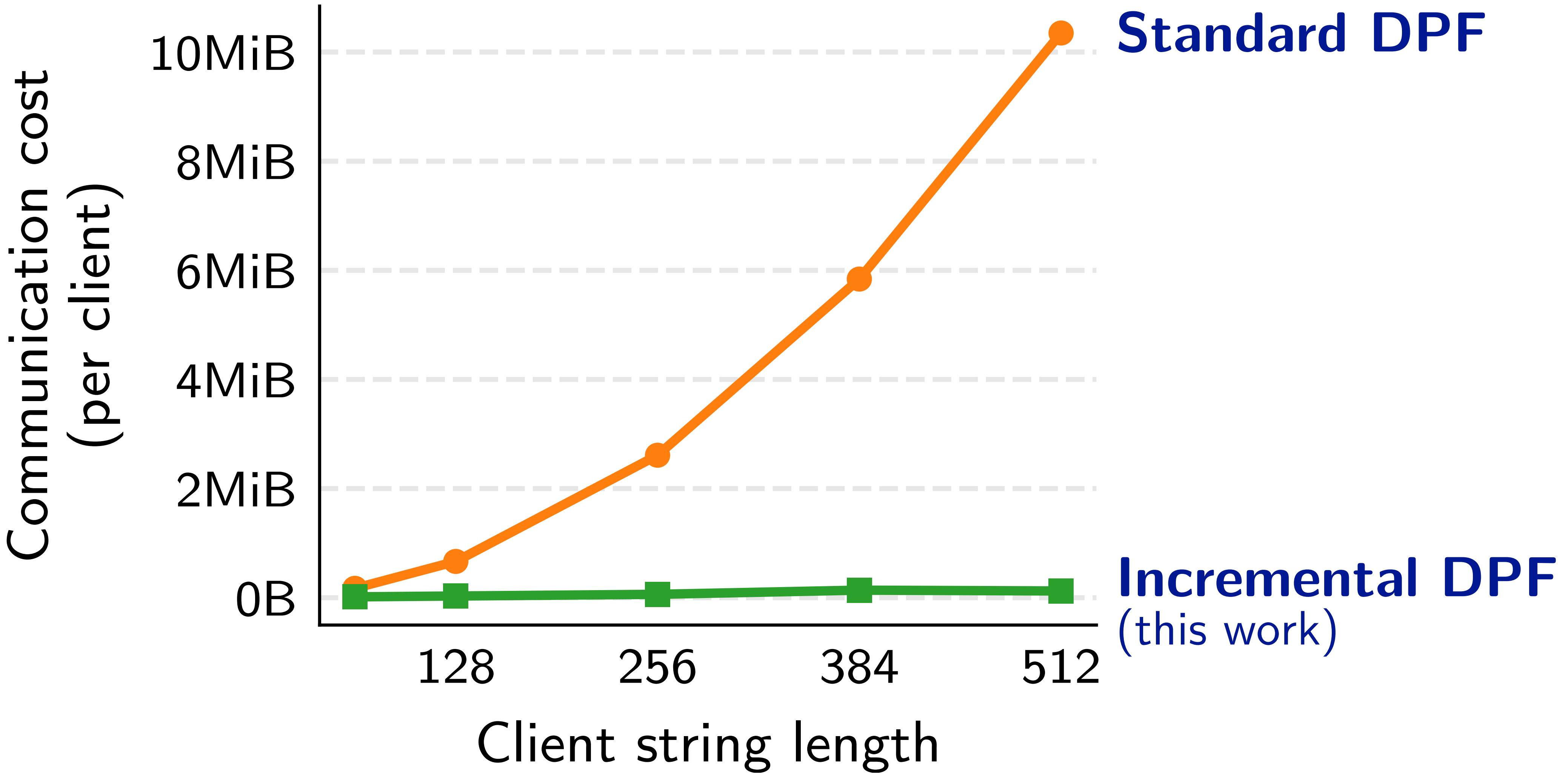

Total cost is manageable for latency-tolerant applications

Searching for top-900 heavy hitters, 256-bit strings

(Strings sampled from Zipf distribution with parameter 1.03 and support 10k. Two c4-8xlarge communicating over WAN.)

Clients	**Computation**	**Bandwidth**
100k	13.8 mins	6.5 GB
200k	27.2 mins	13.1 GB
400k	53.8 mins	26.2 GB

Completely parallelizable

Lightweight Techniques for Private Heavy Hitters

With 400,000 clients, server-side computation takes less than one hour over WAN.

Privacy against **malicious server**, correctness against **malicious clients**
→ MPC-style privacy guarantee (not local differential privacy)

New techniques

- More powerful distributed point functions: incremental & extractable (see paper)
- Tools for malicious security in systems using secret sharing
- Application to other private data-collection problems (see paper)

Paper: https://eprint.iacr.org/2021/017
Code: https://github.com/henrycg/heavyhitters