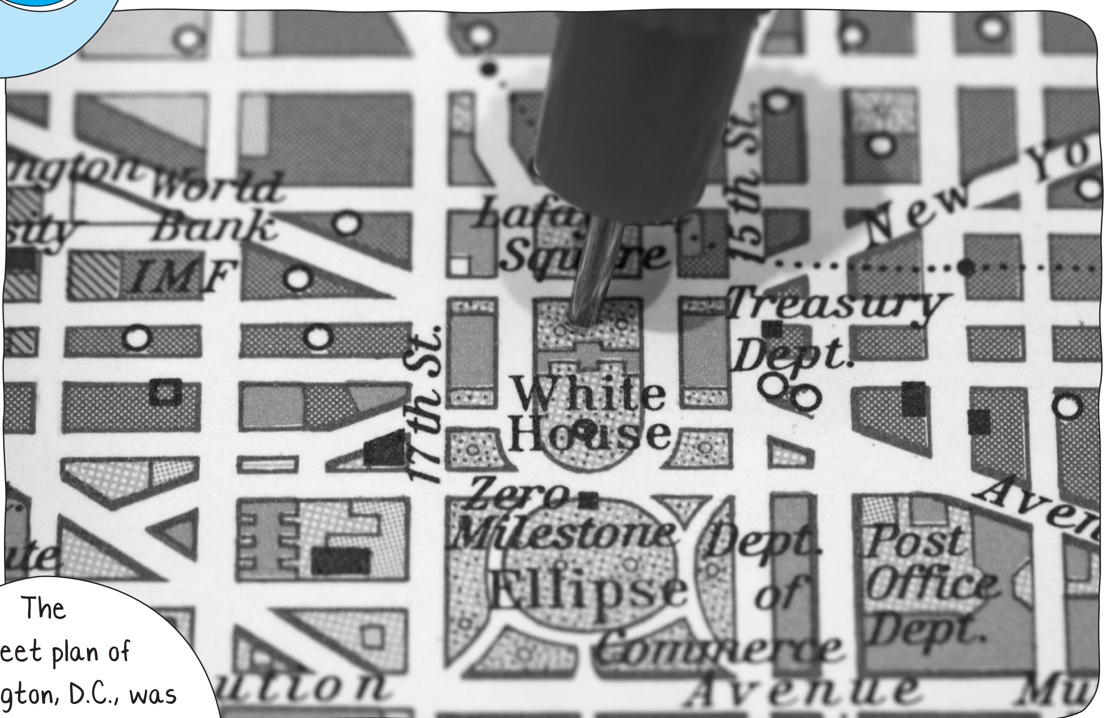
10 Line and Angle Relationships



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10.1 Location, Location, Location!

Line Relationships

Learning Goal

In this lesson, you will:

- Explore possible relationships between two lines in Euclidean geometry.

Key Terms

- intersecting lines
- plane
- perpendicular lines
- parallel lines
- coplanar lines
- skew lines
- coincidental lines

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A Greek mathematician named Euclid of Alexandria has often been called the "Father of Geometry" because of his extremely influential book titled *Elements*, in which Euclid defined much of what you will study in this and other chapters about Euclidean geometry.

So influential was Euclid's geometry that we still learn about it today—more than 2300 years after it was first written down!

Problem 1 Different Pairs of Lines

The words *intersecting lines* describe a specific relationship between two lines. **Intersecting lines** are lines in a plane that intersect, or cross each other. A **plane** extends infinitely in all directions in two dimensions and has no thickness.

1. Sketch an example of intersecting lines.

   a. Are your lines drawn in the same plane, and do they cross each other? Explain your reasoning.

   b. Compare your sketch with your classmates' sketches. Did everyone sketch the same intersecting lines? Explain how the sketches are the same or different.

Perpendicular lines are lines that intersect at a right angle. The symbol for perpendicular is $\perp$. The symbol for not perpendicular is $\not\perp$.

$\ell_1 \perp \ell_2$ is read as "line 1 is perpendicular to line 2."

$\ell_1 \not\perp \ell_2$ is read as "line 1 is not perpendicular to line 2."

2. What is a right angle?

3. Sketch an example of perpendicular lines.

a. Do your lines intersect at a right angle? How do you know?

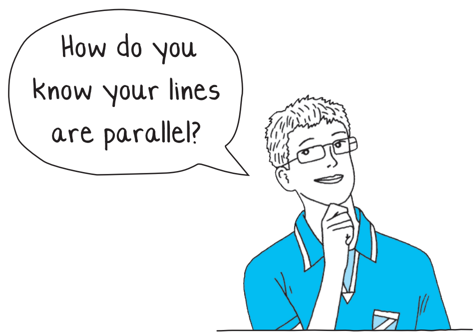
b. Compare your sketch with your classmates' sketches. Did everyone sketch the same perpendicular lines? Explain how the sketches are the same or different.

Parallel lines are lines that lie in the same plane and do not intersect no matter how far they extend. The symbol for parallel is $\parallel$. The symbol for not parallel is $\nparallel$.

$\ell_1 \parallel \ell_2$ is read as "line 1 is parallel to line 2."

$\ell_1 \nparallel \ell_2$ is read as "line 1 is not parallel to line 2."

4. Sketch an example of parallel lines.



5. Compare your sketch with your classmates' sketches. Did everyone sketch the same parallel lines? Explain how the sketches are the same or different.

Coplanar lines are lines that lie in the same plane.

6. Sketch an example of coplanar lines.

   a. Did you sketch lines in the same plane? Explain your reasoning.

   b. Compare your sketch with your classmates' sketches. Did everyone sketch the same coplanar lines? Explain how the sketches are the same or different.

Skew lines are lines that do not lie in the same plane.

7. Sketch an example of skew lines.

8. Compare your sketch with your classmates' sketches. Did everyone sketch the same skew lines? Explain how the sketches are the same or different.

Coincidental lines are lines that have equivalent linear equations and overlap at every point when they are graphed.

9. What is meant by "equivalent linear equations"?

10. Sketch an example of coincidental lines.

11. Compare your sketch with your classmates' sketches. Did everyone sketch the same coincidental lines? Explain how the sketches are the same or different.

Problem 2 Relationships between Lines

Euclidean geometry describes two or more lines as having four possible relationships.

Case 1: Two or more coplanar lines intersect at a single point.

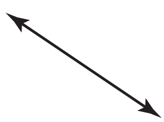
Case 2: Two or more coplanar lines intersect at an infinite number of points.

Case 3: Two or more coplanar lines do not intersect.

Case 4: Two or more lines are not coplanar.

1. Identify the relationship represented by each sketch.

a.



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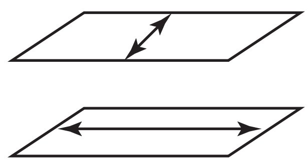

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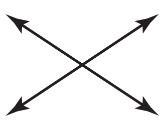
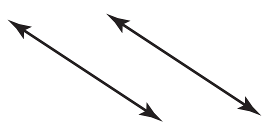


d.



2. If two lines share only a single point, are the lines always coplanar? Explain your reasoning.

3. If two lines share an infinite number of points, are the lines always coplanar? Explain your reasoning.

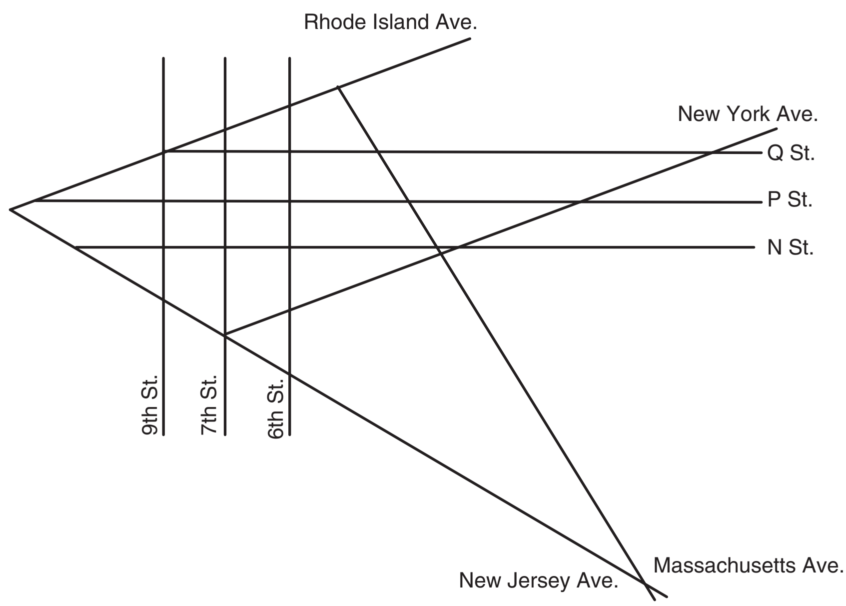
4. Are coplanar lines that do not intersect equidistant? Explain your reasoning.

5. Explain why skew lines cannot possibly intersect.

Problem 3 Maps

The layout of the streets of Washington, D.C., was created by Pierre Charles L'Enfant, a French-born architect. L'Enfant began working on the layout of the city in 1791.

A map of part of Washington, D.C., is shown.



1. Imagine that each street on the map is part of a line. Use the map to give an example of each relationship.

a. intersecting lines

b. perpendicular lines

c. parallel lines

d. coplanar lines

e. skew lines

f. coincidental lines

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2. Imagine that each state border line on the map of the United States is part of a line. Name a state whose border lines appear to show one of the following line relationships.

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a. intersecting lines

b. perpendicular lines

c. parallel lines

d. coplanar lines

e. skew lines

f. coincidental lines

There is a famous place in the United States called the Four Corners. At this location, four state borders intersect at one point.

g. Use the U.S. map in Question 2 to determine the names of the four states. What line relationship closely models this intersection?

3. Sketch a map of the streets in your neighborhood. Include the street you live on and streets near your house.

Imagine that each street on the map is part of a line. Use the map to give an example of each relationship.

a. intersecting lines

b. perpendicular lines

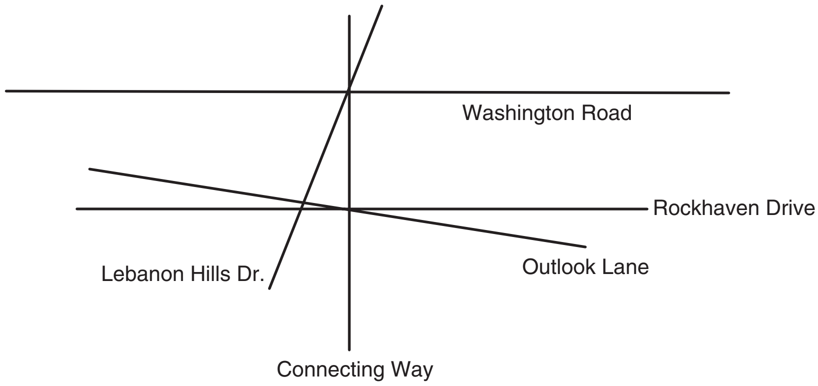
c. parallel lines

d. coplanar lines

e. skew lines

f. coincidental lines

4. Use the map shown to answer each question.



a. The street map shows Washington Road intersecting Lebanon Hills Drive and Connecting Way at the same point. If Washington Road is perpendicular to Connecting Way, is it possible for Lebanon Hills Drive to be perpendicular to Washington Road? Explain your reasoning.

b. The street map shows Washington Road is parallel to Rockhaven Drive. If Outlook Lane intersects Rockhaven Drive, is it possible for Outlook Lane to be parallel to Washington Road? Explain your reasoning.

Be prepared to share your solutions and methods.

10.2 When Lines Come Together

Angle Relationships Formed by Two Intersecting Lines

Learning Goals

In this lesson, you will:

- Explore the angles determined by two intersecting lines.
- Identify congruent angles.
- Identify adjacent angles.
- Identify vertical angles.
- Identify a linear pair of angles.
- Identify supplementary angles.
- Solve for the supplement of an angle.

Key Terms

- supplementary angles
- linear pair of angles

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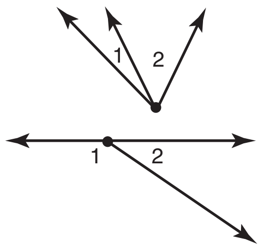
One city in the United States has become famous because of its connection to a very popular board game. Some of the streets in this city are Tennessee Avenue, Mediterranean Avenue, Illinois Avenue, and Boardwalk. Can you name that famous board game? How about the city? In this lesson, you will identify angle relationships on maps.

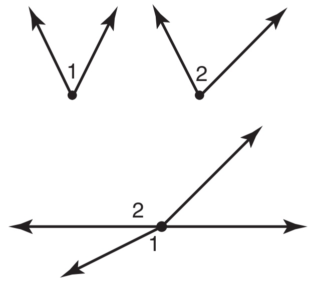
Problem 1 Pairs of Angles

1. Analyze the examples shown.

∠1 and ∠2 are adjacent angles.

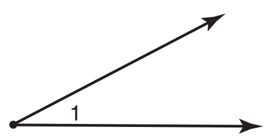


∠1 and ∠2 are NOT adjacent angles.



a. Describe adjacent angles.

b. Draw ∠2 adjacent to ∠1.



c. Is it possible to draw two angles that have a common vertex but do NOT have a common side? If so, draw an example. If not, explain.

d. Is it possible to draw two angles that have a common side but do NOT have a common vertex? If so, draw an example. If not, explain.

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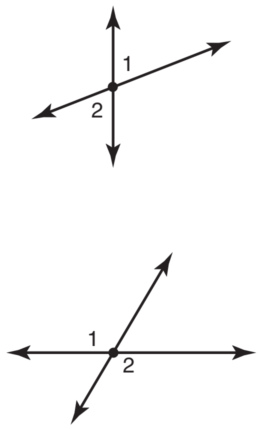
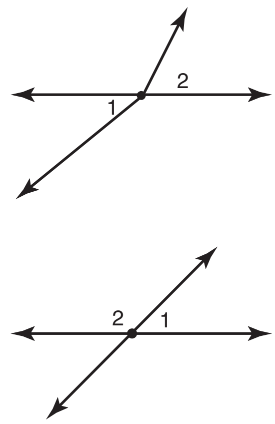
2. Analyze the examples shown.

∠1 and ∠2 are vertical angles.

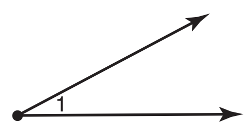


∠1 and ∠2 are NOT vertical angles.

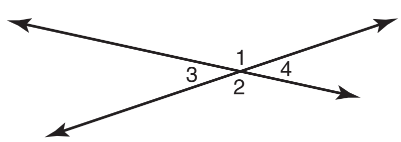


a. Describe vertical angles.

b. Draw ∠2 so that ∠1 and ∠2 are vertical angles. Use a protractor to measure both angles.



c. Name all of the pairs of vertical angles. Use a protractor to measure the four angles.



d. Draw several different pairs of vertical angles and use a protractor to determine if vertical angles are congruent in all situations.

Supplementary angles are two angles whose measures have a sum of 180°.

3. Use a protractor to draw a pair of supplementary angles that have a common side. Measure each angle.

4. Compare your drawing with your classmates' drawings. Did everyone draw the same supplementary angles? Explain how the sketches are the same or different.

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5. Use a protractor to draw a pair of supplementary angles that do NOT have a common side. Measure each angle.

A **linear pair of angles**, or a linear pair, consists of two adjacent angles that form a straight line.

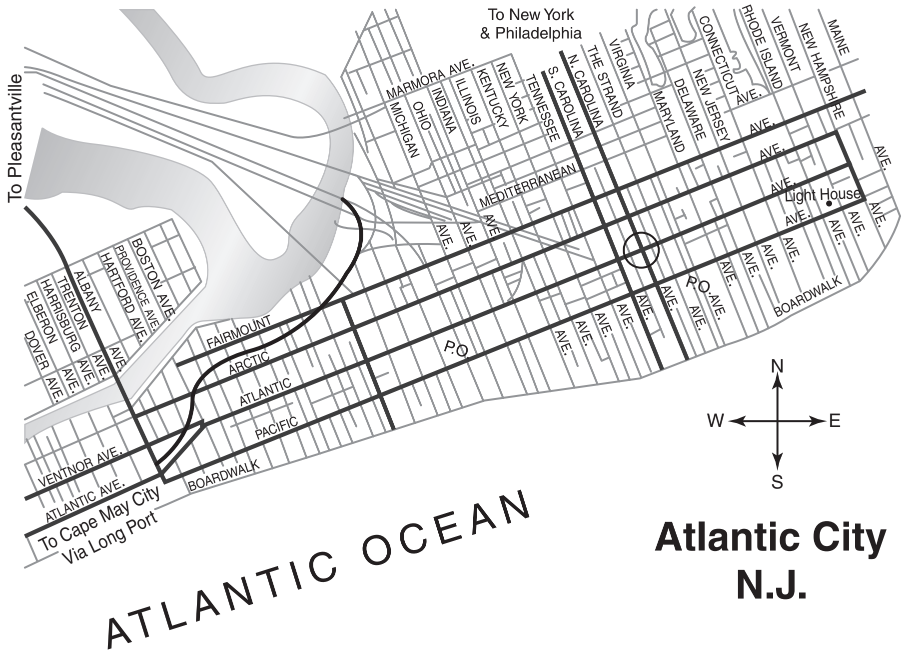
6. Use a protractor to draw a linear pair of angles.

7. Compare your sketch with your classmates' sketches. Did everyone sketch the same linear pair of angles? Explain how the sketches are the same or different.

8. What is the difference between a linear pair of angles and a pair of supplementary angles that have a common side?

Problem 2 Street Maps

Refer to the map of Atlantic City, New Jersey, shown to answer each question. Assume all line segments that appear to be perpendicular are perpendicular. Assume all line segments that appear to be parallel are parallel.



1. Imagine that each street on the map is part of a line. Use the map to give an example of each relationship.

   a. congruent angles

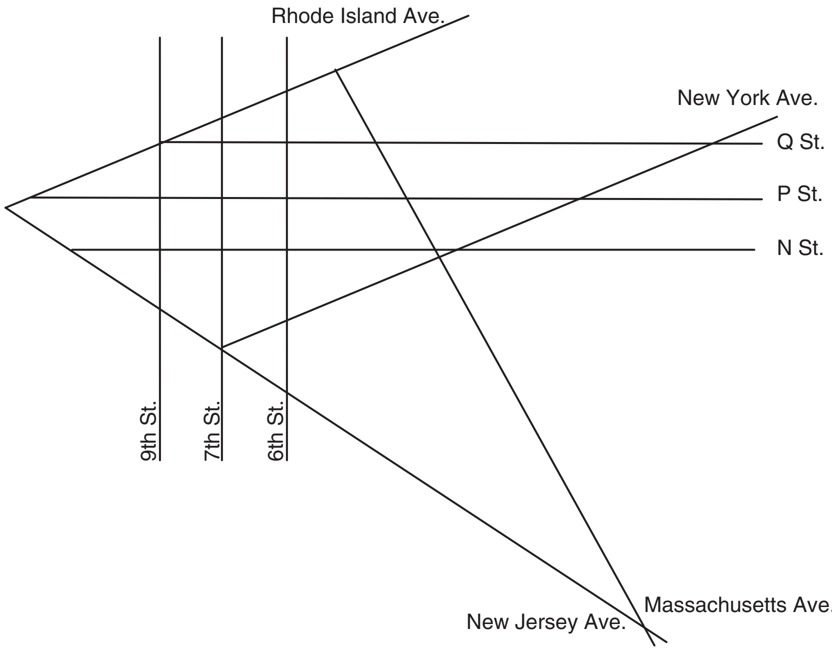
   b. adjacent angles

   c. vertical angles

   d. linear pair of angles

   e. supplementary angles

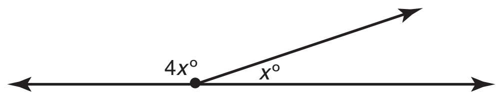
A map of part of Washington, D.C., is shown. Assume all line segments that appear to be perpendicular are perpendicular. Assume all line segments that appear to be parallel are parallel.



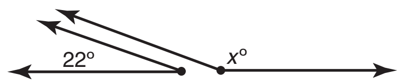
2. Imagine that each street on the map is part of a line. On the map, mark a different location for each pair of angles.
   a. vertical angles: $\angle 1$ and $\angle 2$
   b. supplementary angles: $\angle 3$ and $\angle 4$
   c. adjacent angles: $\angle 5$ and $\angle 6$
   d. linear pair of angles: $\angle 7$ and $\angle 8$
   e. congruent angles: $\angle 9$ and $\angle 10$

Problem 3 Solving for Unknown Measures of Angles

1. The angles shown are a linear pair of angles. Solve for x.



2. The angles shown are supplementary. Solve for x.



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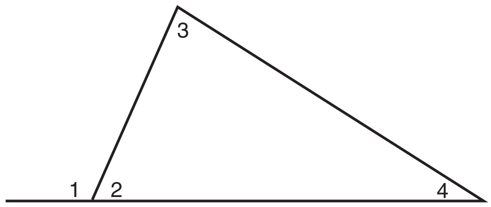
3. If two angles are both congruent and supplementary, what are their measures? Explain your reasoning.

4. If the supplement of an angle is half the measure of the angle, what is the measure of each angle?

5. If the supplement of an angle is 20° more than the measure of the angle, what is the measure of each angle?



6. Sara understands supplementary angles and linear pairs. She said she understands why $m\angle 1 = m\angle 3 + m\angle 4$ in the figure shown. Sara's lab partner, Sean, sees no connection between $m\angle 1$ and $m\angle 3 + m\angle 4$ because those angles aren't next to each other. Explain to Sean what Sara discovered.

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7. If two intersecting lines form congruent adjacent angles, what can you conclude about the lines?

Talk the Talk

1. Two intersecting lines determine how many angles?

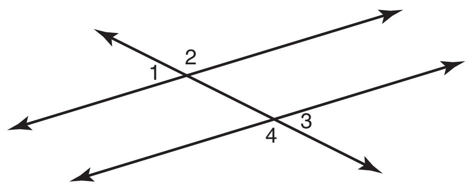
2. Two intersecting lines determine how many pairs of vertical angles?

3. Two intersecting lines determine how many pairs of supplementary angles?

4. Two intersecting lines determine how many linear pairs of angles?

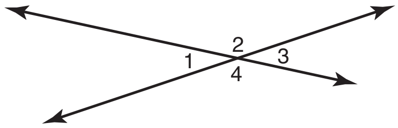
5. Two intersecting lines determine how many pairs of adjacent angles?

6. Suppose two lines intersect. If you are given the measure of one angle, can you determine the measures of the remaining angles without using a protractor? Explain your reasoning.



7. If $\angle 1$ is the supplement of $\angle 2$, $\angle 3$ is the supplement of $\angle 4$, and $\angle 1$ is congruent to $\angle 3$, what can you conclude about the measures of $\angle 2$ and $\angle 4$? Write this conclusion in a general form.

8. Suppose $\angle 1$ is the supplement of $\angle 2$, and $\angle 2$ is the supplement of $\angle 3$.



a. What can you conclude about $m\angle 1$ and $m\angle 3$?

b. Write this conclusion in a general form.

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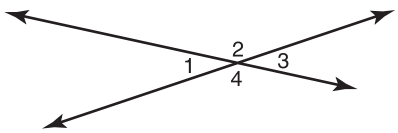
9. When two lines intersect, four different angles are formed as shown.



- Describe the relationship between vertical angles.
- Describe the relationship between adjacent angles.
- Use a protractor to verify your conclusions.

Be prepared to share your solutions and methods.

10.3 Crisscross Applesauce

Angle Relationships Formed by Two Lines Intersected by a Transversal

Learning Goals

In this lesson, you will:

- Explore the angles determined by two lines that are intersected by a transversal.
- Explore the measures of angles determined by two parallel lines that are intersected by a transversal.
- Identify alternate interior angles.
- Identify alternate exterior angles.
- Identify same-side interior angles.
- Identify same-side exterior angles.
- Identify corresponding angles.
- Determine the measure of alternate interior angles, alternate exterior angles, same-side interior angles, same-side exterior angles, and corresponding angles.

Key Terms

- transversal
- alternate interior angles
- alternate exterior angles
- same-side interior angles
- same-side exterior angles

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Take two straws and lay them on your desk. Make them as close to parallel as you can. Then lay a third straw on top of the other two at any angle you like. Tape your entire construction together.

Use your protractor to measure the angles you see. Notice anything interesting? Compare your constructions with your classmates' constructions. What do you notice?

Problem 1 Naming All the Angles

In this lesson, you will explore all the angles that can be formed by *transversals*.

A **transversal** is a line that intersects two or more lines.

1. Sketch an example of a transversal.

2. Compare your sketch with your classmates' sketches. Did everyone sketch the same figure? Explain how the sketches are the same or different.

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Alternate interior angles are angles formed when a line (transversal) intersects two other lines. These angles are on opposite sides of the transversal and are between the other two lines.

3. Sketch an example of alternate interior angles.

4. How many pairs of alternate interior angles are formed by two lines that are intersected by a transversal?

5. Compare your sketch with your classmates' sketches. Did everyone draw the same alternate interior angles? Explain how the sketches are the same or different.

Alternate exterior angles are angles formed when a line (transversal) intersects two other lines. These angles are on opposite sides of the transversal and are outside the other two lines.

6. Sketch an example of alternate exterior angles.

7. How many pairs of alternate exterior angles are formed by two lines that are intersected by a transversal?

8. Compare your sketch with your classmates' sketches. Did everyone draw the same alternate exterior angles? Explain how the sketches are the same or different.

Same-side interior angles are angles formed when a line (transversal) intersects two other lines. These angles are on the same side of the transversal and are between the other two lines.

9. Sketch an example of same-side interior angles.

10. How many pairs of same-side interior angles are formed by two lines that are intersected by a transversal?

11. Compare your sketch with your classmates' sketches. Did everyone draw the same angles? Explain how the sketches are the same or different.

Same-side exterior angles are angles formed when a line (transversal) intersects two other lines. These angles are on the same side of the transversal and are outside the other two lines.

12. Sketch an example of same-side exterior angles.

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13. How many pairs of same-side exterior angles are formed by two lines that are intersected by a transversal?

14. Compare your sketch with your classmates' sketches. Did everyone draw the same angles? Explain how the sketches are the same or different.

Recall that corresponding angles are angles that have the same relative positions in geometric figures.

15. Sketch an example of corresponding angles. Include two lines intersected by a transversal in the sketch.

16. How many pairs of corresponding angles are formed by two lines that are intersected by a transversal?

17. Compare your sketch with your classmates' sketches. Did everyone draw the same corresponding angles? Explain how the sketches are the same or different.

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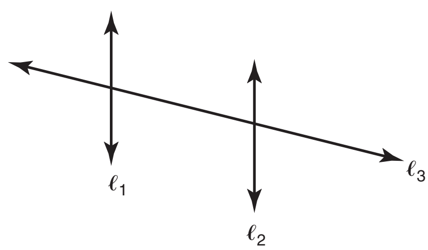
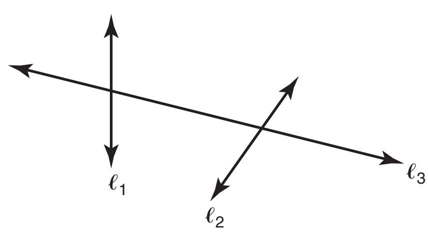
Problem 2 Where Are the Transversals?

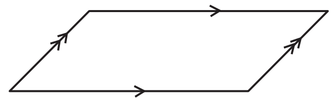
1. Suppose that $\ell_1 \parallel \ell_2$, and both lines intersect ℓ_3. Identify the transversal(s).



2. Suppose that $\ell_1 \nparallel \ell_2$, and both lines intersect ℓ_3. Identify the transversal(s).

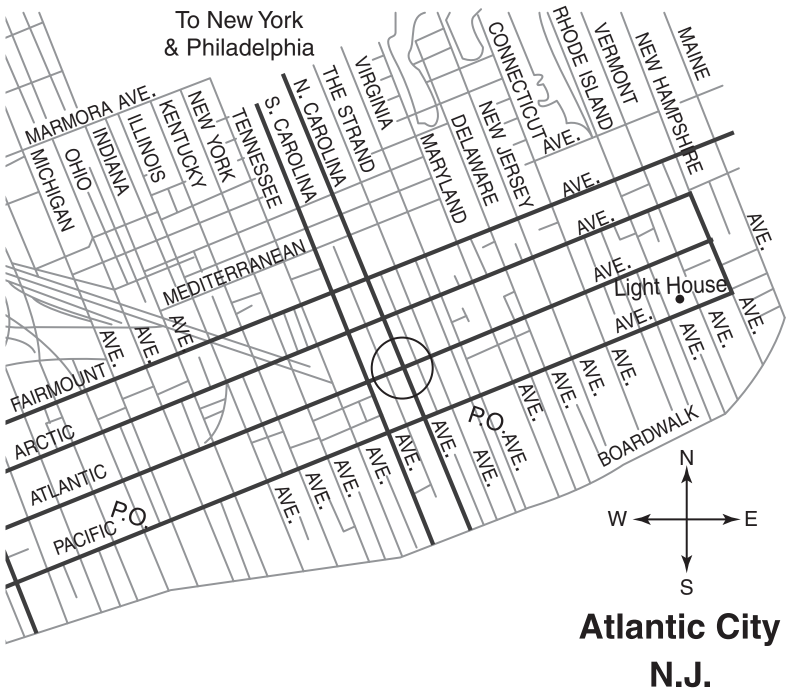


3. The arrowheads on these line segments indicate parallel relationships between opposite sides of the geometric figure. Transversals can be lines or line segments. Does this figure contain transversals? Explain your reasoning.



Problem 3 Street Map of Atlantic City, New Jersey

Refer to the map of part of Atlantic City, New Jersey, to answer each question. Assume all line segments that appear to be perpendicular are perpendicular. Assume all line segments that appear to be parallel are parallel.



1. Is Atlantic Ave. a transversal? Explain your reasoning.

2. Locate the circle drawn on Atlantic Ave. This circle is drawn at the intersection of Atlantic Ave. and what other avenue?

3. How many angles are formed at this intersection?

4. Label each angle.

a. Place a 1 on the angle that would be considered the northwest angle.

b. Place a 2 on the angle that would be considered the northeast angle.

c. Place a 3 on the angle that would be considered the southwest angle.

d. Place a 4 on the angle that would be considered the southeast angle.

5. Using Atlantic Ave. and N. Carolina Ave., choose a third avenue such that Atlantic Ave. is a transversal.

a. Label the four angles at this intersection $\angle 5$, $\angle 6$, $\angle 7$, and $\angle 8$ and describe the location of each angle (northeast, northwest, southeast, or southwest).

b. List all pairs of alternate interior angles.

c. List all pairs of alternate exterior angles.

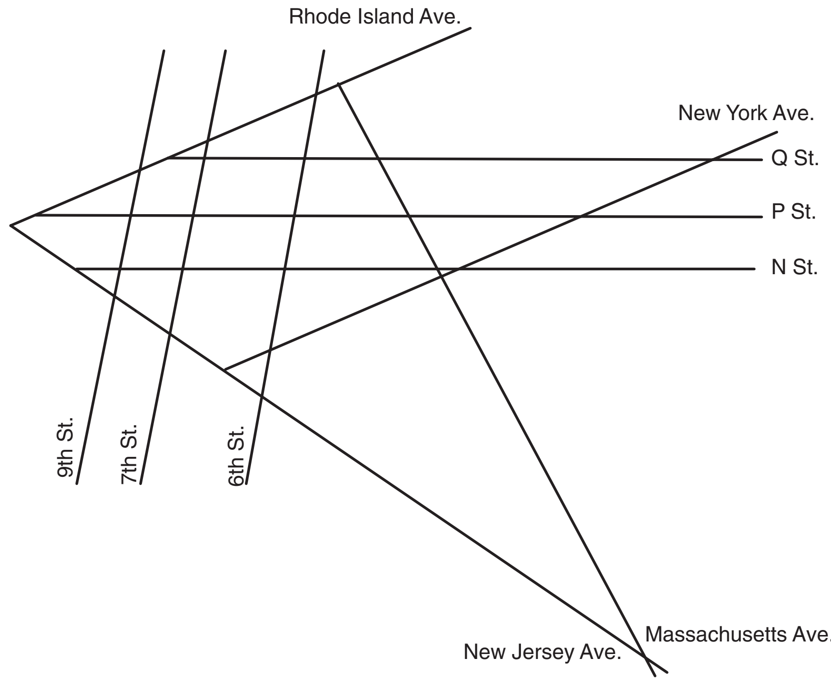
d. List all pairs of same-side interior angles.

e. List all pairs of same-side exterior angles.

f. List all pairs of corresponding angles.

Problem 4 Washington, D.C., Map

Use the map of Washington, D.C., to answer each question. Assume all line segments that appear to be parallel are parallel.



1. Label ∠1, ∠2, ∠3, and ∠4 at the intersection of 7th St. and P St.
2. Label ∠5, ∠6, ∠7, and ∠8 at the intersection of 6th St. and P St.
3. Label ∠9, ∠10, ∠11, and ∠12 at the intersection of Massachusetts Ave. and P St.
4. Use a protractor to measure all 12 angles.
5. Consider only 6th St., 7th St., and P St.

   a. Which of these streets, if any, are transversals?

   b. Name the pairs of alternate interior angles. What do you notice about their angle measures?

c. Name the pairs of alternate exterior angles. What do you notice about their angle measures?

d. Name the pairs of corresponding angles. What do you notice about their angle measures?

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e. Name the pairs of same-side interior angles. What do you notice about their angle measures?

f. Name the pairs of same-side exterior angles. What do you notice about their angle measures?

g. What is the relationship between 6th St. and 7th St.?

6. Consider only 6th Street, Massachusetts Avenue, and P Street.

   a. Which of these streets, if any, are transversals?

   b. Name the pairs of alternate interior angles. What do you notice about their angle measures?

   c. Name the pairs of alternate exterior angles. What do you notice about their angle measures?

   d. Name the pairs of corresponding angles. What do you notice about their angle measures?

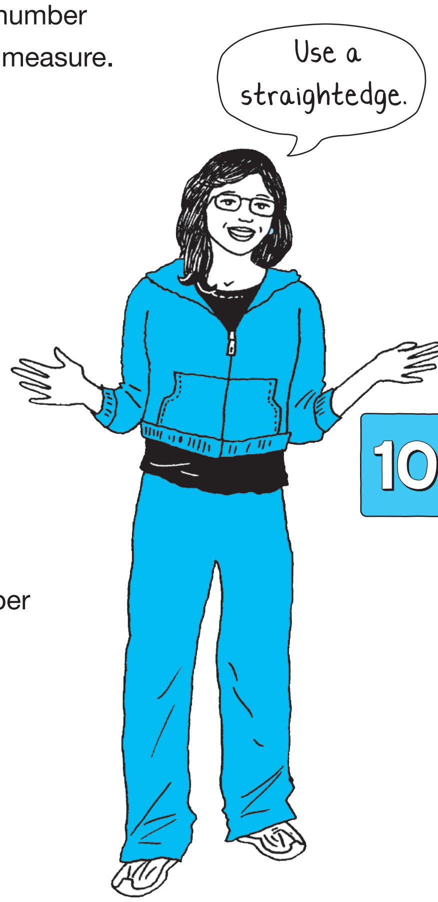
   e. Name the pairs of same-side interior angles. What do you notice about their angle measures?

   f. Name the pairs of same-side exterior angles. What do you notice about their angle measures?

   g. What is the relationship between 6th St. and Massachusetts Ave.?

Problem 5 Measuring Angles Formed by Two Lines and a Transversal

1. Draw a transversal intersecting two non-parallel lines, and number each angle. Then use a protractor to determine each angle measure.



2. Draw a transversal intersecting two parallel lines, and number each angle. Then use a protractor to determine each angle measure.

Use the information from Questions 1 and 2 to answer Questions 3 through 8.

3. What do you notice about the measures of each pair of alternate interior angles when the lines are:

   a. non-parallel?

   b. parallel?

4. What do you notice about the measures of each pair of alternate exterior angles when the lines are:

   a. non-parallel?

   b. parallel?

5. What do you notice about the measures of each pair of corresponding angles when the lines are:

   a. non-parallel?

   b. parallel?

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6. What do you notice about the measures of the same-side interior angles when the lines are:

   a. non-parallel?

   b. parallel?

7. What do you notice about the measures of the same-side exterior angles when the lines are:

   a. non-parallel?

   b. parallel?

8. Summarize your conclusions in the table by writing the relationships of the measures of the angles. The relationships are either congruent or not congruent, supplementary or not supplementary.

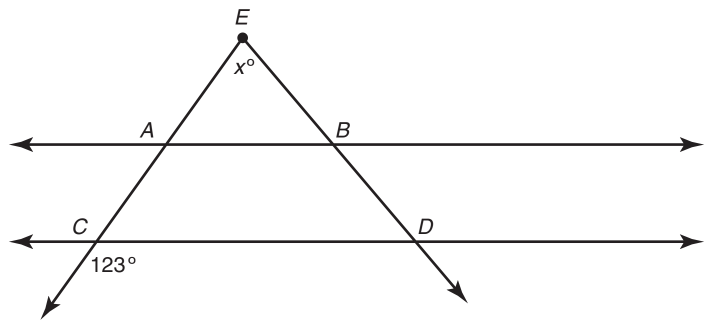
Angles	Two Parallel Lines Intersected by a Transversal	Two Non-Parallel Lines Intersected by a Transversal
Alternate Interior Angles		
Alternate Exterior Angles		
Corresponding Angles		
Same-Side Interior Angles		
Same-Side Exterior Angles		

9. Use your table in Question 8 to compare your conclusions with other groups or classmates. Also, compare the measures of the angles everyone used. What do you notice?

Problem 6 Solving for Unknown Angle Measures

Sylvia and Scott were working together to solve the problem shown.

Given: $\overline{AB} \parallel \overline{CD}$. Solve for x. Show all your work.



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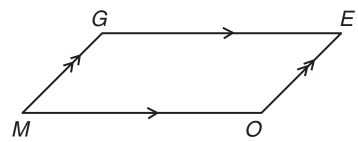
1. Sylvia concluded that $x = 66°$. How did Sylvia get her answer?

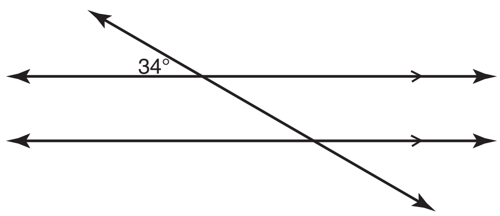
2. Scott does not agree with Sylvia's answer. He thinks there is not enough information to solve the problem. How could Scott alter the figure to explain his reason for disagreeing with Sylvia's answer?

3. Who is correct?

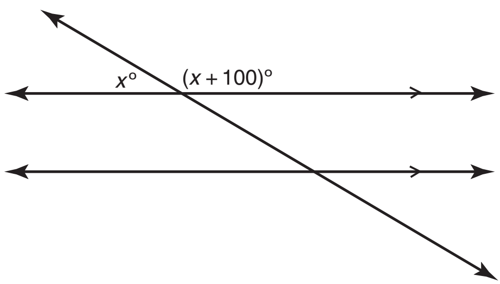
4. Opposite sides of this geometric figure are parallel. Suppose that the measure of angle M is equal to 30°. Solve for the measures of angles G, E, and O. Explain your reasoning.



5. Arrowheads indicate parallel lines. Determine the measures of all angles.



6. Arrowheads indicate parallel lines. Determine the measures of all angles.



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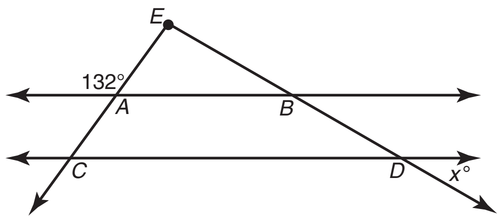
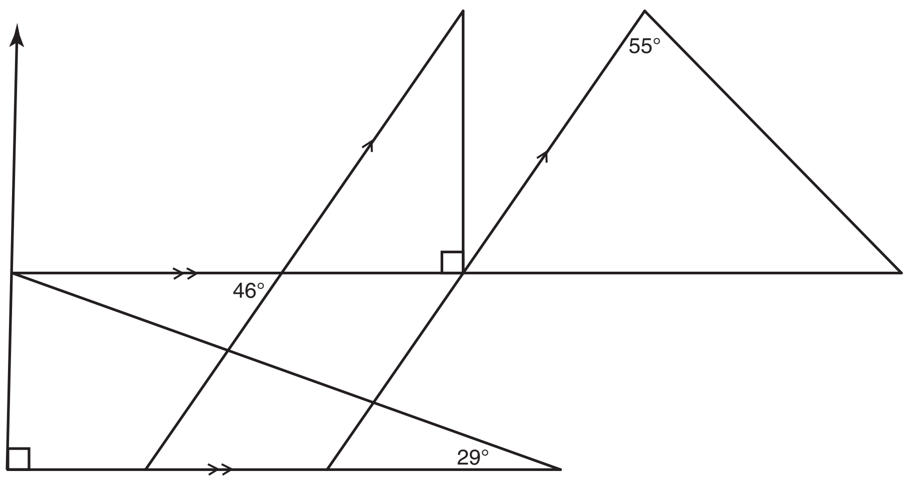
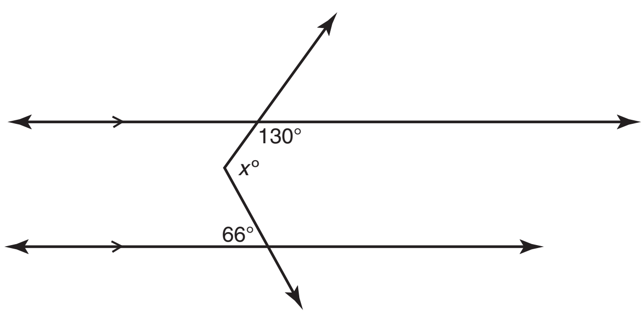
7. In this figure, $\overline{AB} \parallel \overline{CD}$ and $\overrightarrow{CE} \perp \overrightarrow{DE}$. Solve for x. Show all your work.



8. Arrowheads indicate parallel lines, and boxes indicate that the angles are right angles. Determine the measure of each angle in this figure.



9. Solve for x.



Talk the Talk

If two lines are intersected by a transversal…

- … when are alternate interior angles congruent?
- … when are alternate exterior angles congruent?
- … when are corresponding angles congruent?
- … when are vertical angles congruent?
- … when are same-side interior angles supplementary?
- …when are same-side exterior angles supplementary?
- …when are adjacent angles supplementary?

Be prepared to share your solutions and methods.

10

10.4 Parallel or Perpendicular?

Slopes of Parallel and Perpendicular Lines

Learning Goals

In this lesson, you will:

- Determine the slopes of parallel lines.
- Determine the slopes of perpendicular lines.
- Identify parallel lines.
- Identify perpendicular lines.

Key Terms

- reciprocal
- negative reciprocal

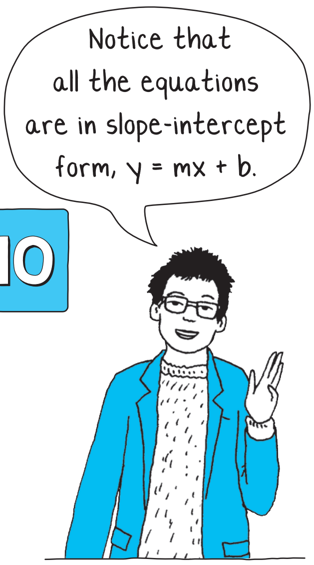
10

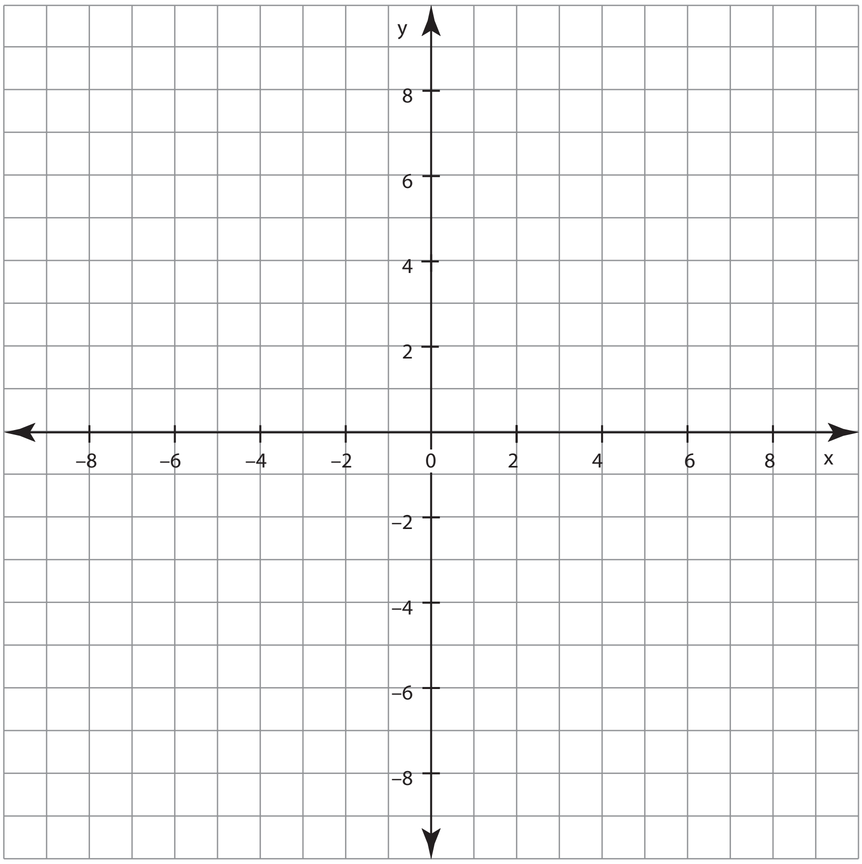
Everything you see around you is made up of atoms—tiny particles (or waves?) that are constantly moving. And most of an atom is actually empty space. So, why is it that you can't walk through walls?

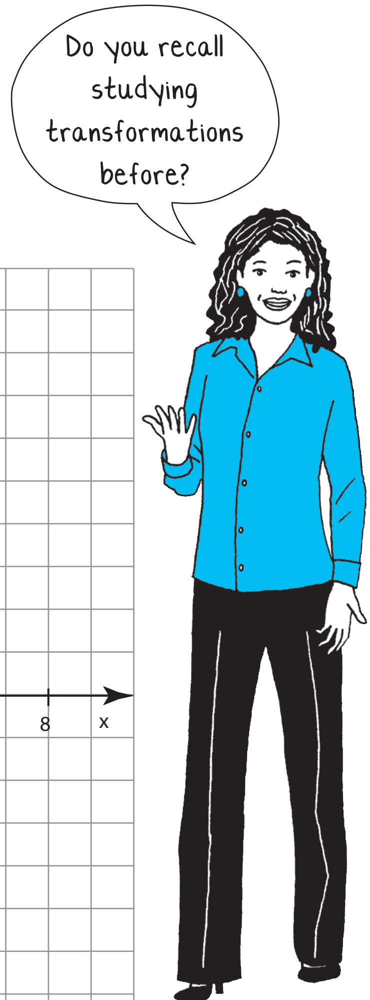
The answer—or at least part of the answer—is the normal force. This force, which is always perpendicular to the surface, is the one that pushes *up* on you. It's the force that keeps you from sinking into the floor—and unfortunately, the force that makes it impossible for you to walk through walls.

Problem 1 Graphing Equations, Part 1

1. Graph each equation on the coordinate plane.
   - $y = 2x$
   - $y = 2x + 3$
   - $y = 2x - 5$
   - $y = 2x + 5$







   a. Describe the relationship between the lines.

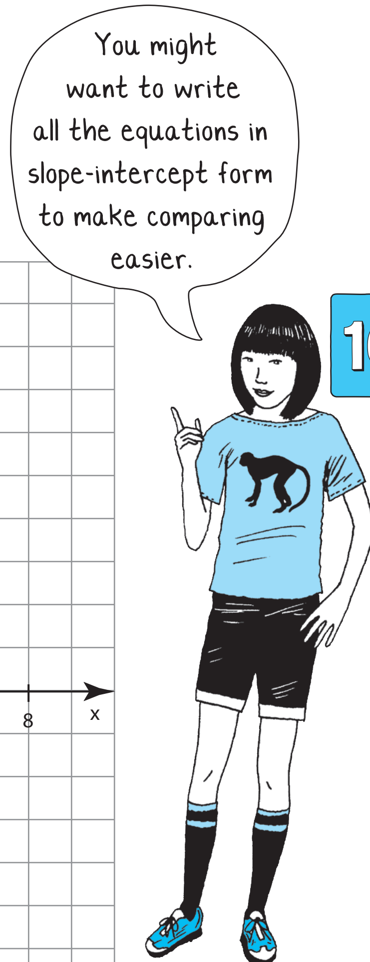
   b. Describe a strategy for verifying the relationship between the lines.

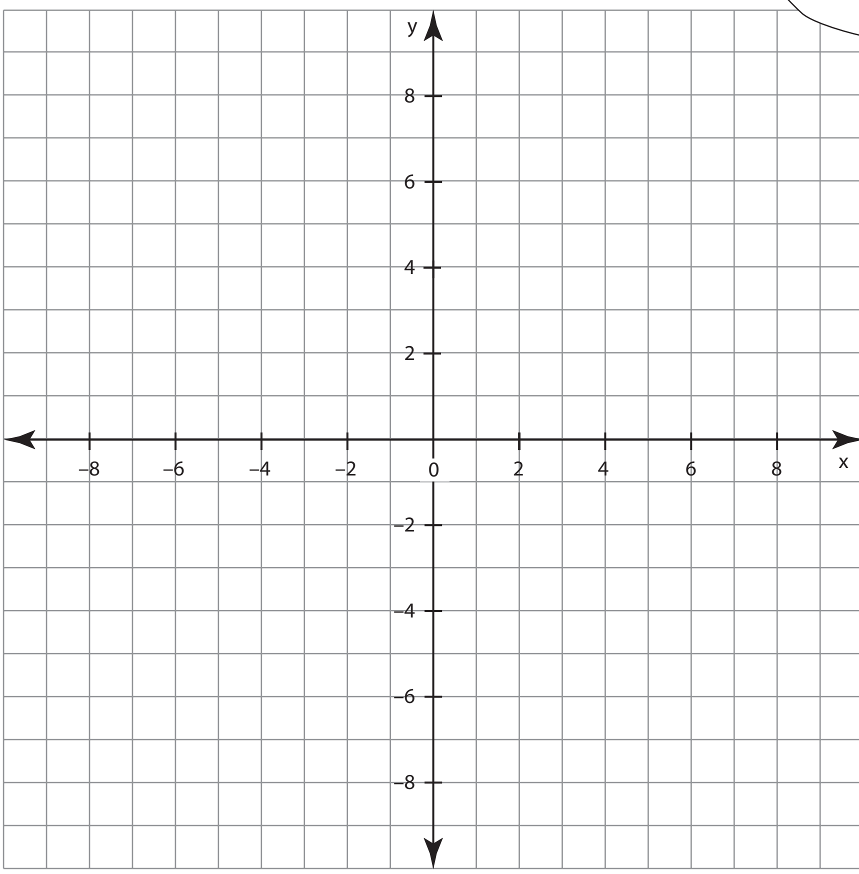
c. Use measuring tools to verify the relationship between the lines.

d. What do all of the equations have in common?

2. Graph and label. each equation on the coordinate plane.

- $y = -3x$
- $y = -3x - 2$
- $y = 5 - 3x$
- $y = -3x - 8$





a. Describe the relationship between the lines.

b. What do all of the equations have in common?

3. Consider these equations.

- $y = 4x$
- $y = 6 + 4x$
- $y = 4x - 3$
- $y = -2 + 4x$

a. Without graphing these equations, describe the relationship between the lines.

b. Explain how you determined the relationship between the lines.

4. Create four linear equations that represent lines with the same slope.

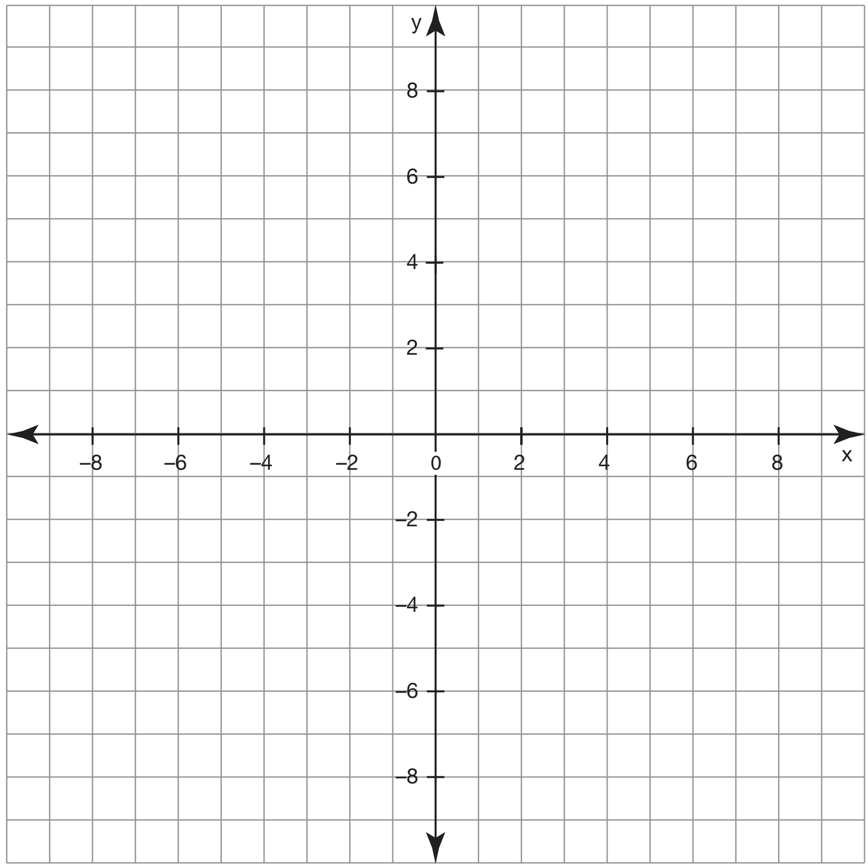
a.

b.

c.

d.

e. Graph and label your equations.



f. Describe the relationship between the lines.

g. Compare the graphs of your equations with those of your classmates. What can you conclude about the slopes of parallel lines?

5. What is the slope of a line that is parallel to the line represented by the equation $y = 200x + 93$?

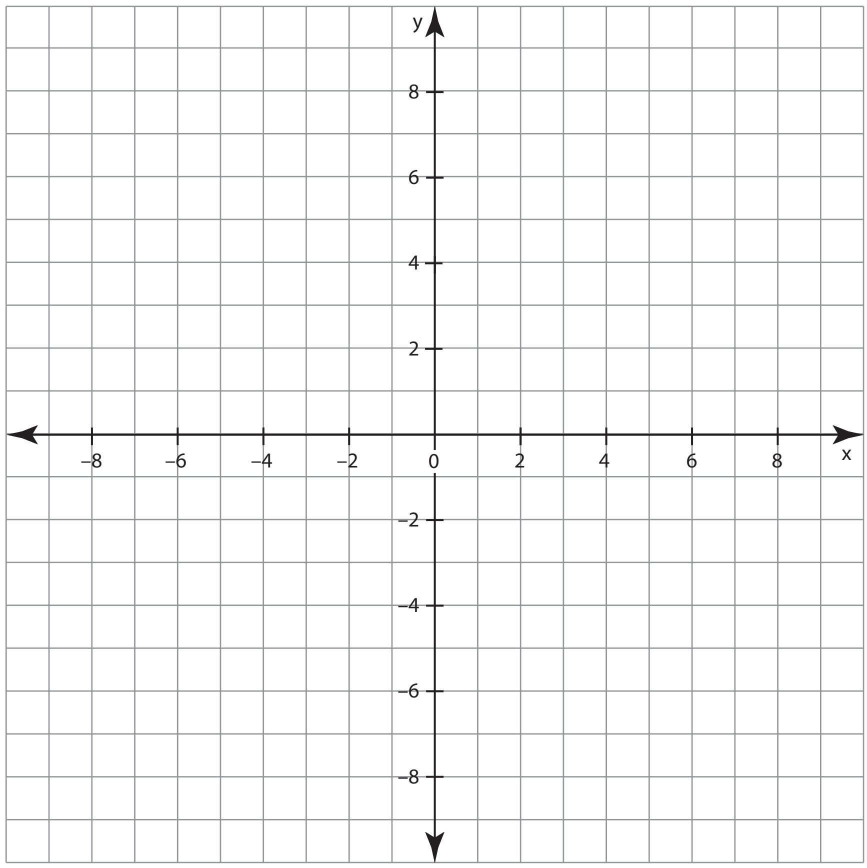
6. What is the slope of a line that is parallel to the line represented by the equation $y = 20 - 7x$?

7. Write equations for four different lines that are parallel to the line $y = x$.

   a.   b.

   c.   d.

   e. Verify that your lines are parallel by graphing them on the coordinate plane.



8. Identify the slope value in each of the equations shown to determine if the lines represented by the equations are parallel to each other.

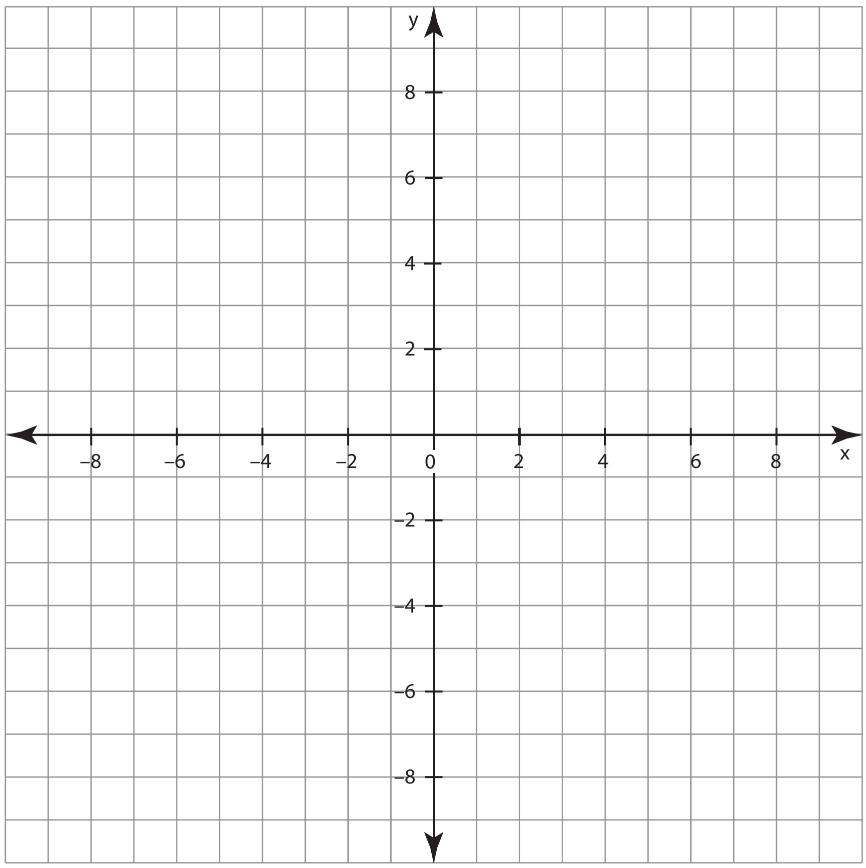
   $y = 5x + 4$

   $y = 7 + 5x$

   $y + 2x = 3x + 10$

Problem 2 Graphing Equations, Part 2

1. Graph and label each equation on the coordinate plane.
   - $y = \frac{2}{3}x$
   - $y = -\frac{3}{2}x$



a. Describe the relationship between the lines.

b. Describe a strategy for verifying the relationship between the lines.

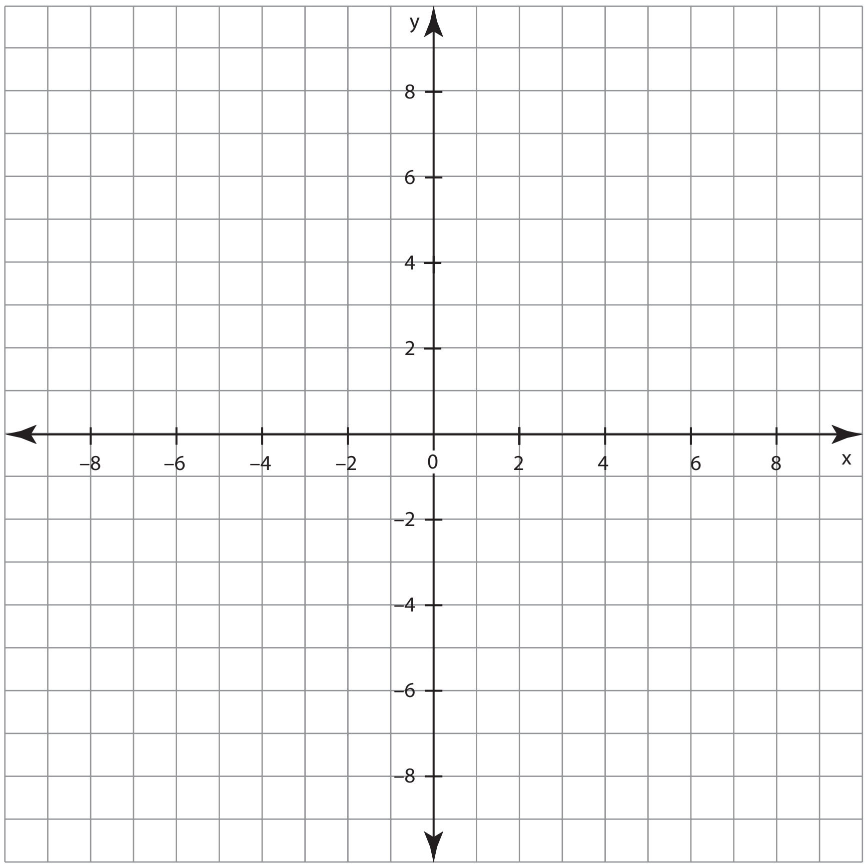
c. Use measuring tools to verify the relationship between the lines.

d. Calculate the product of the slopes.

10

2. Graph and label each equation on the coordinate plane.

- $y = \frac{4}{5}x + 1$
- $y = 2 - \frac{5}{4}x$



a. Describe the relationship between the lines.

b. Calculate the product of the slopes.

3. Consider these equations.

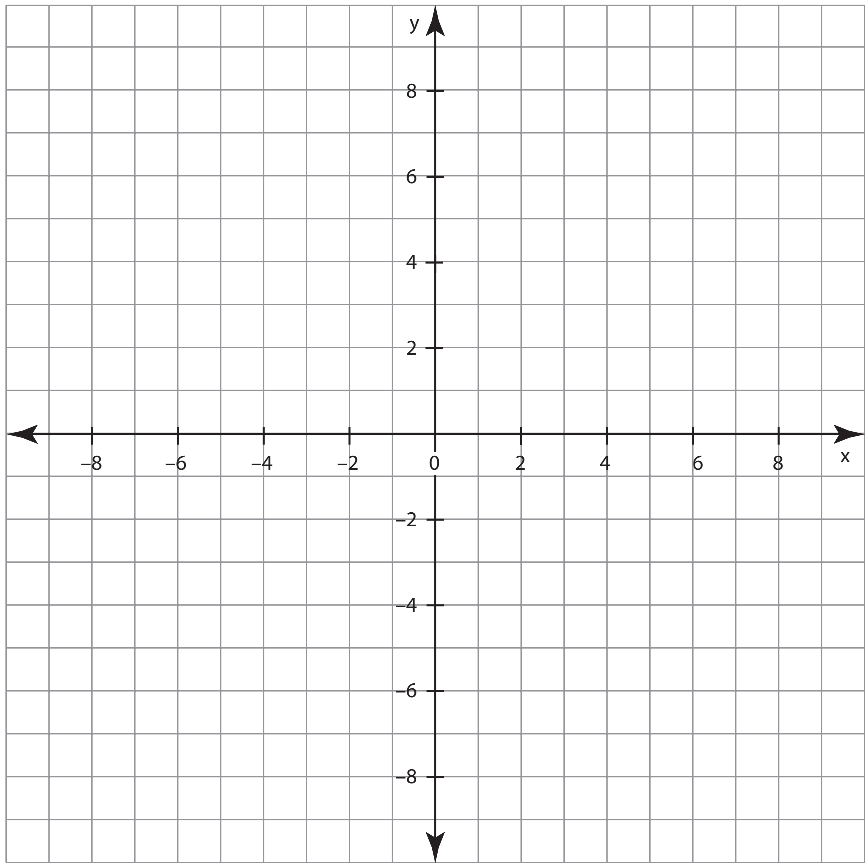
- $y = 6x$
- $y = 2 - \frac{1}{6}x$

a. Without graphing these equations, describe the relationship between the lines.

b. Explain how you determined the relationship between the lines.

4. Write two equations where the product of the slope values is −1.

a. Graph and label your equations.



b. Describe the relationship between the lines.

c. Compare the graphs of your equations with those of your classmates. What do you notice?

5. What can you conclude about the slope values in equations that represent perpendicular lines?

6. What is the slope of a line that is perpendicular to the line $y = 200x + 93$?

7. What is the slope of a line that is perpendicular to the line $y = 20 - 7x$?

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8. Write four different equations for lines that are perpendicular to the line $y = x$.

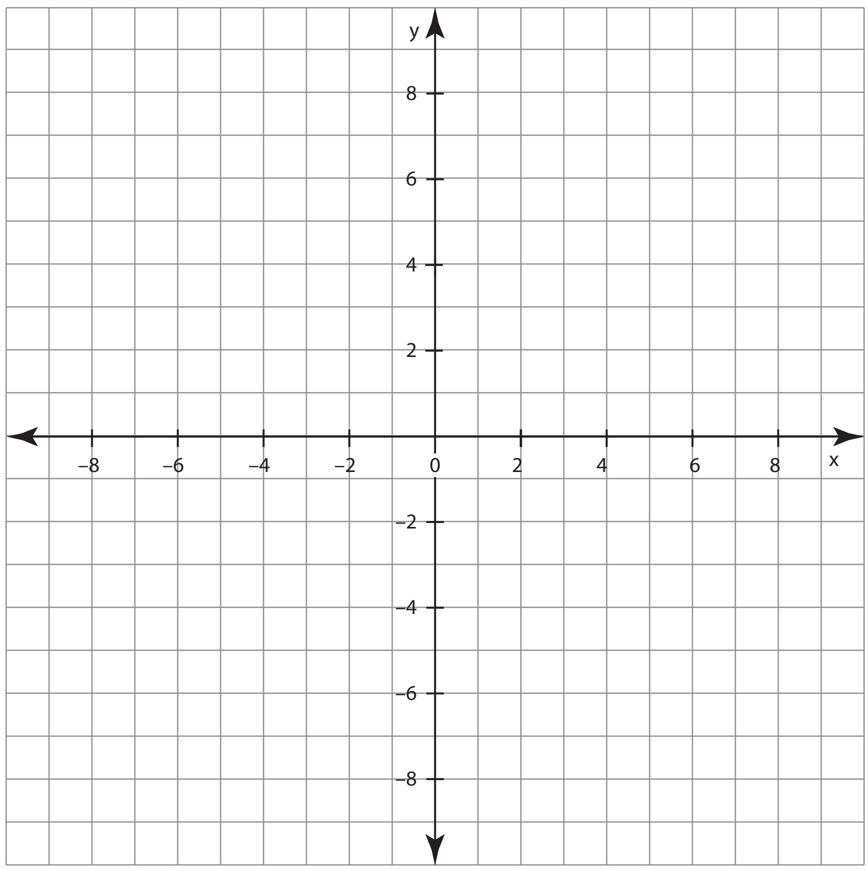
a.

b.

c.

d.

e. Verify that your lines are perpendicular by graphing them on the grid.



9. Identify the slope value in each of the equations shown to determine if the lines they represent are perpendicular.

$y = 3 + 10x$

$y - 7 = \frac{1}{10}x$

When the product of two numbers is 1, the numbers are **reciprocals** of one another. When the product of two numbers is -1, the numbers are **negative reciprocals** of one another. So, the slopes of perpendicular lines are negative reciprocals of each other.

Talk the Talk

1. Determine if the two equations are parallel, perpendicular, or neither.

   a. $y = x + 8$

   $y = 10 + x$

   b. $4y = 12 - x$

   $y = -4x - 5$

   c. $3y = 12 - x$

   $y = 3x + 4$

   d. $-y = x + 8$

   $y = 10 - x$

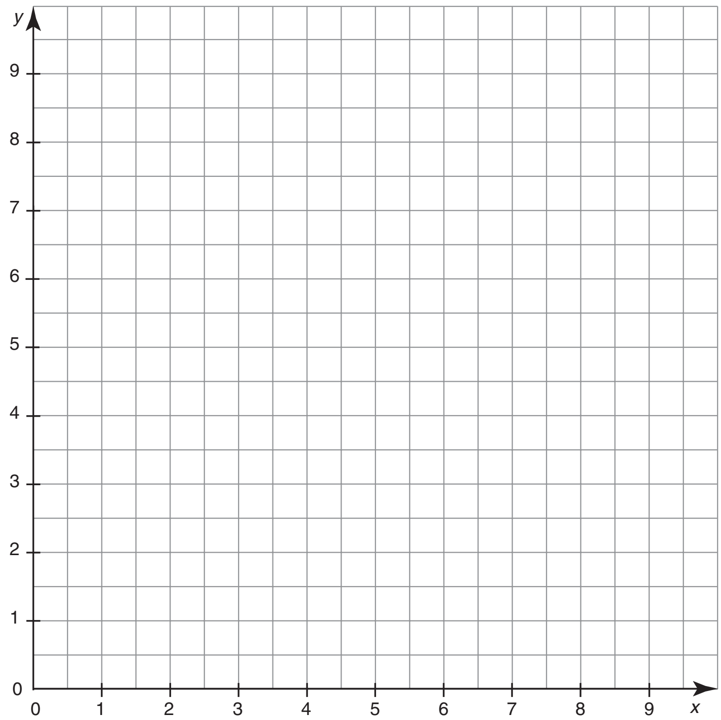
   e. $4y = 12 + x$

   $y = -4x - 5$

2. Graph points A (3, 1), B (8, 1), C (10, 5), and D (5, 5).

   a. Use slopes to determine if opposite sides of the figure are parallel.

   b. Use slopes to determine if the diagonals of the figure are perpendicular.



Be prepared to share your solutions and methods.

10.5 Up, Down, and All Around

Line Transformations

Learning Goals

In this lesson, you will:

- Explore transformations related to parallel lines.
- Explore transformations related to perpendicular lines.

Key Term

- Triangle Sum Theorem

10

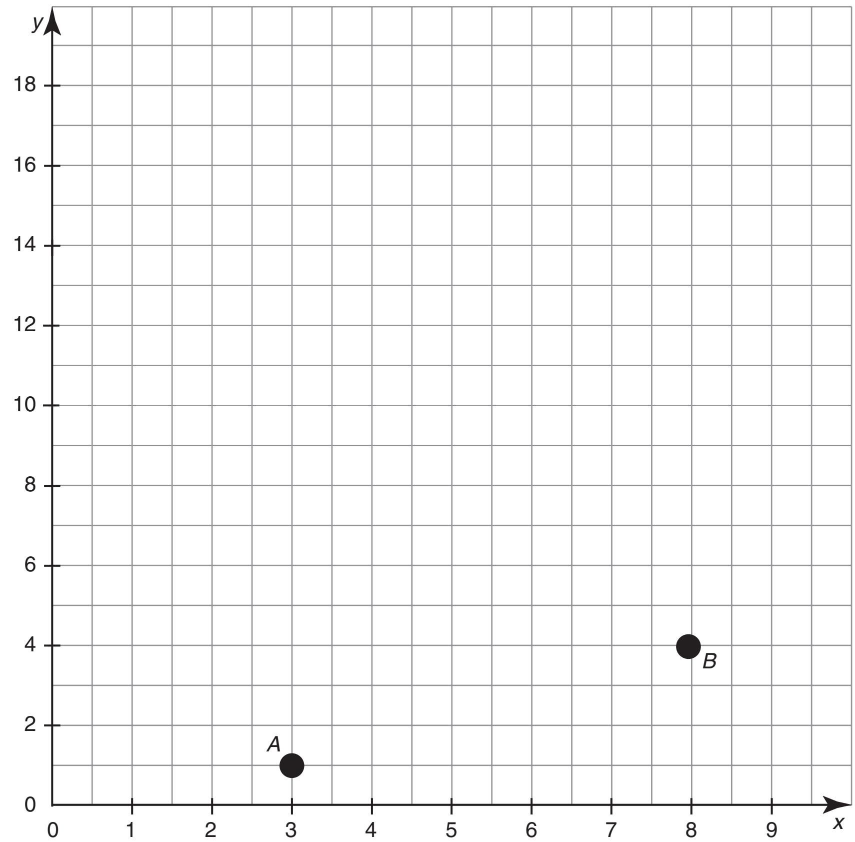
In an earlier lesson, you learned that when you rotate a point (x, y) 90 degrees counterclockwise about the origin, the location of the new point is $(-y, x)$. But what happens when you rotate an entire line 90 degrees?

If you rotate the line described by the equation $y = x$ counterclockwise 90 degrees, what would be the equation for the rotated line?
Can you graph the two lines?

Problem 1 Translating Lines

1. Points A (3, 1) and B (8, 4) are given.

- Connect points A and B to form line AB.
- Create points A' and B' by vertically translating points A and B 10 units.
- Connect points A' and B' to form line $A'B'$.

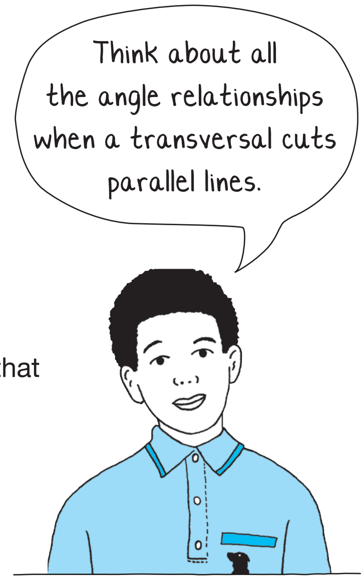


2. Calculate the slope of line AB.

3. Calculate the slope of line $A'B'$.

4. Is line AB parallel to line $A'B'$? Why or why not?

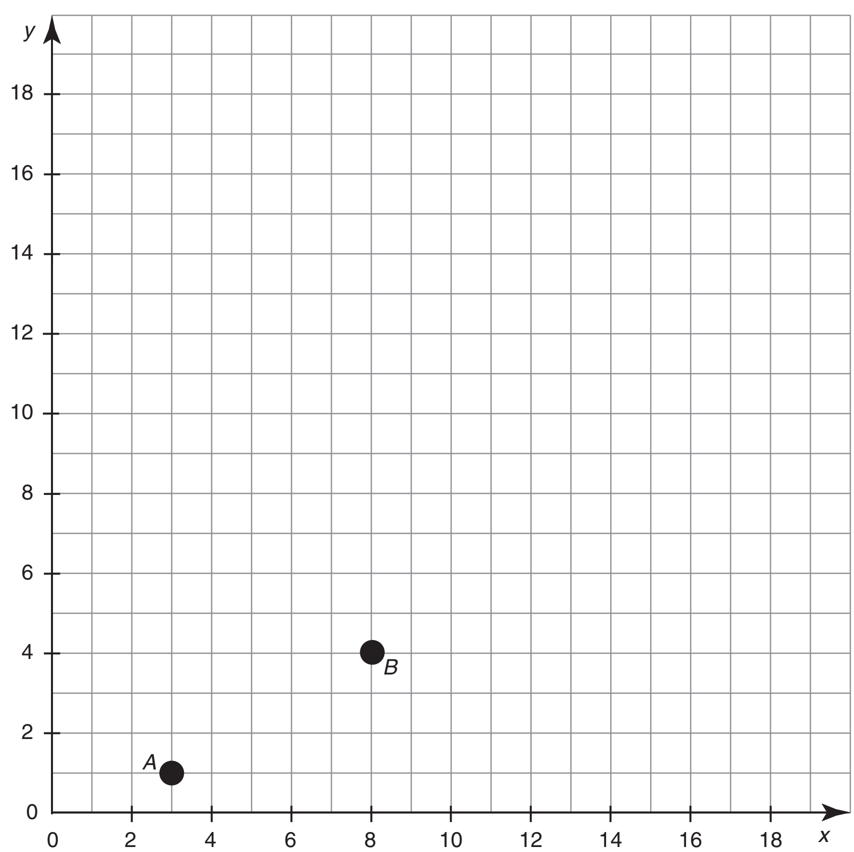
5. How could a transversal help to prove that lines AB and $A'B'$ are parallel?



6. Draw a transversal on the graph and use a protractor to verify that line AB is parallel to line $A'B'$.

Problem 2 Rotating Lines

1. Points A (3, 1) and B (8, 4) are given.
   - Connect points A and B to form line AB.
   - Use point A as the point of rotation and rotate line AB 90° counterclockwise.
   - Sketch this image line and label its y-intercept C.



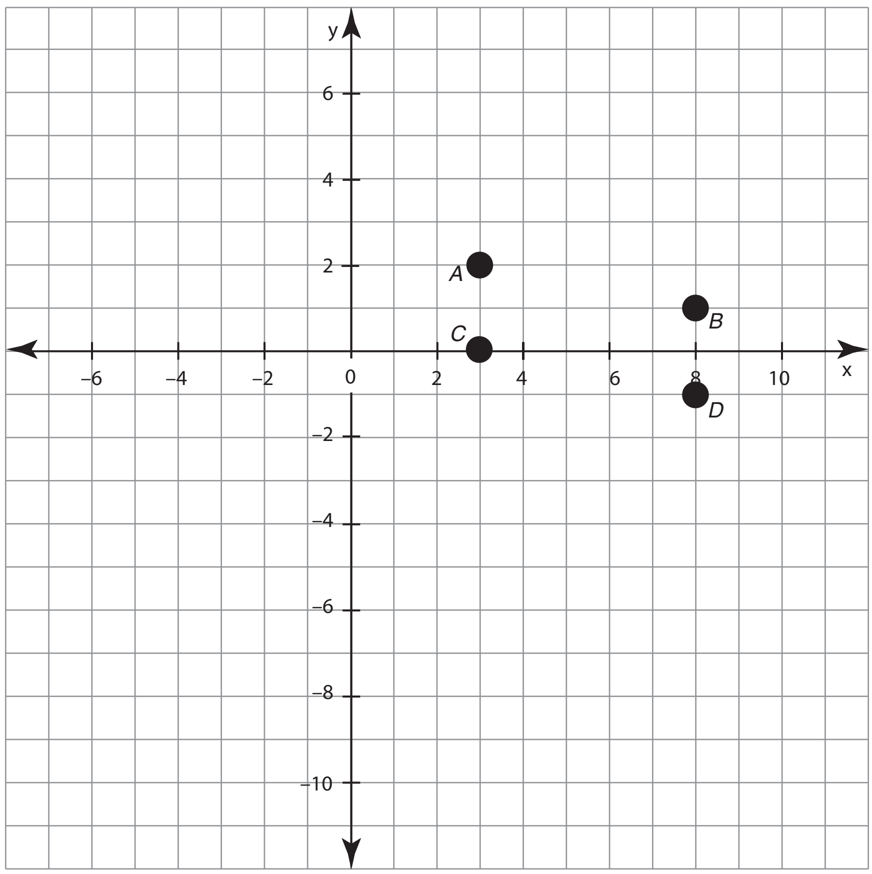
2. Calculate the slope of line *AB*.

3. Calculate the slope of line *AC*.

4. Is line *AB* perpendicular to line *AC*? Why or why not?

Problem 3 Reflecting Lines

1. Points A (3, 2), B (8, 1), C (3, 0), and D (8, -1) are given.
   - Connect point A to point B to form line segment AB, and connect point C to point D to form line segment CD.
   - Graph the reflection line $y = -x$.
   - Reflect point A over the reflection line $y = -x$ to create point A'.
   - Reflect point B over the reflection line $y = -x$ to create point B'.
   - Reflect point C over the reflection line $y = -x$ to create point C'.
   - Reflect point D over the reflection line $y = -x$ to create point D'.
   - Connect point A' to point B' to form line segment $A'B'$, and connect point C' to point D' to form line segment $C'D'$.



2. What are the coordinates of points A', B', C', and D'?

10

3. Calculate the slope of line *AB*.

4. Calculate the slope of line *CD*.

5. Is line *AB* parallel to line *CD*? Why or why not?

6. Calculate the slope of line *A*′*B*′.

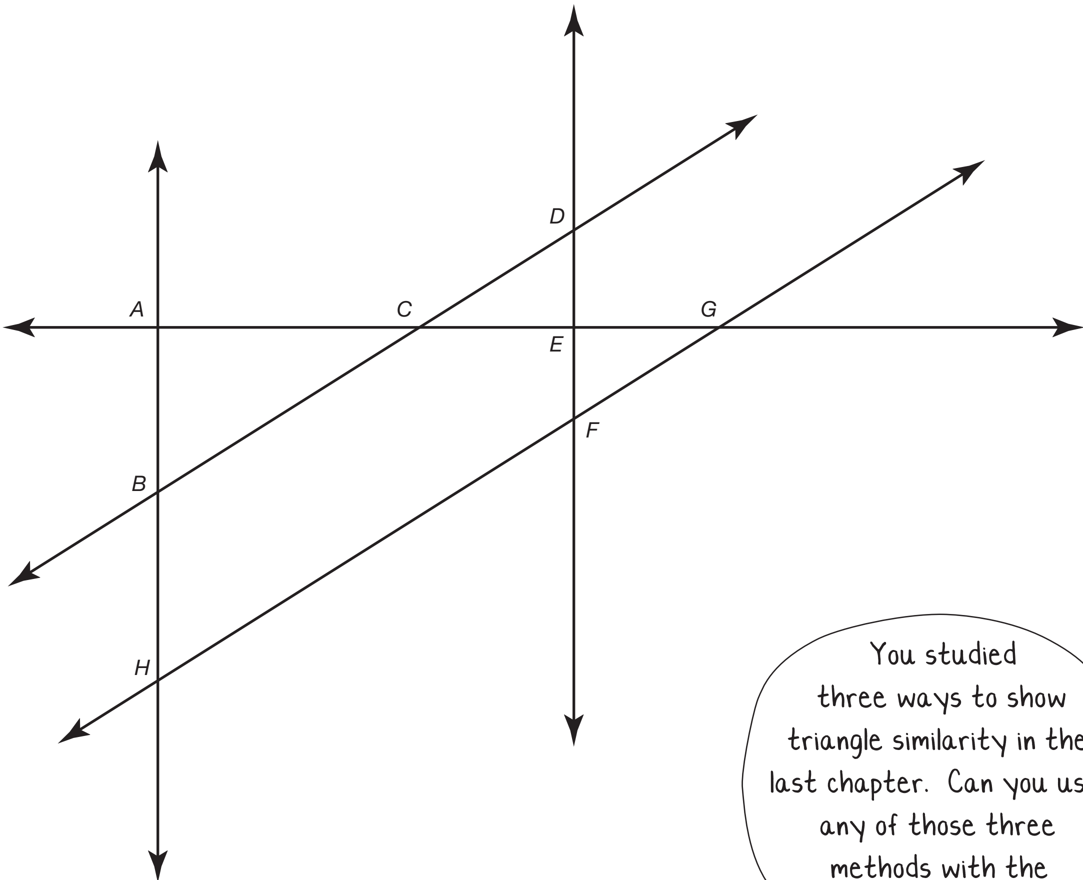
7. Calculate the slope of line *C*′*D*′.

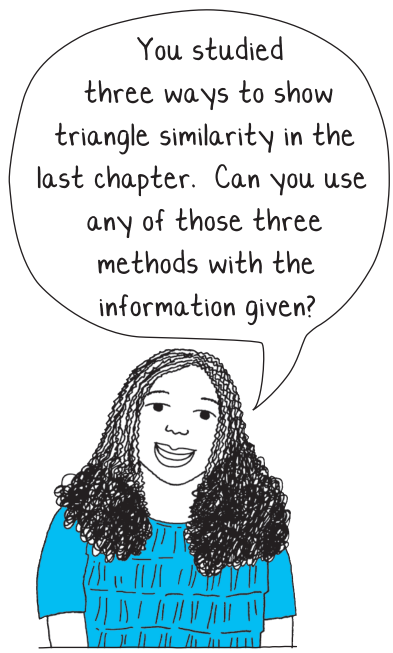
8. Is line *A*′*B*′ parallel to line *C*′*D*′? Why or why not?

Problem 4 Triangle Relationships

Use what you have learned about triangle similarity to answer the following questions.

Given: $\overleftrightarrow{BD} \parallel \overleftrightarrow{HG}$, $\overleftrightarrow{AH} \parallel \overleftrightarrow{DF}$, $\overleftrightarrow{AH} \perp \overleftrightarrow{AG}$, $\overleftrightarrow{DF} \perp \overleftrightarrow{AG}$





1. Identify all of the triangles in the diagram.

2. Is $\triangle ABC \sim \triangle AHG$? Explain your reasoning.

3. Is $\triangle ABC \sim \triangle EDC$? Explain your reasoning.

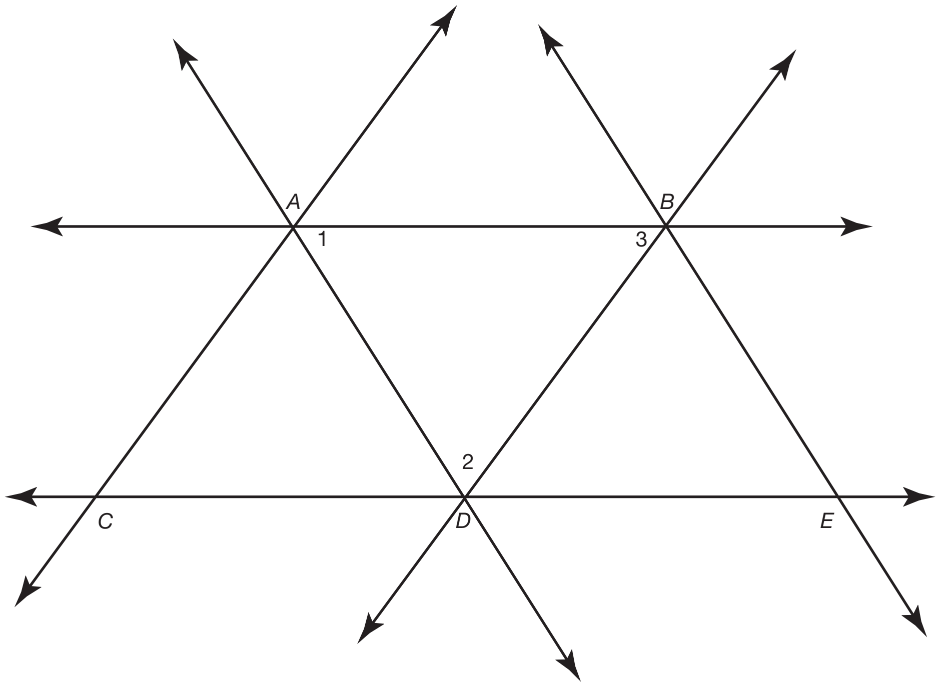
4. Is $\triangle EDC \sim \triangle EFG$? Explain your reasoning.

5. Is $\triangle ABC \sim \triangle EFG$? Explain your reasoning.

6. Is $\triangle AHG \sim \triangle EFG$? Explain your reasoning.

Problem 5 Parallel Lines and the Triangle Sum Theorem

Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CE}$, $\overleftrightarrow{AC} \parallel \overleftrightarrow{BD}$, $\overleftrightarrow{AD} \parallel \overleftrightarrow{BE}$



1. Label all other angles in the diagram congruent to $\angle 1$ by writing a 1 at the location of each angle.
2. Label all other angles in the diagram congruent to $\angle 2$ by writing a 2 at the location of each angle.
3. Label all other angles in the diagram congruent to $\angle 3$ by writing a 3 at the location of each angle.

The **Triangle Sum Theorem** states that the sum of the measures of the three interior angles of a triangle is equal to 180°.

4. Explain how this diagram can be used to justify the Triangle Sum Theorem.

Be prepared to share your solutions and methods.

10

10

Chapter 10 Summary

Key Terms

- intersecting lines (10.1)
- plane (10.1)
- perpendicular lines (10.1)
- parallel lines (10.1)
- coplanar lines (10.1)
- skew lines (10.1)
- coincidental lines (10.1)
- supplementary angles (10.2)
- linear pair of angles (10.2)
- transversal (10.3)
- alternate interior angles (10.3)
- alternate exterior angles (10.3)
- same-side interior angles (10.3)
- same-side exterior angles (10.3)
- reciprocal (10.4)
- negative reciprocal (10.4)
- Triangle Sum Theorem (10.5)

10

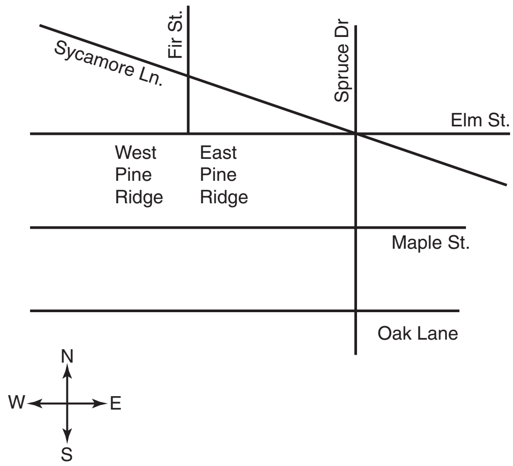
10.1 Defining the Relationship between Two Lines

Each of the following terms can be used to describe the relationship between two lines.

- Intersecting lines are lines in a plane that cross or intersect each other.
- Perpendicular lines are lines that intersect at a right angle.
- Parallel lines are lines that lie on the same plane and do not intersect.
- Coplanar lines are lines that lie on the same plane.
- Skew lines are lines that do not lie on the same plane.
- Coincidental lines are lines that have equivalent linear equations and overlap at every point when they are graphed.

Example

The map shows many examples of each type of relationship.



a. Intersecting lines

There are many pairs of intersecting lines. One pair is Sycamore Lane and Fir Street.

b. Perpendicular lines

There are several pairs of perpendicular lines. One pair is Spruce Drive and Elm Street.

c. Parallel lines

There are several pairs of parallel lines. One pair is Elm Street and Maple Street.

d. Coplanar lines

All of the lines lie on the same plane. Any two of the lines on the map can be labeled coplanar.

e. Skew lines

Because all of the lines lie on the same plane, none of the lines are skew lines.

f. Coincidental lines

West Pine Ridge and East Pine Ridge are the same road, so they are coincidental lines.

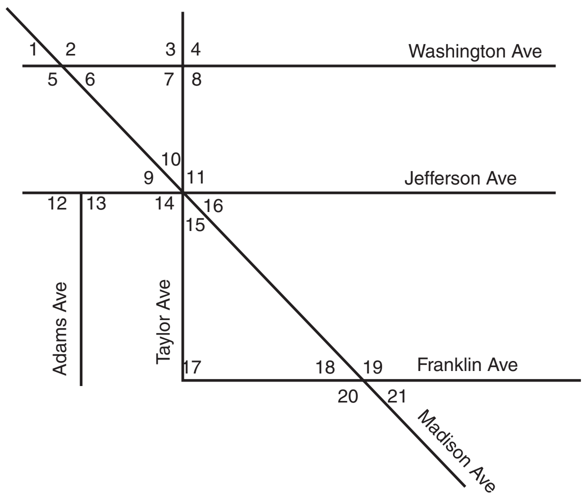
10.2 Defining Angle Relationships Formed by Two Intersecting Lines

Each of the following terms can be used to describe the relationship between two angles.

- Adjacent angles are coplanar angles that have a common vertex and a common side, but no common interior points.
- Vertical angles are two non-adjacent angles formed by intersecting lines or segments.
- A linear pair is two adjacent angles that form a straight line.
- Supplementary angles are two angles whose sum is 180 degrees.

Example

The map shows many examples of each type of relationship.



a. adjacent angles
There are many pairs of adjacent angles. One pair is $\angle 1$ and $\angle 2$.

b. vertical angles
There are many pairs of vertical angles. One pair is $\angle 4$ and $\angle 7$.

c. linear pair
There are many linear pairs of angles. One pair is $\angle 5$ and $\angle 6$.

d. supplementary angles
There are many pairs of supplementary angles. One pair is $\angle 12$ and $\angle 13$.

10.3 Exploring Angle Relationships Formed by Two Lines Intersected by a Transversal

10

A transversal is a line that intersects two or more lines. Each of the following terms describes pairs of angles that are created by a transversal.

Alternate interior angles are pairs of angles formed when a third line (transversal) intersects two other lines. These angles are on opposite sides of the transversal and are in between the other two lines. The alternate interior angles formed when two parallel lines are intersected by a transversal are congruent.

Alternate exterior angles are pairs of angles formed when a third line (transversal) intersects two other lines. These angles are on opposite sides of the transversal and are outside the other two lines. The alternate exterior angles formed when two parallel lines are intersected by a transversal are congruent.

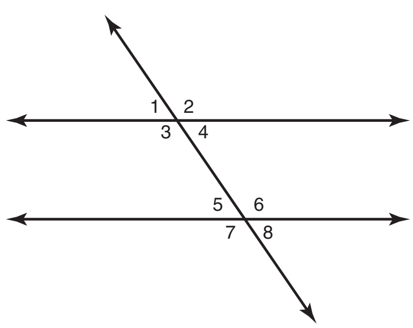
Same-side interior angles are pairs of angles formed when a third line (transversal) intersects two other lines. These angles are on the same side of the transversal and are between the other two lines. The same-side interior angles formed when two parallel lines are intersected by a transversal are supplementary.

Same-side exterior angles are pairs of angles formed when a third line (transversal) intersects two other lines. These angles are on the same side of the transversal and are outside the other two lines. The same-side exterior angles formed when two parallel lines are intersected by a transversal are supplementary.

Corresponding angles are pairs of angles that have the same relative positions in geometric figures. The corresponding angles formed when two parallel lines are intersected by a transversal are congruent.

Example

Given that $m\angle 1 = 72°$:



Because $\angle 1$ and $\angle 3$ are a linear pair of angles, they are supplementary, and $m\angle 3 = 108°$.

Because $\angle 1$ and $\angle 7$ are same-side exterior angles, they are supplementary, and $m\angle 7 = 108°$.

Because $\angle 3$ and $\angle 5$ are same-side interior angles, they are supplementary, and $m\angle 5 = 72°$.

Because $\angle 3$ and $\angle 6$ are alternate interior angles, they are congruent, and $m\angle 6 = 108°$.

Because $\angle 1$ and $\angle 8$ are alternate exterior angles, they are congruent, and $m\angle 8 = 72°$.

Because $\angle 6$ and $\angle 2$ are corresponding angles, they are congruent, and $m\angle 2 = 108°$.

Because $\angle 8$ and $\angle 4$ are corresponding angles, they are congruent, and $m\angle 4 = 72°$.

10.4 Determining Slopes of Parallel and Perpendicular Lines

The equations of parallel lines have equal slope values. The equations of perpendicular lines have slope values that are negative reciprocals of each other. The product of the slope values of perpendicular lines is -1.

10

Example

Consider each linear equation.

a. $16x + 4y = 32$

b. $-20x - 5y = 15$

c. $2x - 8y = 48$

First, determine the slope of each line.

a. $16x + 4y = 32$

$$4y = -16x + 32$$
$$y = -4x + 8$$
$$\text{slope} = -4$$

b. $-20x - 5y = 15$

$$-5y = 20x + 15$$
$$y = -4x - 3$$
$$\text{slope} = -4$$

c. $2x - 8y = 48$

$$-8y = -2x + 48$$
$$y = \frac{2}{8}x - 6$$
$$y = \frac{1}{4}x - 6$$
$$\text{slope} = \frac{1}{4}$$

Next, compare the slopes to determine if the lines are parallel or perpendicular.

The slopes of part (a) and part (b) are equal, so the lines are parallel.

The product of the slopes of part (a) and part (c) is -1, so the lines are perpendicular.

The product of the slopes of part (b) and part (c) is -1, so the lines are perpendicular.

10.5

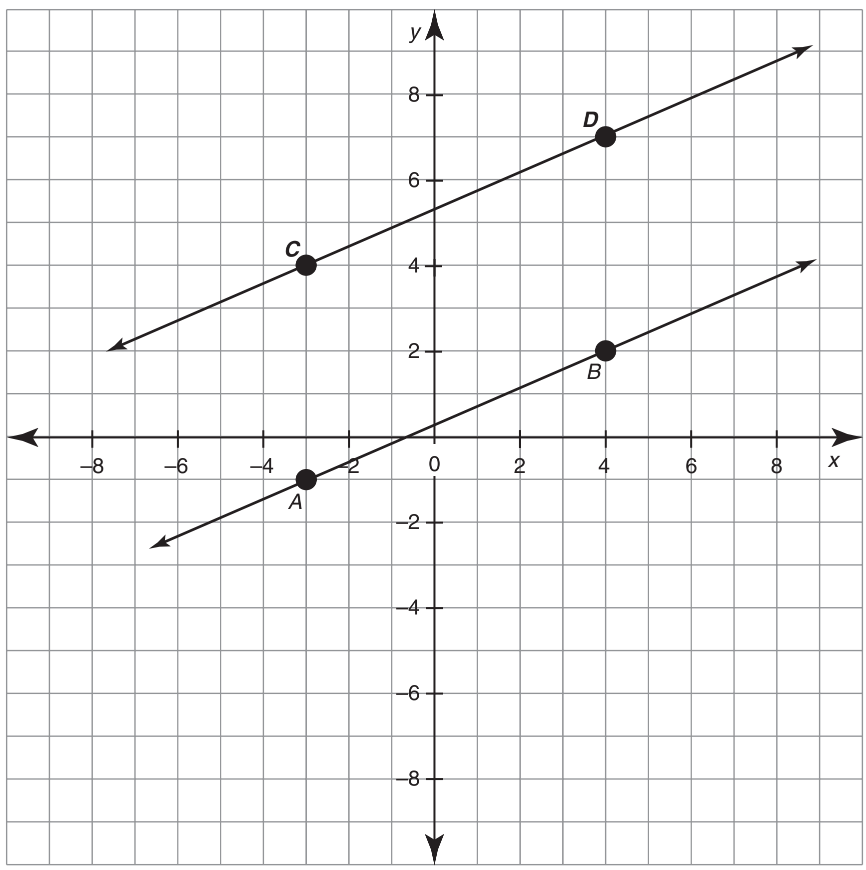
Translating Lines

Translating a line 90° will form a new line that is parallel to the original line.

Example

Line *AB* was translated vertically 5 units to create line *CD*. You can calculate the slope of each line to determine if the lines are parallel.

10



line *AB*:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-1)}{4 - (-3)} = \frac{3}{7}$$

line *CD*:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 4}{4 - (-3)} = \frac{3}{7}$$

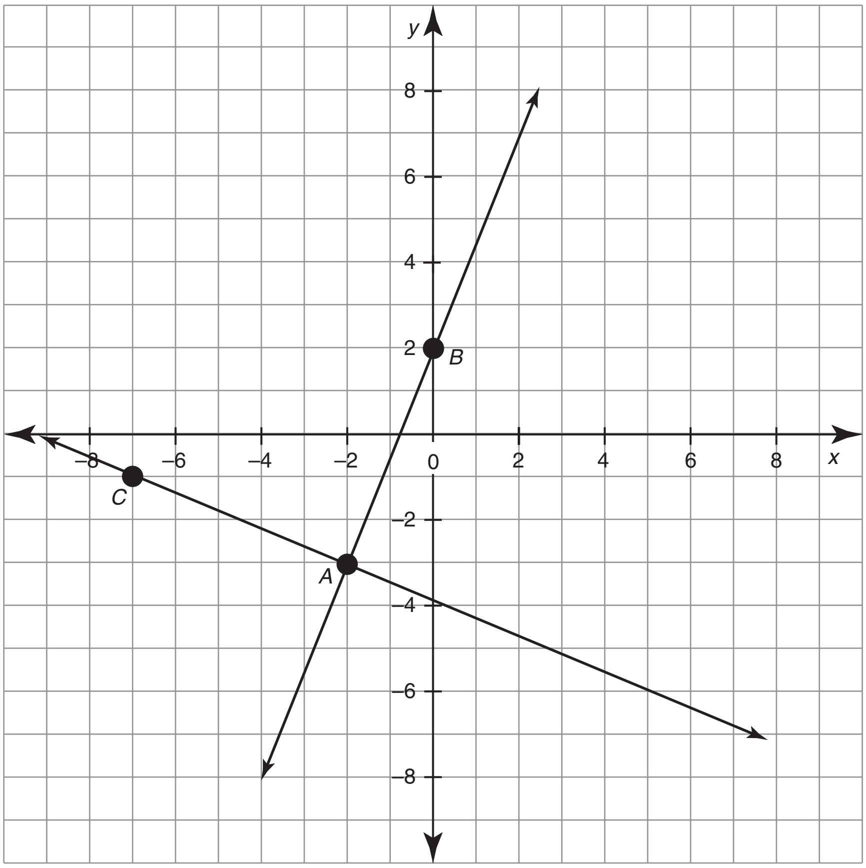
The slope of line *AB* is equal to the slope of line *CD*, so line *AB* is parallel to line *CD*.

10.5 Rotating Lines

Rotating a line will form a new line that is perpendicular to the original line.

Example

Line *AB* was rotated 90°counterclockwise around point *A* to form line *AC*. You can calculate the slope of each line to determine if the lines are perpendicular.



line *AB*:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{2 - (-3)}{0 - (-2)}$$
$$= \frac{5}{2}$$

line *AC*:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{-1 - (-3)}{-7 - (-2)}$$
$$= -\frac{2}{5}$$

The slope of line *AC* is the negative reciprocal of the slope of line *AB*, so line *AC* is perpendicular to line *AB*.

10

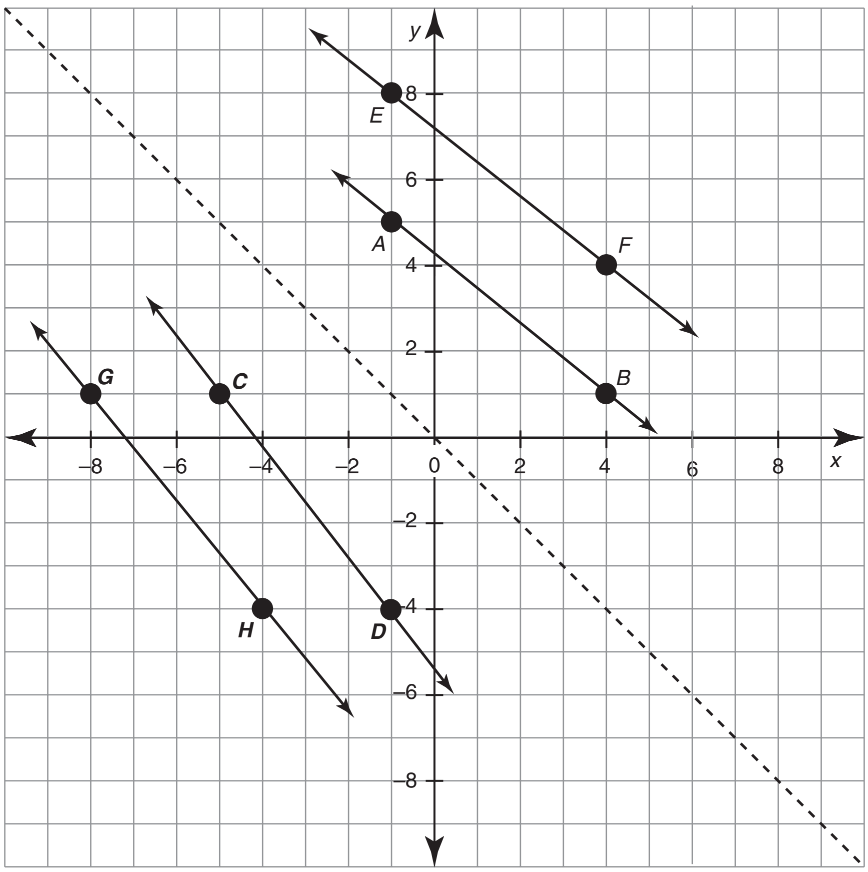
10.5 Reflecting Lines

Reflecting two parallel lines will form two new parallel lines.

Example

Line segment *AB* has been reflected over the reflection line $y = -x$ to form line segment *CD*.

Line segment *EF* has been reflected over the reflection line $y = -x$ to form line segment *GH*.



slope of $\overline{AB} = -\frac{4}{5}$

slope of $\overline{EF} = -\frac{4}{5}$

$\overline{AB} \parallel \overline{EF}$

slope of $\overline{CD} = -\frac{5}{4}$

slope of $\overline{GH} = -\frac{5}{4}$

$\overline{CD} \parallel \overline{GH}$