Line of Sight Guidance

Introduction

Most first generation missiles employed command guidance of some form or other. Using the sensors of the time it was not possible to observe the three dimensional position errors between missile and target accurately enough to achieve small miss distances. The line of sight geometry required just the two lateral position errors to be known, which was feasible with the trackers of the time, notably the human eyeball.

Also, the line of sight geometry was a natural development of the gun systems which were prevalent at the time. Furthermore, the early infra red seekers could only detect the hot jet pipe of the aircraft, and so could only engage after the enemy has over flown the target and done its mischief.

Line of sight guidance has been used in air to surface and surface to surface (anti-tank), but predominantly it is used in surface to air weapon systems. With improvements in seeker technology, it is likely to be abandoned in this role as well.

It has the advantage of a relatively low cost round, as most of the expense is concentrated in the platform sensors and launcher/magazine systems.

In the surface to air role, the object of the exercise is to defend a region about the launch point.

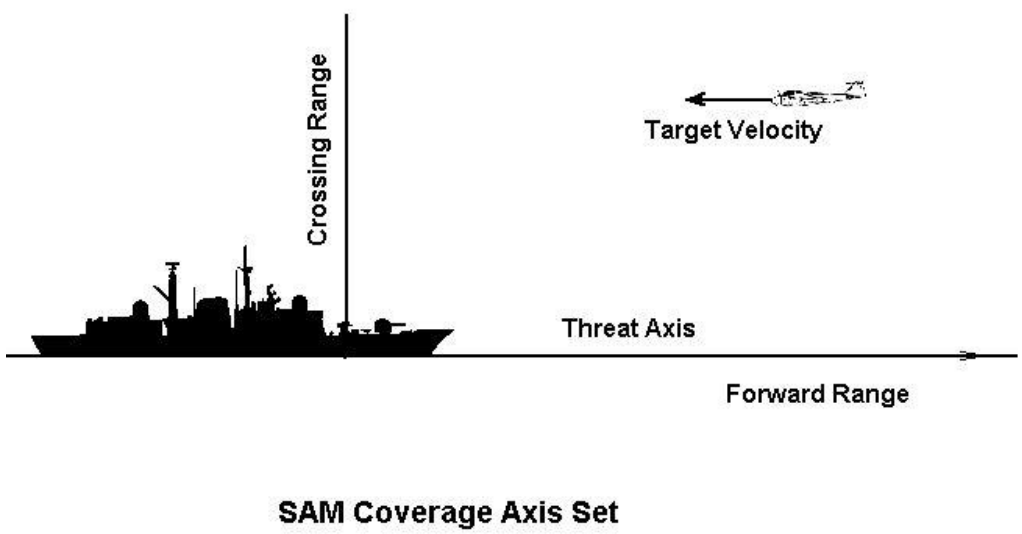


Figure 1 : Coverage Characterisation

We define the performance of a surface to air system in terms of the volume of space it denies to the enemy. To begin with, at least, we treat the target as a constant velocity point. Once we have gained understanding of the constraints under these simple conditions, we may proceed to more complex target behaviour.

The coverage is referred to an axis with missile launch point as origin, with x-axis parallel but in the opposite sense to, the incoming threat. This is the ‘forward range’ axis. Perpendicular to this axis is the ‘crossing range’ axis. Using this as the reference set it is reasonably easy to determine the constraints which limit coverage capability.

Constraints

Kinematic

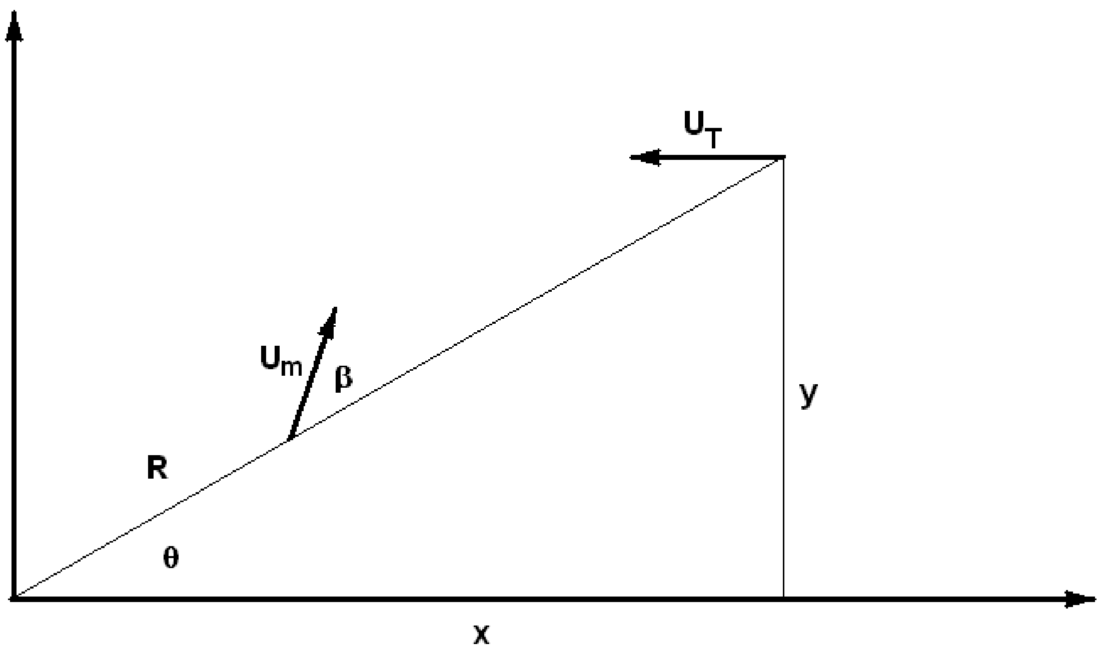


Figure 2 : Engagement Kinematics

The kinematics of the engagement are presented in Figure 2. Unless both target and launch platform are stationary, the missile velocity vector will not be aligned with the sight line. This immediately imposes a constraint, because the angle between the missile velocity vector and the sight line cannot exceed the beam width of the rearward facing sensors of a beam rider or the command link beam width of a command system. These angles will be small primarily to prevent jamming, and in the case of the beam rider to ensure guidance accuracy.

At this level of coarse analysis, we treat the velocity vector as coincident with the body longitudinal axis, so we refer to angle β in Figure 2 as the ‘body to beam’ angle. It is given by:

$$\sin\beta = \frac{R}{U_m}\dot{\theta}$$

Denoting the instantaneous range from launch point to target R_T, we have for the angular velocity of the beam:

$$\dot{\theta} = \frac{U_T}{R_T}\sin\theta$$

At intercept, $R=R_T$, so we have:

$$\sin\beta = \frac{U_T}{U_m}\sin\theta$$

It follows that to avoid this constraint, the missile speed must be considerably greater than the target speed. A body to beam limit of about 5 degrees implies a 10:1 speed advantage of missile over target, which for an air target, is out of the question, but could be achieved with an anti-armour missile. Every line of sight guided surface to air missile is therefore expected to run into body to beam angle limitations, which will limit the crossing range cover.

The other pure kinematic constraint is the lateral acceleration requirement. Unlike a homing missile which flies more or less toward the intercept point, the line of sight missile must apply lateral acceleration just to stay on the beam. Since the body to beam angle must be small, for the reasons given, the body lateral acceleration may be taken as equal to the beam lateral acceleration.

$$f_y = 2U_m\dot{\theta} + R\ddot{\theta}$$

Before considering the feedback control to maintain the missile on the beam it is advisable to include this as a feed forward acceleration into the guidance loop. Note however, this information is not available to a beam rider as it is estimated at the launch platform. Also, it is often difficult to obtain an accurate, noise free estimate of the sight line acceleration and the error in estimating it can easily exceed the miss distance caused by ignoring it, so it is frequently ignored. It is probably better to include integral of error feedback within the loop to eliminate an acceleration bias.

The dominant term in the feed forward is the Coriolis acceleration. This increases as the crossing range reduces, and is a maximum near the crossing range axis.

Additional Constraints

The maximum range depends on many factors, but hopefully it will be specified as an input to the analysis process, presumably from higher level tactical studies, based on how far away from the target area the enemy must be kept.

For example, a ground attack aircraft can toss bomb its target from a range of just over 5km. A UAV fitted with optical sensors is not expected to have a stand-off of greater than between 5 and 10km.

These all lie within the ambit of potential line of sight systems. However, to begin with, all options will be considered, and the study will continue to sufficiently fine resolution to enable a rational basis for down-selection, rather than the geek table, power point and guesswork that seems to be the current fashion.

The missile thrust profile will be chosen such that there is sufficient manoeuvre capability at this maximum range to still hit the target. In some circumstances there may be a constraint on the missile size and weight, such as compatibility with existing magazine handling systems and launchers, or restriction to manual loading, or portability/ logistics constraints in which case maximum range may well be dictated by propulsion constraints, and sensors would be selected to suit.

Sometimes a particular missile may be specified, and work is necessary to determine what the consequences will be on overall system performance of pre-judging the characteristics of such a critical system element.

Minimum range might be dictated by the distance needed to gather into the guidance beam, which depends on the method of launch and the spacing of the launcher and sensor. In extreme cases it may be dictated by warhead safety and arming considerations.

A further issue may arise because the number of fire channels may be limited and the overall time between engaging a target and being ready to engage the next might be greater than the spacing between consecutive targets. The density of the attack expected is another consideration which needs to flow down from higher level considerations.

These issues all interact and need to be addressed for the specific system under consideration, probably requiring several iterations before a viable option is derived, or the data is acquired to form the basis of a worth-while system specification.

Only the kinematic constraints may be considered fundamental. The rest are system-specific.

Lead Guidance

The kinematic limitations arise from the requirement to fly along a moving beam. If, instead, the missile were guided toward the predicted intercept point, these problems would disappear. However if the intercept point were known with such certainty, there would be no need for a missile at all, as an unguided shell would presumably work just as well.

Placing a lead angle on the beam is expected to reduce the severity of the constraints of flying on a rotating beam. A straight forward extrapolation of the target position is usually defeated by the uncertainty in the intercept position, so the idea of flying to a point part way between the current target position and the predicted impact position has been used on certain systems, which will remain nameless. This is called part lead line of sight guidance. The amount of lead angle is a trade off of body to beam angle, manoeuvre capability and uncertainty in intercept point.

In many cases the best compromise has been found to be with an aim point half way between the current target position and the predicted impact point. This result is fortuitous, and believed to be just a feature of the particular systems studied. However, ‘half-lead guidance’, as it is called, is a term which has found its way into the folk lore, so is worth mentioning for completeness.

In guiding up a line other than the line of sight to the target we might alleviate the manoeuvrability problems, but it introduces problems of its own. We can no longer keep both target and missile in the same beam. Either the beam must be broad enough to encompass both, usually with a degradation in resolution and/or signal to noise ratio, or the missile guidance beam must be separate from the target tracker, and we introduce a whole raft of alignment problems. The misalignment of target and missile sensor introduces a guidance error, known as 'collimation error' which can place a further constraint on range.

The use of separate guidance and tracking beams is more frequently associated with beam rider, rather than command to line of sight systems, as body to beam angle must be small to maintain a narrow guidance beam. However, with modern phase-array technology, the possibility of steering the sensor to always point at the launch point appears a viable approach to alleviating the body to beam limitation. Indeed the same technology could be applied to steering the command link antenna of a CLOS (command to line of sight) system.

Commanded Proportional Navigation

The scheduling of lead angle with time to go is likely to be a highly system-specific problem. However, as we have derived proportional navigation as a navigation law for systems in which the principal sensor is carried on the missile, it is interesting to see what kinematic advantage there is to implementing it with a pair of surface based sensors. This type of system is known as 'three-point' guidance, and to date at least, it has usually been incapable of achieving adequate miss distances.

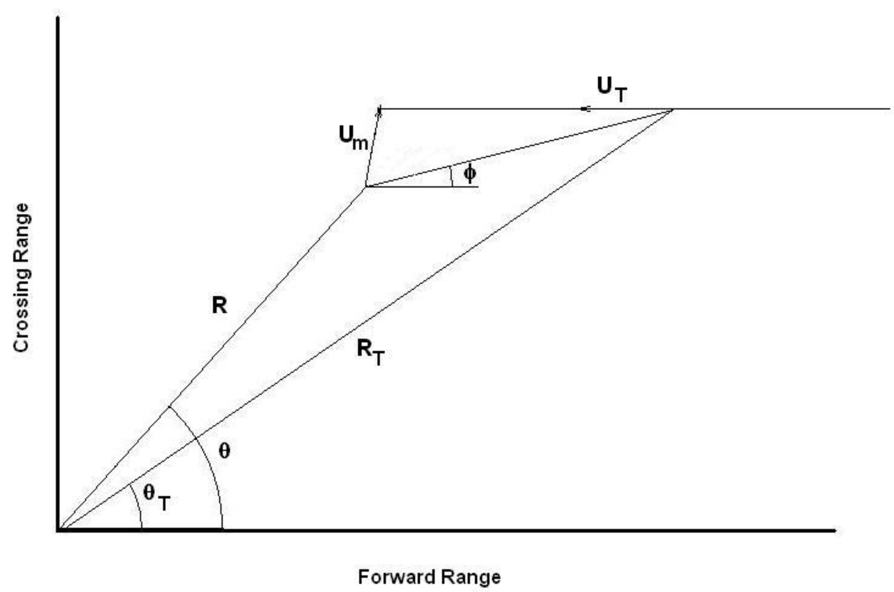


Figure 3 : Three Point Guidance

General three point guidance does not necessarily require missile tracking sensor and target tracking sensor to be co-located. This version restricts itself to the angular alignment problem in isolation, before obscuring the issue with position alignment errors.

We see from the geometry of Figure 3 that the sight line direction is given by:

$$\tan\phi = \frac{R_T \sin\theta_T - R_m \sin\theta_m}{R_T \cos\theta_T - R_m \cos\theta_m}$$

The exact expression is likely to get very messy, so we'll leave that to people who don't get out much. We will concern ourselves with engagements close to the forward range treat axis. If we can't hit the target there, then what happens at greater crossing ranges is really of academic interest only.

So using the small angle approximations:

$$\phi \approx \frac{R_T\theta_T - R_m\theta_m}{R_T - R_m}$$

Differentiating:

$$\dot{\phi} = \frac{\left(\dot{R}_T\theta_T + R_T\dot{\theta}_T - \dot{R}_m\theta_m - R_m\dot{\theta}_m\right)\left(R_T - R_m\right) - \left(R_T\theta_T - R_m\theta_m\right)\left(\dot{R}_T - \dot{R}_m\right)}{\left(R_T - R_m\right)^2}$$

$$\dot{\phi} = \frac{R_T\dot{\theta}_T - R_m\dot{\theta}_m}{R_T - R_m} - \frac{\left(\dot{R}_T R_m - R_T\dot{R}_m\right)}{\left(R_T - R_m\right)^2}\left(\theta_T - \theta_m\right)$$

The missile lateral acceleration command becomes:

$$f_y = N\left(\dot{R}_T - \dot{R}_m\right)\dot{\phi}$$

It appears that proportional navigation does not depend critically on the alignment of the two beams; a fixed collimation error is removed by the process of differentiation.

In three dimensions, the sight line becomes a vector expression:

$$\underline{\hat{s}} = \frac{\left(\underline{R}_T - \underline{R}_m\right)}{\left|\underline{R}_T - \underline{R}_m\right|}$$

Where the underscore denotes a vector quantity and the 'hat' in this context denotes a unit vector.

The missile and target lines of sight define a plane having unit normal:

$$\underline{\hat{n}} = \frac{R_T \times R_m}{\left|R_T \times R_m\right|}$$

The cross product of the sight line and normal yields the third axis of an orthogonal set:

$$\underline{\hat{m}} = \underline{\hat{n}} \times \underline{\hat{s}}$$

The components of sight line spin are given by:

$$\omega_m = \dot{\underline{\hat{s}}} \cdot \underline{\hat{n}}$$

$$\omega_n = -\dot{\underline{\hat{s}}} \cdot \underline{\hat{m}}$$

The lateral acceleration would be calculated with respect to the missile sight line and resolved into missile body axes. The missile would be launched in the direction of the predicted intercept point, drastically reducing the body to beam or lateral acceleration requirements.

The requirement for range implies that the sensor is radar. The range rates are derivable from the Doppler, so it looks like proportional navigation is potentially useable with a command system. However, the author has never encountered a system which uses this method of guidance, and is uncertain whether it would actually work in practice.

Loop Structure

Most line of sight systems make no attempt at lead guidance, so we are typically talking about flying the

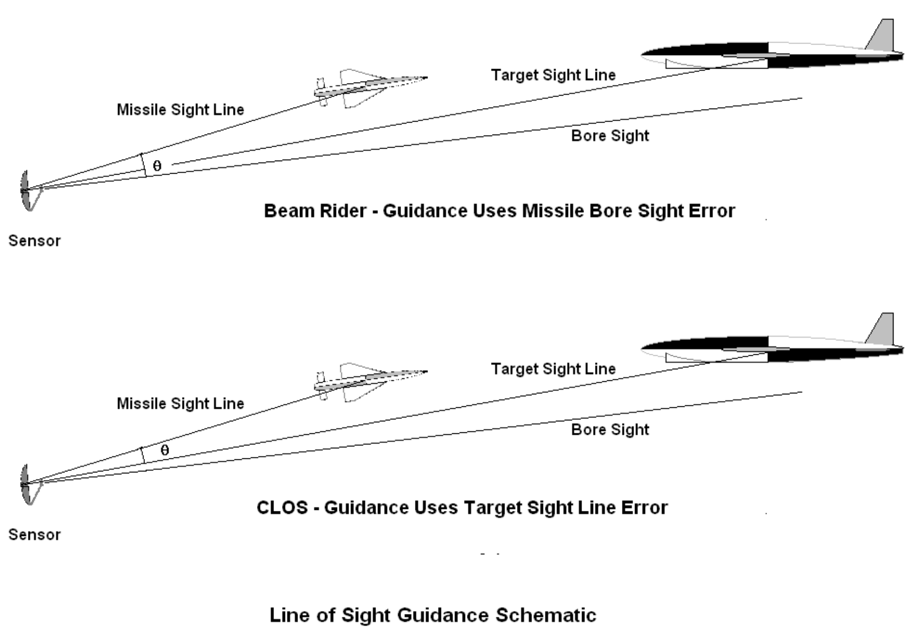


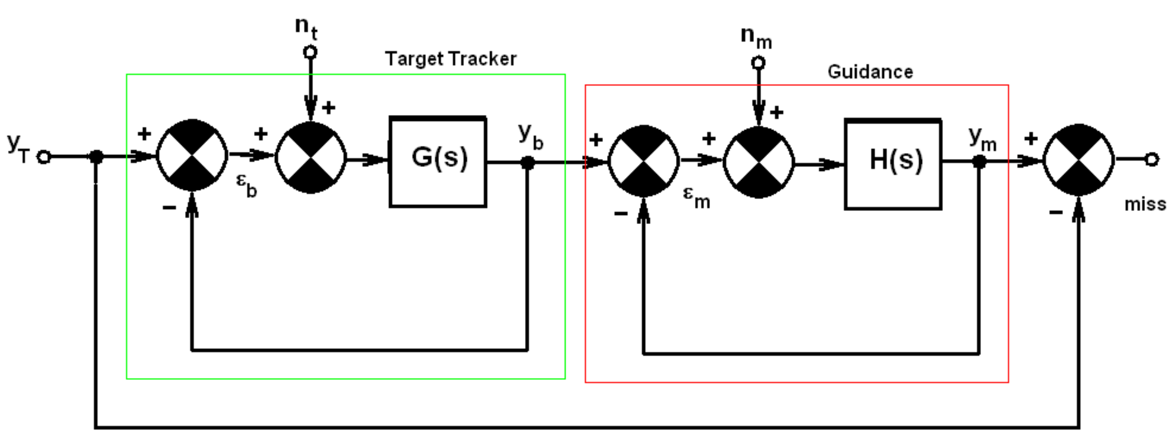
Figure 4 : Fundamental Difference between CLOS and Beam Rider

missile along a straight line joining the tracker and the target. Whether the means of measuring the distance off bore sight is on board the missile or on the launch platform appears immaterial.

The fundamental difference between the two is shown in Figure 4. In both cases a tracker tries to keep its bore sight on the target. We shall assume that the beam rider uses the same beam for guidance, although this is rarely the case. Even without the alignment problems of a separate guidance beam, it is evident that the target tracking and guidance functions are connected in series. In the case of command to line of sight, the bore sight need only be near the true sight line, as the angular error between the target and missile sight lines may be measured directly. The two loops operate in parallel, and not in series.

Guidance up the slewing beam requires a Coriolis acceleration, and possibly also the angular acceleration term. These may be estimated at the sensor and, in the case of CLOS, added to the lateral acceleration command. With no direct communication between the tracker and the missile, this term requires the guidance beam to lead the tracker beam, so that in general, a separate guidance beam is usually required, otherwise crossing range performance is likely to be poor. In principle, integral of error could be introduced within the guidance loop to alleviate this problem.

Beam riders are considered obsolete technology nowadays, so we don't call them 'beam riders' anymore. The modern term is 'information field' guidance. This is just another implementation of the old beam rider. The guidance beam, rather than the missile itself, provides the position of the missile within the beam. As an example, a raster scanning laser transmitting codes corresponding to the position of the missile in the scan pattern is detected by a rearwards facing sensor, the codes may be offset to provide lead when the beam is slewing. The actual kinematics are really no different from the classical beam rider, and the fundamental constraints remain the same.



.Figure 5 : Beam Rider Basic Loop Structure

A beam rider loop at its simplest is presented in .Figure 5. The tracker deals with angles, but these are converted to positions (y) by multiplying by the missile range. As with the homing missile analysis, these are referred to an initial aim direction.

The transfer functions G(s) and H(s) are the open loop transfer functions of the tracker and guidance respectively. The measurements are expected to be corrupted by noise.

If there is a separate guidance beam, its dynamics would be located between the target tracker and missile, adding a third delay. Not only are the loops in series, resulting in a potentially sluggish response, the noise from both tracker and guidance feed through to the miss distance.

Miss distance arises from the kinematic tracking error due to the time-varying nature of the input target position, and the noise in the loop. Both are determined by the return differences of the respective loops. High bandwidth is needed to suppress the kinematic error, whilst low bandwidth is needed to suppress noise.

Usually the tracker and missile are developed by different manufacturers, who are loath to share information with each other. Each has only part of the jigsaw, so that optimisation becomes a major problem. If we wish to hit the target, we cannot afford to leave system integration until after the elements have been developed, the system integration must be addressed from the outset, as it will raise issues which may require changes to the component parts.

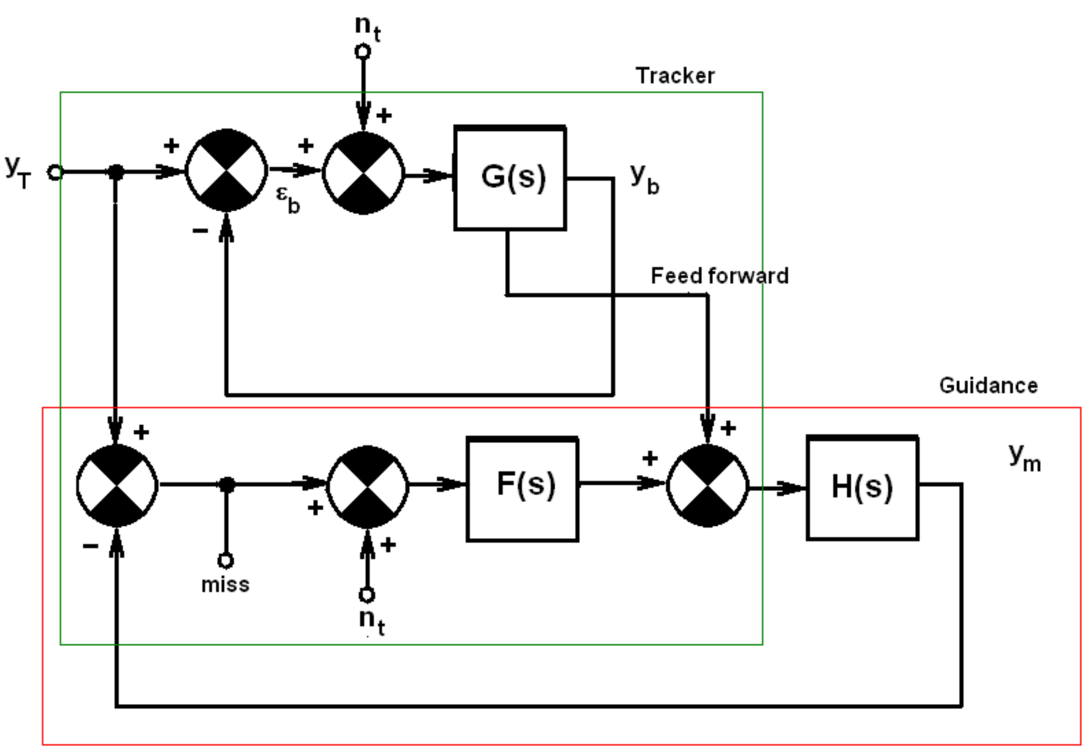


Figure 6 : Basic CLOS Loop Structure

As can be seen from Figure 6, the dynamics of the tracker loop run in parallel with the guidance loop. We have introduced a new transfer function, F(s), representing the filtering in the tracker. Unlike the beam rider which has potential noise sources in both missile and target channels, the CLOS approach only introduces noise in the tracker. Also it is far easier to introduce the feed forward acceleration into the guidance command.

These advantages come at a cost. The tracker and guidance loops are more intimately associated, requiring close co-operation between the teams developing each major component. Needless to say such co-operation is rare between industrial rivals.

But this is a common trend in systems. Higher performance usually requires tighter integration. The modern tendency to try and treat system elements in isolation, ignoring their interaction, is a classic tail-wag-dog approach of putting programme management convenience ahead of achieving system proper function.

The tracker will contain a filter similar to that in the guidance loop. Indeed, it is in all probability the same filter. The difference is in the delay introduced in the tracker pointing loop, which for mechanically steered antennas might be significant. However, modern phased array technology largely eliminates that delay, implying this particular advantage may no longer be very great.

The advantage of the CLOS system over beam riders has to some extent been eroded by technology. In fact, using optical or infra-red sensors, rather than radar, the advantage always was fairly marginal.

Coarse Analysis

Before proceeding to the level at which processor speeds, servo motor power and component bandwidths may be determined, a high level information theoretic analysis is in order. If perfect tracking and guidance is assumed, all the noise sources feed through to the miss distance, so we know from the outset not to select sensors whose resolution is coarser than the required lethal radius at the required maximum range. Finite loop response will make matters worse.

However, although this will eliminate obvious non-starters, the problem is more complex than this. Fine resolution may be achieved at the cost of reduced field of view, and we need the kinematic behaviour to decide on the appropriate compromise. Also sensor noise depends on integration time; very low noise levels are possible with very long integration times. However, such delays introduce undesirable phase lag into the loop.

We can state from the outset whether a given off-the-shelf item stands a chance of working within the loop, but until it is analysed in context, we cannot determine its actual utility.

Loop Analysis/Design

The majority of texts on weapon guidance have a tendency to treat the missile guidance loop in isolation, hence there is a widespread belief that beam rider and CLOS are kinematically equivalent. We hope that the previous section has dispelled that particular myth.

We need to consider both tracker and guidance loop to indicate, in fairly general terms, i.e. what we expect to see in the loop. Any more detailed description is likely to be system-specific and outside the scope of this text.

Tracker

Regarding trackers, there is a range of potential options, and a complete discussion of sensor types and their attributes would be a major work in its own right, and would not really be relevant. We are concerned with how the tracker behaviour influences miss distance, without reference to the details of how it actually works. Indeed, we do not want to pre-judge the type of sensor to employ, but need a means of assessing how the sensor parameters affect miss distance.

Strangely enough, modern object-oriented system analysis actually begins with existing elements, as opposed to finding out what they should be from a flow down from proper function. As the latter approach tends to be highly numerate, it is confused by the ignorant with design.

We shall assume the sensor is mechanically steered, although the modern trend is towards staring arrays, and protected from the environment by a radome or window. A typical functional loop, good enough for initial concept, is presented in Figure 7.

Essentially the tracker has a means of measuring the angles off bore sight of target and missile, and is capable of generating the guidance command and feed forward.

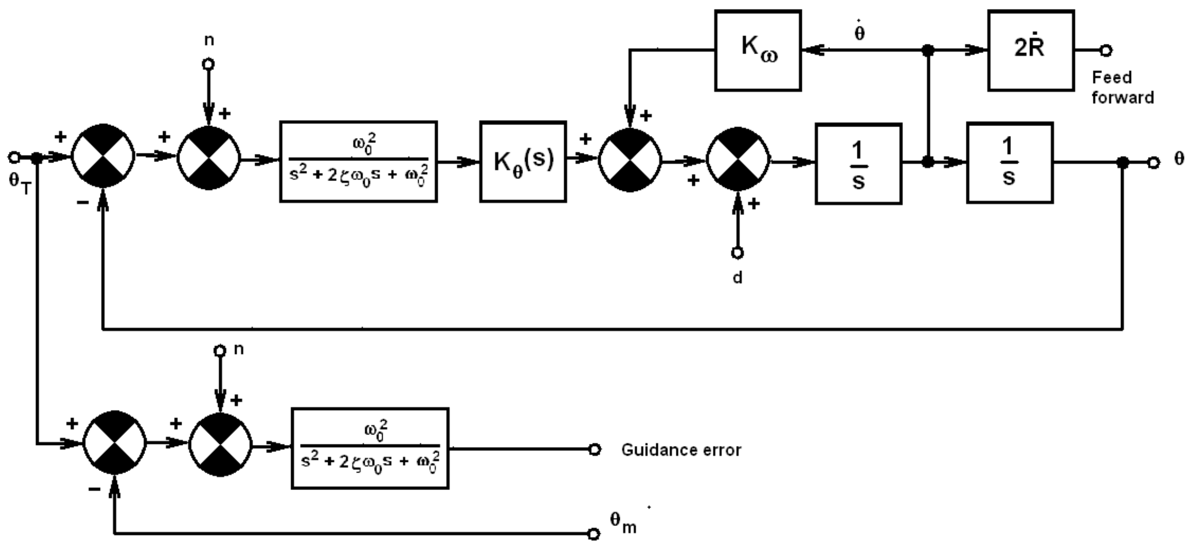


Figure 7 : Conceptual Tracker Loop

We have represented a tracking filter as a second order lag, largely so that some representation of its bandwidth is present in the loop. We can determine the sensitivity of the system to this assumption as part of the analysis. If it proves critical, more detail will obviously be needed.

One important point, by representing everything as continuous time functions we are not pre-supposing an analogue implementation. That is a common misconception of the technically naive. The filter is actually expected to be implemented digitally.

However, we begin with the continuous system representation, as this is the ideal. Introducing the z-transform representation, anti-aliasing filter, processor delay and zero order hold, with the introduction of quantisation noise, degrades the ideal behaviour. The issues of implementing the processing digitally are addressed in the analysis.

The guidance error is assumed extracted from an identical tracking filter as is used to track the target, although this need not be the case.

The input to the tracking filter is the bore sight error corrupted by measurement noise. The filtered estimate of bore sight error is fed via a compensator, which dominantly acts as the loop stiffness, which is combined with a rate feedback to generate the input to the torque motor needed to steer the antenna.

There may be some disturbance torques also acting, e.g. structural vibration, and these are included as the disturbance: ‘d’.

Not all tracker options will have all of the elements depicted, so the consequences of their omission in the particular system context need to be assessed. Also, the ingenious designer may think of extra bells and whistles which may be added, but Figure 7 represents an effective starting point for the analysis.

The world of automatic control furnishes us with a cornucopia of methods which allow us to relate behaviour to system parameter values, and hence form the rational basis for a system specification. This is in stark contrast to the pseudo-scientific *hocus pocus* which has become the current fashion. There are plenty of books on the subject, and courses are offered at universities, for anybody interested in genuine system science.

The tracking filter serves essentially to prevent the (wide band) measurement noise leaking through into the servo, and also into the guidance loop. This implies a low pass characteristic which introduces a de-stabilising phase lag into the loop. This, in turn, may limit the pointing loop bandwidth, potentially introducing greater kinematic tracking error. In the extreme, the target may be lost from the field of view.

Usually, the tracker will require a wide beam for initial gather of the missile on to the bore sight, and loop parameters may be different for this phase of flight. This is important, because gather invariably determines the minimum engagement range and must be done quickly.

There are occasions when, no matter how the loop is tweaked, it cannot be made satisfactory with the values of measurement noise, and measures, such as increasing the signal power, would need to be considered.

Guidance Loop

The actual guidance loop is similar for both beam rider and CLOS. The beam rider believes the guidance beam bore sight is aligned perfectly with the target, except perhaps for a deliberate lead angle. The CLOS system is commanded by differential missile/target error, and is not too concerned where the tracker bore sight is, provided the tracking error is less than the beam width. The target tracking errors lie outside the guidance loop, so the missile sees a guidance error corrupted by tracker noise.

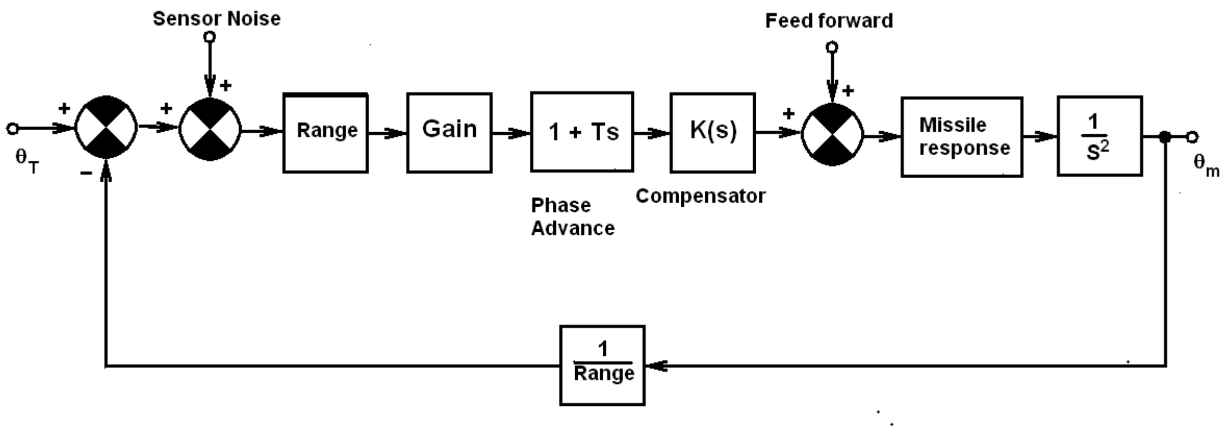


Figure 8 : Typical Guidance Loop Structure

The essential features of the guidance loop are presented in Figure 8. The angular guidance error is converted into a position error using an estimate of the range. This range estimate need not be particularly accurate, as any error amounts to a small variation in loop gain from its intended value. As this is a null-seeking system, it will have no effect on the equilibrium, and only a minor effect on the settling time. A stored value of range versus time is often adequate.

In a radar system, range may be a means of discriminating target from missile, and may well be available.

The phase advance term is part of the compensator, but is presented explicitly because it is fundamental to loop stability. The rate of change of position error must be known to provide damping of the loop, this cannot be measured, so must be estimated. The compensator contains the pole for the phase advance and further phase advance to recover the phase loss in the autopilot.

The phase advance amplifies noise and can cause control saturation, so there is a limit as to how much can be used.

The faster the missile response, the less phase advance is needed, so the noise requirements cannot be known until there is a reasonable estimate of the autopilot bandwidth.

One approach is to generate a population of fly out trajectories of a point moving at constant speed along a line joining the launch point to the target, and taking the Fourier transform of the lateral acceleration histories in order to characterise the set of engagements as a spectrum. The half power width of the spectrum gives an indication of the bandwidth required of the guidance loop, from which the autopilot bandwidth may be derived.

Alternatively, if the missile is 'given' we can use the linear-quadratic optimal control result for guidance loop bandwidth:

$$\omega_n^2 = Loop\ stiffness = \frac{ng}{x_m}$$

Where g is the acceleration due to gravity, n is the lateral acceleration capability in gs and x_m the desired miss distance.

Only in university examinations are things 'given', and the emphasis is on the solution method. In the real world, it is up to the engineer to find a mathematical formulation which is adequate for his/her purposes. Usually, this is what separates the men from the boys.

Miss Distance

After a few iterations we arrive at a few viable tracker/guidance loop options. The transfer function from each noise source to miss distance can be found. The contribution of the *i*th noise source to miss distance is found from the noise integral:

$$\sigma_i^2 = \frac{1}{\pi}\int_0^{\infty} G_i(j\omega)G_i(-j\omega)d\omega$$

Where: $G_i(s)$ is the transfer function from the *i*th noise source to the miss distance, and σ denotes the standard deviation. The miss distance is then found from:

$$\sigma_m^2 = \sum_1^n \sigma_i^2$$

This is not the complete picture, as this is the statistical distribution about the point of closest approach, which is not necessarily coincident with the target. Some residual kinematic error is to be expected, particularly near the edges of the coverage diagram.

Strictly speaking, the noise statistics depend on target range, so are unlikely to be stationary processes as is implicit in the white noise assumption on which the noise integral is based. If the statistics don't change much within the loop settling time, this doesn't really matter, and consequently it is standard practice to treat all noise sources as stationary, and to repeat the calculation over the complete ambit of target ranges.

The purpose of this calculation is to identify which noise sources are dominant, so that their effects may be tackled with the highest priority. It is not usually intended as a means of producing definitive performance predictions. The tendency of managers to cite work in progress estimates as absolute and achievable performance predictions, is probably the greatest inhibitor of proper system analysis which is imposed on the modern designer by those whose experience is limited to the design of databases.

Note that none of the calculations needed to characterise the system involve the detailed fly-out model, so beloved by non-participants. Indeed, until the analysis is done, we actually don't know what to put in the detailed model.

Gather

The two most important phases of flight, are probably the start and the end. What happens as the missile flies past the target is obviously important, so also is the launch and initial gather. The rest of the flight is generally of secondary consideration, yet accounts for most of the run time of a fly out model.

We have considered miss distance analysis, which covers the target end. The principal issue of the launch end is how soon the guidance loop can be closed. It is often impractical to co-locate sensor and launcher. Rocket plumes and radomes do not mix well. Early systems relied on ballistic flight from launcher to beam, which placed a severe constraint on launcher/beam separation. More modern systems may be fitted with a simple inertial navigation system to guide the missile into the gather beam. With low cost solid state sensors and very short IN guidance phases, this is achievable fairly cheaply.

We shall address guidance under inertial navigation in a later section, for the moment we shall consider the line of sight guidance problems specific to the gather phase. Unlike the guidance phase, when the missile speed is roughly constant gather takes place with the missile still accelerating. This introduces a motor thrust term perpendicular to the beam.

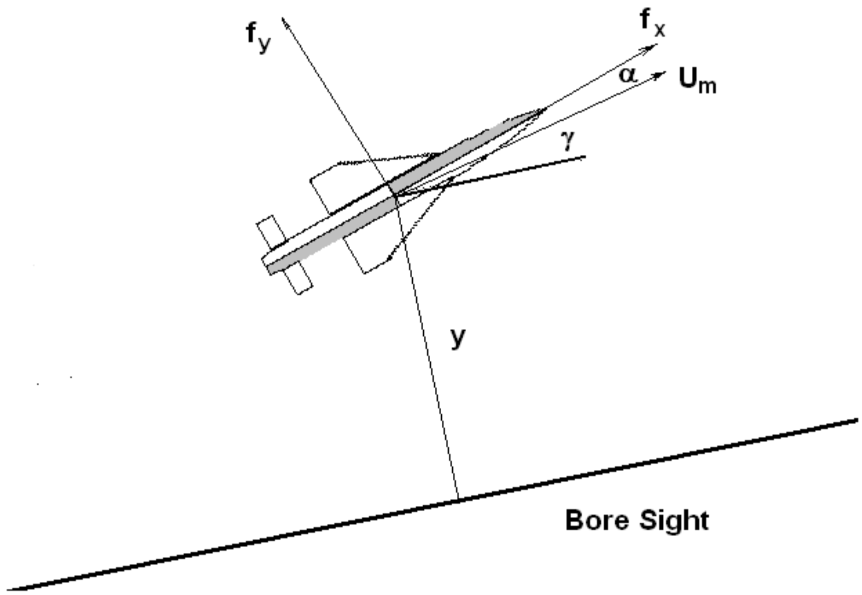


Figure 9 : Kinematics During Boost

From Figure 9, we see that in addition to the lateral acceleration f_y, there is a thrust term perpendicular to the beam:

$$f_2 = f_x(\alpha + \gamma)$$

Assuming the beam is stationary, the direction of the velocity vector relative to the beam is given by:

$$\gamma = \sin^{-1}\left(\frac{\dot{y}}{U_m}\right) \approx \frac{\dot{y}}{U_m}$$

The angle of attack may be estimated from the trim condition, which can be shown to be, approximately:

$$\alpha \approx \left(\frac{m}{Y_\beta}\right) f_y$$

Where m is the missile mass and Y_β is the lateral force per unit angle of attack. This is a stability derivative, which will be discussed when we consider autopilots.

So the lateral acceleration relative to the beam, taking account of the longitudinal acceleration, is:

$$f_2 = f_x\left(\frac{m}{Y_\beta}\right) f_y + f_x \frac{\dot{y}}{U_m}$$

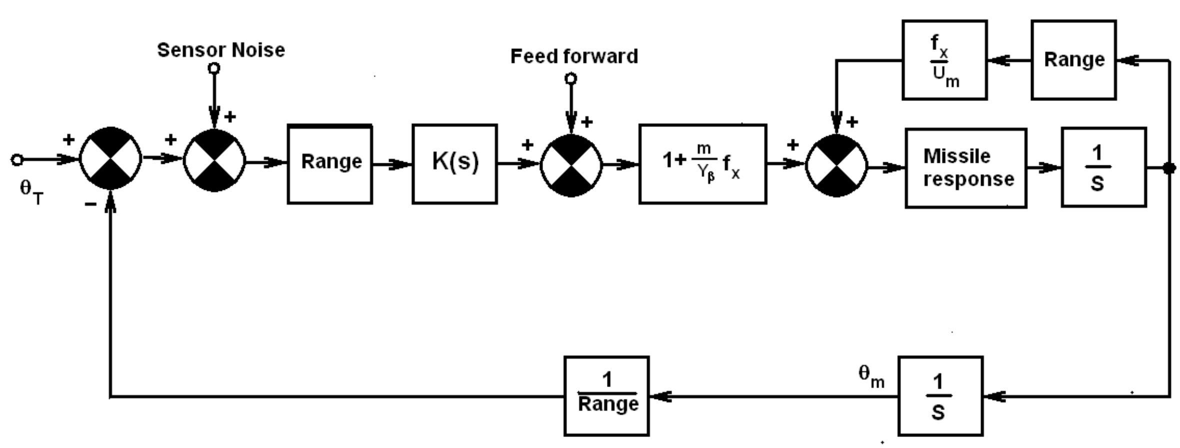


Figure 10 : Longitudinal Acceleration Terms

Note if the beam is rotating, the second term becomes:

$$f_x \frac{(\dot{y} - R\dot{\theta})}{U_m}$$

Where $\dot{\theta}$ is the slew rate and *R* is the range.

The effect of these terms on the guidance loop may be derived from Figure 10. We have lumped the phase advance and loop gain into the compensator in order to avoid clutter. Evidently, if the missile is accelerating, the loop gain is increased significantly due to the extra acceleration component attributable to the motor thrust.

What is worse, there is a positive feedback of lateral velocity, providing negative damping to the loop. This is compounded by the fact that the feedback gain is time-varying - effectively feeding a spurious time varying velocity error into the loop. To some extent this may be offset by increasing the amount of phase advance and reducing the loop gain. Usually, however, the loop cannot be closed until the speed has become large compared with the longitudinal acceleration.

It is undesirable to have a rocket motor burning during the guidance phase. The plume obscures the view of both target and missile and attenuates the radar return and command link power. A boost/coast profile is often employed to avoid this potential problem. During the coast phase the longitudinal acceleration is negative, so the loop gain is reduced and the kinematic feedback of lateral velocity is negative, both these effects improve stability but result in a lower loop bandwidth from that calculated by ignoring the deceleration.

General Line Following

When under inertial navigation control the missile position is no longer measured relative to a sensor beam, instead the sight line is a line in space, the position, direction and slew rates of which may be sent to the missile from the platform sensors at launch. The significant separation between launcher and sensor means the guidance error can no longer be considered small. A more general means of navigation along a line is needed.

The line following geometry is presented in Figure 11. The line segment is defined by its start point vector <u>OA</u> and the vector in the line direction <u>AB</u>.

The position vector of the missile relative to the line start point is:

$$\underline{p} = \underline{r} - \underline{a}$$

Where $\underline{r}$ is the position vector of the missile relative to the origin and $\underline{a}$ is the vector <u>OA</u>. A single character is used for compatibility with the equation editor.

The projection of this on to the line is:

$$|\underline{b}| = \underline{p} \cdot \hat{\underline{b}}$$

Where $\hat{\underline{b}}$ is the unit vector in the direction of the line.

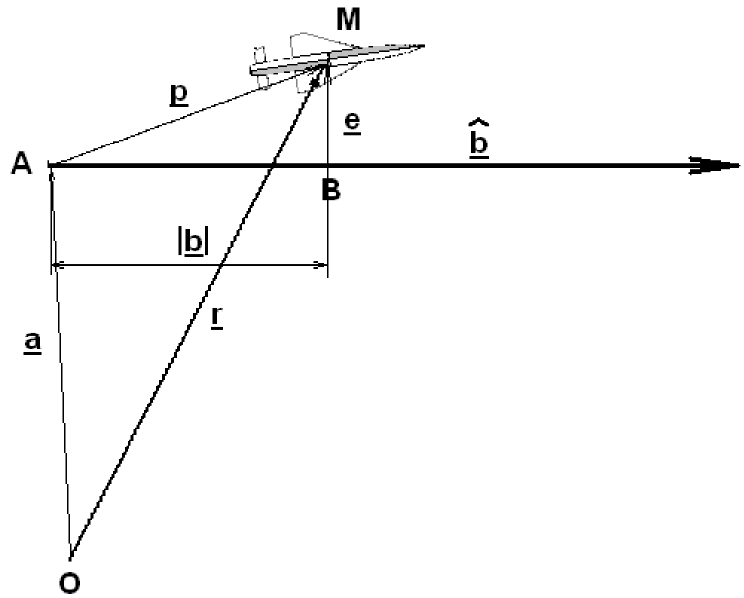


Figure 11 : General Line Geometry

It follows that the perpendicular position error relative to the line is:

$$\underline{e} = \underline{p} - |\underline{b}|\hat{\underline{b}}$$

We cannot assume the missile velocity vector is nearly parallel with the line, as it is when position errors are measured with a guidance beam. So rather than deriving a conventional CLOS acceleration term as a linear combination of position error and its rate of change, we first derive a heading direction from the position error.

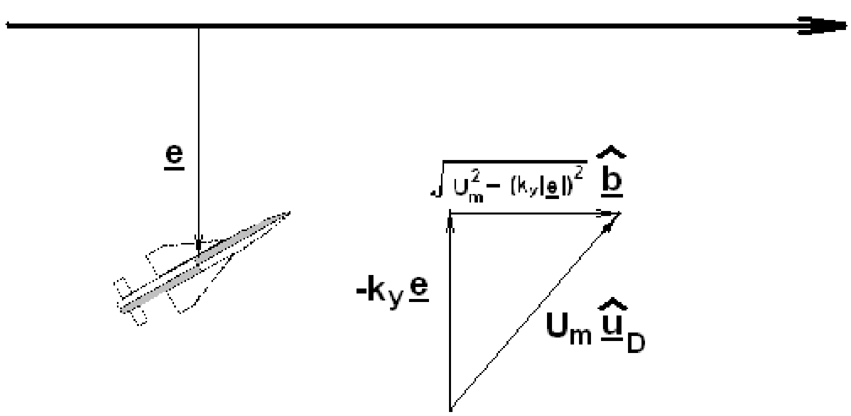


Figure 12 : Calculation of Heading

The desired heading direction is based on the requirement for the velocity demand magnitude never to exceed the actual missile speed. If the velocity perpendicular to the beam is proportional to the position error, it follows that the remaining component of velocity must act along the beam. Hence the heading demand is the unit vector $\underline{\hat{u}}_D$ calculated from the velocity triangle of Figure 12:

$$\underline{\hat{u}}_D = -\frac{k_y}{U_m}\underline{e} + \sqrt{1-\left(\frac{k_y|\underline{e}|}{U_m}\right)^2}\,\underline{\hat{b}}$$

Where k_y is the 'stiffness' gain.

When the term in the square root becomes negative, the required missile heading is perpendicular to the line, and it is impossible to approach it any faster.

The required lateral acceleration is now calculated from the difference between the current heading and the desired heading.

The sine of the heading error is found from the cross product of the desired heading direction with the current heading, so the heading hold would aim to set the turn rate proportional to this error:

$$\underline{\omega}_v = k_v \underline{\hat{u}}_D \times \underline{\hat{u}}_m$$

Where k_v is the velocity gain.

This rate of turn implies a lateral acceleration given by the cross product of the turn rate with the current velocity vector:

$$\underline{f} = \underline{U}_m \times \underline{\omega}_v$$

The result is a vector box product:

$$\underline{f} = k_v U_m \left(\underline{\hat{u}}_m \times \underline{\hat{u}}_D \times \underline{\hat{u}}_m\right)$$

$$= k_v U_m \left(\underline{\hat{u}}_D - \left(\underline{\hat{u}}_D \cdot \underline{\hat{u}}_m\right)\underline{\hat{u}}_m\right)$$

Experience has shown that control divas tend to lack the integrity or honesty to admit to not understanding three dimensional geometry, so this algorithm is protected by self importance, which is much more effective than any patent rights.

We calculate the loop gains from the requirement to settle on the beam when it gets there. Evidently when the position error is small the velocity vector is nearly parallel to the line.

This situation is presented in

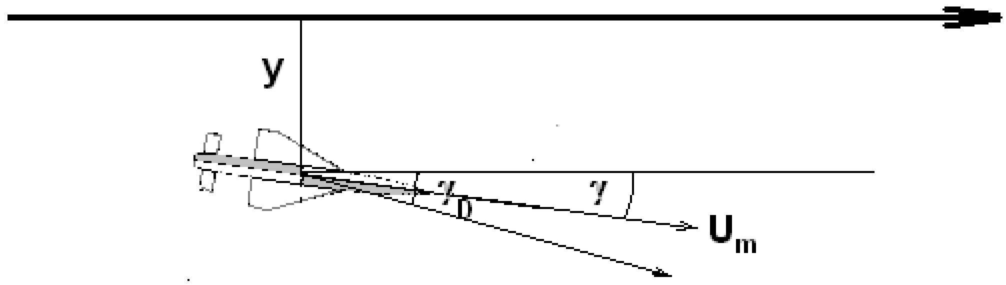


Figure 13 : Small Errors

The component of velocity in the direction of the line is very nearly equal to the missile speed, so we need only consider the perpendicular component. The heading error demand becomes:

$$\gamma_D = -k_y \frac{y}{U_m}$$

We note that:

$$\underline{\hat{u}}_D \cdot \underline{\hat{u}}_m = \cos(\gamma_D - \gamma) \approx 1$$

So the acceleration equation becomes:

$$f_y = k_v U_m (\gamma_D - \gamma) = -k_v k_y y - k_v U_m \gamma = -k_v k_m y - k_v \dot{y}$$

The control scheme reduces to a second order system whose gains are easily calculated.

Thrust Vector Control

The line-following algorithm presented in the previous section using aerodynamic lift to generate lateral acceleration, is probably suitable for mid-course guidance between way points under inertial navigation/GPS sensors. However, during launch, the speed is initially too small for the aerodynamic forces to be effective. The only significant force available is the motor thrust.

In fact, the low speed implies lower centripetal force is needed to steer the missile on to the desired heading. In order to achieve a reasonable minimum engagement range, it is therefore more advisable to exploit the motor thrust to turn the trajectory quickly at the very start of flight, than it is to defer manoeuvre until the speed has built up sufficiently for aerodynamic control to become effective.

By deflecting the thrust perpendicular to the body, or by using ‘bonker’ motors to apply impulsive moments the orientation of the missile body, and hence the direction of thrust in space can be controlled.

The orientation of the missile may be defined as a 3×3 matrix whose columns are the unit vectors along the body x, y and z axes respectively.

$$T = \begin{pmatrix} \underline{\hat{x}} & \underline{\hat{y}} & \underline{\hat{z}} \end{pmatrix}$$

In this form it is used to transform vectors from body axes to fixed. To transform back, the transform is used. The aim of the attitude controller is to point the body x-axis in a desired direction in space; $\underline{\hat{x}}_D$. The attitude error is given by the cross product of the desired orientation with the current orientation:

$$\underline{\varepsilon} = \underline{\hat{x}}_D \times \underline{\hat{x}}$$

Resolving this into body axes:

$$\underline{\varepsilon}_B = T^T \underline{\varepsilon} = (\underline{\hat{x}} \cdot \underline{\varepsilon})\underline{\hat{x}} + (\underline{\hat{y}} \cdot \underline{\varepsilon})\underline{\hat{y}} + (\underline{\hat{z}} \cdot \underline{\varepsilon})\underline{\hat{z}}$$

Evaluating the three scalar triple products, we have:

$$\begin{aligned}
\underline{\hat{x}} \cdot \underline{\varepsilon} &= \underline{\hat{x}} \cdot (\underline{\hat{x}}_D \times \underline{\hat{x}}) = \underline{\hat{x}}_D \cdot (\underline{\hat{x}} \times \underline{\hat{x}}) = 0 \\
\underline{\hat{y}} \cdot \underline{\varepsilon} &= \underline{\hat{y}} \cdot (\underline{\hat{x}}_D \times \underline{\hat{x}}) = \underline{\hat{x}}_D \cdot (\underline{\hat{x}} \times \underline{\hat{y}}) = \underline{\hat{x}}_D \cdot \underline{\hat{z}} \\
\underline{\hat{z}} \cdot \underline{\varepsilon} &= \underline{\hat{z}} \cdot (\underline{\hat{x}}_D \times \underline{\hat{x}}) = \underline{\hat{x}}_D \cdot (\underline{\hat{x}} \times \underline{\hat{z}}) = -\underline{\hat{x}}_D \cdot \underline{\hat{y}}
\end{aligned}$$

The transformation matrix T is central to any inertial navigation system, so will be present. How it is obtained is a matter of implementation.

In essence, we have derived signals equal to the sine of the attitude error expressed in body axes. The attitude control is expected to consist of thrust deflection proportional to this term together with a term proportional to the corresponding angular velocity.

The issue remains as to how we calculate the attitude demand from the desired lateral acceleration.

The problem is identical to that of generating a beam error demand which must respect the finite missile velocity. In this case we are generating an acceleration demand which must take account of the available thrust. The solution is identical.

Our navigation law generates a lateral acceleration vector $\underline{f}$. The available acceleration is:

$$f_T = \frac{T}{m}$$

Where T is the motor thrust, and m is the missile mass. In fact, this term could probably be measured with a longitudinal accelerometer.

By analogy with the line following algorithm, the attitude demand for input to the attitude control loop becomes:

$$f_T \hat{\underline{x}} = \underline{f} + \sqrt{f_T^2 - \left| f \right|^2}\, \hat{\underline{u}}_m$$

In this case, when the square root becomes zero, there is no component of thrust acting along the trajectory, and the missile is actually flying sideways. This represents the maximum possible lateral acceleration under thrust alone.

Considering the boost phase only lasts a few seconds, the inertial reference unit does not appear to require a stringent specification. Rapid response time is to be valued over the usual criteria of drift and biases, which are needed for longer duration flights. The instruments which are optimised for conventional inertial navigation systems may not be optimum in this application, and the delay implicit in introducing filtering and synchronisation with GPS probably cannot be afforded in such a short flight.

As is often the case, the optimum in one application is not necessarily universally the best choice. The turnover autopilot of a vertically launched missile will have quite different requirements from a mid-course cruise application.

Concluding Comment

Line of sight guidance is often viewed as the poor man's homing, yet effective systems are still possible using it, and for short range applications is worth considering at least during the concept phase. Some extremely effective line of sight systems have been fielded and proven under actual combat conditions, so it is probably premature to toll its death knell just yet.