Limitations and applicability of the Lindhard model for few keV nuclear recoils

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see also: PhysRevD 91 083509 (2015)

Caveats

- This is mostly a theory talk
- No theorist has exactly solved this problem (collective many-body scattering)
- I'm no theorist

Motivation

- Measuring low-energy nuclear recoils signals is challenging
- Models can be helpful, if only as guidance
- Literature is littered with statements about how Lindhard model is not applicable at low energy, or below epsilon ~ 0.01

An experimentalist descends from an ivory tower, with the Lindhard model inscribed on two tablets

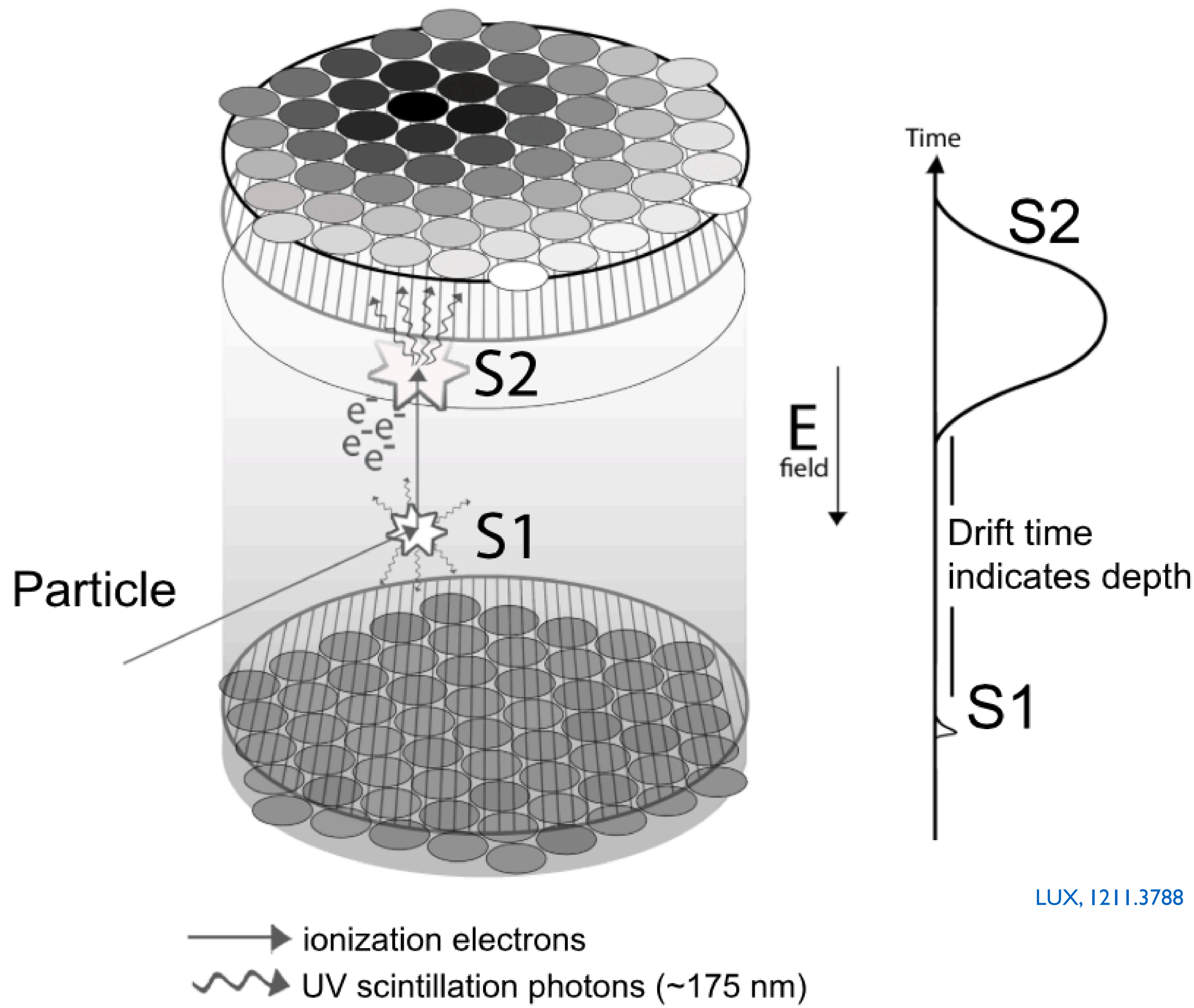
Exodus 34:29

Talk outline

- Description of the Lindhard Model
- Uncertainties
  - in nuclear scattering treatment
  - in electron scattering treatment
- Modification of the model parameterization and solution to account for atomic binding

The big picture tends to gloss over the atomic physics

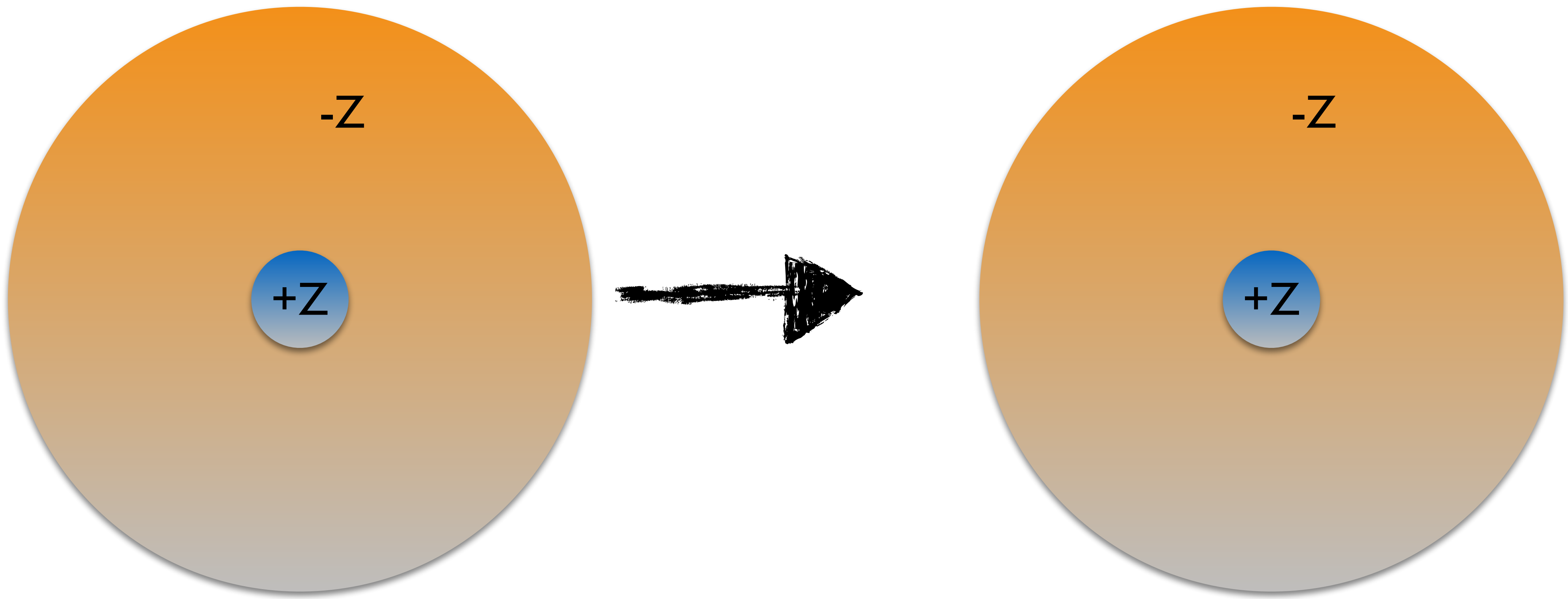
pictures tend to influence our thinking



LUX, 1211.3788

The small picture tends to oversimplify the atomic physics

i.e. this is model, not perfect physical reality

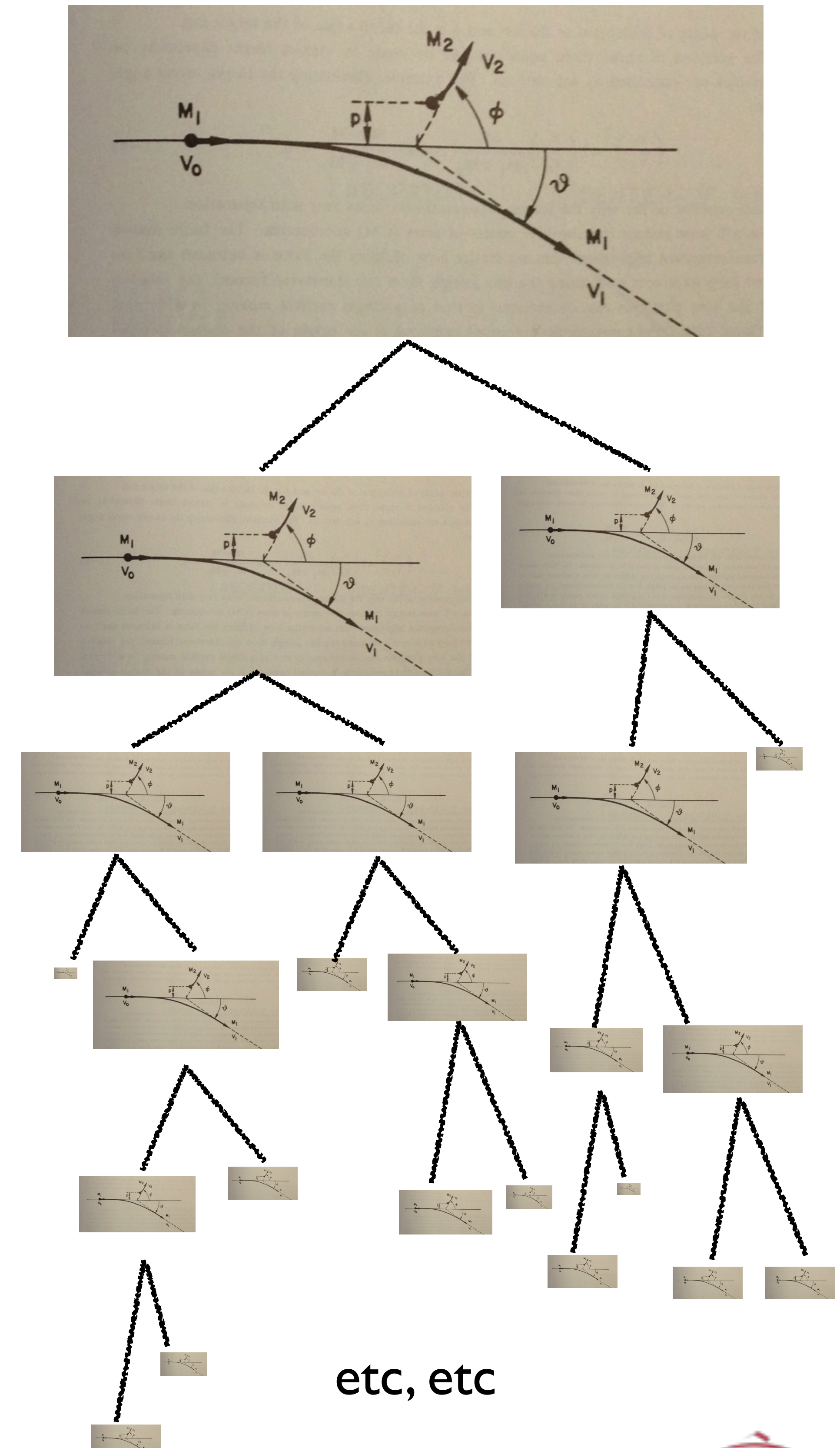


- two body screened Coulomb nuclear scattering
- average electronic scattering (stopping, really: projectile atom perturbs free electron gas)

The scattering problem is simplified to effective two-body kinematics

The origin of electronic signal:

- nucleus gets a kick (from a neutron, a neutrino, dark matter)
- atom recoils
- creates secondary recoils
- cascade continues until atoms are thermalized
- each collision might excite or ionize a target or projectile atom
- but, individual electron collisions?? too complicated. average over electronic energy losses



The Lindhard model, single slide version

reduced energy

nuclear energy loss

$$k\varepsilon^{1/2}\bar{\nu}'(\varepsilon) = \int_0^{\varepsilon^2} \frac{dt}{2t^{3/2}} f(t^{1/2}) \times \left\{ \bar{\nu}\left(\varepsilon - \frac{t}{\varepsilon}\right) - \bar{\nu}(\varepsilon) + \bar{\nu}\left(\frac{t}{\varepsilon}\right) \right\}$$

electronic energy loss

target atom after collision

projectile atom before collision

projectile atom after collision

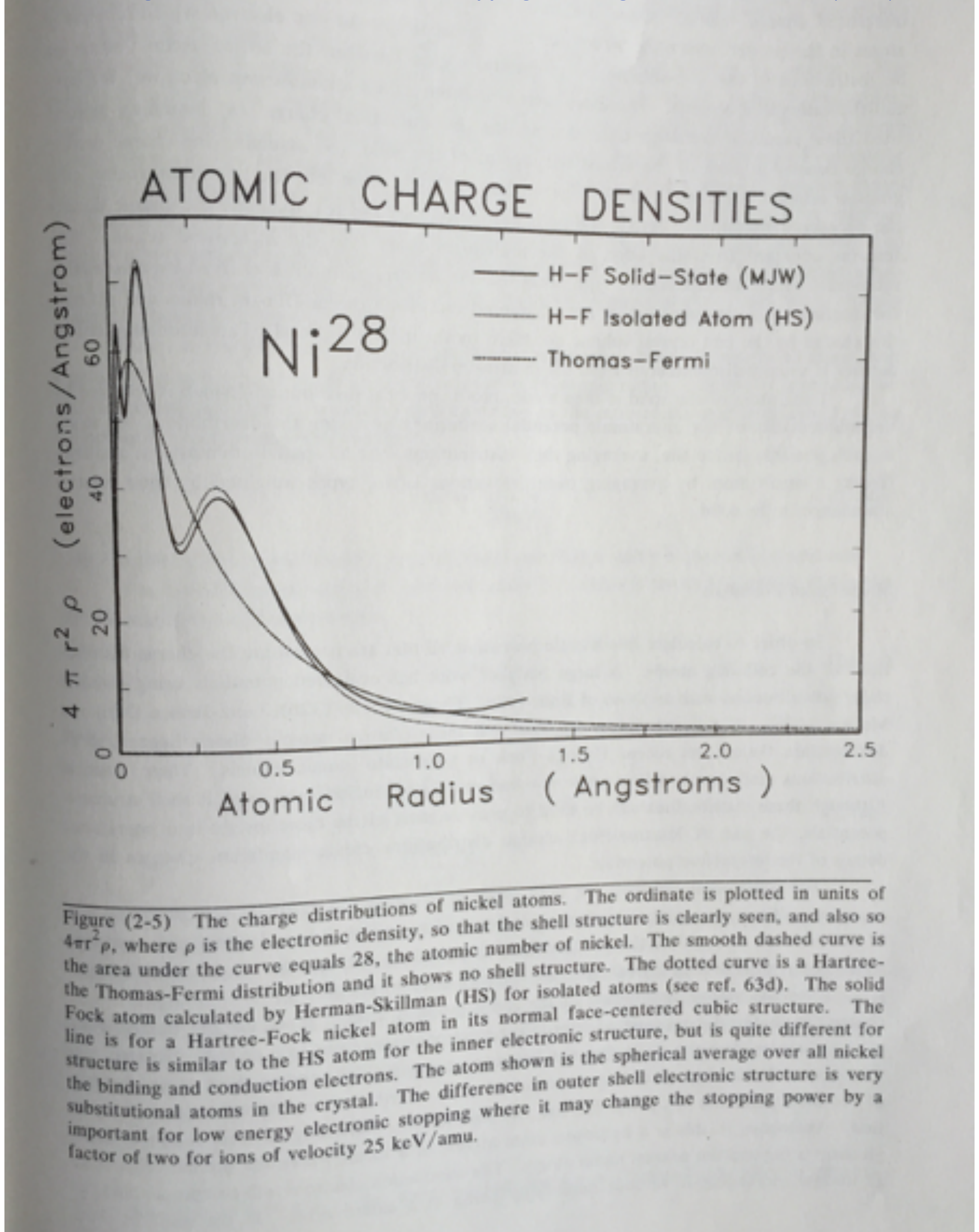
- Integrate over the cascade, obtain a solution for $\bar{\nu}$ (the energy given to atomic motion)
- A parameterization of the solution is

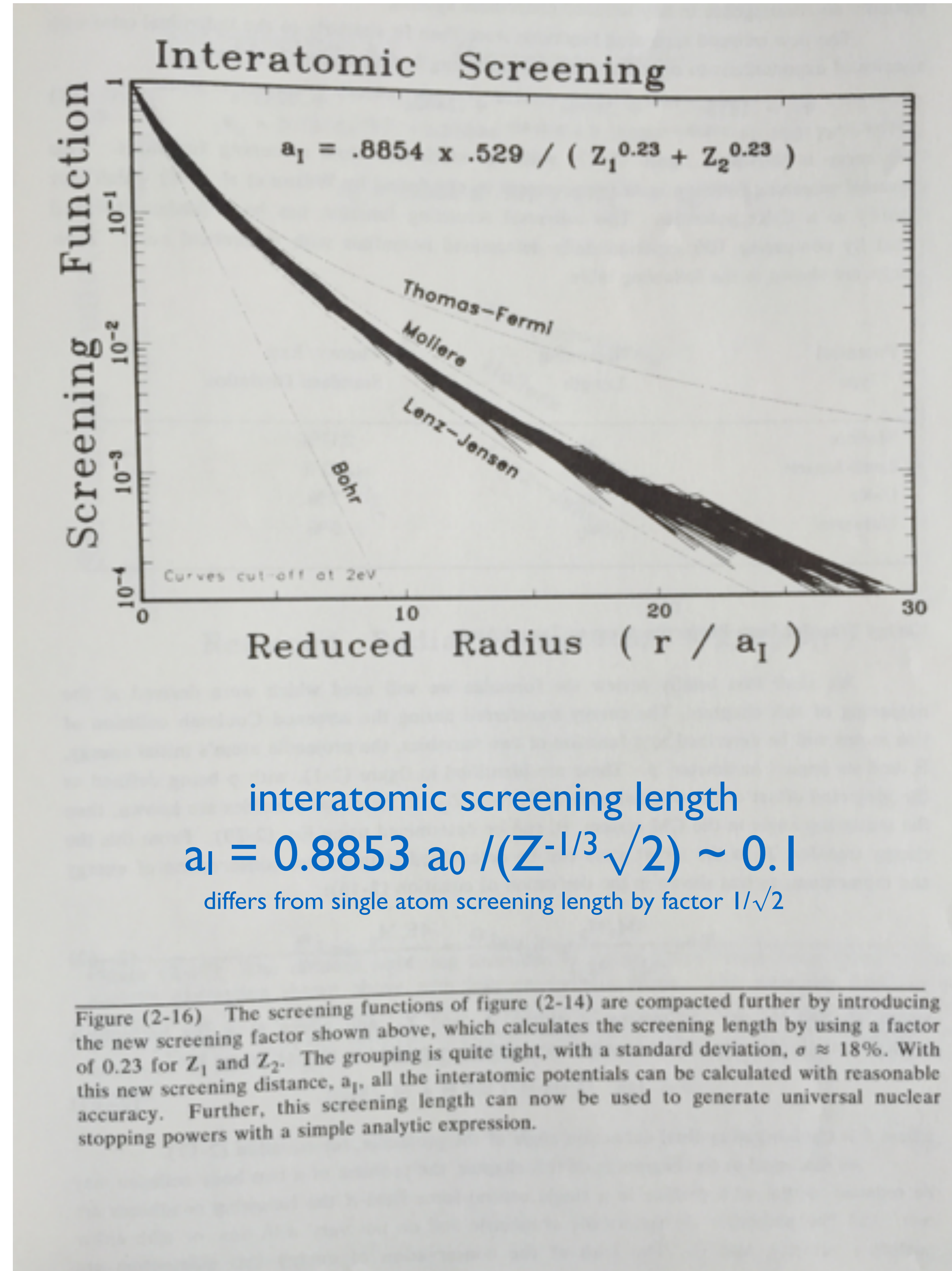
$$\bar{\nu}(\varepsilon) = \frac{\varepsilon}{1 + kg(\varepsilon)}$$ which leads directly to $$f_n \equiv \frac{\varepsilon - \bar{\nu}}{\varepsilon} = \frac{kg(\varepsilon)}{1 + kg(\varepsilon)}$$

f_n is what we usually call the quenching factor

Approximations in nuclear scattering treatment

Ziegler, Biersack, Littmark, "The stopping and range of ions in solids" (1985)





interatomic screening length

$$a_I = 0.8853\, a_0 / (Z^{-1/3} \sqrt{2}) \sim 0.1$$

differs from single atom screening length by factor $1/\sqrt{2}$

What is k?

$$S_e = (8\pi e^2 a_0 \xi Z_1 Z_2 / Z)(v/v_0), \quad = k\varepsilon^{1/2} \qquad (1)$$

where $\xi = Z_1^{1/6}$, $Z = (Z_1^{2/3} + Z_2^{2/3})^{3/2}$, and $a_0 = \hbar^2/me^2$.

- $d\varepsilon/d\rho = k\varepsilon^{1/2}$
- all calculations (there are many) predict this basic behavior

Markin et al, Phys. Rev. Lett 103 113201 (2009)

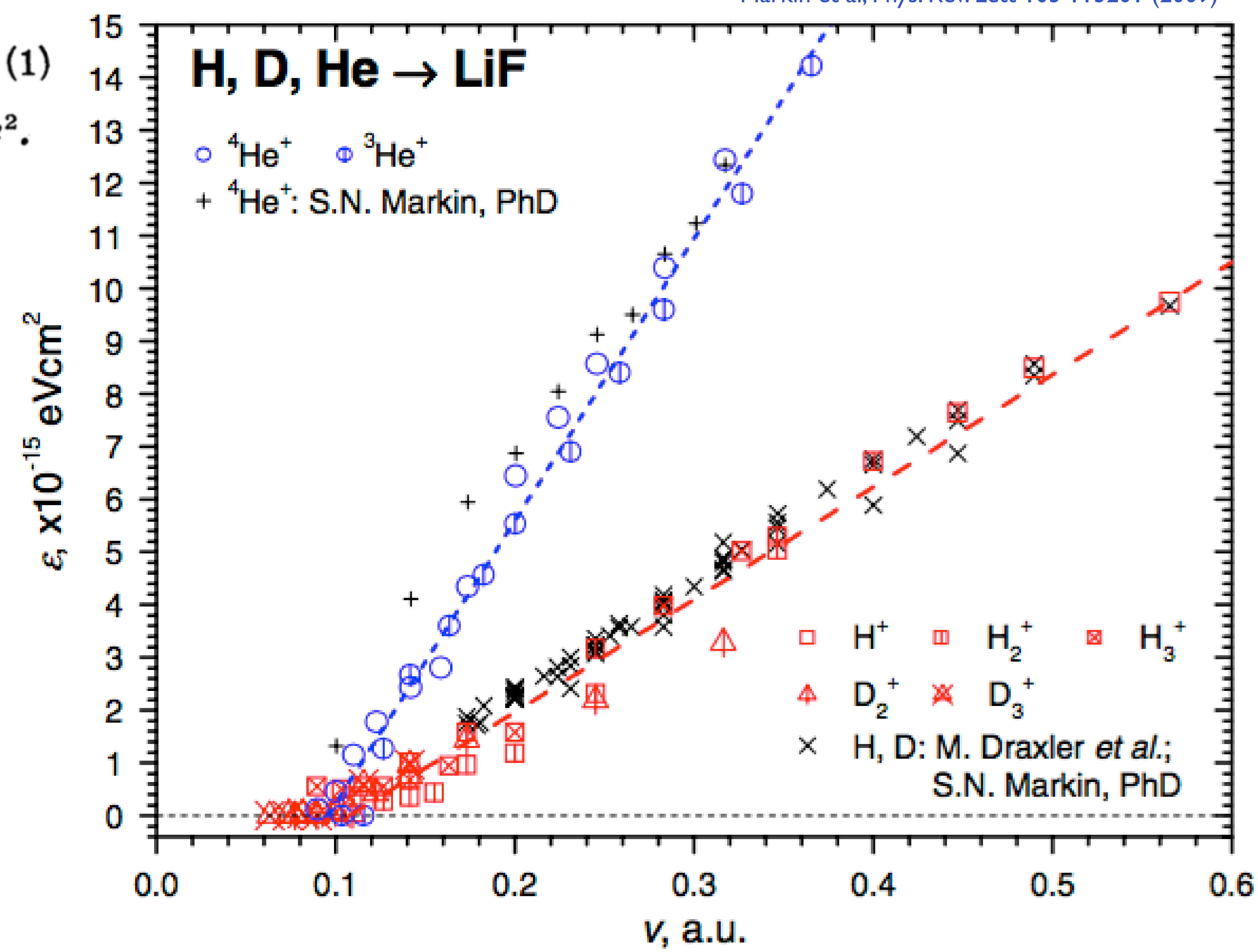


FIG. 2 (color online). Electronic stopping cross section ε of H, D, and He ions in LiF as a function of the projectile velocity v. Also shown are the data for H ions from [13] and for He ions from [24].

Approximations in electron scattering treatment

- Calculations supported by data, but
  1. not a lot of data
  2. non-zero x intercept is often observed
  3. generic expectation for semiconductors to deviate from (drop below) velocity-proportional stopping at low energies, due to band gap **(as observed by DAMIC, see Tiffenberg talk)**
- Should think of liquid nobles as large band gap insulators in this context

Markin et al, Phys. Rev. Lett 103 113201 (2009)

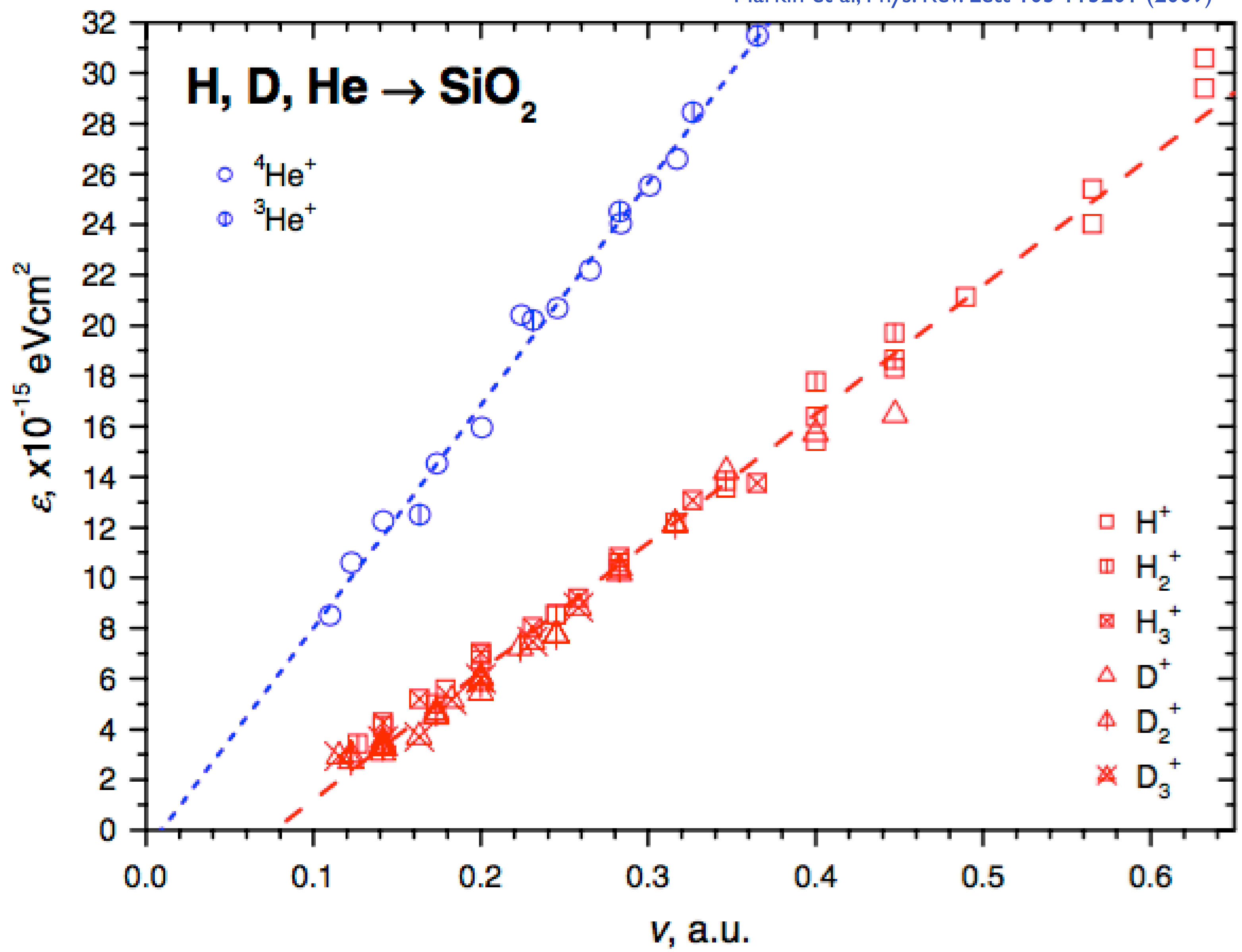


FIG. 4 (color online). Electronic stopping cross section ε of H, D, and He ions in SiO_2 as a function of projectile velocity v.

Variations in electron scattering ("electronic stopping") calculations

- Large uncertainty in k is possible
- Ge happens to be at a sweet spot (all calculations converge)
- Si appears to be approximately sweet
- Liquid nobles may differ (drastically) from naive Lindhard k

Land et al, Phys. Rev. A 16 492 (1977)

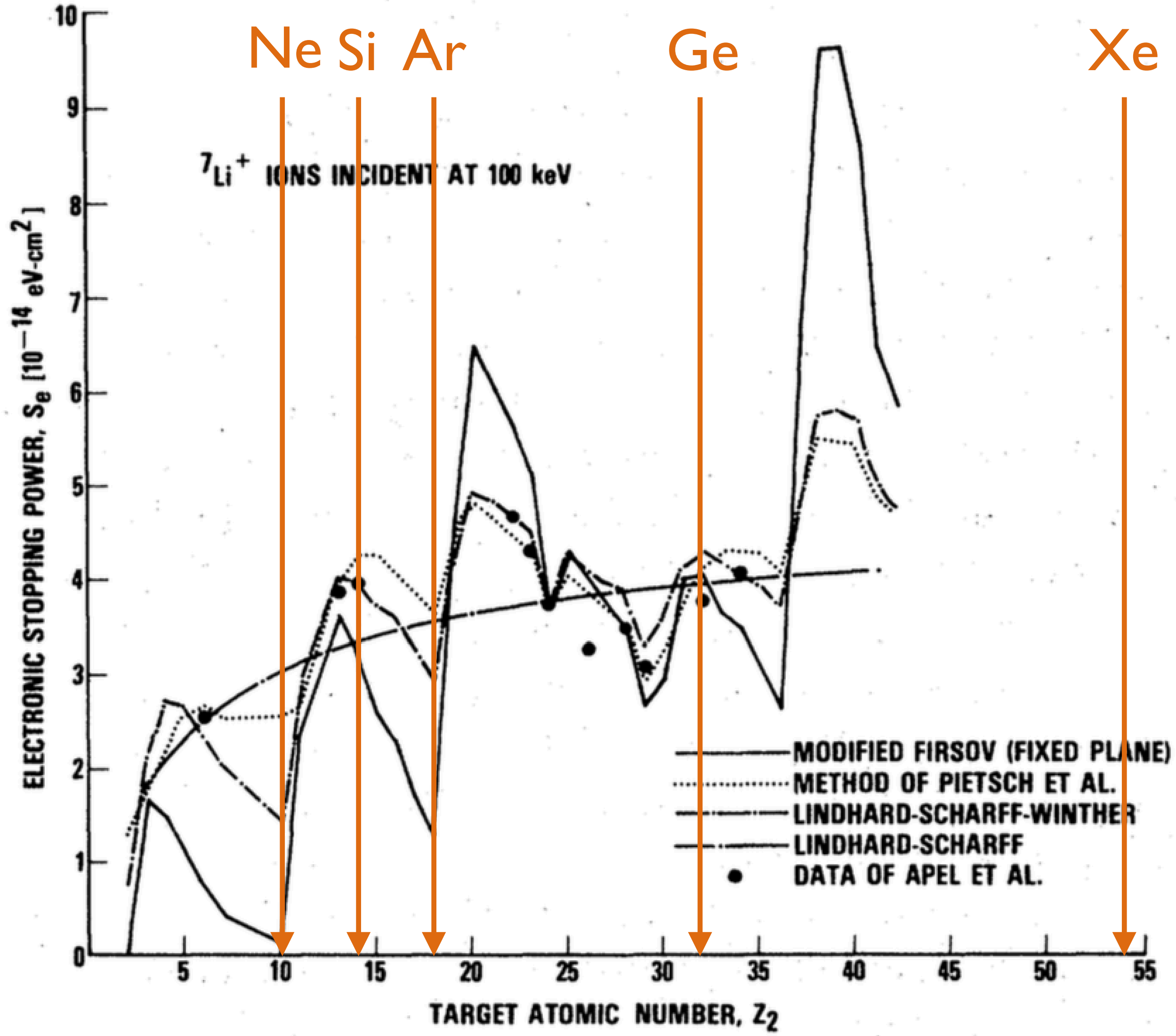


FIG. 4. Comparison of theoretical results for the electronic stopping power of 100-keV $^7Li^+$ ions based upon the modified Firsov method, Lindhard-Scharff-Winther method, and the method of Pietsch *et al.* Experimental data are included.

Recap

- The Lindhard model makes numerous approximations in order to distill solid state atomic scattering into a tractable problem
  - this results in quantitative predictions that appear to agree fairly well for a number of 4 frequently used homogenous targets
  - it is difficult to accurately quantify the uncertainties, but a range can be inferred
- The low velocity behavior of electronic stopping is expected to decrease in materials with a band gap **(i.e. materials from which one might make a detector !!)**
  - this is difficult to quantify and data are sparse
- The model as widely disseminated does not account for atomic binding
  - intuitively this must make a difference at low energy
  - it can be re-instated in model…

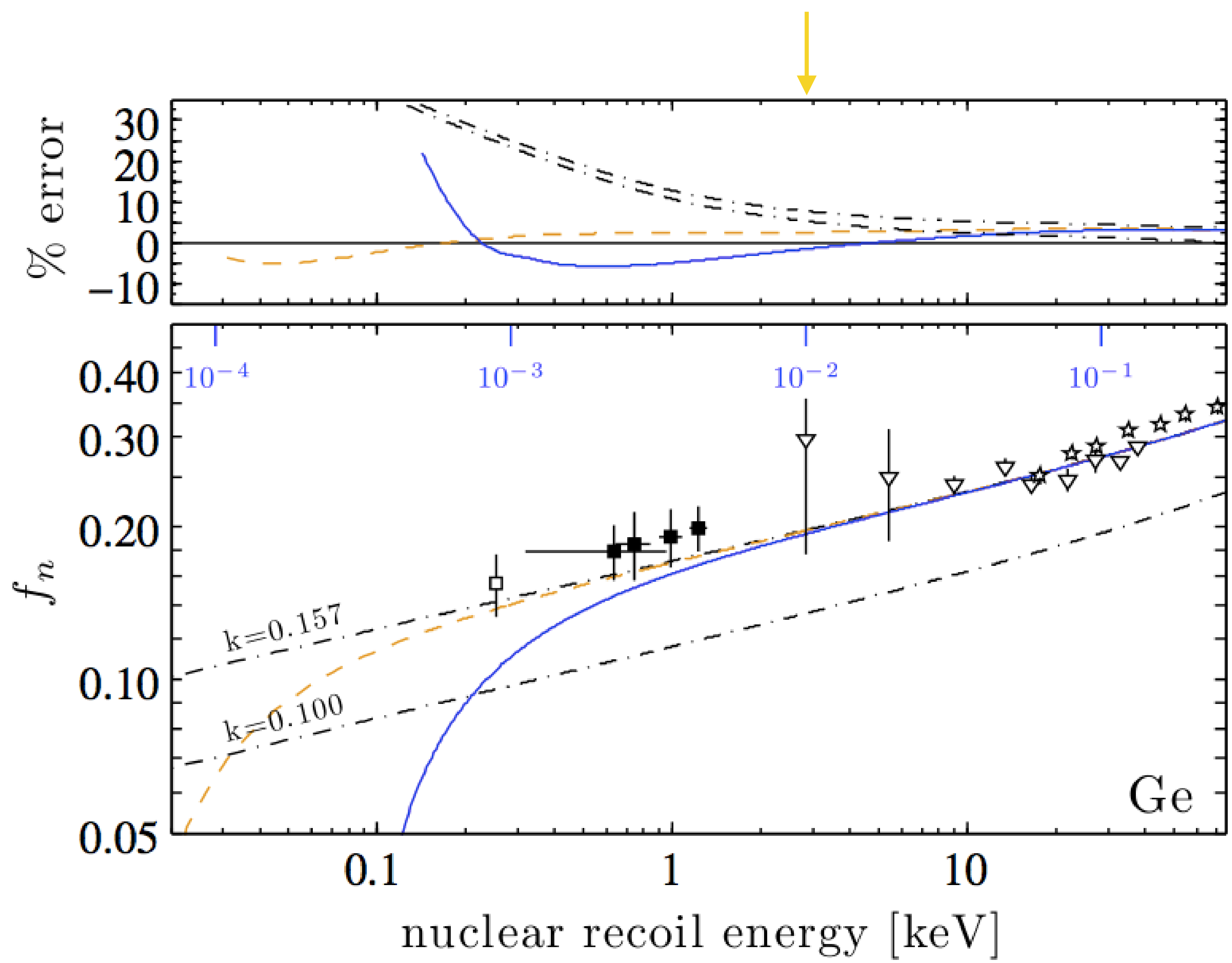
First simple tweak to the model: improve the parameterization

- NB: the % error of the standard solution to the Lindhard model equation increases dramatically below ε ~ 0.01 (arrow)
- hypothesis: at the smallest energies, some irreducible amount of energy must always be wasted in atomic motion
- add a constant energy term q and re-solve the integral equation (cf. slide 9)

$$\bar{\nu}(\varepsilon) = \frac{\varepsilon}{1 + kg(\varepsilon)} + q$$

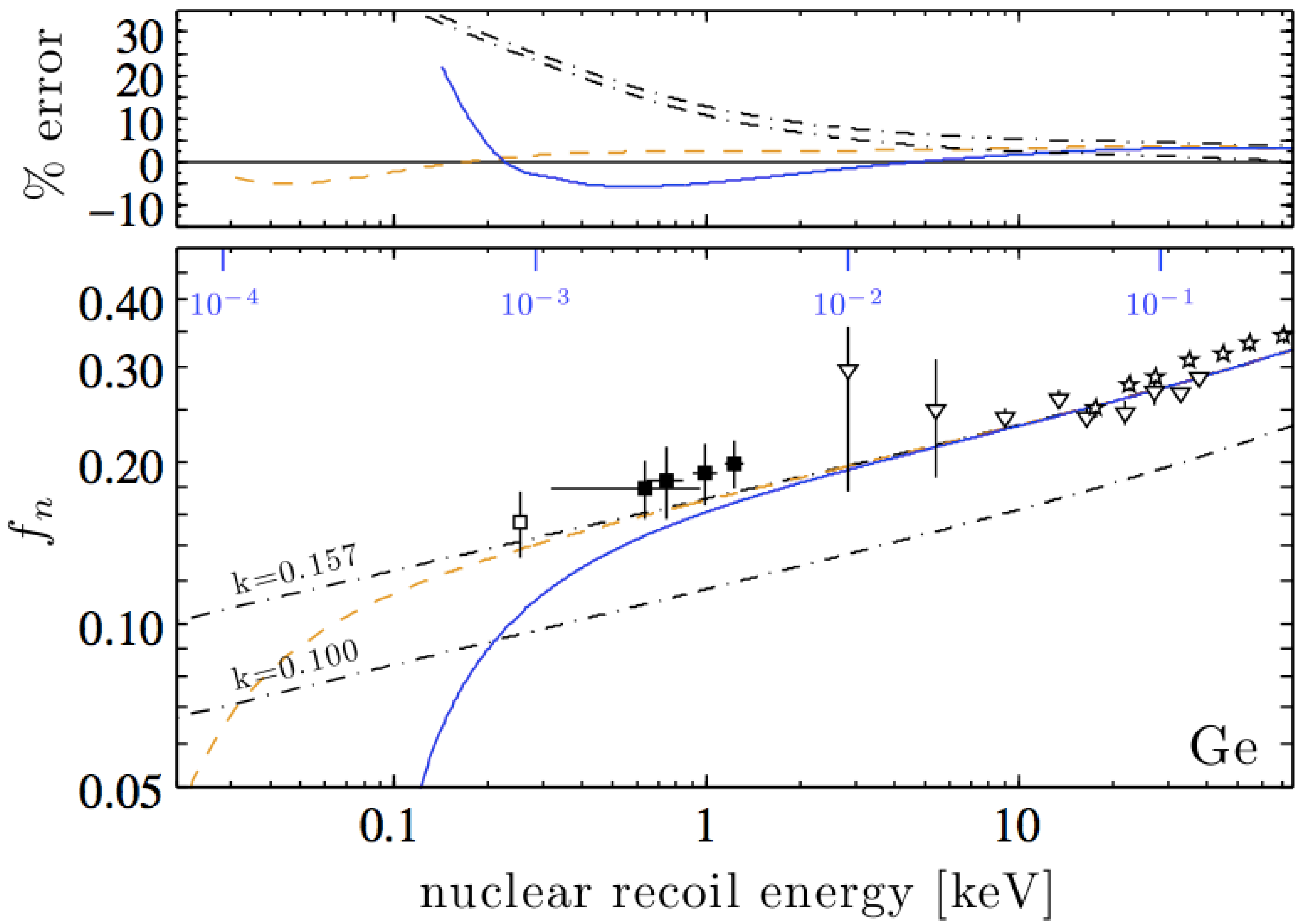
- result is dashed orange curve

$$f_n = \frac{kg(\varepsilon)}{1 + kg(\varepsilon)} - q/\varepsilon$$



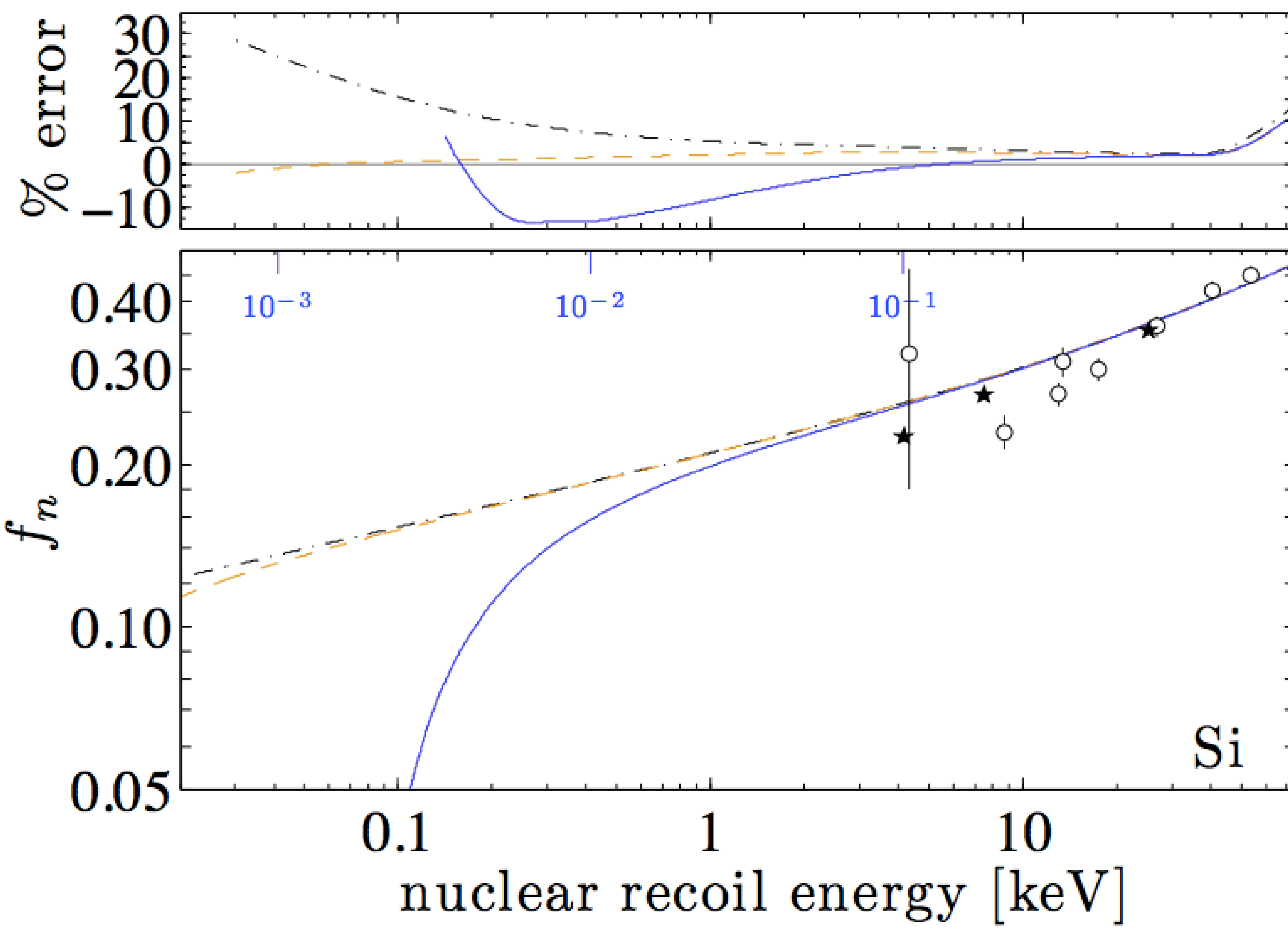
Second simple tweak to the model: account for electron binding energy

- replace the term $\bar{\nu}(t/\varepsilon)$ with $\bar{\nu}(t/\varepsilon - u)$ and re-solve the integral equation (cf. slide 9)
- u is the average energy required to ionize an electron (the w-value)
- result is solid blue curve
- **prediction of a kinematic cutoff**



Result for Si

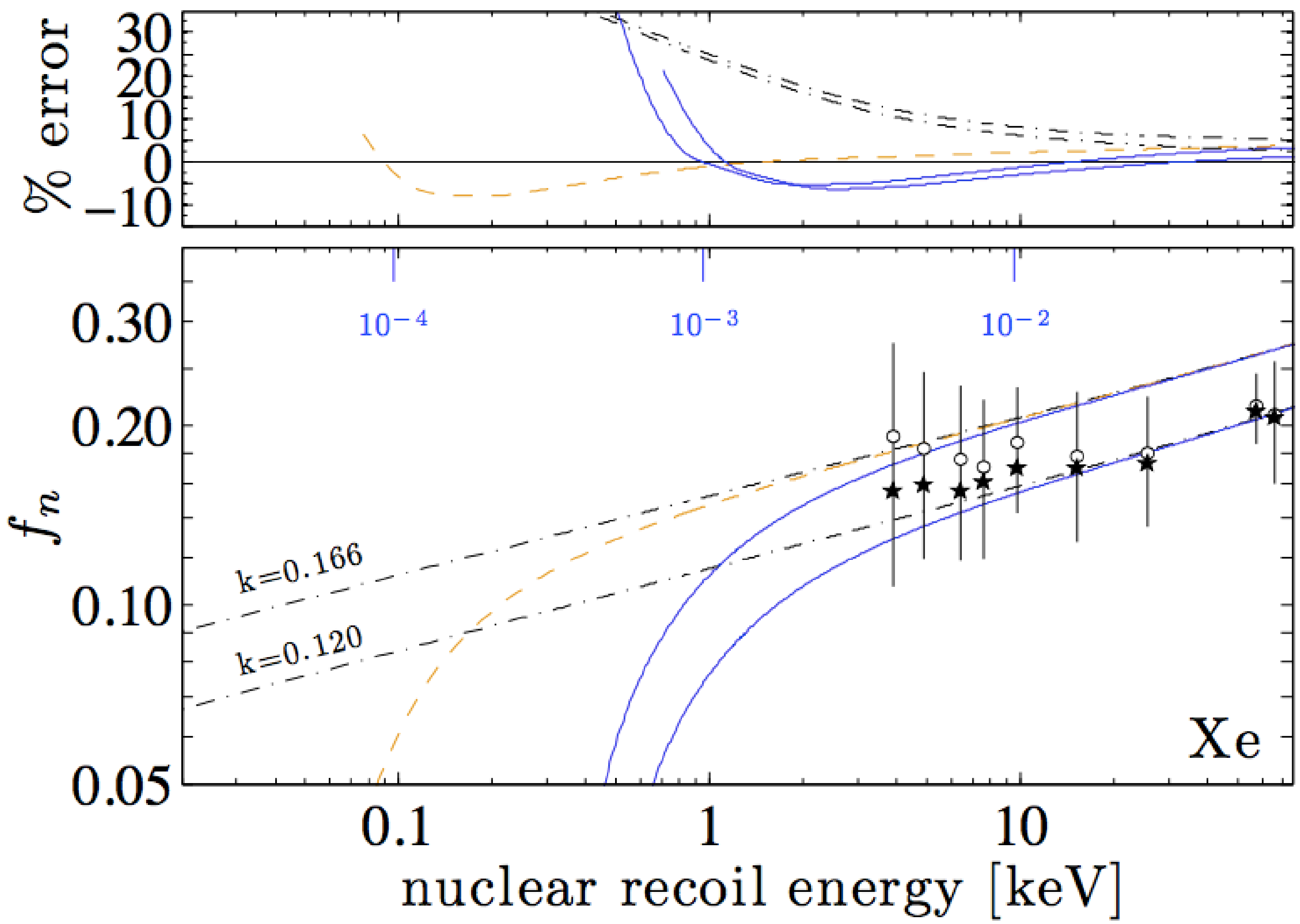
- NB: new data from DAMIC



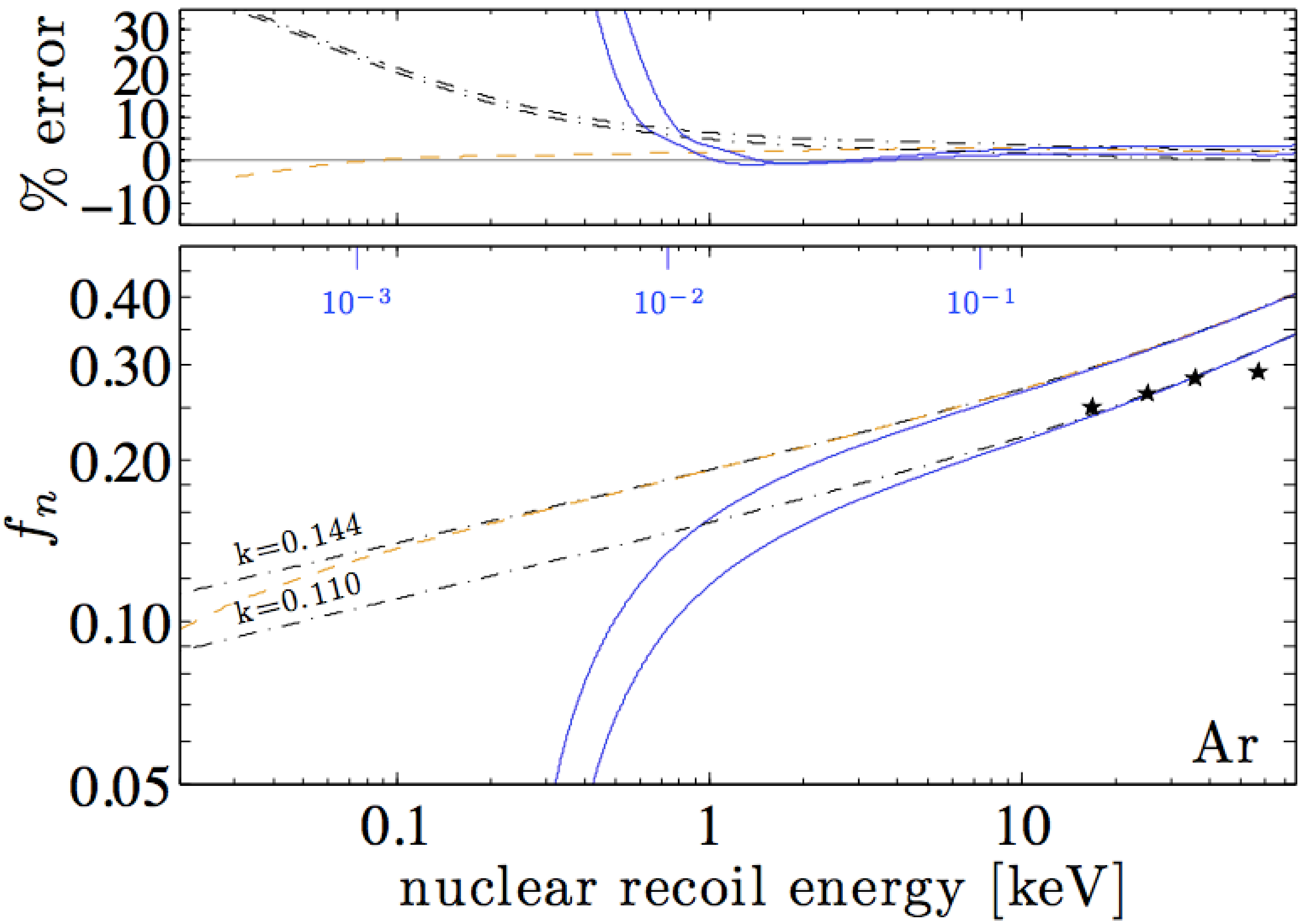
Result for Xe

- NB: new data from LUX

- NB: quenching applies to sum(electrons + photons)

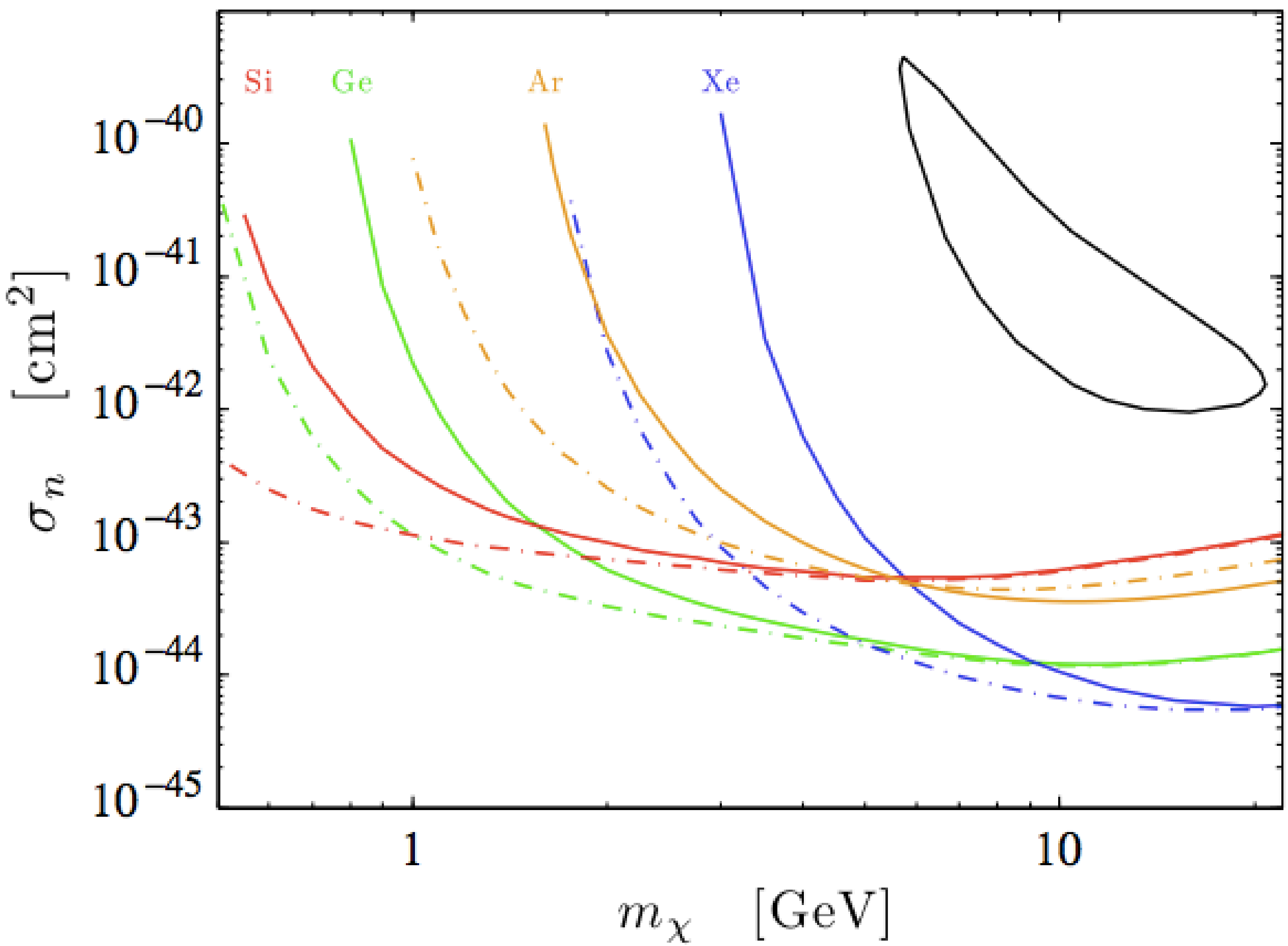


Result for Ar



This matters if you are…

- •Searching for O(1) GeV dark matter via nuclear recoil scattering
- •Searching for CENNS from low-energy (e.g. reactor) neutrinos



Summary

- Lindhard model has plenty of uncertainties, but epsilon<0.01 is not particularly pathological
- Calculations of k vary immensely, may be best to treat it as a free parameter constrained by higher energy data
- Kinematic cutoff is a generic prediction of Lindhard model
- Experimental data in Ge and Xe do not appear to support this prediction… more data are essential

Extra slides follow

The model works pretty well!

$$E_{nr} = \epsilon(n_\gamma + n_e)/f_n$$

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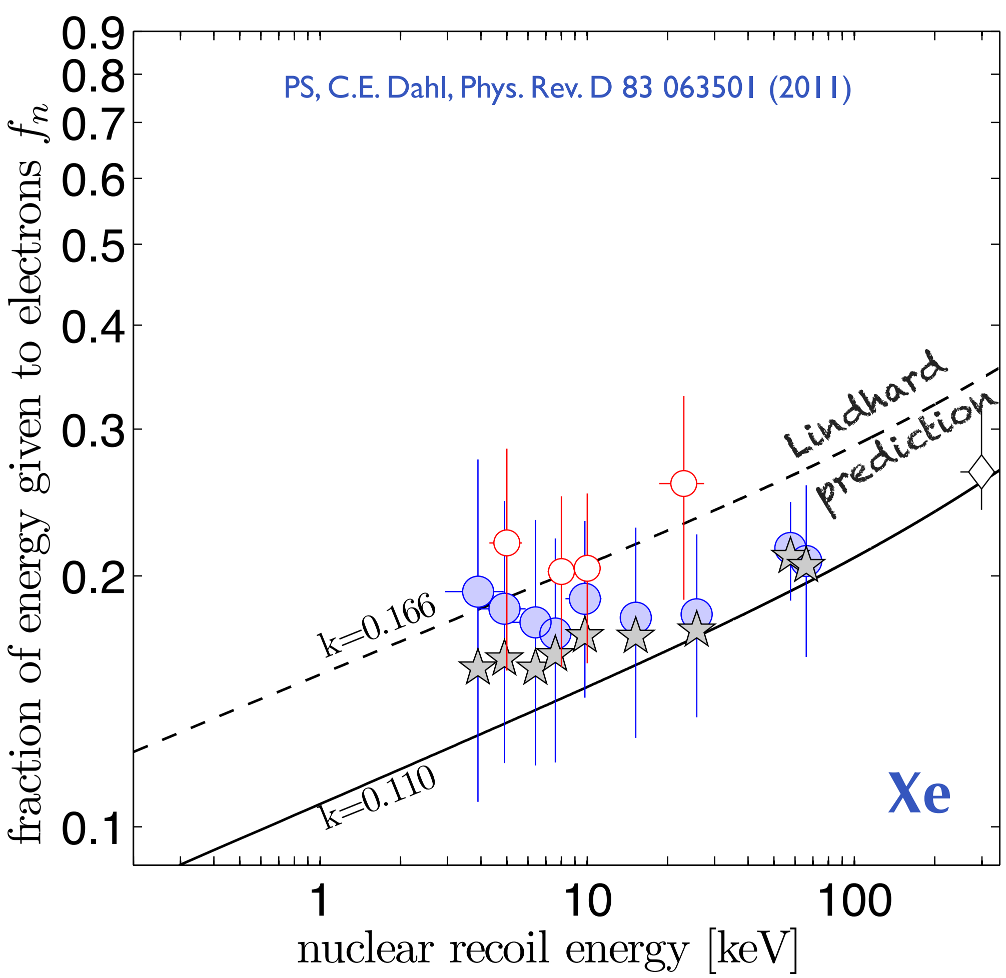

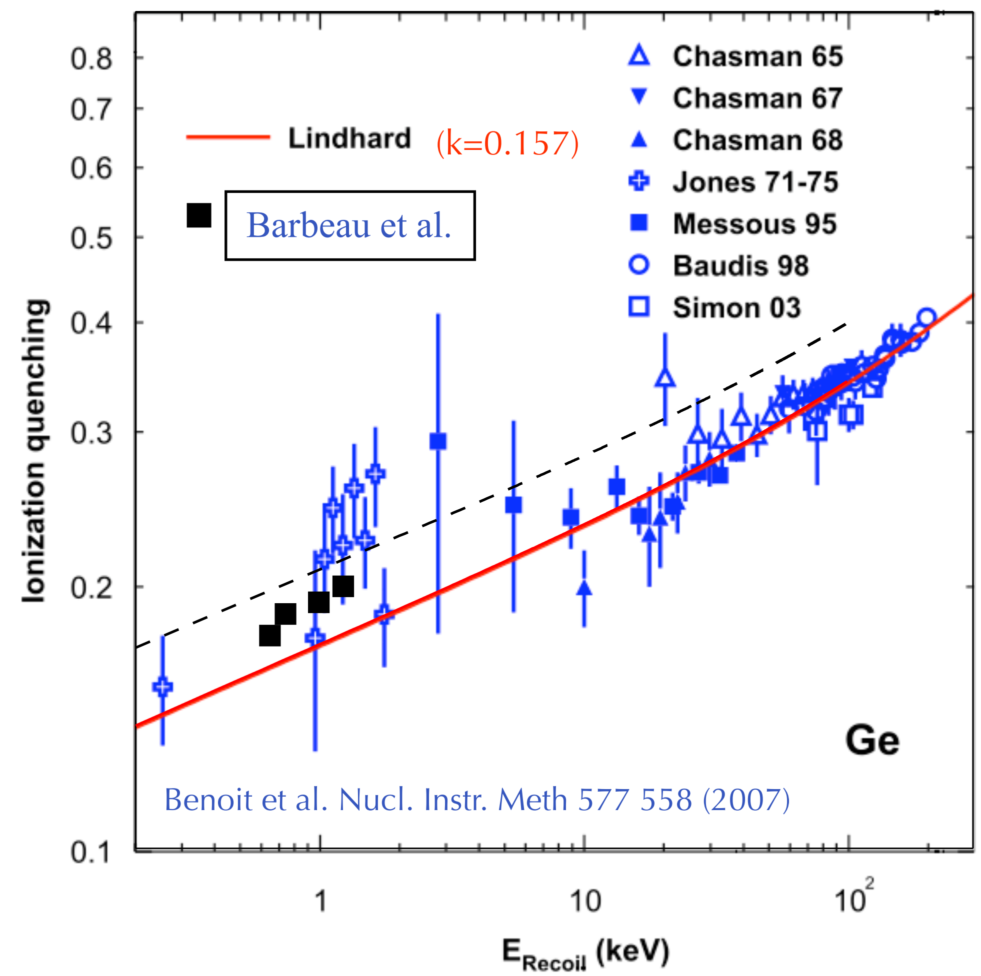

NB: new measurements from LUX extend down to ~1 keV.

Is there a kinematic cutoff?

right idea, wrong physical picture: atomic scattering is not two-body kinematics

quoting from 1005.0838

The marked drop in $\mathcal{L}_{\rm eff}$ at low energies in the experiments that the XENON100 collaboration has ignored may be understood from simple two-body kinematics affecting the energy transfer from a xenon recoil to an atomic electron. As already discussed within the context of the MACRO experiment [10], a kinematic cutoff to the production of scintillation is expected whenever the minimum excitation energy E_g of the system exceeds

[10] Phys. Rev. D 36 311 (1987)

$$E_{\max} = 2m_e v(v + v_e) \qquad V_{cutoff} \approx E_g/2m_e v_F$$

the formulae, applied to nucleus-electron scattering, result in calculated cutoff recoil energies of ~39 keV in Xe and ~0.1 keV in Ge. This is not the right thing to do.

NB: as $E_R \to 0$, atoms are basically standing still, but electrons have $v \sim \alpha$