Linear Combinations of GNSS Phase Observables

to Improve and Assess TEC Estimation Precision

Brian Breitsch

Advisor: Jade Morton

Committee: Charles Rino, Anton Betten

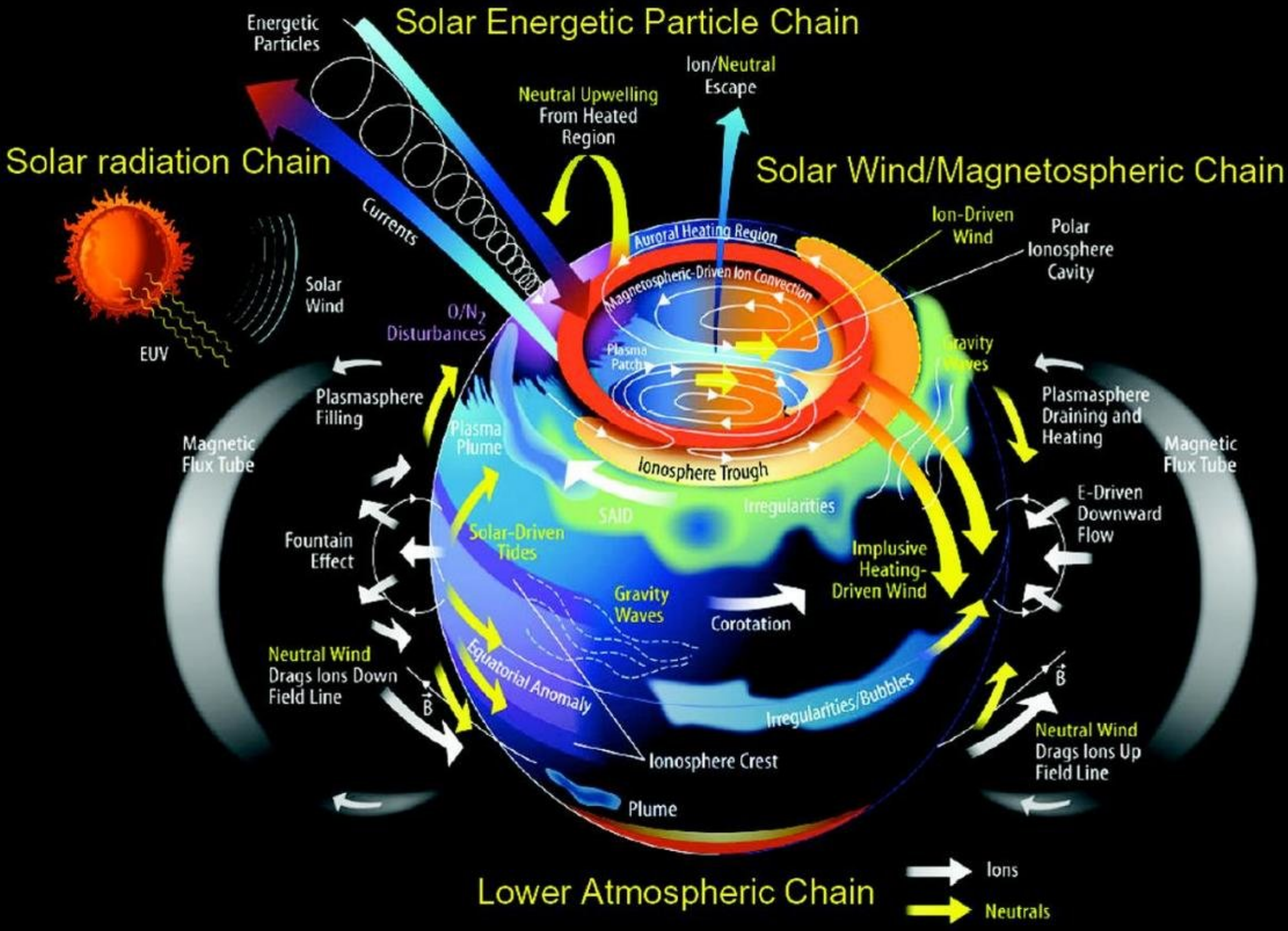
Background and Motivation

Linear Estimation of GNSS Parameters

TEC Estimate Error Residuals

Application to Real GPS Data

Earth's Ionosphere

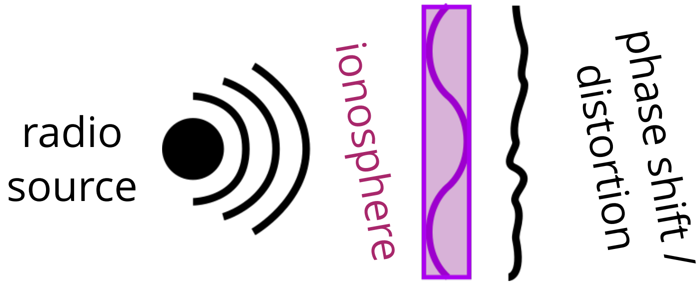


J. Grobowsky / NASA GSFC

Ionosphere Effects on Electromagnetic Propagation

ionosphere = cold, collisionless, magnetized plasma

for L-band frequencies (1-2 GHz) refractive index given by:



$$n = 1 - \frac{1}{2}X \pm \mathcal{O}\left(\frac{1}{f^3}\right)$$

higher-order terms on the order of a few cm

$$X = \frac{\omega_p^2}{\omega^2} \quad \omega = 2\pi f \quad \omega_p = \sqrt{\frac{N_e e^2}{\epsilon_0 m}}$$

f = wave frequency

e = fundamental charge

N_e = plasma density

m = electron rest mass

ϵ_0 = permittivity of free space

Global Navigation Satellite Systems (GNSS)

GPS
GLONASS
Beidou
Galileio
...etc.

" ...a useful everyday radio source for geophysical remote-sensing!

GPS - Global Positioning System

- 32-satellite constellation
- transmit dual-frequency BPSK-moduled signals
- new Block-IIF and next-gen Block-III satellites transmitting triple-frequency signals

Signal	**Frequency (GHz)**
L1CA	1.57542
L2C	1.2276
L5	1.17645

GNSS Carrier Phase Observable

accumulated phase (in meters) of demodulated GNSS signal at receiver for a particular satellite and signal carrier frequency f_i

$$\Phi_i = \boxed{r + c\Delta t + T} + \boxed{I_i} + \boxed{\lambda_i N_i} + \boxed{H_i} + \boxed{S_i} + \boxed{\epsilon_i}$$

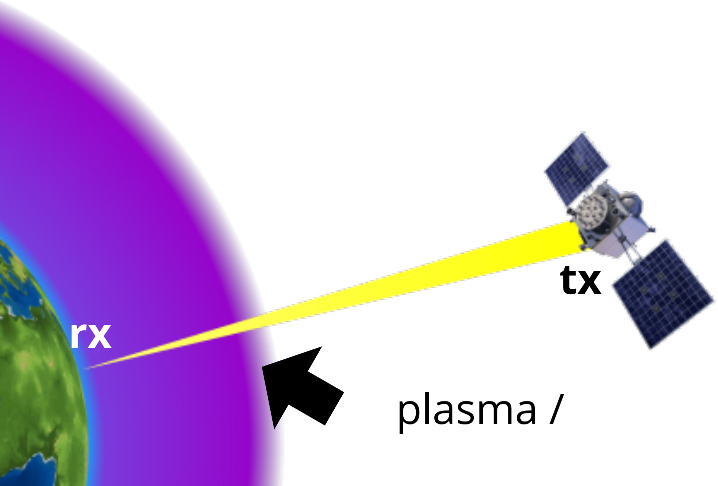
- $r + c\Delta t + T$: FREQUENCY INDEPENDENT EFFECTS
- I_i: IONOSPHERE RANGE ERROR
- $\lambda_i N_i$: CARRIER AMBIGUITY
- H_i: HARDWARE BIAS
- S_i: SYSTEMATIC ERRORS
- ϵ_i: STOCHASTIC ERRORS

Ionosphere Range Error

consider first-order term in ionosphere refractive index

$$n \approx 1 - \frac{1}{2}X = 1 - \frac{\kappa}{f_i^2} N_e \qquad \kappa = \frac{e^2}{8\pi^2 \epsilon_0 m_e} \approx 40.308$$

$$\boxed{I_i} = \int_{\mathrm{rx}}^{\mathrm{tx}} (n-1)\ ds \approx -\frac{\kappa}{f_i^2} \boxed{\int_{\mathrm{rx}}^{\mathrm{tx}} N_e\, ds}$$



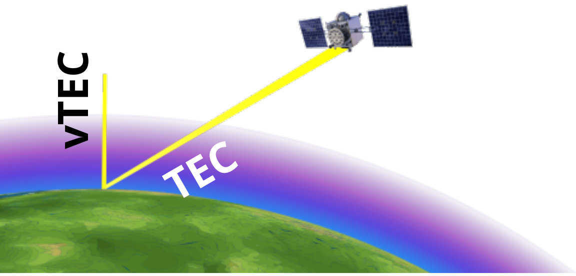
TOTAL ELECTRON CONTENT

units: $\frac{\text{electrons}}{\text{m}^2}$

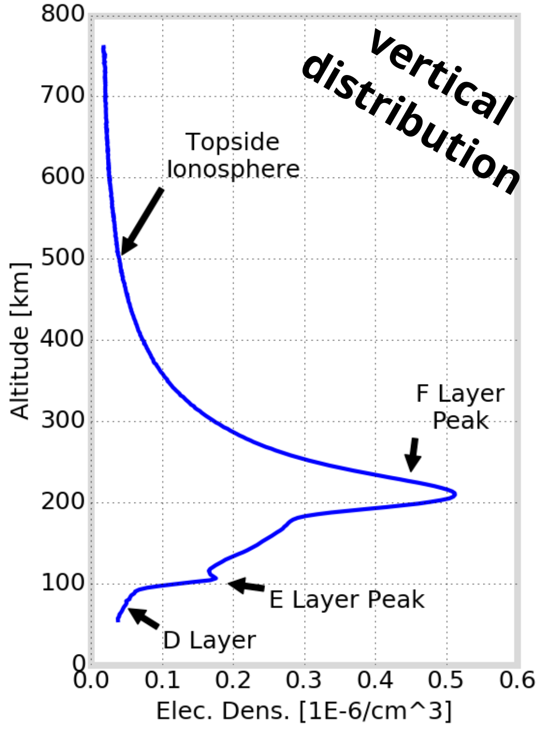
often measured in TEC units:

$$1\text{TECu} = 10^{16}\,\frac{\text{electrons}}{\text{m}^2}$$

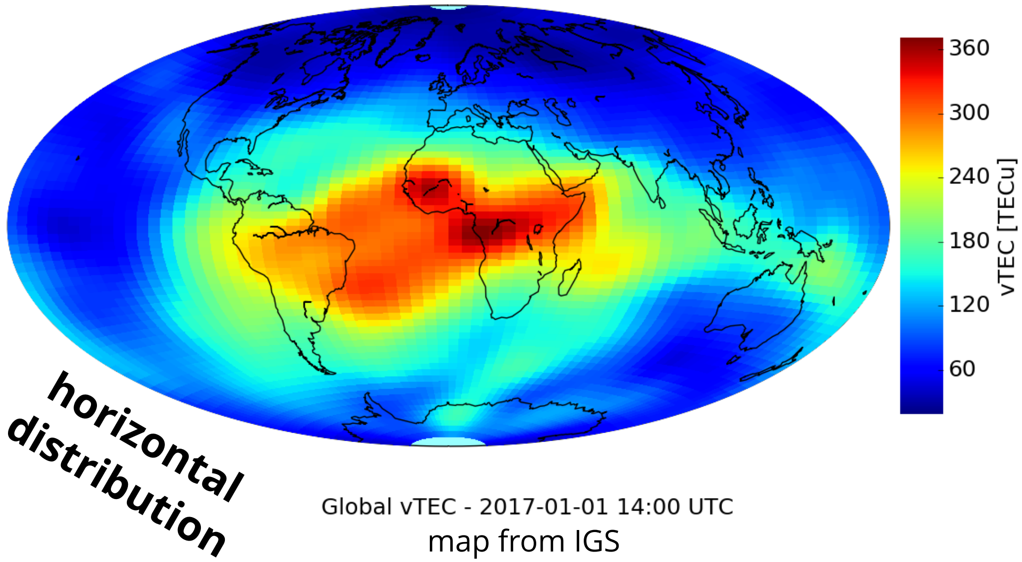
Ionosphere Plasma Density



TEC and vertical TEC (vTEC) used to image plasma density structures



profile from CDAAC



Global vTEC - 2017-01-01 14:00 UTC

map from IGS

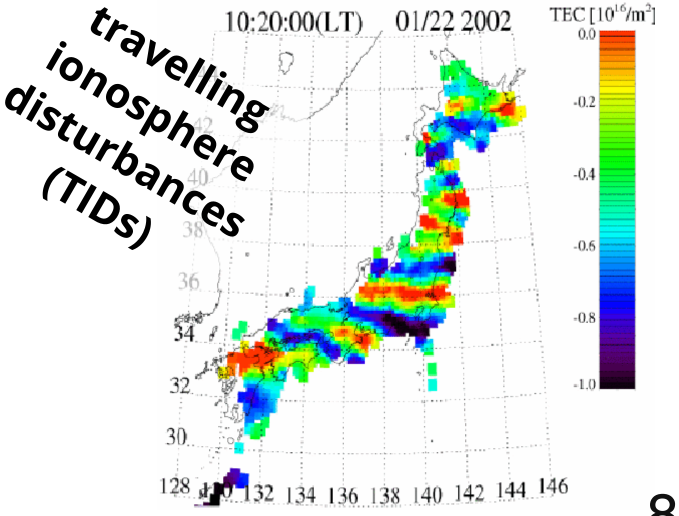


image from Saito et al.

TEC Estimation Using Dual-Frequency GNSS

neglecting systematic and stochastic error terms:

$$\Phi_1 - \Phi_2 = (I_1 - I_2) + (\lambda_1 N_1 - \lambda_2 N_2) + (H_1 - H_2)$$
$$\approx -\kappa \left(\frac{1}{f_1^2} - \frac{1}{f_2^2} \right) \boxed{\mathrm{TEC}} + \boxed{(\lambda_1 N_1 - \lambda_2 N_2)} + \boxed{\Delta H_{1,2}}$$

carrier ambiguities

satellite and receiver inter-frequency hardware biases

bias terms

after resolving bias terms:

$$\boxed{\mathrm{TEC}} = \frac{\Phi_2 - \Phi_1}{\kappa \left(\frac{1}{f_1^2} - \frac{1}{f_2^2} \right)}$$

Resolving Bias Terms

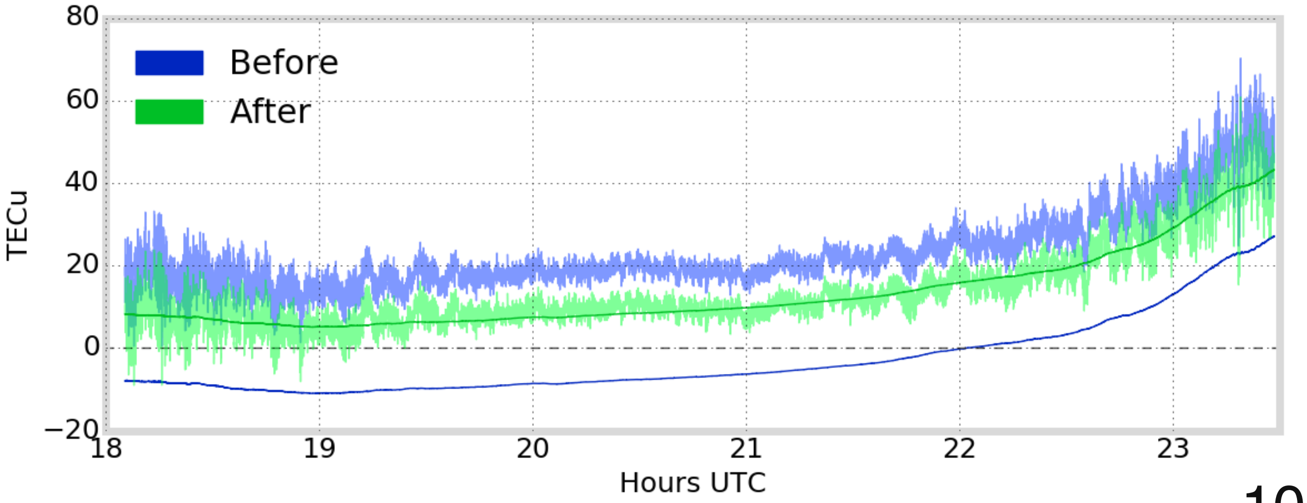
carrier ambiguity resolution

- LAMBDA
- code-carrier-levelling
- [3] derives improved code-carrier leveling / ambiguity resolution using triple-frequency GNSS

hardware bias estimation

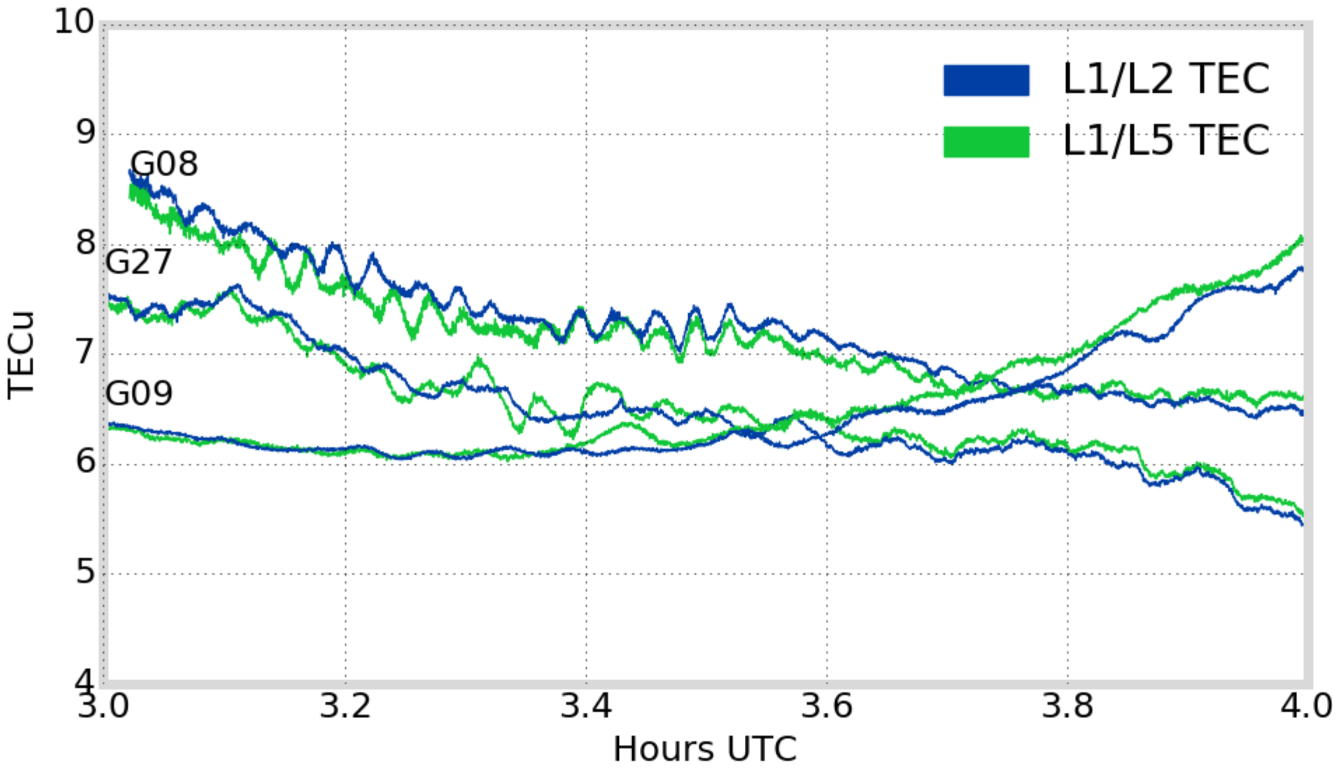
- must apply ionosphere model
  - e.g. global ionosphere model using data assimilation and receiver networks
  - e.g. single receiver and linear 2D-gradient in vTEC (such as work by [2])

Example of L1/L2 TEC before and after code-carrier-levelling / ambiguity estimation, for satellite G01 and receiver at Poker Flat, Alaska.



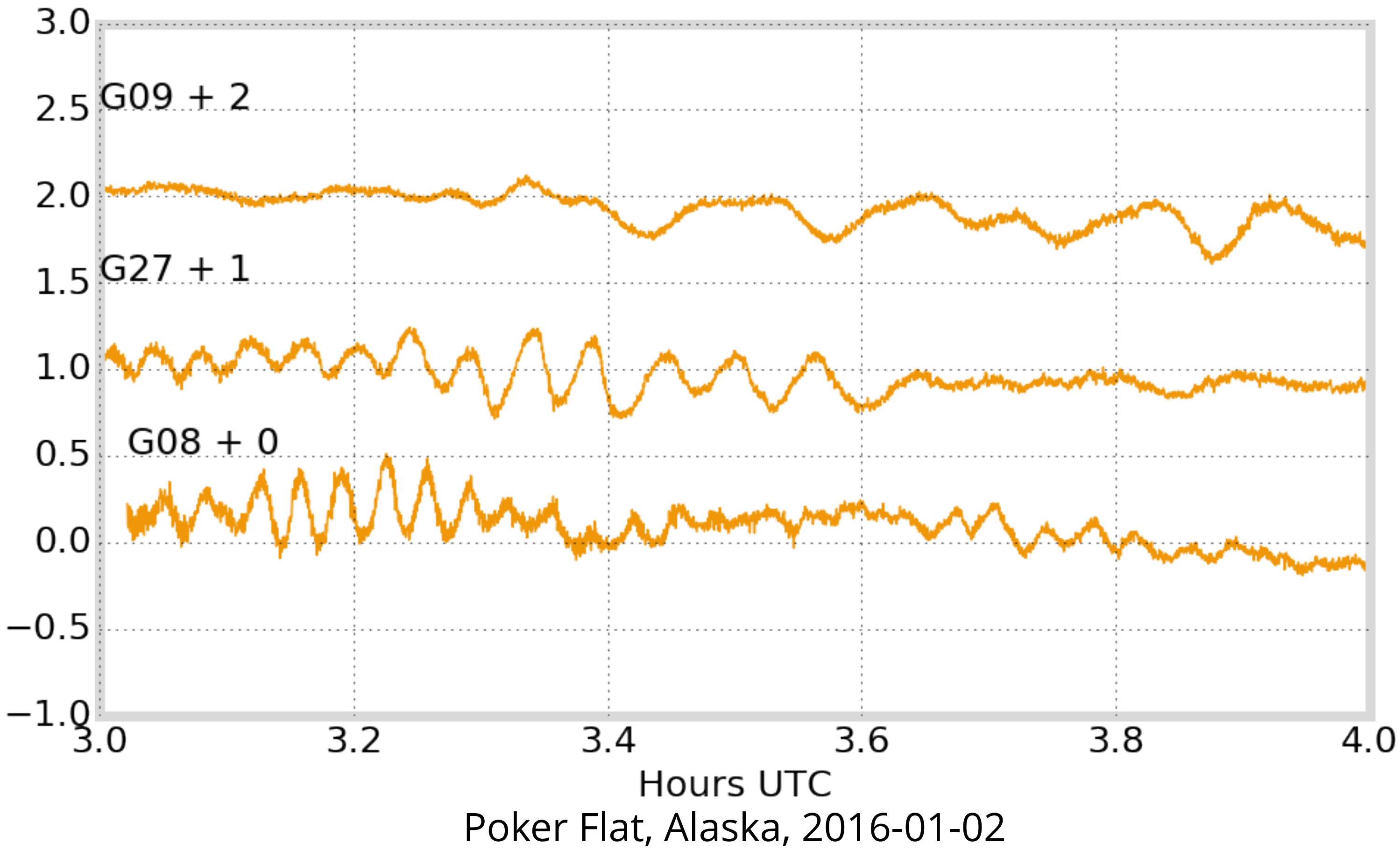
Examples of Dual-Frequency TEC Estimates

Using methods similar to [2] and [3] to solve for bias terms, we compute dual-frequency TEC estimate $TEC_{L1,L2}$ and $TEC_{L1,L5}$



Poker Flat, Alaska, 2016-01-02

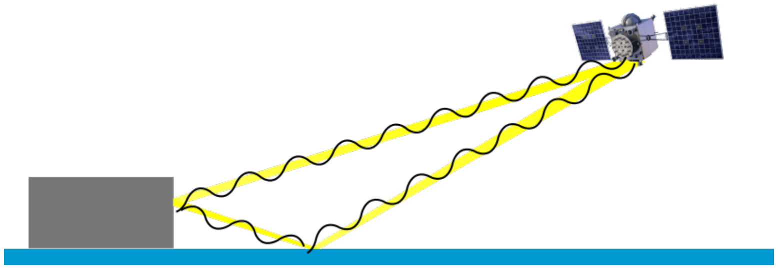
$TEC_{L1,L5} - TEC_{L1,L2}$



Can we characterize / find the source of these discrepancies?

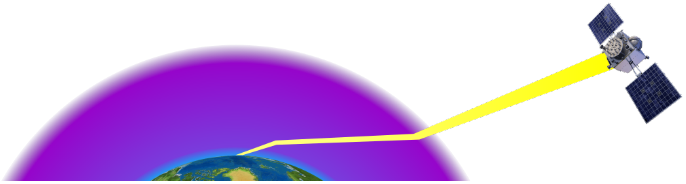
Can we relate them to errors in dual-frequency TEC estimates?

Systematic Errors in GNSS Observations



multipath

reflected signals interfere with primary signal at receiver → causes fluctuations in phase / signal amplitude

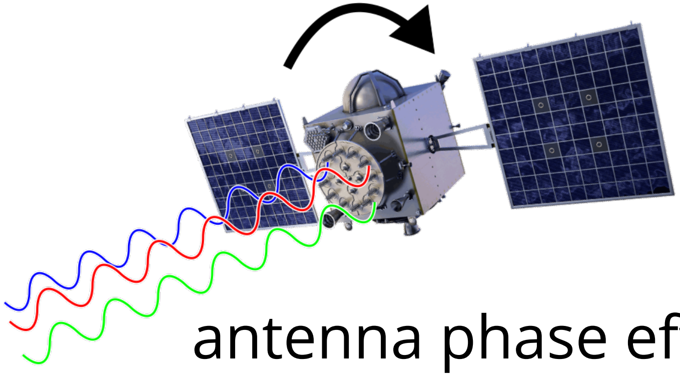


ray-path bending

$r \neq$ line-of-sight range

hardware bias drifts

H_i terms not constant



antenna phase effects

relative displacement of satellite antenna phase centers changes as satellite moves / rotates

higher-order ionosphere terms

need to consider orientation / strength of geomagnetic field

Objectives

Derive optimal triple-frequency estimation of TEC

Investigate the discrepancy in $TEC_{L1,L5} - TEC_{L1,L2}$

Provide a (partial) characterization of TEC estimate residual errors

Motivation

Improve / understand TEC estimate precision

⬇

- Push the boundaries of **TID signature detection** from earthquakes, explosions, etc.
- Understand / address the **errors** in TEC estimates from **low-elevation satellites**
- **Improve user range error** for precise positioning applications

Approach

Develop framework for linear estimation of GNSS parameters

⬇

Apply framework to derive triple-frequency estimates of TEC and systematic errors

⬇

Relate to impact on TEC estimate error residuals

Background and Motivation

Linear Estimation of GNSS Parameters

TEC Estimate Error Residuals

Application to Real GPS Data

Simplified Carrier Phase Model

neglect bias terms

By neglecting bias terms, we address estimation ***precision****, rather than accuracy*

$$\Phi_i = \boxed{r + c\Delta t + T} + \boxed{I_i} + \boxed{\lambda_i N_i} + \boxed{H_i} + \boxed{S_i} + \boxed{\epsilon_i}$$

"geometry" term

$$\Phi_i = \boxed{G} + \boxed{I_i} + \boxed{S_i} + \boxed{\epsilon_i}$$

zero-mean normally-distributed

zero-mean

Linear Inverse Problem

$$\mathbf{\Phi} = \mathbf{Am} + \epsilon$$

$$\mathbf{\Phi} = [\Phi_1, \cdots, \Phi_m]^T$$

observations

$$\mathbf{m} = [G, \mathrm{TEC}, S_1, \cdots, S_m]^T$$

model parameters

$$\mathbf{A} = \begin{bmatrix} 1 & -\frac{\kappa}{f_1^2} & 1 & 0 & \cdots & 0 \\ 1 & -\frac{\kappa}{f_2^2} & 0 & 1 & \cdots & 0 \\ \vdots & & & & \ddots & \\ 1 & -\frac{\kappa}{f_m^2} & 0 & \cdots & & 1 \end{bmatrix}$$

forward model

$$\epsilon = [\epsilon_1, \cdots, \epsilon_m]^T$$

stochastic error

Linear Estimation

$$\hat{\mathbf{m}} \approx \mathbf{A}^* \mathbf{\Phi}$$

$\hat{\mathbf{m}}$

model estimate

$\mathbf{A}^* = ?$

model estimator

$$\mathbf{A}^T \left(\mathbf{A} \mathbf{A}^T \right)^{-1}$$ ➡ Poor results; treats each parameter with equal weight

*We must apply **a priori information** about model parameters*

A Priori Information

Under normal conditions, we know that:

$$|G| \gg |I_i| \gg |S_i|$$

$G \sim$ 20,000 km

$I \sim$ 1 - 150 m

$S \sim$ several cm

Using A Priori Information

We *could* apply $|G| \gg |I_i| \gg |S_i|$ using Gaussian priors

Instead we derive each row separately:

$$\mathbf{C} = [c_1, \cdots, c_m]^T \in \mathbb{R}^m$$

estimator

(written as row vectors here)

$$\mathbf{A}^* = \begin{bmatrix} \mathbf{C}_G \\ \mathbf{C}_{\mathrm{TEC}} \\ \mathbf{C}_{S_1} \\ \vdots \\ \mathbf{C}_{S_m} \end{bmatrix}$$

geometry estimator

TECu estimator

systematic-error estimators

How to Choose Optimal $\mathbf{C}$

Linear combination E given by inner-product:

$$E = \langle \mathbf{C} | \mathbf{\Phi} \rangle$$

Goals:

1. produce desired parameter with unity coefficient

2. remove / reduce all other terms

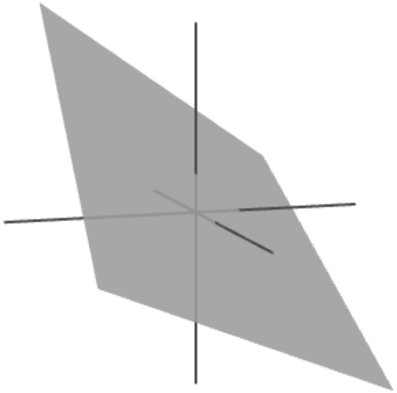
Approach:

First, constrain $\mathbf{C}$ to satisfy Goal 1

Then, constrain / optimize $\mathbf{C}$ to achieve Goal 2

Linear Coefficient Constraints

Use one or two of the following constraints to reduce search space for optimal estimator coefficients:



$$\Phi_i = G + I_i + S_i + \epsilon_i$$

$$\sum_i c_i = 0$$

geometry-free

$$\sum_i c_i = 1$$

geometry-estimator

$$\sum_i \frac{c_i}{f_i^2} = 0$$

ionosphere-free

$$\sum_i -\frac{\kappa}{f_i^2} c_i = 1$$

TEC-estimator

Reduction of Error

Linear combination stochastic error variance:

$$\sigma_\epsilon^2 = \mathbf{C}^T \mathbf{\Sigma}_\epsilon \mathbf{C}$$

where $\mathbf{\Sigma}_\epsilon$ is the covariance matrix between ϵ_i

Optimal $\mathbf{C}$ for minimizing stochastic error variance:

$$\mathbf{C}^* = \arg\min_{\mathbf{C}} \mathbf{C}^T \Sigma_\epsilon \mathbf{C}$$

ϵ_i equal-amplitude and uncorrelated

⬇

$$\mathbf{C}^* = \arg\min_{\mathbf{C}} \sum_i c_i^2$$

TEC Estimator

1. apply **TECu-estimator constraint**
2. apply **geometry-free constraint** (since $|G| \gg |I_i|$)

Dual-Frequency Example

TEC-estimator $$-\frac{\kappa}{f_1^2}c_1 - \frac{\kappa}{f_2^2}c_2 = 1$$

geometry-free $$c_1 + c_2 = 0 \Rightarrow c_1 = -c_2$$

$$\Rightarrow -\kappa c_1 \left(\frac{1}{f_1^2} - \frac{1}{f_2^2}\right) = 1$$

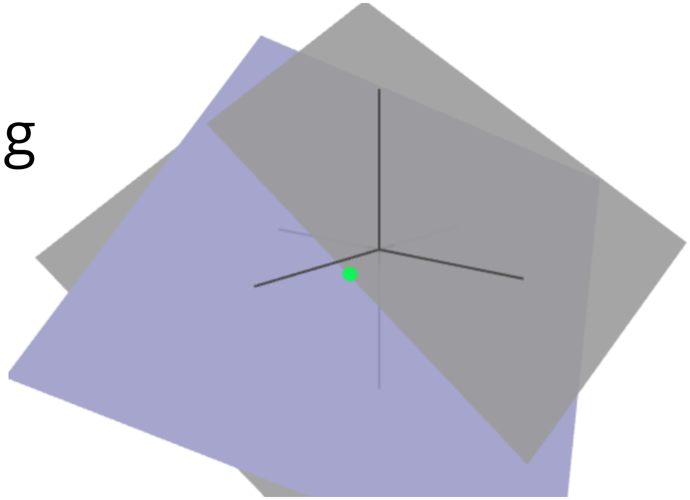
$$\Rightarrow c_1 = -\frac{1}{\kappa\left(\frac{1}{f_1^2} - \frac{1}{f_2^2}\right)}$$

recall:

$$\mathrm{TEC} = \frac{\Phi_2 - \Phi_1}{\kappa\left(\frac{1}{f_1^2} - \frac{1}{f_2^2}\right)}$$

Triple-Frequency TEC Estimator

Applying constraints yields following system of coefficients (with free parameter denoted x):



$$c_1 = \frac{\frac{1}{\kappa} + x\left(\frac{1}{f_3^2} - \frac{1}{f_2^2}\right)}{\frac{1}{f_2^2} - \frac{1}{f_1^2}}$$

$$c_2 = \frac{-\frac{1}{\kappa} - x\left(\frac{1}{f_3^2} - \frac{1}{f_1^2}\right)}{\frac{1}{f_2^2} - \frac{1}{f_1^2}}$$

$$c_3 = x$$

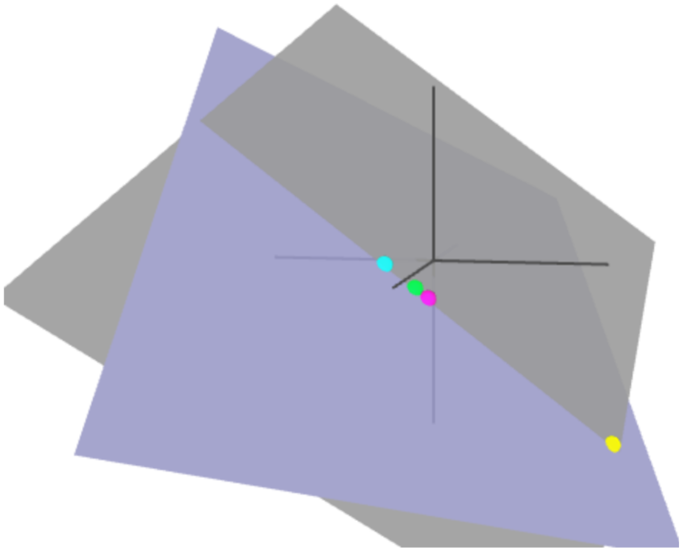
To satisfy $\mathbf{C}^* = \arg\min_{\mathbf{C}} \sum_i c_i^2$, choose

$$x^* = \frac{\frac{1}{\kappa}\left(\frac{2}{f_3^2} - \frac{1}{f_2^2} - \frac{1}{f_1^2}\right)}{\left(\frac{1}{f_1^2} - \frac{1}{f_2^2}\right)^2 + \left(\frac{1}{f_2^2} - \frac{1}{f_3^2}\right)^2 + \left(\frac{1}{f_3^2} - \frac{1}{f_1^2}\right)^2}$$

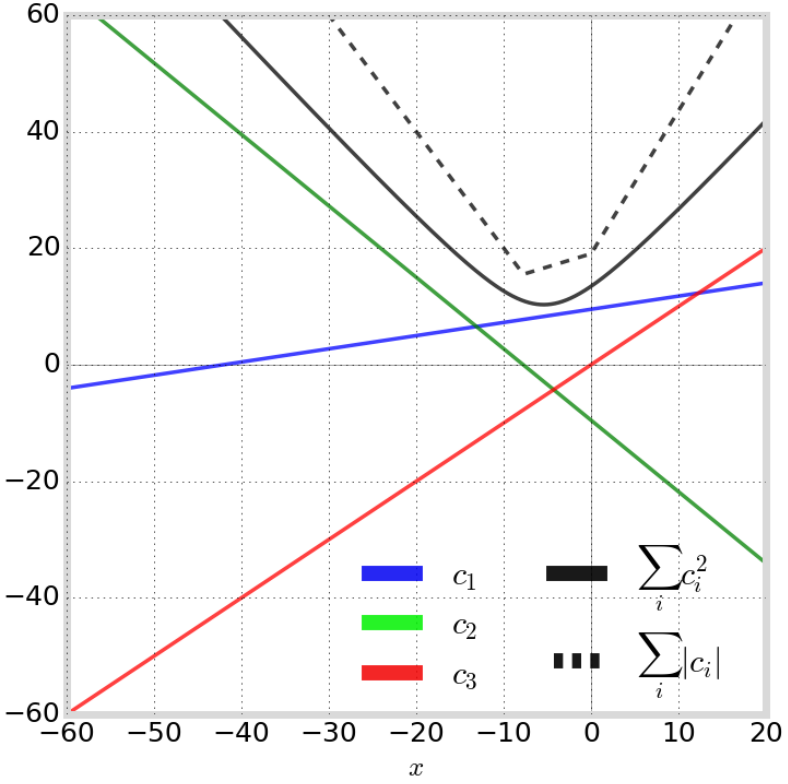
⬇

denote corresponding coefficient vector $\mathbf{C}_{\text{TEC}_{1,2,3}}$ and its corresponding estimate $\text{TEC}_{1,2,3}$

TEC Estimator Using Triple-Frequency GPS



- $\mathbf{C}_{\text{TEC}_{\text{L1,L2,L5}}}$ (green)
- $\mathbf{C}_{\text{TEC}_{\text{L1,L5}}}$ (magenta)
- $\mathbf{C}_{\text{TEC}_{\text{L1,L2}}}$ (cyan)
- $\mathbf{C}_{\text{TEC}_{\text{L2,L5}}}$ (yellow)



Estimate	c_1	c_2	c_3	$\sum_i c_i^2$
$\text{TEC}_{\text{L1,L2,L5}}$	8.294	−2.883	−5.411	10.314
$\text{TEC}_{\text{L1,L5}}$	7.762	0	−7.762	10.977
$\text{TEC}_{\text{L1,L2}}$	9.518	−9.518	0	13.460
$\text{TEC}_{\text{L2,L5}}$	0	42.080	−42.080	59.510

Geometry Estimator

1. apply **geometry-estimator constraint**
2. apply **ionosphere-free constraint** since I_i are the next-largest terms

For triple-frequency GNSS: To satisfy $\mathbf{C}^* = \arg\min_{\mathbf{C}} \sum_i c_i^2$,

$$c_1 = \frac{-\frac{1}{f_2^2} + x\left(\frac{1}{f_2^2} - \frac{1}{f_3^2}\right)}{\frac{1}{f_1^2} - \frac{1}{f_2^2}}$$

$$c_2 = \frac{\frac{1}{f_1^2} - x\left(\frac{1}{f_1^2} - \frac{1}{f_3^2}\right)}{\frac{1}{f_1^2} - \frac{1}{f_2^2}}$$

$$c_3 = x$$

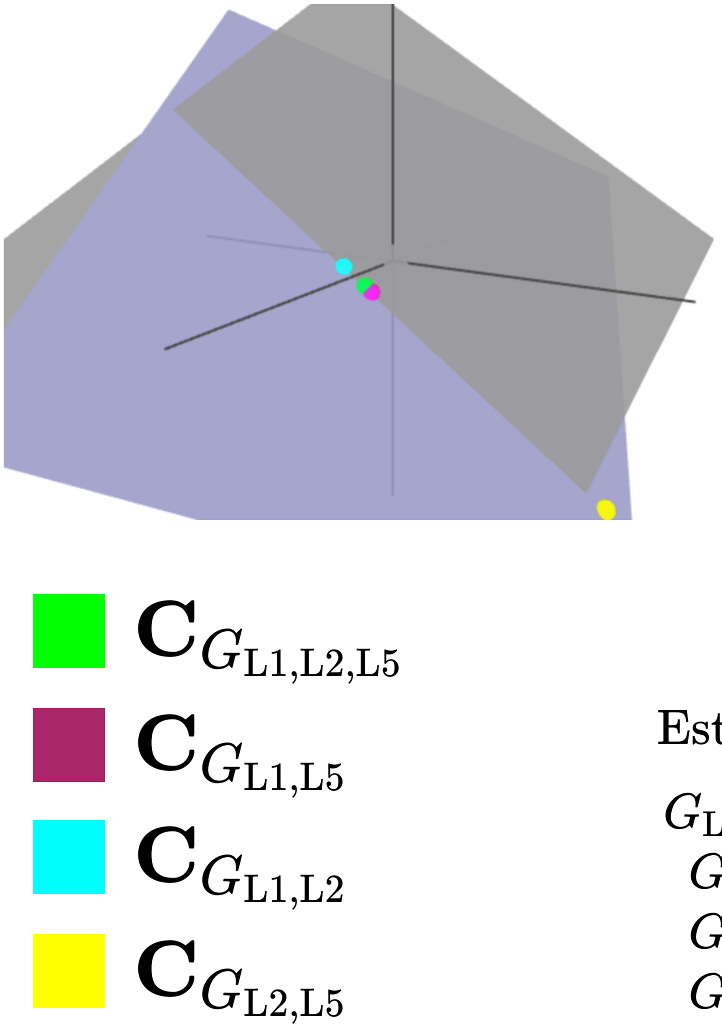
$$x^* = \frac{\frac{1}{\kappa}\left(\frac{2}{f_3^2} - \frac{1}{f_2^2} - \frac{1}{f_1^2}\right)}{\left(\frac{1}{f_1^2} - \frac{1}{f_2^2}\right)^2 + \left(\frac{1}{f_2^2} - \frac{1}{f_3^2}\right)^2 + \left(\frac{1}{f_3^2} - \frac{1}{f_1^2}\right)^2}$$

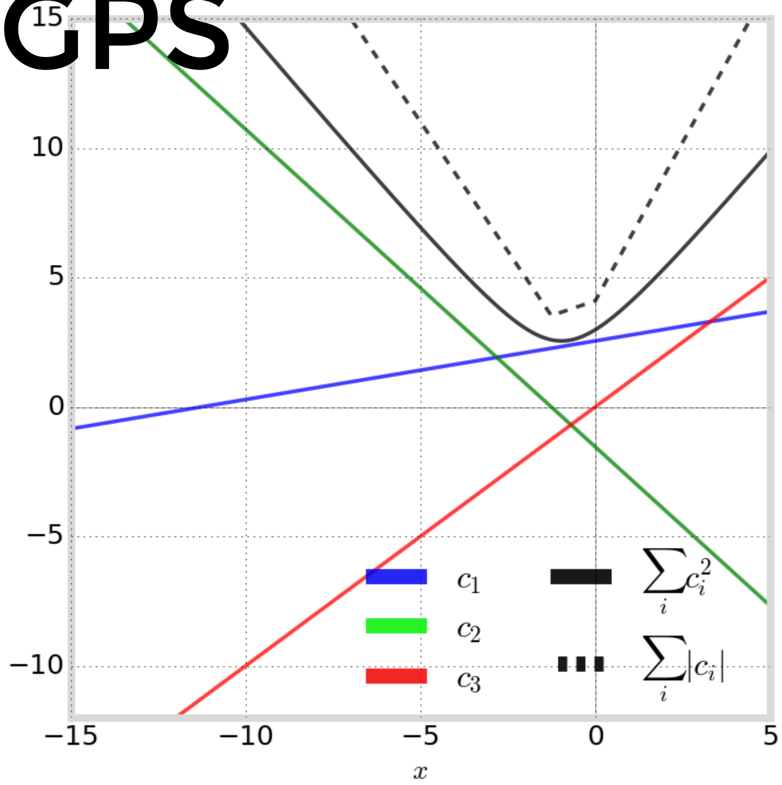
We call this coefficient vector $\mathbf{C}_{G_{1,2,3}}$ and its corresponding estimate $G_{1,2,3}$

↑

the optimal "ionosphere-free combination"

Geometry Estimator Using Triple-Frequency GPS



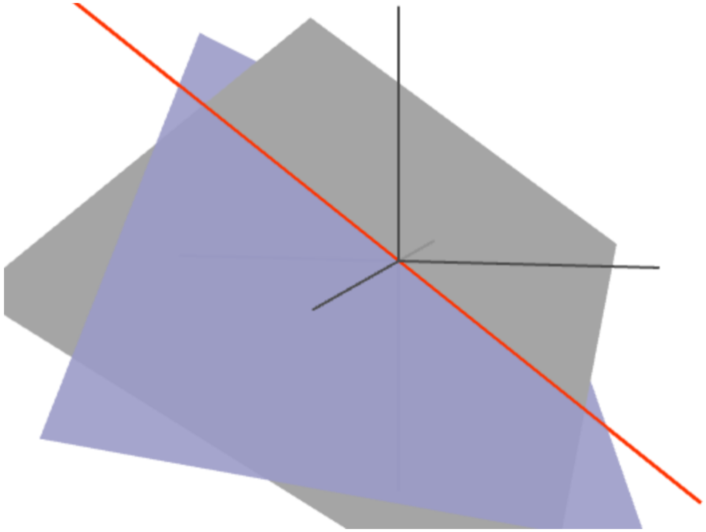


Estimate	c_1	c_2	c_3	$\sum_i c_i^2$
$G_{\mathrm{L1,L2,L5}}$	2.327	−0.360	−0.967	2.546
$G_{\mathrm{L1,L5}}$	2.261	0	−1.261	2.588
$G_{\mathrm{L1,L2}}$	2.546	−1.546	0	2.978
$G_{\mathrm{L2,L5}}$	0	12.255	−11.255	16.639

Systematic Error Estimator

Since $|G| \gg |I_i| \gg |S_i|$, must apply both **geometry-free** and **ionosphere-free** constraints

note this requires $m \geq 3$



For triple-frequency GNSS:

$$c_1 = \ \ x \frac{\frac{1}{f_3^2} - \frac{1}{f_2^2}}{\frac{1}{f_2^2} - \frac{1}{f_1^2}}$$

$$c_2 = -x \frac{\frac{1}{f_3^2} - \frac{1}{f_1^2}}{\frac{1}{f_2^2} - \frac{1}{f_1^2}}$$

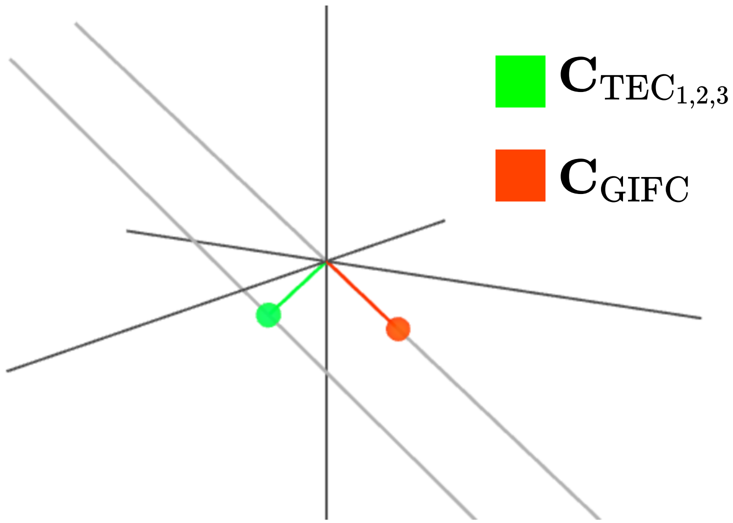
$$c_3 = \ \ x$$

system is linear subspace

Geometry-Ionosphere-Free Combination

We call the linear combination that applies both **geometry-free** and **ionosphere-free** constraints the geometry-ionosphere-free combination (**GIFC**)

FACT: The difference between any two TEC estimates produces some scaling of the GIFC



FACT: $\mathbf{C}_{\mathrm{GIFC}}$ and $\mathbf{C}_{\mathrm{TEC}_{1,2,3}}$ are perpendicular, i.e.

$$\langle \mathbf{C}_{\mathrm{GIFC}} | \mathbf{C}_{\mathrm{TEC}_{1,2,3}} \rangle = 0$$

FACT: $\frac{\langle \mathbf{C}_{\mathrm{TEC}} | \mathbf{C}_{\mathrm{TEC}_{1,2,3}} \rangle}{||\mathbf{C}_{\mathrm{TEC}_{1,2,3}}||} = ||\mathbf{C}_{\mathrm{TEC}_{1,2,3}}||$

i.e. $\mathbf{C}_{\mathrm{TEC}}$ projected onto direction $\mathbf{C}_{\mathrm{TEC}_{1,2,3}}$ lands at $\mathbf{C}_{\mathrm{TEC}_{1,2,3}}$

GIFC Triple-Frequency GPS

We (arbitrarily) choose:

$$\mathbf{C}_{\mathrm{GIFC}_{\mathrm{L1,L2,L5}}} = \mathbf{C}_{\mathrm{TEC}_{\mathrm{L1,L5}}} - \mathbf{C}_{\mathrm{TEC}_{\mathrm{L1,L2}}}$$

$$= [-1.756, 9.520, -7.764]^T$$

Note: the triple-frequency GIFC **does not have** a **well-defined unit**.

GIFC in our results section have the scaling shown here.

Background and Motivation

Linear Estimation of GNSS Parameters

TEC Estimate Error Residuals

Application to Real GPS Data

Estimate Residual Error

Define the error residual vector $\mathbf{R}$ with components:

$$R_i = S_i + \epsilon_i$$

The residual error impacting the TEC estimate is:

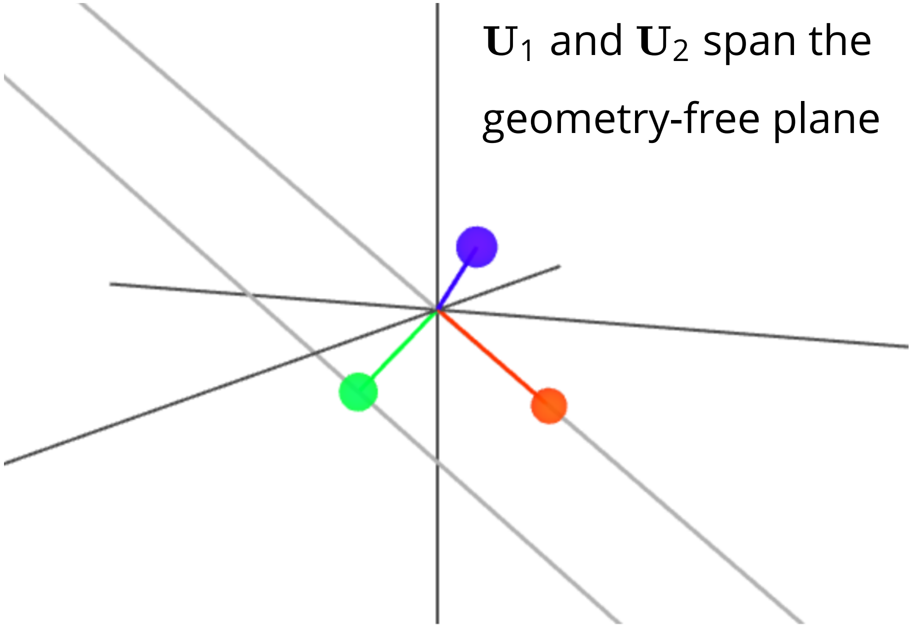
$$R_{\mathrm{TEC}} = \langle \mathbf{C}_{\mathrm{TEC}} | \mathbf{R} \rangle$$

Note that:

$$\mathrm{GIFC} = \langle \mathbf{C}_{\mathrm{GIFC}} | \mathbf{R} \rangle$$

A Convenient Basis

We transform $\mathbf{R}$ using the orthonormal basis:



Note that $\mathbf{U}_3 \perp \mathbf{C}_{\text{TEC}}$ since $\mathbf{U}_1$ and $\mathbf{U}_2$ span the geometry-free plane

$$\mathbf{U}_1 = \frac{\mathbf{C}_{\text{TEC}_{1,2,3}}}{||\mathbf{C}_{\text{TEC}_{1,2,3}}||}$$

$$\mathbf{U}_2 = \frac{\mathbf{C}_{\text{GIFC}}}{||\mathbf{C}_{\text{GIFC}}||}$$

$$\mathbf{U}_3 = \mathbf{U}_1 \times \mathbf{U}_2$$

$$\mathbf{U} = \begin{bmatrix} \mathbf{U}_1 \\ \mathbf{U}_2 \\ \mathbf{U}_3 \end{bmatrix}$$

$$\mathbf{R}' = \mathbf{U}\mathbf{R}$$

$$R'_i = \langle \mathbf{U}_i | \mathbf{R} \rangle$$

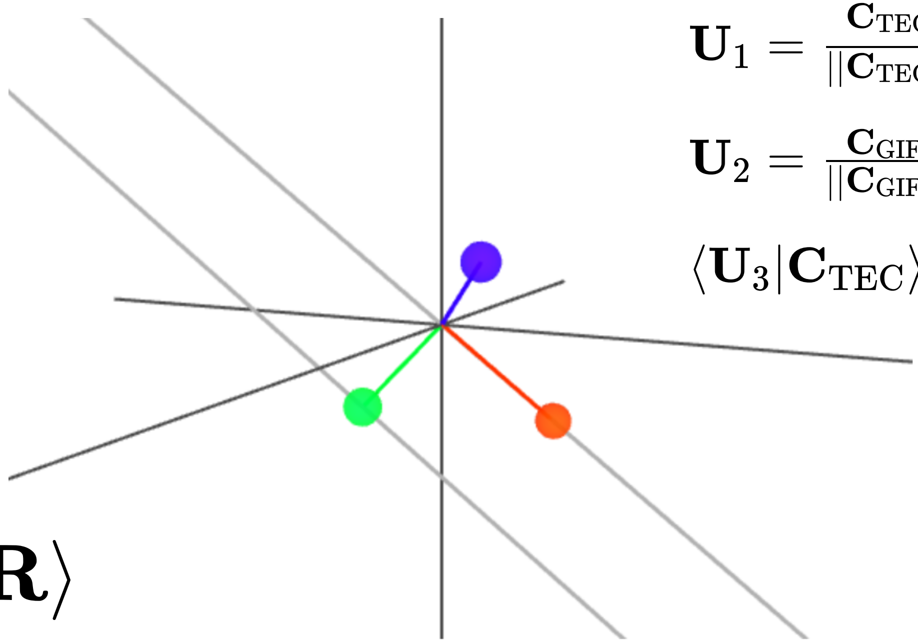
Note that:

$$R'_1 = \frac{R_{\text{TEC}_{1,2,3}}}{||\mathbf{C}_{\text{TEC}_{1,2,3}}||}$$

$$R'_2 = \frac{\text{GIFC}}{||\mathbf{C}_{\text{GIFC}}||}$$

TEC Estimate Residual Error

Express R_{TEC} as residual error components in transformed coordinate system:



$$\mathbf{U}_1 = \frac{\mathbf{C}_{\text{TEC}_{1,2,3}}}{||\mathbf{C}_{\text{TEC}_{1,2,3}}||}$$

$$\mathbf{U}_2 = \frac{\mathbf{C}_{\text{GIFC}}}{||\mathbf{C}_{\text{GIFC}}||}$$

$$\langle \mathbf{U}_3 | \mathbf{C}_{\text{TEC}} \rangle = 0$$

$$R_{\text{TEC}} = \langle \mathbf{U}\mathbf{C}_{\text{TEC}} | \mathbf{U}\mathbf{R} \rangle$$

$$= \langle \mathbf{U}_1 | \mathbf{C}_{\text{TEC}} \rangle R'_1 + \langle \mathbf{U}_2 | \mathbf{C}_{\text{TEC}} \rangle R'_2$$

$$= R_{\text{TEC}_{1,2,3}} + \frac{\langle \mathbf{C}_{\text{GIFC}} | \mathbf{C}_{\text{TEC}} \rangle}{||\mathbf{C}_{\text{GIFC}}||^2} R_{\text{GIFC}}$$

common TEC estimate residual error component

GIFC residual error component

TEC Estimate Residual Error Discussion

Term $\frac{\langle \mathbf{C}_{\text{GIFC}} | \mathbf{C}_{\text{TEC}} \rangle}{||\mathbf{C}_{\text{GIFC}}||^2}$ = amplitude of **GIFC residual error** component in TEC estimate

$\text{TEC}_{1,2,3}$ is optimal in the sense that it **completely removes the GIFC component** of residual error

Term $R_{\text{TEC}_{1,2,3}}$ = unobservable **"TEC-like" residual error** component

But can we say anything about the ***overall*** *TEC estimate residual error?*

Argument for Using GIFC to Assess Overall Residual Error

Assume $\mathbf{R}$ has an **overall distribution** that is **joint symmetric** about the origin with distribution function $f_R(x)$

R_i equal amplitude and uncorrelated

By definition, $\mathbf{UR}$ ~ symmetric with $f_R(x)$ for any orthonormal transformation $\mathbf{U}$

The distribution of a scaled version $a\mathbf{R}$ for some scalar a is $f_R(\frac{x}{a})$

$$f_{\text{GIFC}}(x) = f_R\left(\frac{x}{||\mathbf{C}_{\text{GIFC}}||}\right)$$

$$f_{R_{\text{TEC}}}(x) = f_R\left(\frac{x}{||\mathbf{C}_{\text{TEC}}||}\right)$$

➡

$$f_{R_{\text{TEC}}}(x) = f_{\text{GIFC}}\left(\frac{||\mathbf{C}_{\text{GIFC}}||}{||\mathbf{C}_{\text{TEC}}||}x\right)$$

Overall TEC Residual Error Discussion

The assumption that $\mathbf{R}$ has joint symmetric distribution is wrong

We can do better by carefully assessing a priori knowledge about the error components in each Φ_i

- investigating GIFC is first-step in this process

$f_{R_{\text{TEC}}}(x) = f_{\text{GIFC}}\left(\frac{||\mathbf{C}_{\text{GIFC}}||}{||\mathbf{C}_{\text{TEC}}||}x\right)$ is a coarse approximation

- relates deviations as: dev$R_{\text{TEC}} \approx \frac{||\mathbf{C}_{\text{TEC}}||}{||\mathbf{C}_{\text{GIFC}}||}$dev GIFC
- could be very wrong if $R_{\text{TEC}_{1,2,3}} \gg GIFC$

Relation Between GIFC and TEC Estimate Residual Errors

amplitude of GIFC error signal in TEC residual

relates deviation in GIFC and TEC residual

Estimate	$\frac{\langle \mathbf{C}_{\mathrm{GIFC}} \vert \mathbf{C}_{\mathrm{TEC}} \rangle}{\vert\vert \mathbf{C}_{\mathrm{GIFC}} \vert\vert^2}$	$\frac{\vert\vert \mathbf{C}_{\mathrm{TEC}} \vert\vert}{\vert\vert \mathbf{C}_{\mathrm{GIFC}} \vert\vert}$
$\mathrm{TEC}_{\mathrm{L1,L2,L5}}$	0	0.831
$\mathrm{TEC}_{\mathrm{L1,L5}}$	0.303	0.885
$\mathrm{TEC}_{\mathrm{L1,L2}}$	-0.697	1.085
$\mathrm{TEC}_{\mathrm{L2,L5}}$	4.723	4.796

Background and Motivation

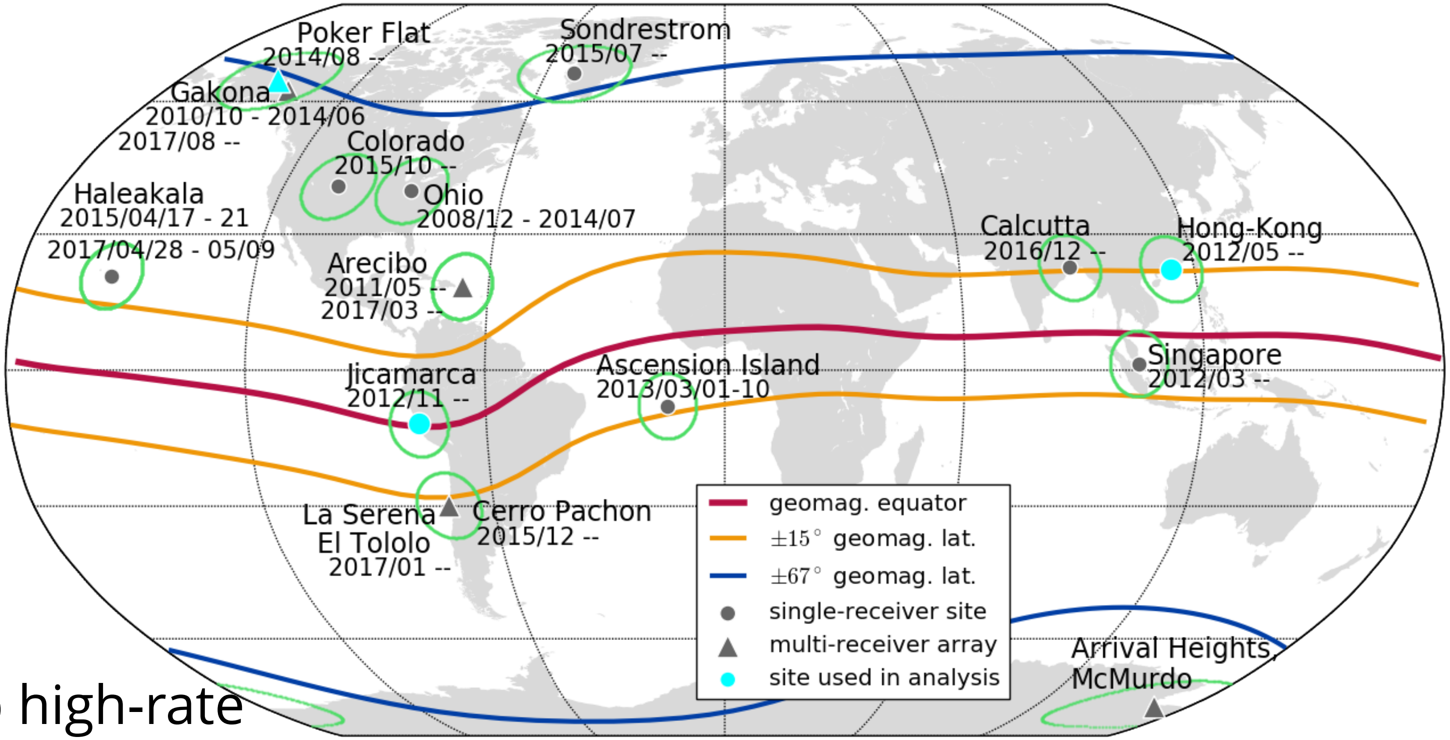
Linear Estimation of GNSS Parameters

TEC Estimate Error Residuals

Application to Real GPS Data

Experiment Data

- Alaska, Hong Kong, Peru
- 2013, 2014, 2015, 2016
- Septentrio PolarXs
- 1 Hz GPS L1/L2/L5 measurements



GPS Lab high-rate
GNSS data
collection network

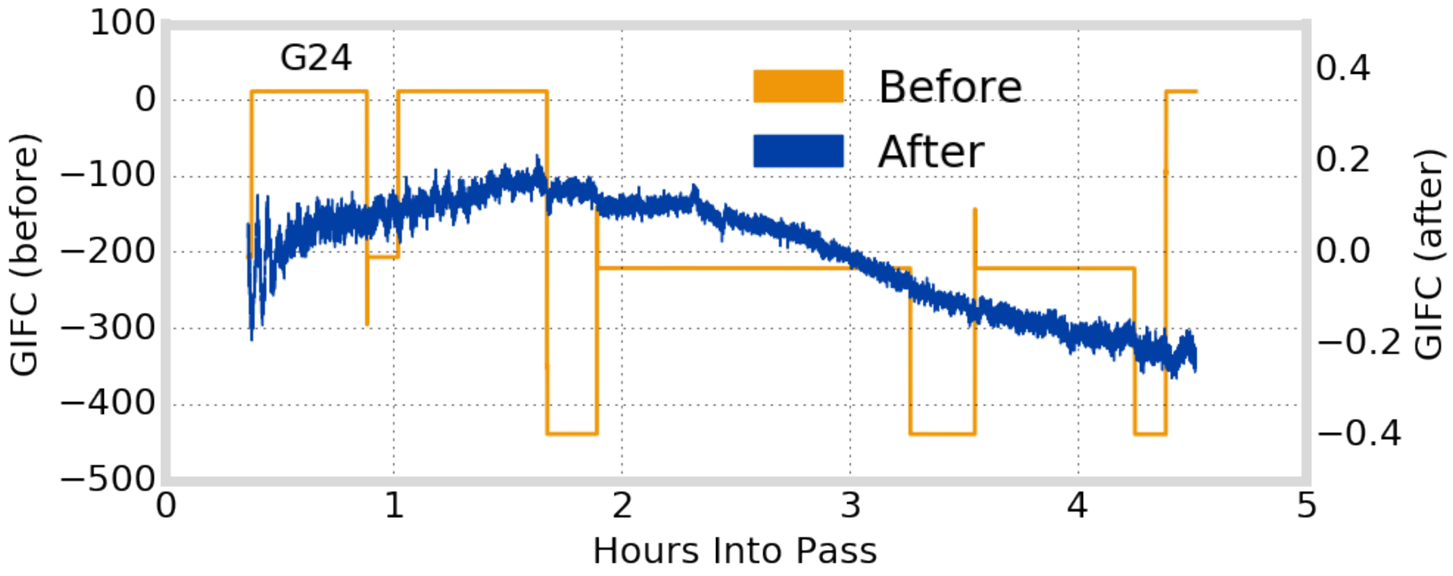
Data Alignment and Correction

align data by sidereal day
= 23h 55m 54.2 s

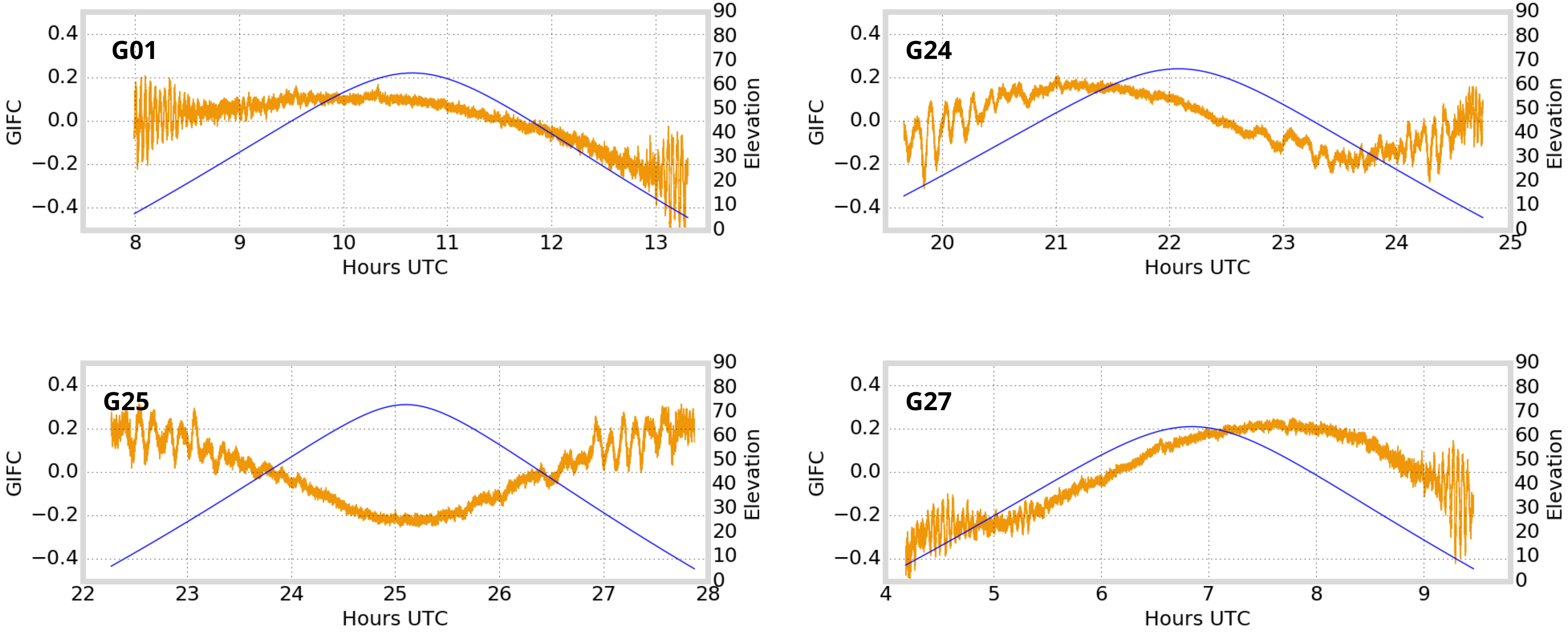
GPS orbital period ≈ 1/2 sidereal day

must remove jumps in GIFC data due to ionosphere activity / multipath / interference

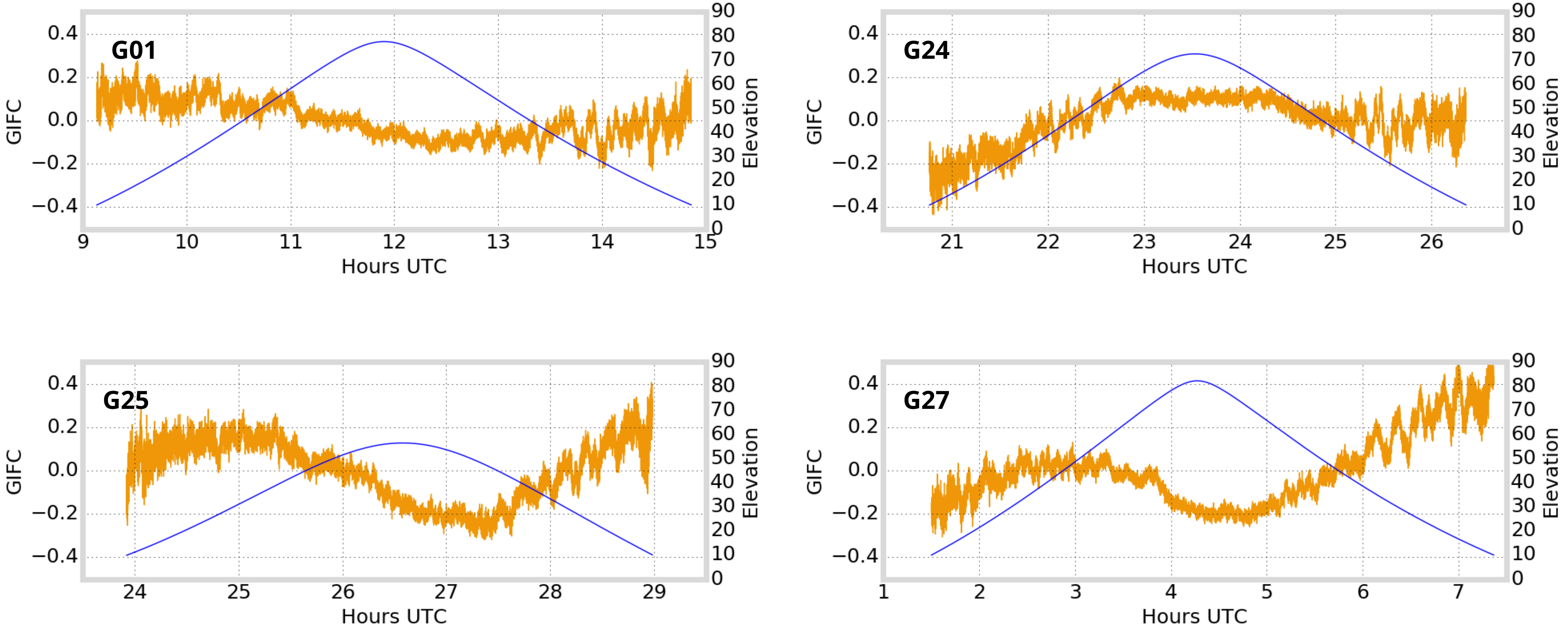
Outlier segments (|GIFC| > 2) are removed from analysis



GIFC Examples
Alaska

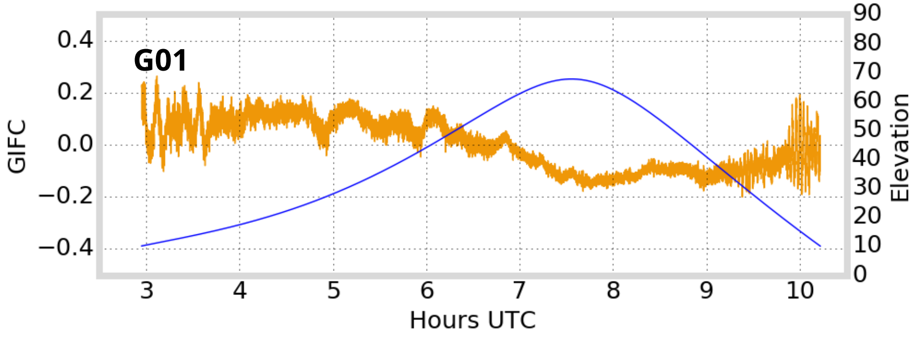


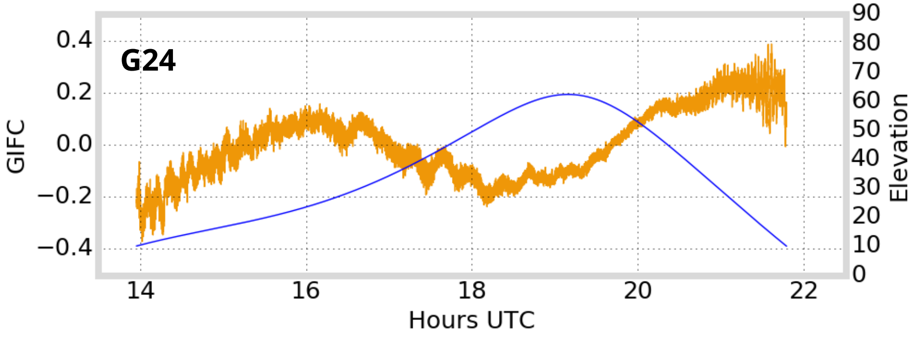
GIFC Examples
Hong Kong

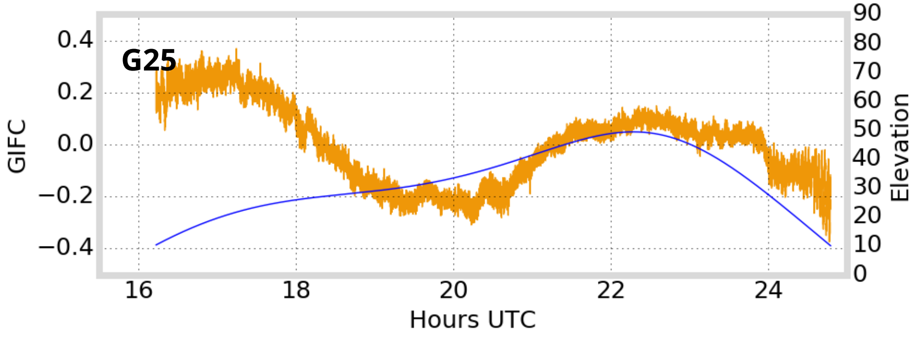


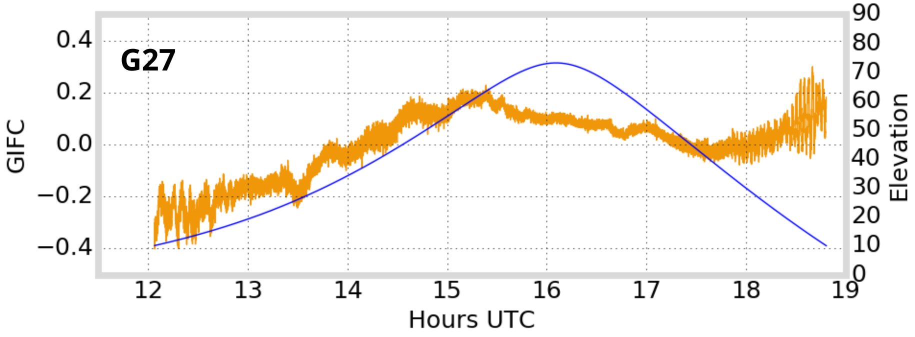
GIFC Examples

Peru

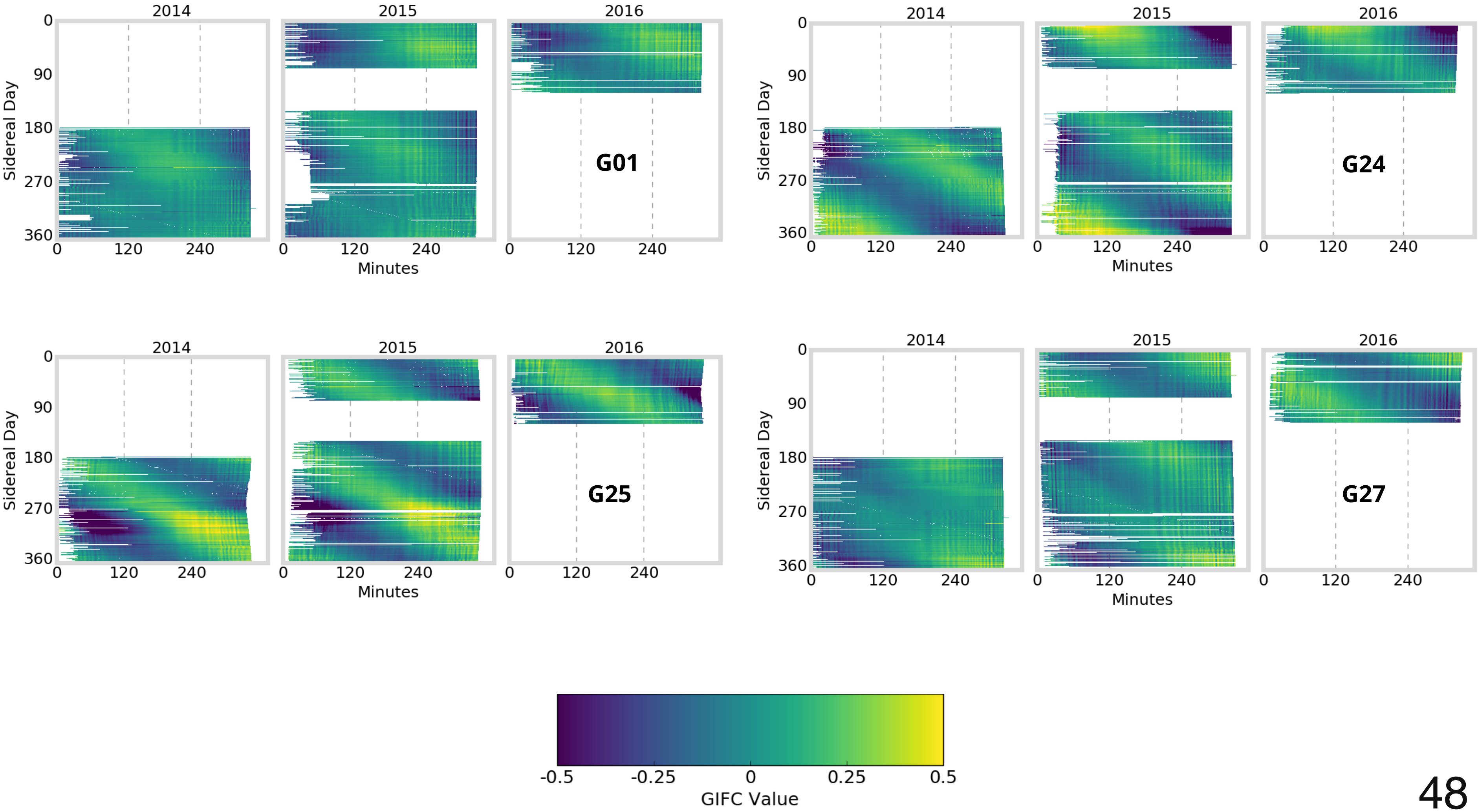




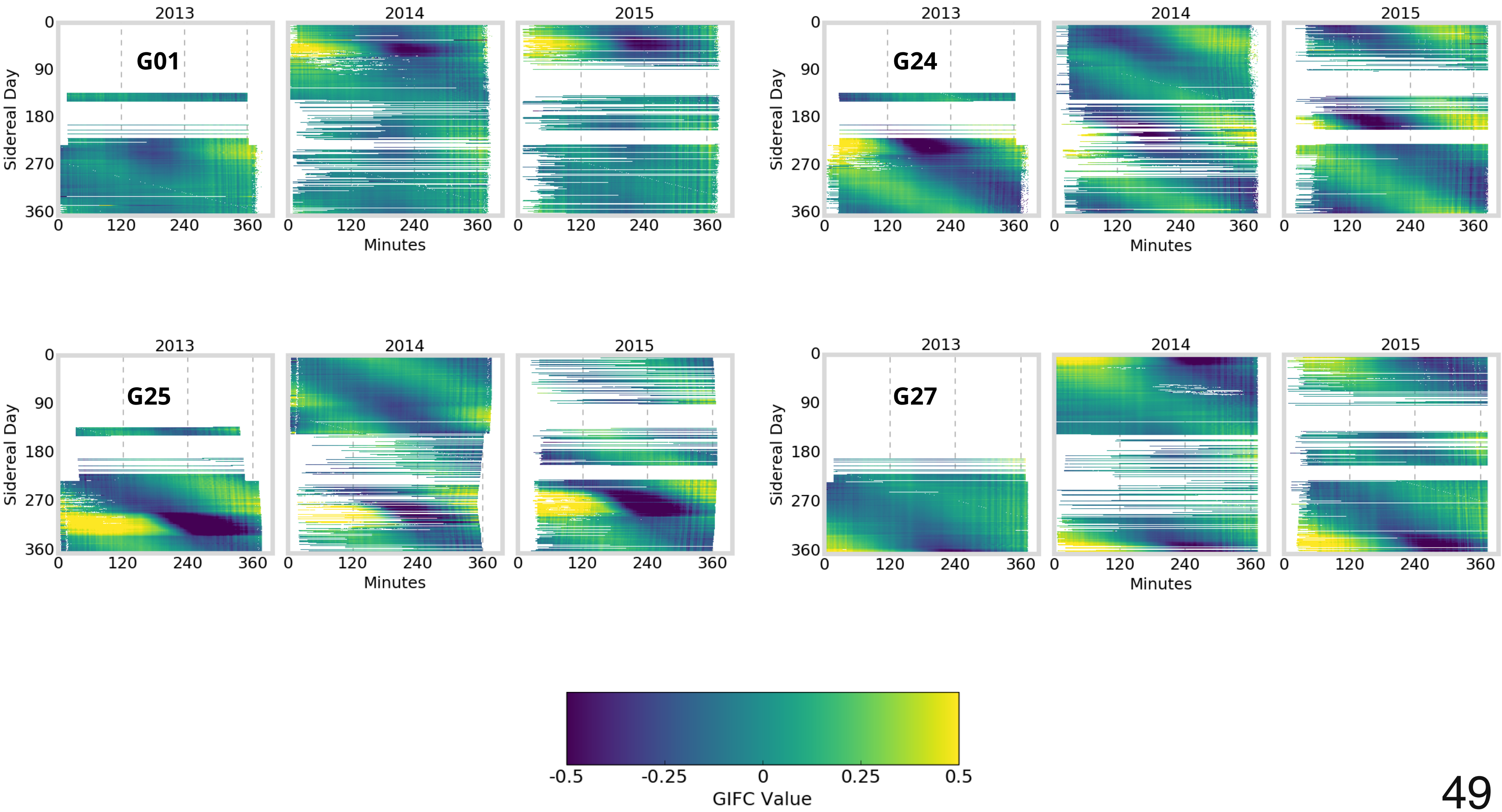




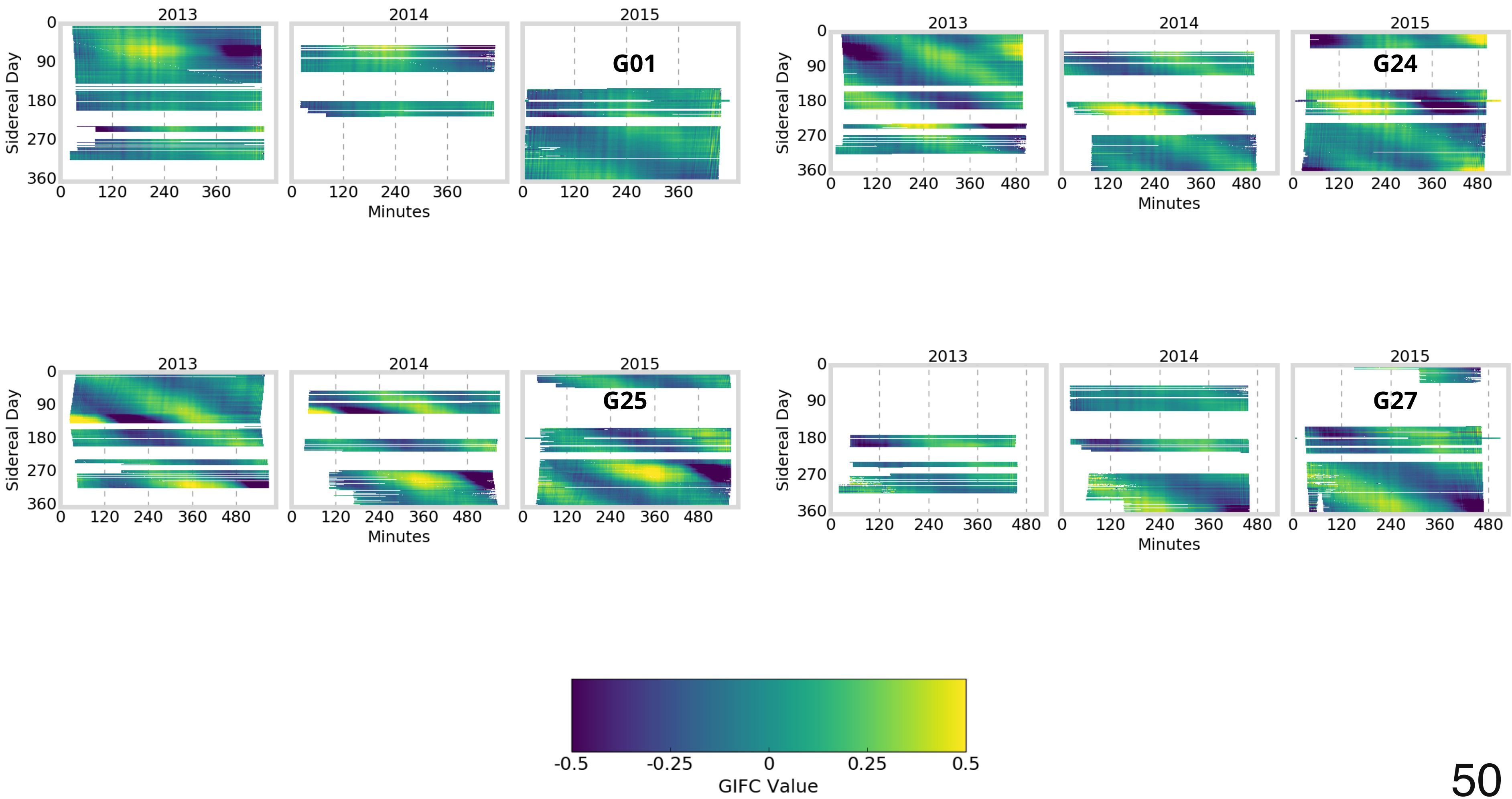
GIFC Calendar
Alaska



GIFC Calendar
Hong Kong

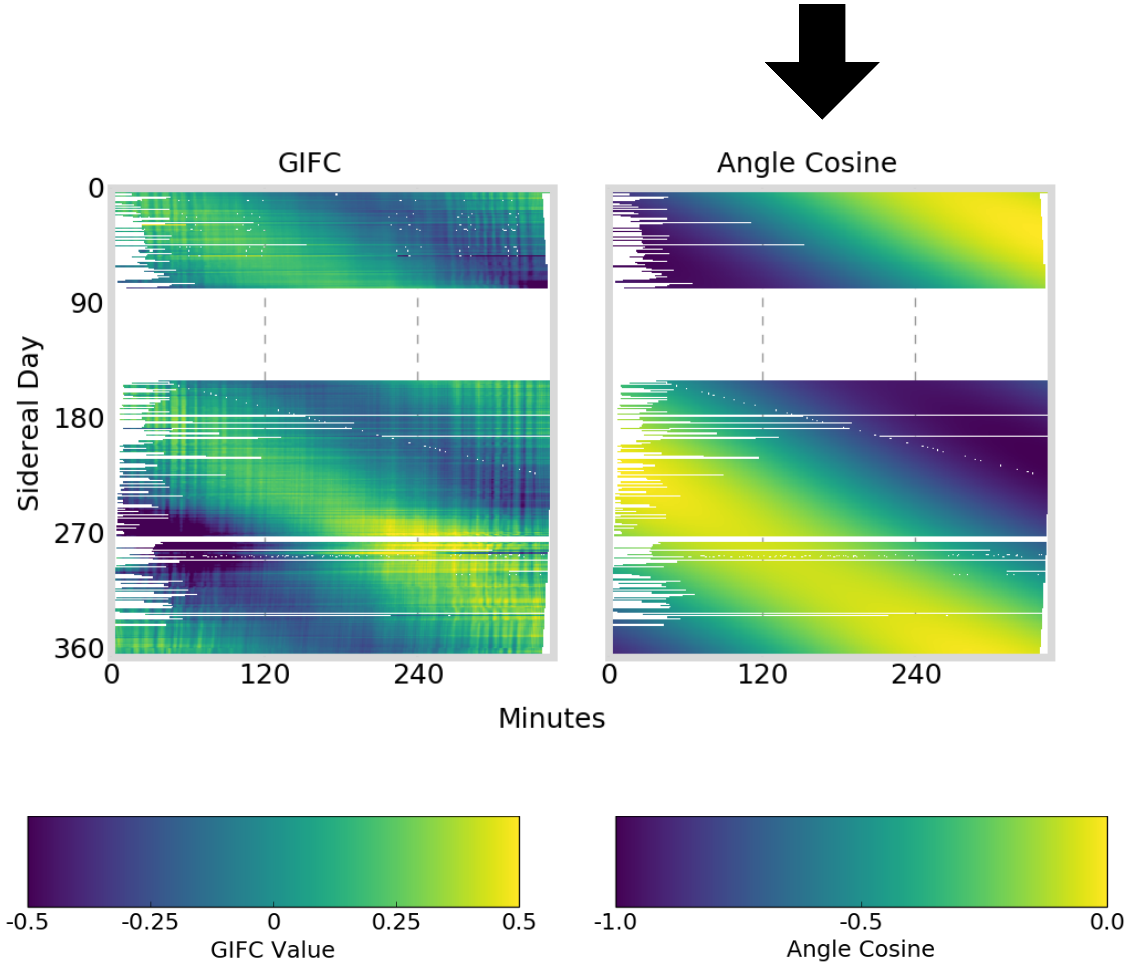


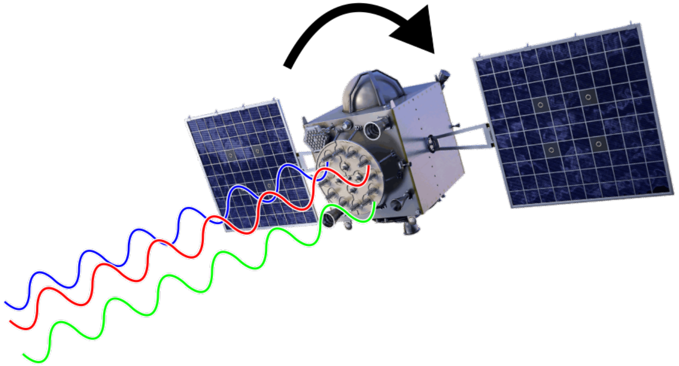
GIFC Calendar
Peru



Satellite Antenna Phase Effects?

angle cosine between Earth center, satellite, and Sun



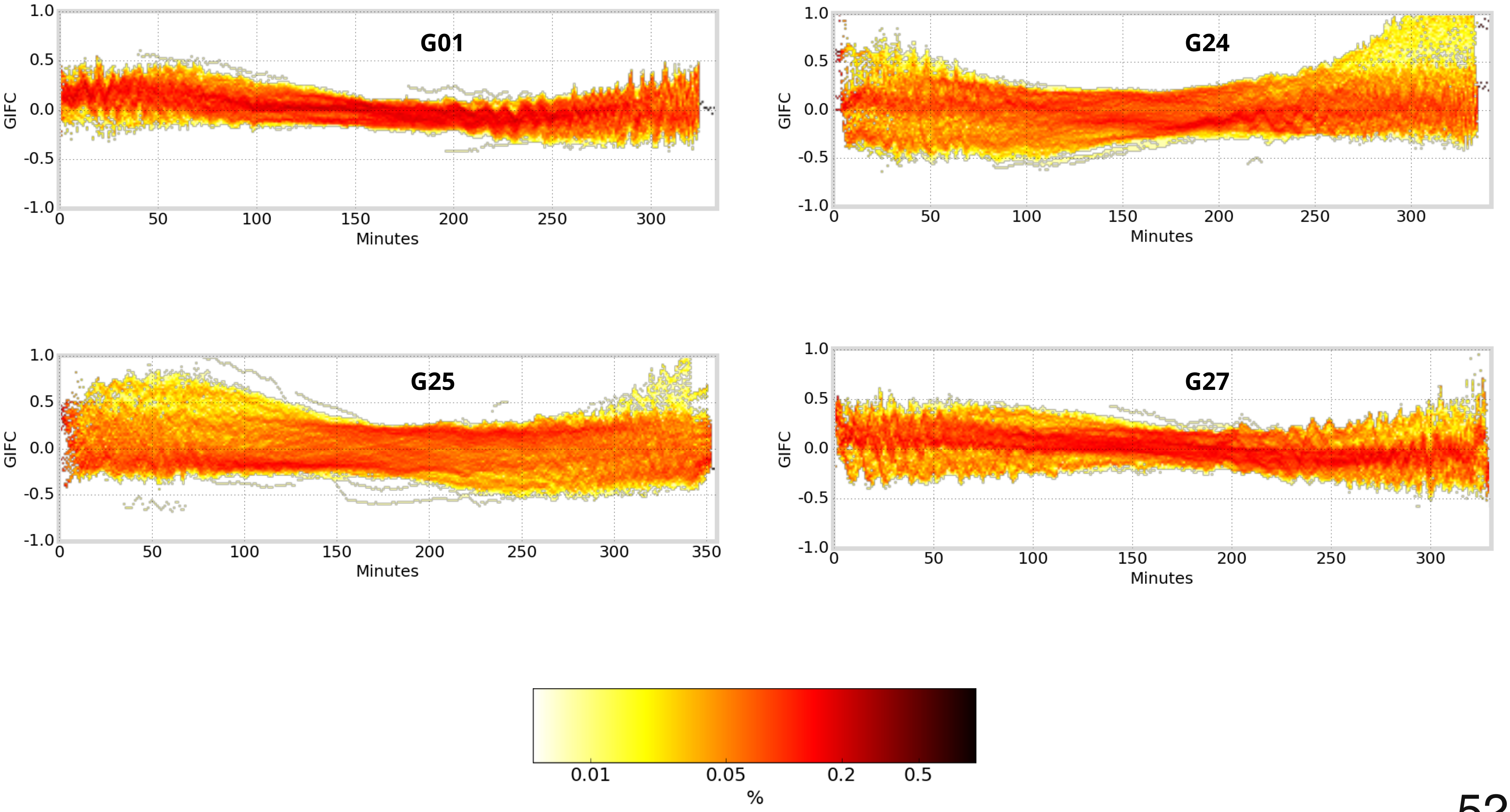


antenna phase effects

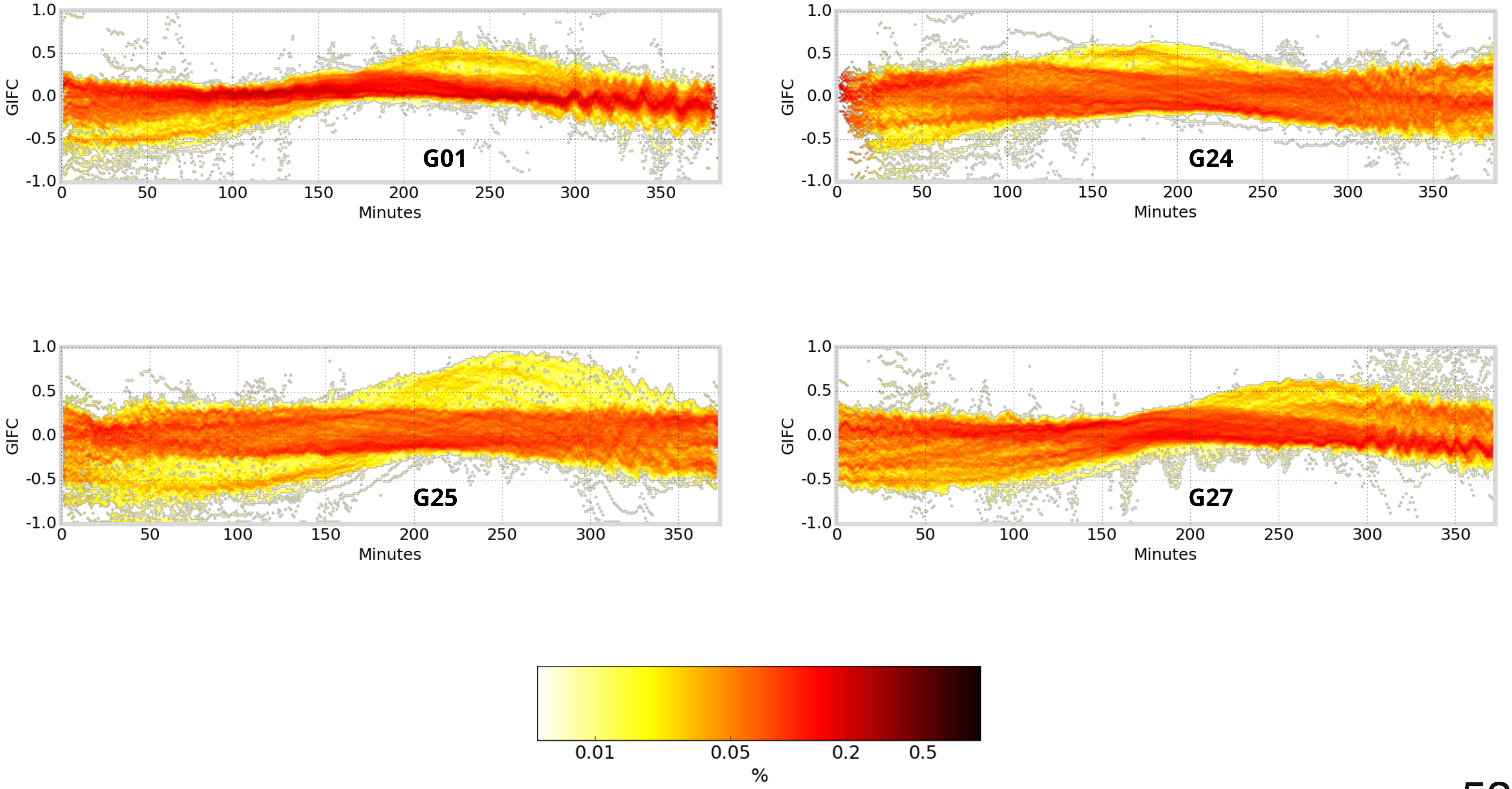
relative displacement of satellite antenna phase centers changes as satellite moves / rotates

GIFC Heatmap

Alaska

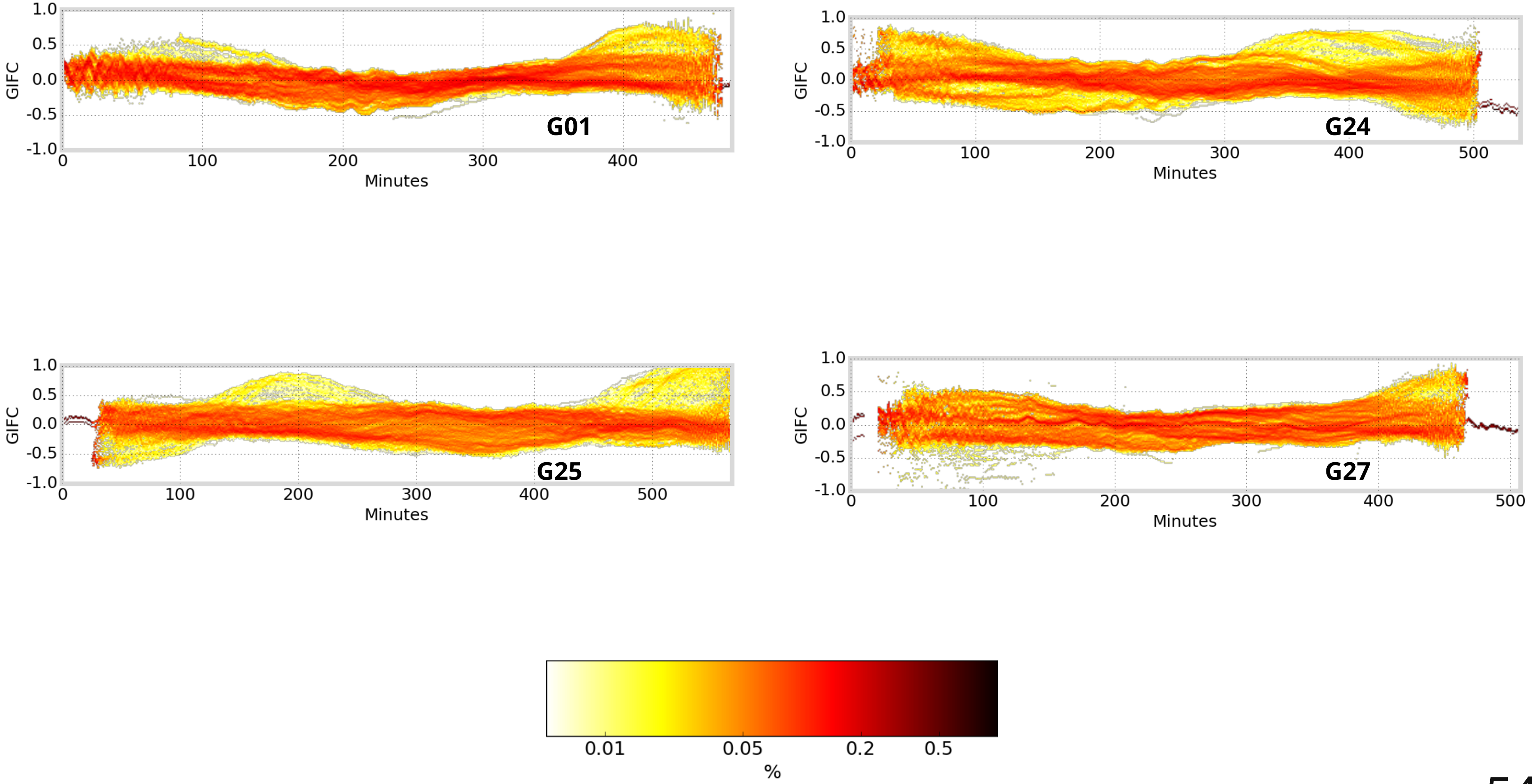


GIFC Heatmap
Hong Kong



GIFC Heatmap

Peru



GIFC Deviations and TEC Residual Error Estimates

GIFC percentile deviations computed over aggregate of all data

Percentile	Overall
50	0.11
75	0.19
90	0.21

GIFC deviation multiplied by scaling factor

	Percentile		$TEC_{L1,L5}$	$TEC_{L1,L2}$	$TEC_{L2,L5}$
$\frac{\langle \mathbf{C}_{GIFC} \| \mathbf{C}_{TEC} \rangle}{\|\|\mathbf{C}_{GIFC}\|\|^2}$	50		0.033	0.077	0.520
	75		0.058	0.132	0.897
	90		0.064	0.146	0.992

	Percentile	$TEC_{L1,L2,L5}$	$TEC_{L1,L5}$	$TEC_{L1,L2}$	$TEC_{L2,L5}$
$\frac{\|\|\mathbf{C}_{TEC}\|\|}{\|\|\mathbf{C}_{GIFC}\|\|}$	50	0.091	0.097	0.119	0.528
	75	0.158	0.168	0.206	0.911
	90	0.175	0.186	0.228	1.007

[TECu]

Recap

simple phase observation model (ignore biases)

⬇

methodology for **choosing optimal linear estimators**

⬇

triple-frequency

↙ ↘

optimal? $\text{TEC}_{1,2,3}$

GIFC

↙

characterize / relate to R_{TEC}

Discussion

$\text{TEC}_{\text{L1,L2}}$ residual error on order of 0.2 TECu

- includes large-scale trend → for TID detection, trend is removed and precision improves
- [4] cites 0.05 TECu fluctuations to be above noise for TID detection

Improvement of $\text{TEC}_{\text{L1,L2,L5}}$ over $\text{TEC}_{\text{L1,L5}}$ seems minor:

$$||\mathbf{C}_{\text{TEC}_{\text{L1,L2,L5}}}|| = 10.314$$
$$||\mathbf{C}_{\text{TEC}_{\text{L1,L5}}}|| = 10.977$$
$$||\mathbf{C}_{\text{TEC}_{\text{L1,L2}}}|| = 13.460$$

...but it does eliminate GIFC component in TEC residual error

Next Steps

Use characterization of GIFC to address residual errors

Is the GIFC trend variation due to satellite antenna phase effects?

Can we obtain and apply better information on residual error components R_i*?*

Can we use GIFC to validate mitigation techniques for multipath, higher-order ionosphere terms, ray-path bending, antenna phase effects?

→ enable TEC estimation from **low-elevation satellites**

Acknowledgements

This research was supported by the Air Force Research Laboratory and NASA.

Thank you to my advisor, committee members, and all who provided me with feedback and criticism!

References

[1] Saito A., S. Fukao, and S. Miyazaki, *High resolution mapping of TEC perturbations with the GSI GPS network over Japan*, Geophys. Res. Lett., 25, 3079-3082, 1998.

[2] Bourne, Harrison W. *An algorithm for accurate ionospheric total electron content and receiver bias estimation using GPS measurements*. Diss. Colorado State University. Libraries, 2016.

[3] Spits, Justine. *Total Electron Content reconstruction using triple frequency GNSS signals*. Diss. Université de Liège, Belgique, 2012.

[4] M. Nishioka, A. Saito, and T. Tsugawa, “Occurrence characteristics of plasma bubble derived from global ground-based GPS receiver networks,” Journal of Geophysical