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Procedia Engineering 40 (2012) 205 – 210
Procedia Engineering
www.elsevier.com/locate/procedia

Steel Structures and Bridges 2012

Linear analytical solution of an active tensegrity unit

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Abstract

Results of the linear closed form solution of an active or adaptive tensegrity unit, as well as its numerical analysis using finite element method are presented. The shape of the unit is an octahedral cell and it is formed by thirteen members (eight cables, four edge struts and one central strut). The central strut is designed as an actuator that allows for an adjustment of the shape of the unit which leads to changes of tensile forces in the cables. Since the unit is diagonally symmetrical, it may be simply solved as planar biconvex cable system with one central strut.

Keywords: struts; cables; tensegrity unit; active structure; planar biconvex cable system; linear closed-form solution;

1. Introduction

In general, a structural response to external loads (internal forces, stresses, deflection) can be solved analytically or numerically. Analytical methods provide linear or non-linear closed-form solutions and they are particularly suitable for the simple structures with explicitly defined geometry and simple boundary conditions. For more complicated structures or load cases the results obtained by analytical solutions are less accurate and they are suitable only for preliminary design [1].

The paper presents a linear analytical close-form solution of an active or adaptive tensegrity unit and its numerical analysis using finite element method (FEM).

The active tensegrity unit presented, as well as the whole test facility was developed at the Institute of Structural Engineering of the Faculty of Civil Engineering in Košice. Its production was performed in cooperation with INOVA Praha Ltd. This active tensegrity unit was developed and manufactured in order to test the possibility of active control of tensegrity systems through an activator or action member [2].

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doi:10.1016/j.proeng.2012.07.081

2. Prototype of active tensegrity structure

The chosen tensegrity unit consists of a strut that is centered in the rectangular base and stiffened by crossed cables. This unit is also known as a tensegric unit cell of type I [3], or like a crystal pyramid [4] and it is suitable for the generation of line structures or plate structures with a straight or curved central line.

The theoretical dimensions of the square base of the active tensegrity unit are 2.000 x 2.000 mm and its theoretical height is 800 mm. The unit consists of thirteen members (four circumferential compressed members, eight cables and one central strut) as is introduced in Table 1. The unit is also equipped with six strain gauges SG1 - SG6 and four load cells FT1 - FT4 (Fig. 1, 2).

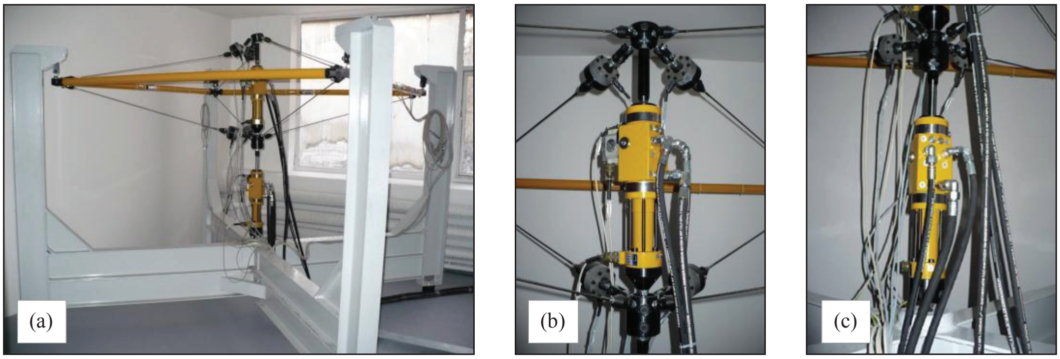

Fig. 1. (a) the active tensegrity unit suspended on the self-supporting frame - inactive state; (b) detailed view of the central strut - actuator; (c) hydraulic load cylinder

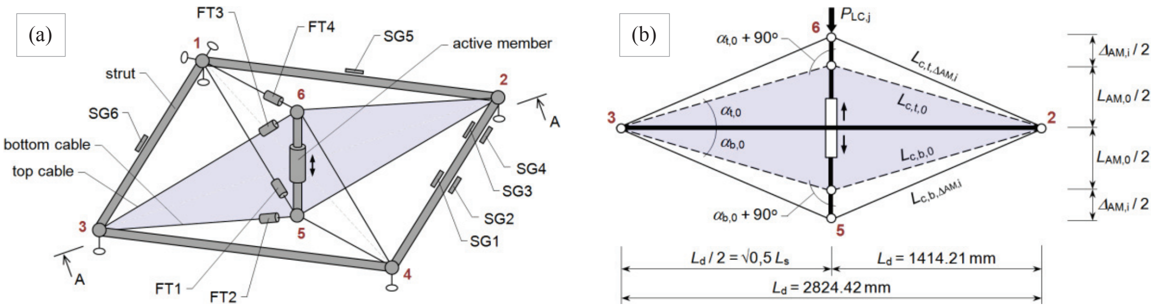

Fig. 2. (a) isometric diagram of the unit; (b) diagonal A-A section

Table 1. Members of the active tensegrity unit and their properties

Member	Cross-section	A (mm^2)	Material	E (MPa)
Compressed members - struts	∅ 51 / 3.2 mm	$A_s = 475.9$	steel S 235	$E_s = 210\,000$
Upper and bottom cables	∅ 6 mm	$A_c = 15.14$	steel cable 7x7	$E_c = 120\,000$
Central or active member	-	-	steel S235	$E_s = 210\,000$

The central strut is designed as an actuator or active member that allows to adjust the shape of the unit which leads to changes of tensile forces in the cables. All members are mutually connected in nodes by hinge joints.

3. Linear analytical solution

The above tensegrity unit can be simply solved as planar biconvex cable system with one central strut. This system is symmetrical along longitudinal x-axis (if we neglect the asymmetry caused by the own weight of the members), therefore $d = d_b = d_t$ and its geometry is given by Fig. 3. Another assumption is that the cables are perfectly flexible, working only in tension and have zero stiffness in compression and bending.

$$z_1 = 2d(x/L) \qquad \text{for } x \in \langle 0;L/2 \rangle, \tag{1a}$$

$$z_2 = 2d(L - x/L) \qquad \text{for } x \in \langle L/2;L \rangle. \tag{1b}$$

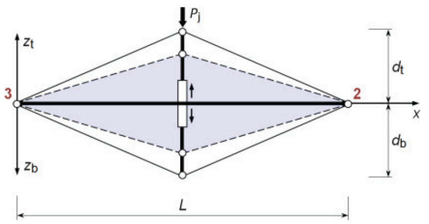

Fig. 3. Geometry of the simplified biconvex cable system

Following Irvine [5], vertical equilibrium equations at a cross section of the planar biconvex cable system are in the forms

$$2H\frac{dw_1}{dx} + (\Delta H_b + \Delta H_t)\frac{dz_1}{dx} = \frac{P}{2} \qquad \text{for } x \in \langle 0;L/2 \rangle, \tag{2a}$$

$$2H\frac{dw_2}{dx} + (\Delta H_b + \Delta H_t)\frac{dz_2}{dx} = -\frac{P}{2} \qquad \text{for } x \in \langle L/2;L \rangle. \tag{2b}$$

Movement of the action member (central strut) to value $\Delta_{AM,i} \geq 0$ causes a change of its height, then

$$d = d_b = d_t = L_{AM,0}/2 + \Delta_{AM,i}/2. \tag{3}$$

The tensile forces in the top and bottom cables then increase to value (4) and horizontal component of these forces can be solved as (5)

$$N_{c,i,} = N_{b,i} = N_{t,i}, \tag{4}$$

$$H_{c,i,} = N_{c,i} \cos \alpha_{c,i}, \tag{5}$$

where $\alpha_{c,i}$ is deflection of the cables from the horizontal plane formed by the peripheral struts. If the system is loaded with a nodal vertical load $P_{j,} = P_{LC,j}/2$, applied in the middle of the span, change of the horizontal component of tension forces in the top and bottom cables is calculated from the following equation

$$\Delta H_{c,i,j} = \Delta H_{b,i,j} = \Delta H_{t,i,j} = \frac{1}{2}\frac{P_j d}{H_{c,i} L_e / E_c A_c + 4d^2/L}, \tag{6}$$

where $L_e = L_{eb} = L_{et} = L + 6d^2 / L$. Vertical deflection in the middle of the span (for $x = L/2$) is

$$w_{1,i,j} = \frac{1}{2H_{c,i}}\left(\frac{P_j L}{4} - 2\Delta H_{c,i,j} d\right). \tag{7}$$

Assume that the tensegrity unit system is in a prestressed state i (total length of the central strut is $L_{AM,i} > L_{AM,0}$), then it is loaded with a nodal vertical load P_j. That results to the increase of the tensile forces in the bottom cables and to the decrease of the tensile forces in the top cables. Their horizontal components are

$$H_{b,i,j} = H_{c,i} + \Delta H_{c,i,j}, \quad \text{and} \quad H_{t,i,j} = H_{c,i} - \Delta H_{c,i,j}. \tag{8}$$

The resulting values of the tensile forces in the bottom and top cables are given by

$$N_{b,i,j} = H_{b,i,j} / cos\,\alpha_{b,i,j}, \quad \text{and} \quad N_{t,i,j} = H_{t,i,j} / cos\,\alpha_{t,i,j}, \tag{9}$$

where the final rotation of the bottom and top cables are

$$\alpha_{b,i,j} = arctan\left((d + w_{1,i,j})/(L/2)\right), \quad \text{and} \quad \alpha_{t,i,j} = arctan\left((d - w_{1,i,j})/(L/2)\right). \tag{10}$$

4. Results of analytical and numerical solution

4.1. Linear closed-form solution

If the prestressed active tensegrity unit is not loaded by a nodal vertical load P_j a movement of the action member with a value of $\Delta_{AM,i} \geq 0$ causes a change of its geometry and a change of the lengths of cable members. The initial lengths of cables $L_{c,0}$ are changed into $L_{c,i}$ and corresponding strain is calculated from the equation (11). From a known initial geometry (assuming that nodes 3 and 2 are unmoved and Hooke's law, hence $\sigma = N_c / A_c = E_c \varepsilon_c$ is valid) the change of the tensile force in the cable can by determined from the simplified expression (12).

$$\varepsilon_{c,i} = L_{c,i} / L_{c,0} - 1 \tag{11}$$

$$N_{c,i} = E_c A_c \varepsilon_{c,i} = E_c A_c = \left[\frac{1}{L_{c,0}}\sqrt{L_{c,0}^{\;2} + 0{,}25\Delta_{AM,i}^{\;2} - L_{c,0}\Delta_{AM,i}\,cos(\alpha_c + 90^o)} - 1\right] \tag{12}$$

The linear closed-form solution, as well as finite element analysis was carried out for various values of the initial prestressing forces in the top and bottom cables (Table 2).

Table 2. Movement of the action member and the initial prestressing forces

$\Delta_{AM,i}$ (mm)	$L_{AM,i}$ (mm)	$d_b = d_c$ (mm)	$N_{c,i}$ (kN)	$\varepsilon_{c,i}$ (N)
17.65	817.65	408.83	3.0	0.001651
34.96	834.96	417.48	6.0	0.003303
51.95	851.95	425.98	9.0	0.004954

4.2. FEM analysis

To verify results obtained by linear closed-form solution, the finite element analysis (geometrically nonlinear and physically linear) of the active tensegrity unit in ANSYS 12 Classic software was performed. The following types of finite elements were used [6]:

- LINK10 tension-only spar for the top and bottom cables,
- LINK10 compression-only spar for the compressed members,
- LINK11 linear actuator for the active member (AM).

The finite element model was supported at the nodes 1, 2, 3 a 4 as is shown in Fig. 2. The real constants and material properties of the members are shown in Table 1.

4.3. Comparison of the results

Comparison of the analytically (linear closed-form solution) and numerically (finite element method analysis) obtained values of the change of the tensile forces in the top cables and in the bottom cables and vertical deflection of the node 5 for various values of the nodal loads and initial prestressing forces are shown in Fig. 4, Fig 5 and Fig 6.

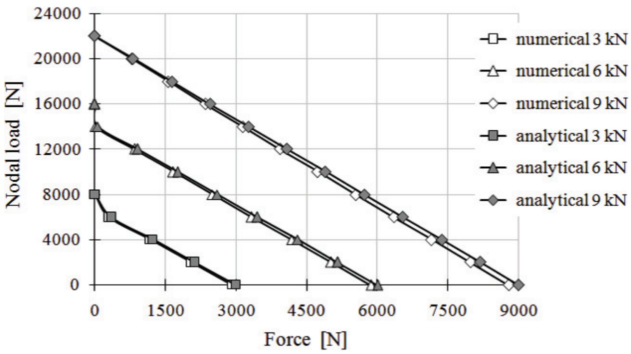

Fig. 4. Comparison of the analytically and numerically obtained values of the change of the tensile forces in the top cables for various values of the nodal loads and initial prestressing forces

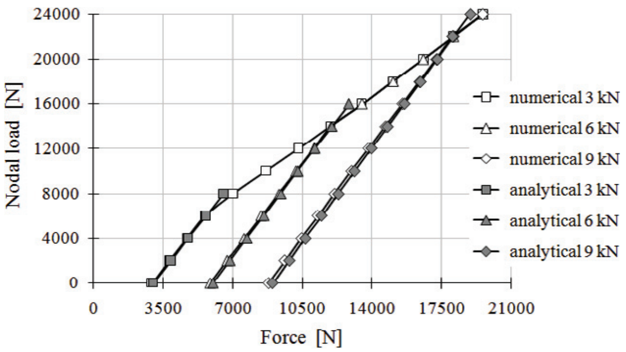

Fig. 5. Comparison of the analytically and numerically obtained values of the change of the tensile forces in the bottom cables for various values of the nodal loads and initial prestressing forces

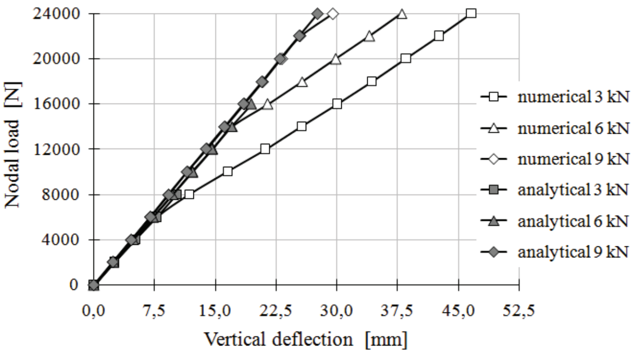

Fig. 6. Comparison of the analytically and numerically obtained values of the vertical deflection of the node 5 for various values of the nodal loads and initial prestressing forces

5. Conclusion

In this paper the linear close-form solution of the active or adaptive tensegrity unit has been presented. This method offers a relatively simple and effective tool to analyze nodal loaded structural systems in the shape of a crystal pyramid with simple boundary conditions.

Fig. 4, Fig. 5 and Fig. 6 shown that the results (tensile forces in the top and bottom cables and vertical deflection in the mid-span of the simplified planar biconvex cable system) obtained by the presented linear closed-form solution are in a very good agreement with those obtained by the geometrically nonlinear and physically linear finite element analysis when ANSYS 12 Classic software was used.

The obtained results confirmed the correctness of the derived equations and their mathematical and physical importance (at the given geometry and the load range).

Acknowledgements

This research has been carried out in terms of the projects VEGA No. 1/0321/12 and NFP26220120037 Centre of excellent research of the progressive building structures, materials and technologies, supported from the European Union Structural funds.

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