Linear Algebra and TI 89

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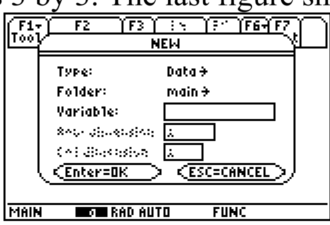
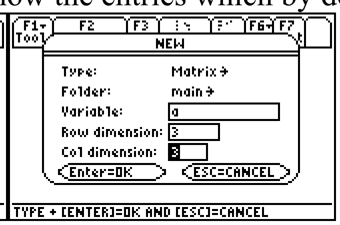
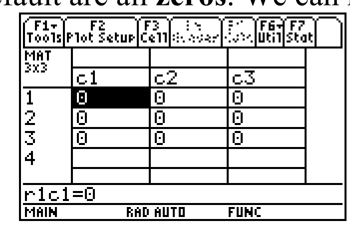
This short manual is a quick guide to the use of TI89 for Linear Algebra. We do this in two sections. In the first section, we will go over the editing of matrices and vectors. The second section will address the algebraic operations of Linear Algebra. To make the manual short and useful, we have assumed that the reader is familiar with the keyboard and the functions of the **SECOND, ALPHA, DIAMOND GREEN** keys as well as the use of **CATALOG.** The reader should also be familiar with the usages of the "cursor" keys to move form one option to another in a given menu. Note also that some menus have several submenus, which in turn may have many options.

1. EDITING MATRICES AND VECTORS

APPS 6:DATA/MATRIX Editor provides the format for editing matrix. To edit a new matrix, simply enter **APPS 6 3** and then select **2:matrix** for **Type**. In the box for **variable**, type in a name for your matrix. The next two boxes are to be filled with the dimension of the matrix. Note that you should use the cursor key to move from a box to another. At the end press **ENTER**. This way of editing a matrix has several advantages as you can see from the tool bars at the top of your screen. The figures below show these steps. The first figure shows the result of **APPS 6 3**, the second shows that we selected **matrix** for **Type,** we called our matrix **a,** and it is 3 by 3. The last figure show the entries which by default are all **zeros**. We can now start editing.





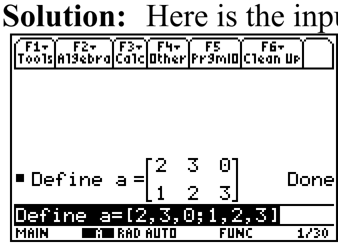


Notice the top of the screen in the last figure has **F1, F2, F3, F6,** and **F7** are highlighted. This means that you can press these keys and apply the options provided in them. For example, **F1 9** allows you to resize the width of the columns. **F6** has many commands you might want to explore.

There is however a shorter way of editing a matrix. From the home screen, we can enter a matrix by using **Define**(which can be accessed by **F4 1** or could be typed in). Use the square bracket **[ ]** to enclose the matrix. We enter the matrix by typing the first row and then the second and so on. Use commas to separate entries and semicolons to separate rows. Here is an example.

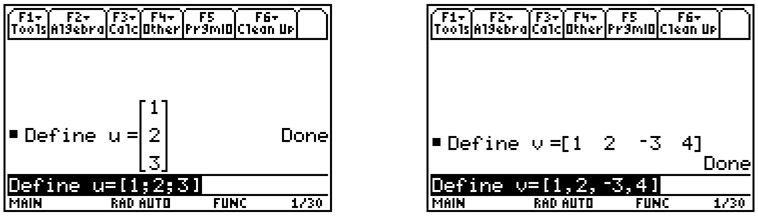
Example 1: Enter the matrix $A=\begin{bmatrix}2 & 3 & 0\\ 1 & 2 & 3\end{bmatrix}$

Solution: Here is the input and output of the calculator.



Example 2: Edit the vectors $u=\begin{bmatrix}1\\ 2\\ 3\end{bmatrix}$ and $v=\begin{bmatrix}1, & 2, & -3, & 4\end{bmatrix}$

Solution: Note that *u* can be regarded as a 3 by 1 matrix while *v* is a 1 by 4. Here are the two vectors.



2. OPERATIONS ON MATRICES

We begin with examples of matrix operations.

Example 3: Consider the following matrices.

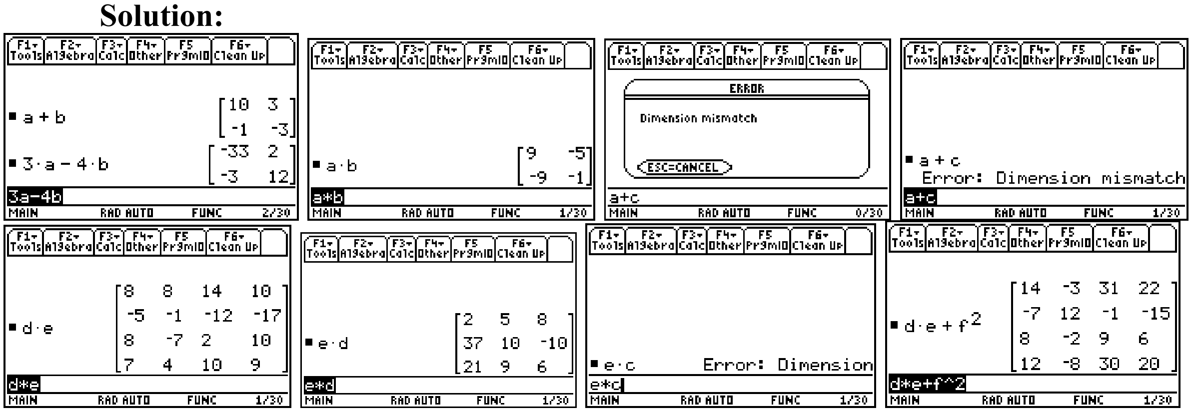
$$A=\begin{bmatrix}1 & 2\\ -1 & 0\end{bmatrix}\qquad B=\begin{bmatrix}9 & 1\\ 0 & -3\end{bmatrix}\qquad C=\begin{bmatrix}1 & 2 & -3\\ 0 & 2 & 1\\ -2 & 0 & 3\end{bmatrix}$$

$$D=\begin{bmatrix}1 & 3 & 1\\ 1 & 0 & -6\\ 7 & 0 & 1\\ 2 & 2 & 1\end{bmatrix}\qquad E=\begin{bmatrix}1 & -1 & 0 & 1\\ 2 & 3 & 4 & 2\\ 1 & 0 & 2 & 3\end{bmatrix}\qquad F=\begin{bmatrix}1 & 2 & 3 & 2\\ 1 & -3 & 0 & 2\\ 1 & -3 & 2 & 0\\ 0 & 1 & 4 & 3\end{bmatrix}$$

If possible, compute each of the following.

a) $A+B$ b) $3A$ - $4B$ c) $A\cdot B$ d) $A+C$
e) $D\cdot E$ f) $E\cdot D$ g) $E\cdot C$ h) $D\cdot E+F^2$
i) $A\cdot B^T$ j)

Solution:



Example 4: Find the inverse of the matrices $A=\begin{bmatrix}1 & 2\\ -1 & 0\end{bmatrix}$, $B=\begin{bmatrix}1 & 2 & -3\\ 0 & 2 & 1\\ -2 & 0 & 3\end{bmatrix}$ and $C=\begin{bmatrix}1 & 2 & 3 & 2\\ 1 & -3 & 0 & 2\\ 1 & -3 & 0 & 2\\ 0 & 1 & 4 & 3\end{bmatrix}$

Solution: Enter the matrices and then from the **HOME** screen evaluate $A\,^\wedge(-1)$, $B\,^\wedge(-1)$ and $C\,^\wedge(-1)$. Note that for $C\,^\wedge(-1)$ the output is **error: Singular matrix.**

Example 5: Let B be as in Example 4. Find the 2-3 entry, the diagonal entries, first row, and second column of B.

Solution: Enter the matrix as ***b.*** For the ***j-k*** entry use ***b[j,k]***, for first row use ***b[1],*** and for the second column use $b^T[2]$, where the ***T*** in the exponent is for the transpose of ***b.***

*Example 6**(Special Matrices)*** (a) Generate a 2 by 3 random matrix. b) Generate a 3 by 3 identity matrix.

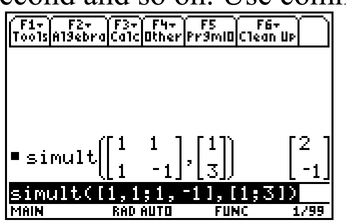
Solution. a) Use **MATH 4 E** to display **randMat(** and then type **2,3)** and evaluate. For b) either type or use **MATH 4 6** to display **identity(** and then type **3** and evaluate.

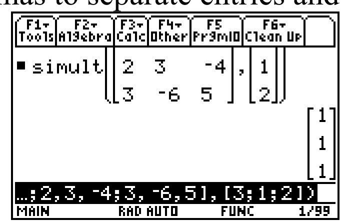
3 SOLVING SYSTEMS OF LINEAR EQUATIONS

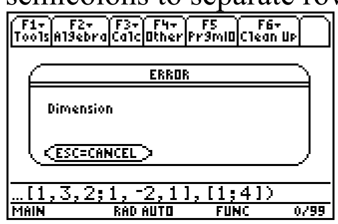
Example 7 Solve the system of linear equations.

a) $\begin{cases} x+y=1 \\ x-y=3 \end{cases}$ b) $\begin{cases} x+y+z=3 \\ 2x+3y-4z=1 \\ 3x-6y+5z=2 \end{cases}$ c) $\begin{cases} x+3y+2z=1 \\ x-2y+z=4 \end{cases}$

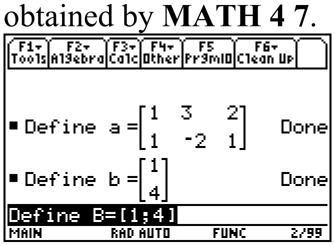
Soution: From the home screen press **MATH 4 5.** This gives **simult(.** Next we enter the coefficients and the constant matrix as below.(Recall that we enter a matrix by typing the first row and then the second and so on. Use commas to separate entries and semicolons to separate rows.)

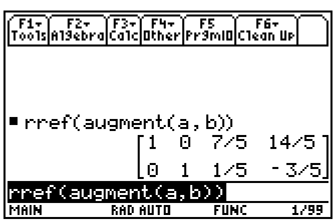






Note that for c) we got an error message as we notice above. We might want to try the elementary row operations to solve this problem. To do this, we define the coefficient matrix as **A** and the constant matrix as **B** and then apply **rref(augment(A,B)).** The **rref** command can be accessed by **Math 4 4** while **augment** can be obtained by **MATH 4 7**.



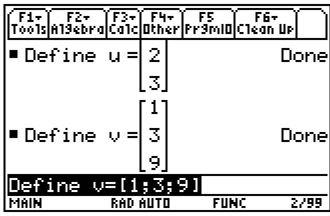


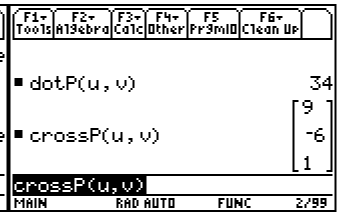
We leave it to the reader to interpret this output.

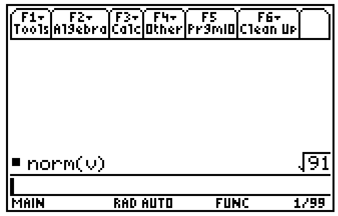
4. Vector Operations

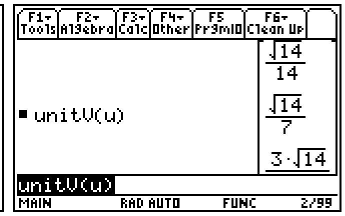
Example 8: Let $u=\begin{bmatrix}1\\2\\3\end{bmatrix}$ and $v=\begin{bmatrix}1\\3\\9\end{bmatrix}$. Find $u\cdot v$, $u\times v$, $||v||$ and the unit vector in the direction of u.

Solution: Enter the vectors as in Example 2 above. Then **MATH 4 L 3** will display **dotP(.** Now type **u,v)** and **ENTER.** The result is the dot product; in this case it is 34. Similarly **MATH 4 L 2 u,v) ENTER** will give the cross product. For the norm of v use **MATH 4 H 1 v) ENTER**. Finally, to find the unit vector in the direction of u, we use **MATH 4 L 1 u) ENTER**. The first figure below shows the result of defining the vectors, the second shows the dot and the cross products and the last one shows the norm of v and the unit vector in the direction of u.







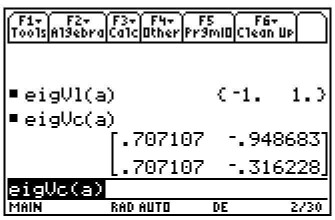


5. Eigenvalues and Eigenvectors

Example 9 Find the eigenvalues and eigenvectors for the following matrices

$$A = \begin{bmatrix} 2 & -3 \\ 1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 1 \\ -2 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 2 & 1 \\ -2 & 0 & 3 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 1 & -3 & 0 & 2 \\ 1 & -3 & 2 & 0 \\ 0 & 1 & 4 & 3 \end{bmatrix}$$

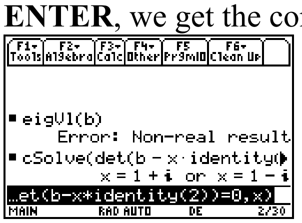
Solution: We use the define option to enter the matrices as **a, b, c,** and **d**, respectively. To find the eigenvalues of the matrix **A,** use **Math 4 9 a) ENTER** or type **eigvl(a)** and **ENTER**. To find the eigenvectors of the matrix **a** use **Math 4 A a) ENTER** or type **eigvc(a).** The figure below shows the eigenvalues and eigenvectors of the matrix **A**.



Remark:

1. The first number given by **eigVl(a)** is the first eigenvalue which in this case is -1 and second eigenvalue is 1. The first column of the **eigVc(a)** is an eigenvector corresponding to the first eigenvalue of **a**. Note that TI 89 is normalizing the vectors, that is the eigenvectors are unit vectors.
2. For most purposes and easier notations, it is convenient to rewrite the eigenvectors with integer entries. This is usually possible. One possible method is to replace the smallest number in the columns by 1 and divide the other entries in that column by the smallest value you just replaced. Use the command **eigVc(a)[j,k]** to refer to the j-k entry of the matrix **eigVc(a)**. It is clear that the entries in the first column are equal. Thus for an eigenvector corresponding to the eigenvalue -1, we may take $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$. The second one may not be clear so we replace -.316228 by 1. Note then that -.96683/-.316228 is 3.062. Thus it is highly recommended that you compute **eigVc(a)[2,1]/ eigVc(a)[2,2]**. We find that this is 3. Thus we may take $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ as the second eigenvector.

If we evaluating **eigvl(b)**, we will get the message **Non-real result**. This means that the characteristic equation of the matrix **B** has complex roots. Note that we could use the command **cSolve(det(b – x*identity(2))=0,x) ENTER**, we get the complex eigenvalues, namely, $x = 1 + i$ or $x = 1 - i$ as shown below.



6. Applications

Example 10. Let v_1, v_2, v_3, v_4 be the vertices of the complete graph on four vertices. Find the determinant and eigenvalues of the graph.

Solution: Note that the determinant and eigenvalues of a graph are the determinant and eigenvalues of the adjacency matrix. The adjacency matrix is defined as the matrix $A = \left[a_{ij}\right]$, where

$$a_{ij} = \begin{cases} 1, \ if \ \{v_i, v_j\} \text{ is an edge of the graph} \\ 0, \text{ otherwise} \end{cases}$$

For the complete graph on four vertices, the adjacency matrix is given by $A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$. We enter this matrix as **a** and evaluate **det(a)** to get -3 as the determinant of the graph and evaluate **eigvl(a)** to get $\{3, -1, -1, -1\}$ as the eigenvalues of the graph.

Example 11: Find the number of walks of length 3 from v_1 to v_3.

Solution: Again we use the adjacency matrix as defined in example 10 above. The number of walks of length 3 from v_1 to v_3 is given by the (1,3) entry of the cube of the adjacency matrix A. As in example 10, we enter the adjacency matrix as **a** and evaluate a^3 to obtain the matrix $A^3 = \begin{bmatrix} 6 & 7 & 7 & 7 \\ 7 & 6 & 7 & 7 \\ 7 & 7 & 6 & 7 \\ 7 & 7 & 7 & 6 \end{bmatrix}$. Thus the (1,3) entry is 7. Hence there are 7 walks of length 3 from v_1 to v_3 in the complete graph on four vertices. We leave it to the reader to list the seven walks.