9

Linear Momentum and Collisions

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q9.1 No. Impulse, $F\Delta t$, depends on the force and the time for which it is applied.

Q9.2 The momentum doubles since it is proportional to the speed. The kinetic energy quadruples, since it is proportional to the speed-squared.

Q9.3 The momenta of two particles will only be the same if the masses of the particles of the same.

Q9.4 (a) It does not carry force, for if it did, it could accelerate itself.

(b) It cannot deliver more kinetic energy than it possesses. This would violate the law of energy conservation.

(c) It can deliver more momentum in a collision than it possesses in its flight, by bouncing from the object it strikes.

Q9.5 Provided there is some form of potential energy in the system, the parts of an isolated system can move if the system is initially at rest. Consider two air-track gliders on a horizontal track. If you compress a spring between them and then tie them together with a string, it is possible for the system to start out at rest. If you then burn the string, the potential energy stored in the spring will be converted into kinetic energy of the gliders.

Q9.6 No. Only in a precise head-on collision with momenta with equal magnitudes and opposite directions can both objects wind up at rest. Yes. Assume that ball 2, originally at rest, is struck squarely by an equal-mass ball 1. Then ball 2 will take off with the velocity of ball 1, leaving ball 1 at rest.

Q9.7 Interestingly, mutual gravitation brings the ball and the Earth together. As the ball moves downward, the Earth moves upward, although with an acceleration 10^{25} times smaller than that of the ball. The two objects meet, rebound, and separate. Momentum of the ball-Earth system is conserved.

Q9.8 (a) Linear momentum is conserved since there are no external forces acting on the system.

(b) Kinetic energy is not conserved because the chemical potential energy initially in the explosive is converted into kinetic energy of the pieces of the bomb.

Q9.9 Momentum conservation is not violated if we make our system include the Earth along with the clay. When the clay receives an impulse backwards, the Earth receives the same size impulse forwards. The resulting acceleration of the Earth due to this impulse is significantly smaller than the acceleration of the clay, but the planet absorbs all of the momentum that the clay loses.

Q9.10 Momentum conservation is not violated if we choose as our system the planet along with you. When you receive an impulse forward, the Earth receives the same size impulse backwards. The resulting acceleration of the Earth due to this impulse is significantly smaller than your acceleration forward, but the planet's backward momentum is equal in magnitude to your forward momentum.

Q9.11 As a ball rolls down an incline, the Earth receives an impulse of the same size and in the opposite direction as that of the ball. If you consider the Earth-ball system, momentum conservation is not violated.

Q9.12 Suppose car and truck move along the same line. If one vehicle overtakes the other, the faster-moving one loses more energy than the slower one gains. In a head-on collision, if the speed of the truck is less than $\frac{m_T + 3m_c}{3m_T + m_c}$ times the speed of the car, the car will lose more energy.

Q9.13 The rifle has a much lower speed than the bullet and much less kinetic energy. The butt distributes the recoil force over an area much larger than that of the bullet.

Q9.14 His impact speed is determined by the acceleration of gravity and the distance of fall, in $v_f^2 = v_i^2 - 2g(0 - y_i)$. The force exerted by the pad depends also on the unknown stiffness of the pad.

Q9.15 The product of the mass flow rate and velocity of the water determines the force the firefighters must exert.

Q9.16 The sheet stretches and pulls the two students toward each other. These effects are larger for a faster-moving egg. The time over which the egg stops is extended so that the force stopping it is never too large.

Q9.17 (c) In this case, the impulse on the Frisbee is largest. According to Newton's third law, the impulse on the skater and thus the final speed of the skater will also be largest.

Q9.18 Usually but not necessarily. In a one-dimensional collision between two identical particles with the same initial speed, the kinetic energy of the particles will not change.

Q9.19 g downward.

Q9.20 As one finger slides towards the center, the normal force exerted by the sliding finger on the ruler increases. At some point, this normal force will increase enough so that static friction between the sliding finger and the ruler will stop their relative motion. At this moment the other finger starts sliding along the ruler towards the center. This process repeats until the fingers meet at the center of the ruler.

Q9.21 The planet is in motion around the sun, and thus has momentum and kinetic energy of its own. The spacecraft is directed to cross the planet's orbit behind it, so that the planet's gravity has a component pulling forward on the spacecraft. Since this is an elastic collision, and the velocity of the planet remains nearly unchanged, the probe must both increase speed and change direction for both momentum and kinetic energy to be conserved.

Q9.22 No—an external force of gravity acts on the moon. Yes, because its speed is constant.

Q9.23 The impulse given to the egg is the same regardless of how it stops. If you increase the impact time by dropping the egg onto foam, you will decrease the impact force.

Q9.24 Yes. A boomerang, a kitchen stool.

Q9.25 The center of mass of the balls is in free fall, moving up and then down with the acceleration due to gravity, during the 40% of the time when the juggler's hands are empty. During the 60% of the time when the juggler is engaged in catching and tossing, the center of mass must accelerate up with a somewhat smaller average acceleration. The center of mass moves around in a little circle, making three revolutions for every one revolution that one ball makes. Letting T represent the time for one cycle and F_g the weight of one ball, we have $F_J 0.60T = 3F_g T$ and $F_J = 5F_g$. The average force exerted by the juggler is five times the weight of one ball.

Q9.26 In empty space, the center of mass of a rocket-plus-fuel system does not accelerate during a burn, because no outside force acts on this system. According to the text's 'basic expression for rocket propulsion,' the change in speed of the rocket body will be larger than the speed of the exhaust relative to the rocket, if the final mass is less than 37% of the original mass.

Q9.27 The gun recoiled.

Q9.28 Inflate a balloon and release it. The air escaping from the balloon gives the balloon an impulse.

Q9.29 There was a time when the English favored position (a), the Germans position (b), and the French position (c). A Frenchman, Jean D'Alembert, is most responsible for showing that each theory is consistent with the others. All are equally correct. Each is useful for giving a mathematically simple solution for some problems.

SOLUTIONS TO PROBLEMS

Section 9.1 **Linear Momentum and Its Conservation**

P9.1 $m = 3.00 \text{ kg}$, $\quad \mathbf{v} = \left(3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}}\right) \text{ m/s}$

(a) $\mathbf{p} = m\mathbf{v} = \left(9.00\hat{\mathbf{i}} - 12.0\hat{\mathbf{j}}\right) \text{ kg}\cdot\text{m/s}$

Thus, $\boxed{p_x = 9.00 \text{ kg}\cdot\text{m/s}}$

and $\boxed{p_y = -12.0 \text{ kg}\cdot\text{m/s}}$

(b) $p = \sqrt{p_x^2 + p_y^2} = \sqrt{(9.00)^2 + (12.0)^2} = \boxed{15.0 \text{ kg}\cdot\text{m/s}}$

$\theta = \tan^{-1}\left(\dfrac{p_y}{p_x}\right) = \tan^{-1}(-1.33) = \boxed{307°}$

P9.2 (a) At maximum height $\mathbf{v}=0$, so $\mathbf{p}=\boxed{0}$.

(b) Its original kinetic energy is its constant total energy,

$$K_i=\frac{1}{2}mv_i^2=\frac{1}{2}(0.100)\text{kg}(15.0\text{ m/s})^2=11.2\text{ J}.$$

At the top all of this energy is gravitational. Halfway up, one-half of it is gravitational and the other half is kinetic:

$$K=5.62\text{ J}=\frac{1}{2}(0.100\text{ kg})v^2$$
$$v=\sqrt{\frac{2\times5.62\text{ J}}{0.100\text{ kg}}}=10.6\text{ m/s}$$

Then $\mathbf{p}=m\mathbf{v}=(0.100\text{ kg})(10.6\text{ m/s})\hat{\mathbf{j}}$

$$\mathbf{p}=\boxed{1.06\text{ kg}\cdot\text{m/s}\,\hat{\mathbf{j}}}.$$

P9.3 I have mass 85.0 kg and can jump to raise my center of gravity 25.0 cm. I leave the ground with speed given by

$$v_f^2-v_i^2=2a(x_f-x_i):\qquad 0-v_i^2=2(-9.80\text{ m/s}^2)(0.250\text{ m})$$
$$v_i=2.20\text{ m/s}$$

Total momentum of the system of the Earth and me is conserved as I push the earth down and myself up:

$$0=(5.98\times10^{24}\text{ kg})v_e+(85.0\text{ kg})(2.20\text{ m/s})$$
$$v_e\sim\boxed{10^{-23}\text{ m/s}}$$

P9.4 (a) For the system of two blocks $\Delta p=0$,

or $p_i=p_f$

Therefore, $0=Mv_m+(3M)(2.00\text{ m/s})$

Solving gives $v_m=\boxed{-6.00\text{ m/s}}$ (motion toward the left).

(b) $\frac{1}{2}kx^2=\frac{1}{2}Mv_M^2+\frac{1}{2}(3M)v_{3M}^2=\boxed{8.40\text{ J}}$

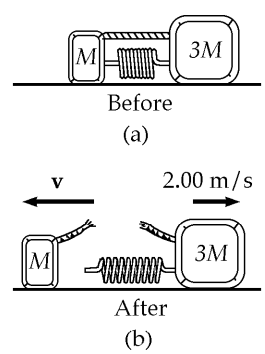

FIG. P9.4

P9.5 (a) The momentum is $p = mv$, so $v = \frac{p}{m}$ and the kinetic energy is $K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \boxed{\frac{p^2}{2m}}$.

(b) $K = \frac{1}{2}mv^2$ implies $v = \sqrt{\frac{2K}{m}}$, so $p = mv = m\sqrt{\frac{2K}{m}} = \boxed{\sqrt{2mK}}$.

Section 9.2 **Impulse and Momentum**

***P9.6** From the impulse-momentum theorem, $\overline{F}(\Delta t) = \Delta p = mv_f - mv_i$, the average force required to hold onto the child is

$$\overline{F} = \frac{m(v_f - v_i)}{(\Delta t)} = \frac{(12\text{ kg})(0 - 60\text{ mi/h})}{0.050\text{ s} - 0}\left(\frac{1\text{ m/s}}{2.237\text{ mi/h}}\right) = -6.44 \times 10^3\text{ N}.$$

Therefore, the magnitude of the needed retarding force is $\boxed{6.44 \times 10^3\text{ N}}$, or 1 400 lb. A person cannot exert a force of this magnitude and a safety device should be used.

P9.7 (a) $I = \int F dt =$ area under curve

$$I = \frac{1}{2}(1.50 \times 10^{-3}\text{ s})(18\,000\text{ N}) = \boxed{13.5\text{ N}\cdot\text{s}}$$

(b) $F = \frac{13.5\text{ N}\cdot\text{s}}{1.50 \times 10^{-3}\text{ s}} = \boxed{9.00\text{ kN}}$

(c) From the graph, we see that $F_{\max} = \boxed{18.0\text{ kN}}$

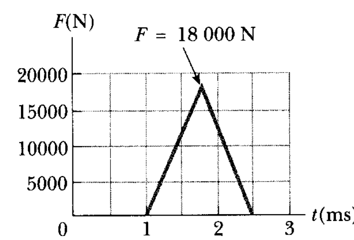

FIG. P9.7

***P9.8** The impact speed is given by $\frac{1}{2}mv_1^2 = mgy_1$. The rebound speed is given by $mgy_2 = \frac{1}{2}mv_2^2$. The impulse of the floor is the change in momentum,

$$\begin{aligned} mv_2\text{ up} - mv_1\text{ down} &= m(v_2 + v_1)\text{ up} \\ &= m\left(\sqrt{2gh_2} + \sqrt{2gh_1}\right)\text{ up} \\ &= 0.15\text{ kg}\sqrt{2(9.8\text{ m/s}^2)}\left(\sqrt{0.960\text{ m}} + \sqrt{1.25\text{ m}}\right)\text{ up} \\ &= \boxed{1.39\text{ kg}\cdot\text{m/s upward}} \end{aligned}$$

P9.9

$$\Delta \mathbf{p} = \mathbf{F}\Delta t$$

$$\Delta p_y = m\left(v_{fy} - v_{iy}\right) = m(v\cos 60.0°) - mv\cos 60.0° = 0$$

$$\Delta p_x = m(-v\sin 60.0° - v\sin 60.0°) = -2mv\sin 60.0°$$

$$= -2(3.00\ \text{kg})(10.0\ \text{m/s})(0.866)$$

$$= -52.0\ \text{kg}\cdot\text{m/s}$$

$$F_{\text{ave}} = \frac{\Delta p_x}{\Delta t} = \frac{-52.0\ \text{kg}\cdot\text{m/s}}{0.200\ \text{s}} = \boxed{-260\ \text{N}}$$

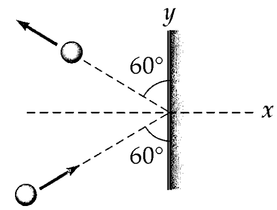

FIG. P9.9

P9.10 Assume the initial direction of the ball in the $-x$ direction.

(a) Impulse, $\mathbf{I} = \Delta\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i = (0.060\,0)(40.0)\hat{\mathbf{i}} - (0.060\,0)(50.0)\left(-\hat{\mathbf{i}}\right) = \boxed{5.40\hat{\mathbf{i}}\ \text{N}\cdot\text{s}}$

(b) Work $= K_f - K_i = \frac{1}{2}(0.060\,0)\left[(40.0)^2 - (50.0)^2\right] = \boxed{-27.0\ \text{J}}$

P9.11 Take x-axis toward the pitcher

(a) $p_{ix} + I_x = p_{fx}$: $(0.200\ \text{kg})(15.0\ \text{m/s})(-\cos 45.0°) + I_x = (0.200\ \text{kg})(40.0\ \text{m/s})\cos 30.0°$

$I_x = 9.05\ \text{N}\cdot\text{s}$

$p_{iy} + I_y = p_{fy}$: $(0.200\ \text{kg})(15.0\ \text{m/s})(-\sin 45.0°) + I_y = (0.200\ \text{kg})(40.0\ \text{m/s})\sin 30.0°$

$$\mathbf{I} = \boxed{\left(9.05\hat{\mathbf{i}} + 6.12\hat{\mathbf{j}}\right)\ \text{N}\cdot\text{s}}$$

(b) $\mathbf{I} = \frac{1}{2}(0 + \mathbf{F}_m)(4.00\ \text{ms}) + \mathbf{F}_m(20.0\ \text{ms}) + \frac{1}{2}\mathbf{F}_m(4.00\ \text{ms})$

$\mathbf{F}_m \times 24.0 \times 10^{-3}\ \text{s} = \left(9.05\hat{\mathbf{i}} + 6.12\hat{\mathbf{j}}\right)\ \text{N}\cdot\text{s}$

$\mathbf{F}_m = \boxed{\left(377\hat{\mathbf{i}} + 255\hat{\mathbf{j}}\right)\ \text{N}}$

P9.12 If the diver starts from rest and drops vertically into the water, the velocity just before impact is found from

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}mv_{\text{impact}}^2 + 0 = 0 + mgh \Rightarrow v_{\text{impact}} = \sqrt{2gh}$$

With the diver at rest after an impact time of Δt, the average force during impact is given by

$$\overline{F} = \frac{m\left(0 - v_{\text{impact}}\right)}{\Delta t} = \frac{-m\sqrt{2gh}}{\Delta t} \text{ or } \overline{F} = \frac{m\sqrt{2gh}}{\Delta t} \text{ (directed upward).}$$

Assuming a mass of 55 kg and an impact time of $\approx 1.0\ \text{s}$, the magnitude of this average force is

$$\left|\overline{F}\right| = \frac{(55\ \text{kg})\sqrt{2\left(9.8\ \text{m/s}^2\right)(10\ \text{m})}}{1.0\ \text{s}} = 770\ \text{N, or } \boxed{\sim 10^3\ \text{N}}.$$

P9.13 The force exerted on the water by the hose is

$$F = \frac{\Delta p_{\text{water}}}{\Delta t} = \frac{mv_f - mv_i}{\Delta t} = \frac{(0.600\text{ kg})(25.0\text{ m/s}) - 0}{1.00\text{ s}} = \boxed{15.0\text{ N}}.$$

According to Newton's third law, the water exerts a force of equal magnitude back on the hose. Thus, the gardener must apply a 15.0 N force (in the direction of the velocity of the exiting water stream) to hold the hose stationary.

***P9.14** (a) Energy is conserved for the spring-mass system:

$$K_i + U_{si} = K_f + U_{sf}: \qquad 0 + \frac{1}{2}kx^2 = \frac{1}{2}mv^2 + 0$$

$$v = x\sqrt{\frac{k}{m}}$$

(b) From the equation, a $\boxed{\text{smaller}}$ value of m makes $v = x\sqrt{\frac{k}{m}}$ larger.

(c) $I = \left|\mathbf{p}_f - \mathbf{p}_i\right| = mv_f = 0 = mx\sqrt{\frac{k}{m}} = x\sqrt{km}$

(d) From the equation, a $\boxed{\text{larger}}$ value of m makes $I = x\sqrt{km}$ larger.

(e) For the glider, $W = K_f - K_i = \frac{1}{2}mv^2 - 0 = \frac{1}{2}kx^2$

The mass makes $\boxed{\text{no difference}}$ to the work.

Section 9.3 **Collisions in One Dimension**

P9.15 $(200\text{ g})(55.0\text{ m/s}) = (46.0\text{ g})v + (200\text{ g})(40.0\text{ m/s})$

$v = \boxed{65.2\text{ m/s}}$

***P9.16** $(m_1v_1 + m_2v_2)_i = (m_1v_1 + m_2v_2)_f$

$22.5\text{ g}(35\text{ m/s}) + 300\text{ g}(-2.5\text{ m/s}) = 22.5\text{ g}v_{1f} + 0$

$$v_{1f} = \frac{37.5\text{ g}\cdot\text{m/s}}{22.5\text{ g}} = \boxed{1.67\text{ m/s}}$$

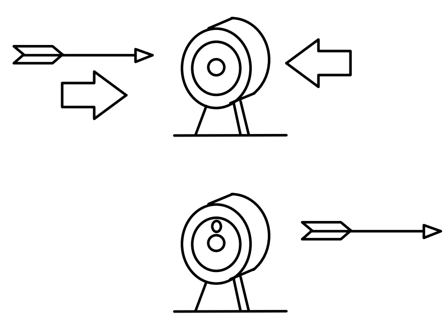

FIG. P9.16

P9.17 Momentum is conserved

$$\left(10.0\times10^{-3}\ \text{kg}\right)v=(5.01\ \text{kg})(0.600\ \text{m/s})$$

$$v=\boxed{301\ \text{m/s}}$$

P9.18 (a) $mv_{1i}+3mv_{2i}=4mv_f$ where $m=2.50\times10^4$ kg

$$v_f=\frac{4.00+3(2.00)}{4}=\boxed{2.50\ \text{m/s}}$$

(b) $$K_f-K_i=\frac{1}{2}(4m)v_f^2-\left[\frac{1}{2}mv_{1i}^2+\frac{1}{2}(3m)v_{2i}^2\right]=\left(2.50\times10^4\right)(12.5-8.00-6.00)=\boxed{-3.75\times10^4\ \text{J}}$$

P9.19 (a) The internal forces exerted by the actor do not change the total momentum of the system of the four cars and the movie actor

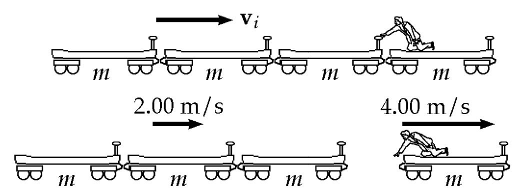

FIG. P9.19

$$(4m)v_i=(3m)(2.00\ \text{m/s})+m(4.00\ \text{m/s})$$

$$v_i=\frac{6.00\ \text{m/s}+4.00\ \text{m/s}}{4}=\boxed{2.50\ \text{m/s}}$$

(b) $$W_{\text{actor}}=K_f-K_i=\frac{1}{2}\left[(3m)(2.00\ \text{m/s})^2+m(4.00\ \text{m/s})^2\right]-\frac{1}{2}(4\ \text{m})(2.50\ \text{m/s})^2$$

$$W_{\text{actor}}=\frac{\left(2.50\times10^4\ \text{kg}\right)}{2}(12.0+16.0-25.0)(\text{m/s})^2=\boxed{37.5\ \text{kJ}}$$

(c) The event considered here is the time reversal of the perfectly inelastic collision in the previous problem. The same momentum conservation equation describes both processes.

P9.20 v_1, speed of m_1 at B before collision.

$$\frac{1}{2}m_1v_1^2=m_1gh$$

$$v_1=\sqrt{2(9.80)(5.00)}=9.90\ \text{m/s}$$

v_{1f}, speed of m_1 at B just after collision.

$$v_{1f}=\frac{m_1-m_2}{m_1+m_2}v_1=-\frac{1}{3}(9.90)\ \text{m/s}=-3.30\ \text{m/s}$$

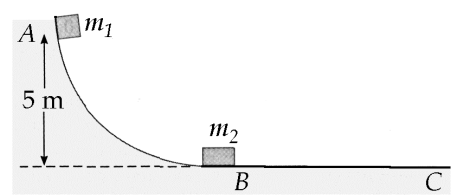

FIG. P9.20

At the highest point (after collision)

$$m_1gh_{\max}=\frac{1}{2}m_1(-3.30)^2 \qquad h_{\max}=\frac{(-3.30\ \text{m/s})^2}{2\left(9.80\ \text{m/s}^2\right)}=\boxed{0.556\ \text{m}}$$

P9.21 (a), (b) Let v_g and v_p be the velocity of the girl and the plank relative to the ice surface. Then we may say that $v_g - v_p$ is the velocity of the girl relative to the plank, so that

$$v_g - v_p = 1.50 \qquad (1)$$

But also we must have $m_g v_g + m_p v_p = 0$, since total momentum of the girl-plank system is zero relative to the ice surface. Therefore

$$45.0v_g + 150v_p = 0\text{, or } v_g = -3.33v_p$$

Putting this into the equation (1) above gives

$$-3.33v_p - v_p = 1.50 \text{ or } v_p = \boxed{-0.346 \text{ m/s}}$$

Then $v_g = -3.33(-0.346) = \boxed{1.15 \text{ m/s}}$

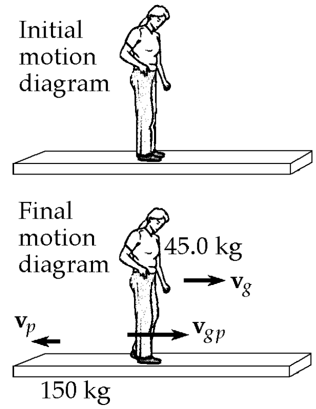

FIG. P9.21

***P9.22** For the car-truck-driver-driver system, momentum is conserved:

$\mathbf{p}_{1i} + \mathbf{p}_{2i} = \mathbf{p}_{1f} + \mathbf{p}_{2f}$: $\quad (4\,000 \text{ kg})(8 \text{ m/s})\hat{\mathbf{i}} + (800 \text{ kg})(8 \text{ m/s})(-\hat{\mathbf{i}}) = (4\,800 \text{ kg})v_f\hat{\mathbf{i}}$

$$v_f = \frac{25\,600 \text{ kg}\cdot\text{m/s}}{4\,800 \text{ kg}} = 5.33 \text{ m/s}$$

For the driver of the truck, the impulse-momentum theorem is

$\mathbf{F}\Delta t = \mathbf{p}_f - \mathbf{p}_i$: $\quad \mathbf{F}(0.120 \text{ s}) = (80 \text{ kg})(5.33 \text{ m/s})\hat{\mathbf{i}} - (80 \text{ kg})(8 \text{ m/s})\hat{\mathbf{i}}$

$$\mathbf{F} = \boxed{1.78 \times 10^3 \text{ N}(-\hat{\mathbf{i}}) \text{ on the truck driver}}$$

For the driver of the car, $\quad \mathbf{F}(0.120 \text{ s}) = (80 \text{ kg})(5.33 \text{ m/s})\hat{\mathbf{i}} - (80 \text{ kg})(8 \text{ m/s})(-\hat{\mathbf{i}})$

$$\mathbf{F} = \boxed{8.89 \times 10^3 \text{ N}\hat{\mathbf{i}} \text{ on the car driver}}\text{, 5 times larger.}$$

P9.23 (a) According to the Example in the chapter text, the fraction of total kinetic energy transferred to the moderator is

$$f_2 = \frac{4m_1m_2}{(m_1 + m_2)^2}$$

where m_2 is the moderator nucleus and in this case, $m_2 = 12m_1$

$$f_2 = \frac{4m_1(12m_1)}{(13m_1)^2} = \frac{48}{169} = \boxed{0.284 \text{ or } 28.4\%}$$

of the neutron energy is transferred to the carbon nucleus.

(b) $K_C = (0.284)(1.6 \times 10^{-13} \text{ J}) = \boxed{4.54 \times 10^{-14} \text{ J}}$

$K_n = (0.716)(1.6 \times 10^{-13} \text{ J}) = \boxed{1.15 \times 10^{-13} \text{ J}}$

P9.24 Energy is conserved for the bob-Earth system between bottom and top of swing. At the top the stiff rod is in compression and the bob nearly at rest.

$$K_i + U_i = K_f + U_f: \qquad \frac{1}{2}Mv_b^2 + 0 = 0 + Mg2\ell$$
$$v_b^2 = g4\ell \text{ so } v_b = 2\sqrt{g\ell}$$

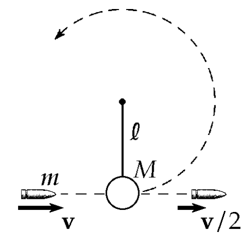

FIG. P9.24

Momentum of the bob-bullet system is conserved in the collision:

$$mv = m\frac{v}{2} + M\left(2\sqrt{g\ell}\right) \qquad \boxed{v = \frac{4M}{m}\sqrt{g\ell}}$$

P9.25 At impact, momentum of the clay-block system is conserved, so:

$$mv_1 = (m_1 + m_2)v_2$$

After impact, the change in kinetic energy of the clay-block-surface system is equal to the increase in internal energy:

$$\frac{1}{2}(m_1 + m_2)v_2^2 = f_f d = \mu(m_1 + m_2)gd$$
$$\frac{1}{2}(0.112 \text{ kg})v_2^2 = 0.650(0.112 \text{ kg})\left(9.80 \text{ m/s}^2\right)(7.50 \text{ m})$$
$$v_2^2 = 95.6 \text{ m}^2/\text{s}^2 \qquad v_2 = 9.77 \text{ m/s}$$
$$\left(12.0\times10^{-3} \text{ kg}\right)v_1 = (0.112 \text{ kg})(9.77 \text{ m/s}) \qquad v_1 = \boxed{91.2 \text{ m/s}}$$

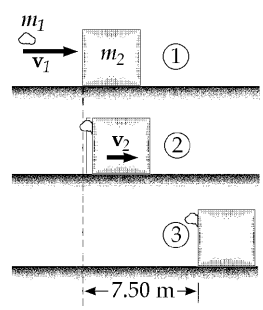

FIG. P9.25

P9.26 We assume equal firing speeds v and equal forces F required for the two bullets to push wood fibers apart. These equal forces act backward on the two bullets.

For the first, $\quad K_i + \Delta E_{\text{mech}} = K_f \quad \frac{1}{2}\left(7.00\times10^{-3} \text{ kg}\right)v^2 - F\left(8.00\times10^{-2} \text{ m}\right) = 0$

For the second, $\quad p_i = p_f \quad \left(7.00\times10^{-3} \text{ kg}\right)v = (1.014 \text{ kg})v_f$

$$v_f = \frac{\left(7.00\times10^{-3}\right)v}{1.014}$$

Again, $\quad K_i + \Delta E_{\text{mech}} = K_f: \quad \frac{1}{2}\left(7.00\times10^{-3} \text{ kg}\right)v^2 - Fd = \frac{1}{2}(1.014 \text{ kg})v_f^2$

Substituting for v_f, $\quad \frac{1}{2}\left(7.00\times10^{-3} \text{ kg}\right)v^2 - Fd = \frac{1}{2}(1.014 \text{ kg})\left(\frac{7.00\times10^{-3}\,v}{1.014}\right)^2$

$$Fd = \frac{1}{2}\left(7.00\times10^{-3}\right)v^2 - \frac{1}{2}\frac{\left(7.00\times10^{-3}\right)^2}{1.014}v^2$$

Substituting for v, $\quad Fd = F\left(8.00\times10^{-2} \text{ m}\right)\left(1 - \frac{7.00\times10^{-3}}{1.014}\right) \qquad d = \boxed{7.94 \text{ cm}}$

***P9.27** (a) Using conservation of momentum, $\left(\sum \mathbf{p}\right)_{\text{after}} = \left(\sum \mathbf{p}\right)_{\text{before}}$, gives

$$\left[(4.0+10+3.0)\text{ kg}\right]v = (4.0\text{ kg})(5.0\text{ m/s}) + (10\text{ kg})(3.0\text{ m/s}) + (3.0\text{ kg})(-4.0\text{ m/s}).$$

Therefore, $v = +2.24\text{ m/s}$, or $\boxed{2.24\text{ m/s toward the right}}$.

(b) $\boxed{\text{No}}$. For example, if the 10-kg and 3.0-kg mass were to stick together first, they would move with a speed given by solving

$$(13\text{ kg})v_1 = (10\text{ kg})(3.0\text{ m/s}) + (3.0\text{ kg})(-4.0\text{ m/s}),\text{ or } v_1 = +1.38\text{ m/s}.$$

Then when this 13 kg combined mass collides with the 4.0 kg mass, we have

$$(17\text{ kg})v = (13\text{ kg})(1.38\text{ m/s}) + (4.0\text{ kg})(5.0\text{ m/s}),\text{ and } v = +2.24\text{ m/s}$$

just as in part (a). Coupling order makes no difference.

Section 9.4 **Two-Dimensional Collisions**

P9.28 (a) First, we conserve momentum for the system of two football players in the x direction (the direction of travel of the fullback).

$$(90.0\text{ kg})(5.00\text{ m/s}) + 0 = (185\text{ kg})V\cos\theta$$

where θ is the angle between the direction of the final velocity V and the x axis. We find

$$V\cos\theta = 2.43\text{ m/s} \qquad (1)$$

Now consider conservation of momentum of the system in the y direction (the direction of travel of the opponent).

$$(95.0\text{ kg})(3.00\text{ m/s}) + 0 = (185\text{ kg})(V\sin\theta)$$

which gives, $V\sin\theta = 1.54\text{ m/s}$ (2)

Divide equation (2) by (1) $\tan\theta = \dfrac{1.54}{2.43} = 0.633$

From which $\boxed{\theta = 32.3°}$

Then, either (1) or (2) gives $V = \boxed{2.88\text{ m/s}}$

(b) $K_i = \frac{1}{2}(90.0\text{ kg})(5.00\text{ m/s})^2 + \frac{1}{2}(95.0\text{ kg})(3.00\text{ m/s})^2 = 1.55\times10^3\text{ J}$

$K_f = \frac{1}{2}(185\text{ kg})(2.88\text{ m/s})^2 = 7.67\times10^2\text{ J}$

Thus, the kinetic energy lost is $\boxed{\text{783 J into internal energy.}}$

P9.29 $p_{xf} = p_{xi}$

$$mv_O \cos 37.0° + mv_Y \cos 53.0° = m(5.00 \text{ m/s})$$

$$0.799v_O + 0.602v_Y = 5.00 \text{ m/s} \qquad (1)$$

$$p_{yf} = p_{yi}$$

$$mv_O \sin 37.0° - mv_Y \sin 53.0° = 0$$

$$0.602v_O = 0.799v_Y \qquad (2)$$

Solving (1) and (2) simultaneously,

$$\boxed{v_O = 3.99 \text{ m/s}} \text{ and } \boxed{v_Y = 3.01 \text{ m/s}}.$$

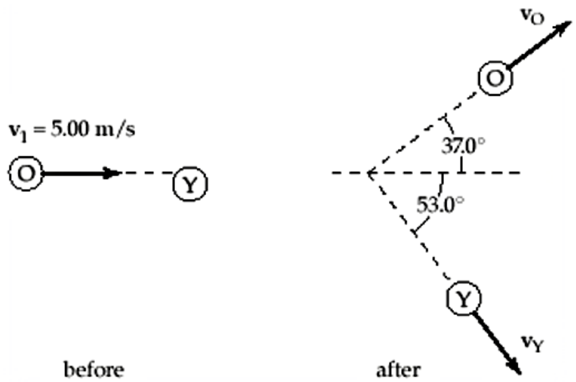

FIG. P9.29

P9.30 $p_{xf} = p_{xi}$: $\quad mv_O \cos\theta + mv_Y \cos(90.0° - \theta) = mv_i$

$$v_O \cos\theta + v_Y \sin\theta = v_i \qquad (1)$$

$p_{yf} = p_{yi}$: $\quad mv_O \sin\theta - mv_Y \sin(90.0° - \theta) = 0$

$$v_O \sin\theta = v_Y \cos\theta \qquad (2)$$

From equation (2),

$$v_O = v_Y\left(\frac{\cos\theta}{\sin\theta}\right) \qquad (3)$$

Substituting into equation (1),

$$v_Y\left(\frac{\cos^2\theta}{\sin\theta}\right) + v_Y \sin\theta = v_i$$

so $\quad v_Y\left(\cos^2\theta + \sin^2\theta\right) = v_i \sin\theta$, and $\boxed{v_Y = v_i \sin\theta}$.

Then, from equation (3), $\boxed{v_O = v_i \cos\theta}$.

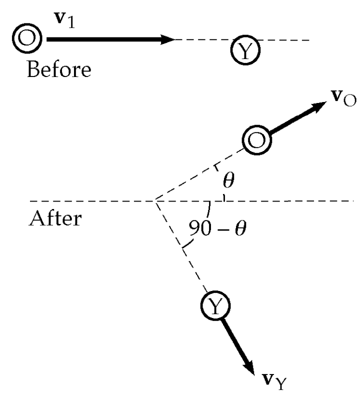

FIG. P9.30

We did not need to write down an equation expressing conservation of mechanical energy. In the problem situation, the requirement of perpendicular final velocities is equivalent to the condition of elasticity.

P9.31 The initial momentum of the system is 0. Thus,

$$(1.20m)v_{Bi} = m(10.0 \text{ m/s})$$

and $v_{Bi} = 8.33 \text{ m/s}$

$$K_i = \frac{1}{2}m(10.0 \text{ m/s})^2 + \frac{1}{2}(1.20m)(8.33 \text{ m/s})^2 = \frac{1}{2}m\left(183 \text{ m}^2/\text{s}^2\right)$$

$$K_f = \frac{1}{2}m(v_G)^2 + \frac{1}{2}(1.20m)(v_B)^2 = \frac{1}{2}\left(\frac{1}{2}m\left(183 \text{ m}^2/\text{s}^2\right)\right)$$

or $v_G^2 + 1.20v_B^2 = 91.7 \text{ m}^2/\text{s}^2$ (1)

From conservation of momentum,

$$mv_G = (1.20m)v_B$$

or $v_G = 1.20v_B$ (2)

Solving (1) and (2) simultaneously, we find

$\boxed{v_G = 7.07 \text{ m/s}}$ (speed of green puck after collision)

and $\boxed{v_B = 5.89 \text{ m/s}}$ (speed of blue puck after collision)

P9.32 We use conservation of momentum for the system of two vehicles for both northward and eastward components.

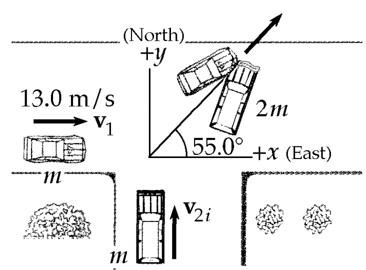

FIG. P9.32

For the eastward direction:

$$M(13.0 \text{ m/s}) = 2MV_f \cos 55.0°$$

For the northward direction:

$$Mv_{2i} = 2MV_f \sin 55.0°$$

Divide the northward equation by the eastward equation to find:

$$v_{2i} = (13.0 \text{ m/s})\tan 55.0° = 18.6 \text{ m/s} = \boxed{41.5 \text{ mi/h}}$$

Thus, the driver of the north bound car was untruthful.

P9.33 By conservation of momentum for the system of the two billiard balls (with all masses equal),

$$5.00\ \text{m/s} + 0 = (4.33\ \text{m/s})\cos 30.0° + v_{2fx}$$
$$v_{2fx} = 1.25\ \text{m/s}$$
$$0 = (4.33\ \text{m/s})\sin 30.0° + v_{2fy}$$
$$v_{2fy} = -2.16\ \text{m/s}$$
$$\mathbf{v}_{2f} = \boxed{2.50\ \text{m/s at } -60.0°}$$

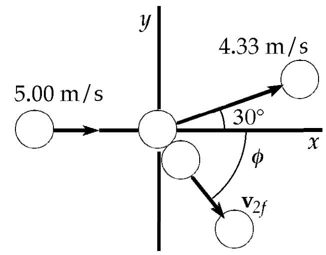

FIG. P9.33

Note that we did not need to use the fact that the collision is perfectly elastic.

P9.34 (a) $\mathbf{p}_i = \mathbf{p}_f$ so $p_{xi} = p_{xf}$

and $p_{yi} = p_{yf}$

$$mv_i = mv\cos\theta + mv\cos\phi \quad (1)$$
$$0 = mv\sin\theta + mv\sin\phi \quad (2)$$

From (2), $\sin\theta = -\sin\phi$

so $\theta = -\phi$

Furthermore, energy conservation for the system of two protons requires

$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

so $\boxed{v = \frac{v_i}{\sqrt{2}}}$

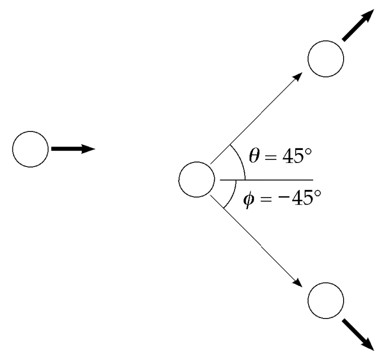

FIG. P9.34

(b) Hence, (1) gives $v_i = \frac{2v_i\cos\theta}{\sqrt{2}}$ $\theta = \boxed{45.0°}$ $\phi = \boxed{-45.0°}$

P9.35 $m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = (m_1 + m_2)\mathbf{v}_f$: $3.00(5.00)\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}} = 5.00\mathbf{v}$

$$\mathbf{v} = \boxed{\left(3.00\hat{\mathbf{i}} - 1.20\hat{\mathbf{j}}\right)\ \text{m/s}}$$

P9.36 x-component of momentum for the system of the two objects:

$p_{1ix} + p_{2ix} = p_{1fx} + p_{2fx}$: $-mv_i + 3mv_i = 0 + 3mv_{2x}$

y-component of momentum of the system: $0 + 0 = -mv_{1y} + 3mv_{2y}$

by conservation of energy of the system: $+\frac{1}{2}mv_i^2 + \frac{1}{2}3mv_i^2 = \frac{1}{2}mv_{1y}^2 + \frac{1}{2}3m\left(v_{2x}^2 + v_{2y}^2\right)$

we have $v_{2x} = \frac{2v_i}{3}$

also $v_{1y} = 3v_{2y}$

So the energy equation becomes $4v_i^2 = 9v_{2y}^2 + \frac{4v_i^2}{3} + 3v_{2y}^2$

$$\frac{8v_i^2}{3} = 12v_{2y}^2$$

or $v_{2y} = \frac{\sqrt{2}v_i}{3}$

continued on next page

(a) The object of mass m has final speed $v_{1y} = 3v_{2y} = \boxed{\sqrt{2}v_i}$

and the object of mass 3 m moves at $\sqrt{v_{2x}^2 + v_{2y}^2} = \sqrt{\frac{4v_i^2}{9} + \frac{2v_i^2}{9}}$

$$\sqrt{v_{2x}^2 + v_{2y}^2} = \boxed{\sqrt{\frac{2}{3}}v_i}$$

(b) $\theta = \tan^{-1}\left(\frac{v_{2y}}{v_{2x}}\right)$ $\qquad \theta = \tan^{-1}\left(\frac{\sqrt{2}v_i}{3}\frac{3}{2v_i}\right) = \boxed{35.3°}$

P9.37 $m_0 = 17.0 \times 10^{-27}$ kg $\qquad \mathbf{v}_i = 0$ (the parent nucleus)

$m_1 = 5.00 \times 10^{-27}$ kg $\qquad \mathbf{v}_1 = 6.00 \times 10^6 \hat{\mathbf{j}}$ m/s

$m_2 = 8.40 \times 10^{-27}$ kg $\qquad \mathbf{v}_2 = 4.00 \times 10^6 \hat{\mathbf{i}}$ m/s

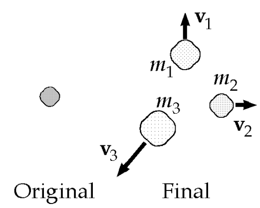

FIG. P9.37

(a) $m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + m_3\mathbf{v}_3 = 0$

where $m_3 = m_0 - m_1 - m_2 = 3.60 \times 10^{-27}$ kg

$$\left(5.00 \times 10^{-27}\right)\left(6.00 \times 10^6 \hat{\mathbf{j}}\right) + \left(8.40 \times 10^{-27}\right)\left(4.00 \times 10^6 \hat{\mathbf{i}}\right) + \left(3.60 \times 10^{-27}\right)\mathbf{v}_3 = 0$$

$$\mathbf{v}_3 = \boxed{\left(-9.33 \times 10^6 \hat{\mathbf{i}} - 8.33 \times 10^6 \hat{\mathbf{j}}\right) \text{ m/s}}$$

(b) $E = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2$

$$E = \frac{1}{2}\left[\left(5.00 \times 10^{-27}\right)\left(6.00 \times 10^6\right)^2 + \left(8.40 \times 10^{-27}\right)\left(4.00 \times 10^6\right)^2 + \left(3.60 \times 10^{-27}\right)\left(12.5 \times 10^6\right)^2\right]$$

$$\boxed{E = 4.39 \times 10^{-13} \text{ J}}$$

Section 9.5 **The Center of Mass**

P9.38 The x-coordinate of the center of mass is

$$x_{CM} = \frac{\sum m_i x_i}{\sum m_i} = \frac{0 + 0 + 0 + 0}{(2.00 \text{ kg} + 3.00 \text{ kg} + 2.50 \text{ kg} + 4.00 \text{ kg})}$$

$$\boxed{x_{CM} = 0}$$

and the y-coordinate of the center of mass is

$$y_{CM} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(2.00 \text{ kg})(3.00 \text{ m}) + (3.00 \text{ kg})(2.50 \text{ m}) + (2.50 \text{ kg})(0) + (4.00 \text{ kg})(-0.500 \text{ m})}{2.00 \text{ kg} + 3.00 \text{ kg} + 2.50 \text{ kg} + 4.00 \text{ kg}}$$

$$\boxed{y_{CM} = 1.00 \text{ m}}$$

P9.39 Take x-axis starting from the oxygen nucleus and pointing toward the middle of the **V**.

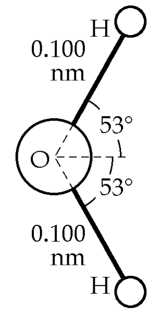

FIG. P9.39

Then $\boxed{y_{CM} = 0}$

and $x_{CM} = \dfrac{\sum m_i x_i}{\sum m_i} =$

$$x_{CM} = \frac{0 + 1.008\text{ u}(0.100\text{ nm})\cos 53.0° + 1.008\text{ u}(0.100\text{ nm})\cos 53.0°}{15.999\text{ u} + 1.008\text{ u} + 1.008\text{ u}}$$

$$\boxed{x_{CM} = 0.006\,73\text{ nm from the oxygen nucleus}}$$

***P9.40** Let the x axis start at the Earth's center and point toward the Moon.

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{5.98 \times 10^{24}\text{ kg }0 + 7.36 \times 10^{22}\text{ kg}\left(3.84 \times 10^{8}\text{ m}\right)}{6.05 \times 10^{24}\text{ kg}}$$

$$= \boxed{4.67 \times 10^{6}\text{ m from the Earth's center}}$$

The center of mass is within the Earth, which has radius 6.37×10^{6} m.

P9.41 Let A_1 represent the area of the bottom row of squares, A_2 the middle square, and A_3 the top pair.

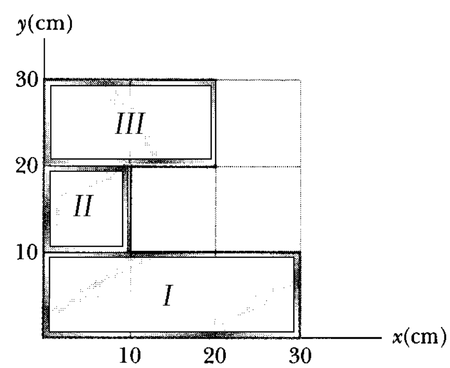

FIG. P9.41

$A = A_1 + A_2 + A_3$

$M = M_1 + M_2 + M_3$

$\dfrac{M_1}{A_1} = \dfrac{M}{A}$

$A_1 = 300\text{ cm}^2,\ A_2 = 100\text{ cm}^2,\ A_3 = 200\text{ cm}^2,\ A = 600\text{ cm}^2$

$$M_1 = M\left(\frac{A_1}{A}\right) = \frac{300\text{ cm}^2}{600\text{ cm}^2}M = \frac{M}{2}$$

$$M_2 = M\left(\frac{A_2}{A}\right) = \frac{100\text{ cm}^2}{600\text{ cm}^2}M = \frac{M}{6}$$

$$M_3 = M\left(\frac{A_3}{A}\right) = \frac{200\text{ cm}^2}{600\text{ cm}^2}M = \frac{M}{3}$$

$$x_{CM} = \frac{x_1 M_1 + x_2 M_2 + x_3 M_3}{M} = \frac{15.0\text{ cm}\left(\frac{1}{2}M\right) + 5.00\text{ cm}\left(\frac{1}{6}M\right) + 10.0\text{ cm}\left(\frac{1}{3}M\right)}{M}$$

$$x_{CM} = \boxed{11.7\text{ cm}}$$

$$y_{CM} = \frac{\frac{1}{2}M(5.00\text{ cm}) + \frac{1}{6}M(15.0\text{ cm}) + \left(\frac{1}{3}M\right)(25.0\text{ cm})}{M} = 13.3\text{ cm}$$

$$y_{CM} = \boxed{13.3\text{ cm}}$$

***P9.42** (a) Represent the height of a particle of mass dm within the object as y. Its contribution to the gravitational energy of the object-Earth system is $(dm)gy$. The total gravitational energy is $U_g = \int_{\text{all mass}} gy\,dm = g\int y\,dm$. For the center of mass we have $y_{CM} = \frac{1}{M}\int y\,dm$, so $U_g = gMy_{CM}$.

(b) The volume of the ramp is $\frac{1}{2}(3.6\text{ m})(15.7\text{ m})(64.8\text{ m}) = 1.83\times10^3\text{ m}^3$. Its mass is $\rho V = \left(3\,800\text{ kg/m}^3\right)\left(1.83\times10^3\text{ m}^3\right) = 6.96\times10^6\text{ kg}$. Its center of mass is above its base by one-third of its height, $y_{CM} = \frac{1}{3}15.7\text{ m} = 5.23\text{ m}$. Then

$$U_g = Mgy_{CM} = 6.96\times10^6\text{ kg}\left(9.8\text{ m/s}^2\right)5.23\text{ m} = \boxed{3.57\times10^8\text{ J}}.$$

P9.43 (a)

$$M = \int_0^{0.300\text{ m}} \lambda dx = \int_0^{0.300\text{ m}} \left[50.0\text{ g/m} + 20.0x\text{ g/m}^2\right]dx$$

$$M = \left[50.0x\text{ g/m} + 10.0x^2\text{ g/m}^2\right]_0^{0.300\text{ m}} = \boxed{15.9\text{ g}}$$

(b)

$$x_{CM} = \frac{\int_{\text{all mass}} xdm}{M} = \frac{1}{M}\int_0^{0.300\text{ m}} \lambda\,xdx = \frac{1}{M}\int_0^{0.300\text{ m}}\left[50.0x\text{ g/m} + 20.0x^2\text{ g/m}^2\right]dx$$

$$x_{CM} = \frac{1}{15.9\text{ g}}\left[25.0x^2\text{ g/m} + \frac{20x^3\text{ g/m}^2}{3}\right]_0^{0.300\text{ m}} = \boxed{0.153\text{ m}}$$

***P9.44** Take the origin at the center of curvature. We have $L = \frac{1}{4}2\pi r$, $r = \frac{2L}{\pi}$. An incremental bit of the rod at angle θ from the x axis has mass given by $\frac{dm}{rd\theta} = \frac{M}{L}$, $dm = \frac{Mr}{L}d\theta$ where we have used the definition of radian measure. Now

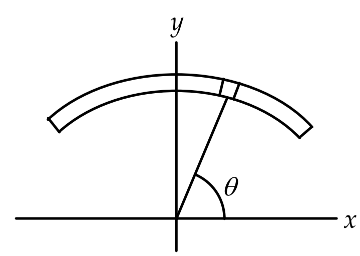

FIG. P9.44

$$y_{CM} = \frac{1}{M}\int_{\text{all mass}} y\,dm = \frac{1}{M}\int_{\theta=45°}^{135°} r\sin\theta\frac{Mr}{L}d\theta = \frac{r^2}{L}\int_{45°}^{135°}\sin\theta\,d\theta$$

$$= \left(\frac{2L}{\pi}\right)^2\frac{1}{L}(-\cos\theta)\Big|_{45°}^{135°} = \frac{4L}{\pi^2}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{4\sqrt{2}L}{\pi^2}$$

The top of the bar is above the origin by $r = \frac{2L}{\pi}$, so the center of mass is below the middle of the bar by $\frac{2L}{\pi} - \frac{4\sqrt{2}L}{\pi^2} = \frac{2}{\pi}\left(1 - \frac{2\sqrt{2}}{\pi}\right)L = \boxed{0.063\,5L}$.

Section 9.6 **Motion of a System of Particles**

P9.45 (a) $$\mathbf{v}_{CM} = \frac{\sum m_i \mathbf{v}_i}{M} = \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{M}$$

$$= \frac{(2.00\text{ kg})(2.00\hat{\mathbf{i}}\text{ m/s} - 3.00\hat{\mathbf{j}}\text{ m/s}) + (3.00\text{ kg})(1.00\hat{\mathbf{i}}\text{ m/s} + 6.00\hat{\mathbf{j}}\text{ m/s})}{5.00\text{ kg}}$$

$$\mathbf{v}_{CM} = \boxed{(1.40\hat{\mathbf{i}} + 2.40\hat{\mathbf{j}})\text{ m/s}}$$

(b) $$\mathbf{p} = M\mathbf{v}_{CM} = (5.00\text{ kg})(1.40\hat{\mathbf{i}} + 2.40\hat{\mathbf{j}})\text{ m/s} = \boxed{(7.00\hat{\mathbf{i}} + 12.0\hat{\mathbf{j}})\text{ kg}\cdot\text{m/s}}$$

P9.46 (a) See figure to the right.

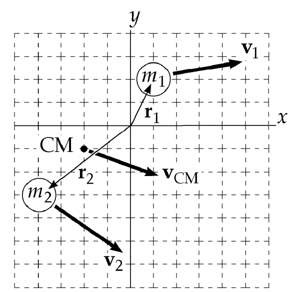

FIG. P9.46

(b) Using the definition of the position vector at the center of mass,

$$\mathbf{r}_{CM} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{m_1 + m_2}$$

$$\mathbf{r}_{CM} = \frac{(2.00\text{ kg})(1.00\text{ m, } 2.00\text{ m}) + (3.00\text{ kg})(-4.00\text{ m, } -3.00\text{ m})}{2.00\text{ kg} + 3.00\text{ kg}}$$

$$\mathbf{r}_{CM} = \boxed{(-2.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}})\text{ m}}$$

(c) The velocity of the center of mass is

$$\mathbf{v}_{CM} = \frac{\mathbf{P}}{M} = \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1 + m_2} = \frac{(2.00\text{ kg})(3.00\text{ m/s, } 0.50\text{ m/s}) + (3.00\text{ kg})(3.00\text{ m/s, } -2.00\text{ m/s})}{(2.00\text{ kg} + 3.00\text{ kg})}$$

$$\mathbf{v}_{CM} = \boxed{(3.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}})\text{ m/s}}$$

(d) The total linear momentum of the system can be calculated as $\mathbf{P} = M\mathbf{v}_{CM}$

or as $\mathbf{P} = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$

Either gives $\mathbf{P} = \boxed{(15.0\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}})\text{ kg}\cdot\text{m/s}}$

P9.47 Let $x =$ distance from shore to center of boat
$\ell =$ length of boat
$x' =$ distance boat moves as Juliet moves toward Romeo
The center of mass stays fixed.

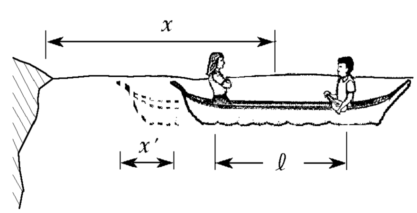

FIG. P9.47

Before: $$x_{CM} = \frac{\left[M_b x + M_J\left(x - \frac{\ell}{2}\right) + M_R\left(x + \frac{\ell}{2}\right)\right]}{(M_B + M_J + M_R)}$$

After: $$x_{CM} = \frac{\left[M_B(x - x') + M_J\left(x + \frac{\ell}{2} - x'\right) + M_R\left(x + \frac{\ell}{2} - x'\right)\right]}{(M_B + M_J + M_R)}$$

$$\ell\left(-\frac{55.0}{2} + \frac{77.0}{2}\right) = x'(-80.0 - 55.0 - 77.0) + \frac{\ell}{2}(55.0 + 77.0)$$

$$x' = \frac{55.0\ell}{212} = \frac{55.0(2.70)}{212} = \boxed{0.700\text{ m}}$$

P9.48 (a) Conservation of momentum for the two-ball system gives us:

$$0.200\text{ kg}(1.50\text{ m/s})+0.300\text{ kg}(-0.400\text{ m/s})=0.200\text{ kg }v_{1f}+0.300\text{ kg }v_{2f}$$

Relative velocity equation:

$$v_{2f}-v_{1f}=1.90\text{ m/s}$$

Then $0.300-0.120=0.200v_{1f}+0.300(1.90+v_{1f})$

$v_{1f}=-0.780\text{ m/s}$ $\qquad v_{2f}=1.12\text{ m/s}$

$\boxed{\mathbf{v}_{1f}=-0.780\hat{\mathbf{i}}\text{ m/s}}$ $\qquad \boxed{\mathbf{v}_{2f}=1.12\hat{\mathbf{i}}\text{ m/s}}$

(b) Before, $\mathbf{v}_{CM}=\dfrac{(0.200\text{ kg})(1.50\text{ m/s})\hat{\mathbf{i}}+(0.300\text{ kg})(-0.400\text{ m/s})\hat{\mathbf{i}}}{0.500\text{ kg}}$

$\boxed{\mathbf{v}_{CM}=(0.360\text{ m/s})\hat{\mathbf{i}}}$

Afterwards, the center of mass must move at the same velocity, as momentum of the system is conserved.

Section 9.7 **Rocket Propulsion**

P9.49 (a) Thrust $=\left|v_e\dfrac{dM}{dt}\right|$ $\qquad$ Thrust $=(2.60\times10^3\text{ m/s})(1.50\times10^4\text{ kg/s})=\boxed{3.90\times10^7\text{ N}}$

(b) $\sum F_y=\text{Thrust}-Mg=Ma$: $\qquad 3.90\times10^7-(3.00\times10^6)(9.80)=(3.00\times10^6)a$

$a=\boxed{3.20\text{ m/s}^2}$

***P9.50** (a) The fuel burns at a rate $\qquad \dfrac{dM}{dt}=\dfrac{12.7\text{ g}}{1.90\text{ s}}=6.68\times10^{-3}\text{ kg/s}$

Thrust $=v_e\dfrac{dM}{dt}$: $\qquad 5.26\text{ N}=v_e(6.68\times10^{-3}\text{ kg/s})$

$v_e=\boxed{787\text{ m/s}}$

(b) $v_f-v_i=v_e\ln\left(\dfrac{M_i}{M_f}\right)$: $\qquad v_f-0=(797\text{ m/s})\ln\left(\dfrac{53.5\text{ g}+25.5\text{ g}}{53.5\text{ g}+25.5\text{ g}-12.7\text{ g}}\right)$

$v_f=\boxed{138\text{ m/s}}$

P9.51 $v=v_e\ln\dfrac{M_i}{M_f}$

(a) $M_i=e^{v/v_e}M_f$ $\qquad M_i=e^5(3.00\times10^3\text{ kg})=4.45\times10^5\text{ kg}$

The mass of fuel and oxidizer is $\qquad \Delta M=M_i-M_f=(445-3.00)\times10^3\text{ kg}=\boxed{442\text{ metric tons}}$

(b) $\Delta M=e^2(3.00\text{ metric tons})-3.00\text{ metric tons}=\boxed{19.2\text{ metric tons}}$

Because of the exponential, a relatively small increase in fuel and/or engine efficiency causes a large change in the amount of fuel and oxidizer required.

P9.52 (a) From Equation 9.41, $v - 0 = v_e \ln\left(\frac{M_i}{M_f}\right) = -v_e \ln\left(\frac{M_f}{M_i}\right)$

Now, $M_f = M_i - kt$, so $v = -v_e \ln\left(\frac{M_i - kt}{M_i}\right) = -v_e \ln\left(1 - \frac{k}{M_i}t\right)$

With the definition, $T_p \equiv \frac{M_i}{k}$, this becomes

$$v(t) = \boxed{-v_e \ln\left(1 - \frac{t}{T_p}\right)}$$

(b) With $v_e = 1\,500$ m/s, and $T_p = 144$ s, $v = -(1\,500 \text{ m/s})\ln\left(1 - \frac{t}{144 \text{ s}}\right)$

$t(s)$	$v(\text{m/s})$
0	0
20	224
40	488
60	808
80	1220
100	1780
120	2690
132	3730

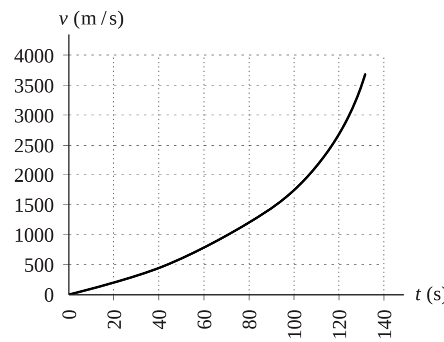

FIG. P9.52(b)

(c) $a(t) = \frac{dv}{dt} = \frac{d\left[-v_e \ln\left(1 - \frac{t}{T_p}\right)\right]}{dt} = -v_e\left(\frac{1}{1 - \frac{t}{T_p}}\right)\left(-\frac{1}{T_p}\right) = \left(\frac{v_e}{T_p}\right)\left(\frac{1}{1 - \frac{t}{T_p}}\right)$, or

$$a(t) = \boxed{\frac{v_e}{T_p - t}}$$

(d) With $v_e = 1\,500$ m/s, and $T_p = 144$ s, $a = \frac{1\,500 \text{ m/s}}{144 \text{ s} - t}$

$t(s)$	$a(\text{m/s}^2)$
0	10.4
20	12.1
40	14.4
60	17.9
80	23.4
100	34.1
120	62.5
132	125

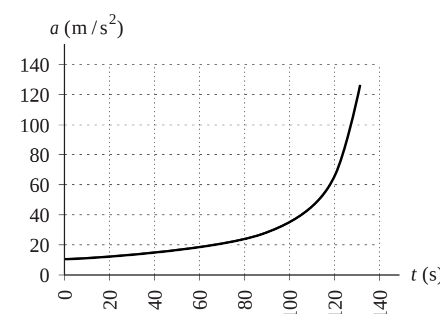

FIG. P9.52(d)

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(e)
$$x(t) = 0 + \int_0^t v\,dt = \int_0^t \left[-v_e \ln\left(1 - \frac{t}{T_p}\right) \right] dt = v_e T_p \int_0^t \ln\left[1 - \frac{t}{T_p}\right]\left(-\frac{dt}{T_p}\right)$$

$$x(t) = v_e T_p \left[\left(1 - \frac{t}{T_p}\right) \ln\left(1 - \frac{t}{T_p}\right) - \left(1 - \frac{t}{T_p}\right) \right]_0^t$$

$$x(t) = \boxed{v_e\left(T_p - t\right)\ln\left(1 - \frac{t}{T_p}\right) + v_e t}$$

(f) With $v_e = 1\,500\ \text{m/s} = 1.50\ \text{km/s}$, and $T_p = 144\ \text{s}$,

$$x = 1.50(144 - t)\ln\left(1 - \frac{t}{144}\right) + 1.50t$$

t(s)	x(km)
0	0
20	2.19
40	9.23
60	22.1
80	42.2
100	71.7
120	115
132	153

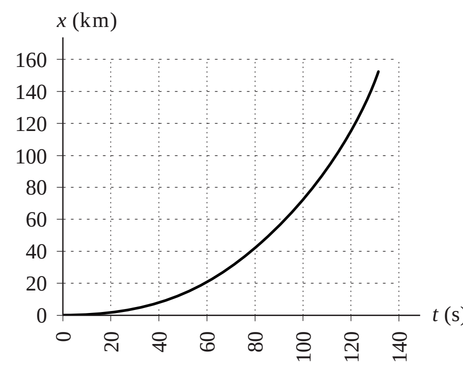

FIG. P9.52(f)

***P9.53** The thrust acting on the spacecraft is

$\sum F = ma$: $\sum F = (3\,500\ \text{kg})\left(2.50 \times 10^{-6}\right)\left(9.80\ \text{m/s}^2\right) = 8.58 \times 10^{-2}\ \text{N}$

thrust $= \left(\frac{dM}{dt}\right) v_e$: $8.58 \times 10^{-2}\ \text{N} = \left(\frac{\Delta M}{3\,600\ \text{s}}\right)(70\ \text{m/s})$

$\Delta M = \boxed{4.41\ \text{kg}}$

Additional Problems

P9.54 (a) When the spring is fully compressed, each cart moves with same velocity **v**. Apply conservation of momentum for the system of two gliders

$$\mathbf{p}_i = \mathbf{p}_f: \qquad m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = (m_1 + m_2)\mathbf{v} \qquad \boxed{\mathbf{v} = \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1 + m_2}}$$

(b) Only conservative forces act, therefore $\Delta E = 0$. $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}kx_m^2$

Substitute for v from (a) and solve for x_m.

$$x_m^2 = \frac{(m_1 + m_2)m_1v_1^2 + (m_1 + m_2)m_2v_2^2 - (m_1v_1)^2 - (m_2v_2)^2 - 2m_1m_2v_1v_2}{k(m_1 + m_2)}$$

$$x_m = \sqrt{\frac{m_1m_2\left(v_1^2 + v_2^2 - 2v_1v_2\right)}{k(m_1 + m_2)}} = \boxed{(v_1 - v_2)\sqrt{\frac{m_1m_2}{k(m_1 + m_2)}}}$$

(c) $m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f}$

Conservation of momentum: $m_1\left(\mathbf{v}_1 - \mathbf{v}_{1f}\right) = m_2\left(\mathbf{v}_{2f} - \mathbf{v}_2\right)$ (1)

Conservation of energy: $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$

which simplifies to: $m_1\left(v_1^2 - v_{1f}^2\right) = m_2\left(v_{2f}^2 - v_2^2\right)$

Factoring gives $m_1\left(\mathbf{v}_1 - \mathbf{v}_{1f}\right)\cdot\left(\mathbf{v}_1 + \mathbf{v}_{1f}\right) = m_2\left(\mathbf{v}_{2f} - \mathbf{v}_2\right)\cdot\left(\mathbf{v}_{2f} + \mathbf{v}_2\right)$

and with the use of the momentum equation (equation (1)),

this reduces to $\left(\mathbf{v}_1 + \mathbf{v}_{1f}\right) = \left(\mathbf{v}_{2f} + \mathbf{v}_2\right)$

or $\mathbf{v}_{1f} = \mathbf{v}_{2f} + \mathbf{v}_2 - \mathbf{v}_1$ (2)

Substituting equation (2) into equation (1) and simplifying yields:

$$\mathbf{v}_{2f} = \boxed{\left(\frac{2m_1}{m_1 + m_2}\right)\mathbf{v}_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\mathbf{v}_2}$$

Upon substitution of this expression for $\mathbf{v}_{2f}$ into equation 2, one finds

$$\mathbf{v}_{1f} = \boxed{\left(\frac{m_1 - m_2}{m_1 + m_2}\right)\mathbf{v}_1 + \left(\frac{2m_2}{m_1 + m_2}\right)\mathbf{v}_2}$$

Observe that these results are the same as Equations 9.20 and 9.21, which should have been expected since this is a perfectly elastic collision in one dimension.

P9.55 (a) $(60.0\text{ kg})4.00\text{ m/s} = (120 + 60.0)\text{ kg}v_f$

$\mathbf{v}_f = \boxed{1.33\text{ m/s}\hat{\mathbf{i}}}$

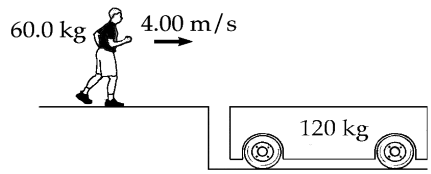

FIG. P9.55

(b) $\sum F_y = 0$: $\quad n - (60.0\text{ kg})9.80\text{ m/s}^2 = 0$

$f_k = \mu_k n = 0.400(588\text{ N}) = 235\text{ N}$

$\mathbf{f}_k = \boxed{-235\text{ N}\hat{\mathbf{i}}}$

(c) For the person, $p_i + I = p_f$

$mv_i + Ft = mv_f$

$(60.0\text{ kg})4.00\text{ m/s} - (235\text{ N})t = (60.0\text{ kg})1.33\text{ m/s}$

$t = \boxed{0.680\text{ s}}$

(d) person: $\quad m\mathbf{v}_f - m\mathbf{v}_i = 60.0\text{ kg}(1.33 - 4.00)\text{ m/s} = \boxed{-160\text{ N}\cdot\text{s}\hat{\mathbf{i}}}$

cart: $\quad 120\text{ kg}(1.33\text{ m/s}) - 0 = \boxed{+160\text{ N}\cdot\text{s}\hat{\mathbf{i}}}$

(e) $x_f - x_i = \frac{1}{2}(v_i + v_f)t = \frac{1}{2}[(4.00 + 1.33)\text{ m/s}]0.680\text{ s} = \boxed{1.81\text{ m}}$

(f) $x_f - x_i = \frac{1}{2}(v_i + v_f)t = \frac{1}{2}(0 + 1.33\text{ m/s})0.680\text{ s} = \boxed{0.454\text{ m}}$

(g) $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}60.0\text{ kg}(1.33\text{ m/s})^2 - \frac{1}{2}60.0\text{ kg}(4.00\text{ m/s})^2 = \boxed{-427\text{ J}}$

(h) $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}120.0\text{ kg}(1.33\text{ m/s})^2 - 0 = \boxed{107\text{ J}}$

(i) The force exerted by the person on the cart must equal in magnitude and opposite in direction to the force exerted by the cart on the person. The changes in momentum of the two objects must be equal in magnitude and must add to zero. Their changes in kinetic energy are different in magnitude and do not add to zero. The following represent two ways of thinking about 'why.' The distance the cart moves is different from the distance moved by the point of application of the friction force to the cart. The total change in mechanical energy for both objects together, -320 J, becomes $+320$ J of additional internal energy in this perfectly inelastic collision.

P9.56 The equation for the horizontal range of a projectile is $R = \frac{v_i^2 \sin 2\theta}{g}$. Thus, with $\theta = 45.0°$, the initial velocity is

$$v_i = \sqrt{Rg} = \sqrt{(200\text{ m})(9.80\text{ m/s}^2)} = 44.3\text{ m/s}$$

$$I = \overline{F}(\Delta t) = \Delta p = mv_i - 0$$

Therefore, the magnitude of the average force acting on the ball during the impact is:

$$\overline{F} = \frac{mv_i}{\Delta t} = \frac{(46.0 \times 10^{-3}\text{ kg})(44.3\text{ m/s})}{7.00 \times 10^{-3}\text{ s}} = \boxed{291\text{ N}}.$$

P9.57 We hope the momentum of the wrench provides enough recoil so that the astronaut can reach the ship before he loses life support! We might expect the elapsed time to be on the order of several minutes based on the description of the situation.
No external force acts on the system (astronaut plus wrench), so the total momentum is constant. Since the final momentum (wrench plus astronaut) must be zero, we have final momentum = initial momentum = 0.

$$m_{\text{wrench}}v_{\text{wrench}} + m_{\text{astronaut}}v_{\text{astronaut}} = 0$$

Thus $v_{\text{astronaut}} = -\dfrac{m_{\text{wrench}}v_{\text{wrench}}}{m_{\text{astronaut}}} = -\dfrac{(0.500\text{ kg})(20.0\text{ m/s})}{80.0\text{ kg}} = -0.125\text{ m/s}$

At this speed, the time to travel to the ship is

$$t = \frac{30.0\text{ m}}{0.125\text{ m/s}} = \boxed{240\text{ s}} = 4.00\text{ minutes}$$

The astronaut is fortunate that the wrench gave him sufficient momentum to return to the ship in a reasonable amount of time! In this problem, we were told that the astronaut was not drifting away from the ship when he threw the wrench. However, this is not quite possible since he did not encounter an external force that would reduce his velocity away from the ship (there is no air friction beyond earth's atmosphere). If this were a real-life situation, the astronaut would have to throw the wrench hard enough to overcome his momentum caused by his original push away from the ship.

P9.58 Using conservation of momentum from just before to just after the impact of the bullet with the block:

$$mv_i = (M+m)v_f$$

or $\quad v_i = \left(\dfrac{M+m}{m}\right)v_f \qquad (1)$

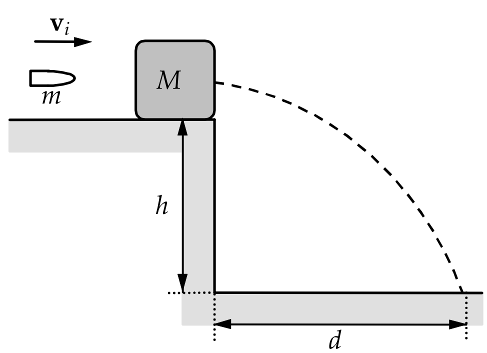

FIG. P9.58

The speed of the block and embedded bullet just after impact may be found using kinematic equations:

$$d = v_f t \text{ and } h = \frac{1}{2}gt^2$$

Thus, $t = \sqrt{\dfrac{2h}{g}}$ and $v_f = \dfrac{d}{t} = d\sqrt{\dfrac{g}{2h}} = \sqrt{\dfrac{gd^2}{2h}}$

Substituting into (1) from above gives $v_i = \boxed{\left(\dfrac{M+m}{m}\right)\sqrt{\dfrac{gd^2}{2h}}}$.

***P9.59** (a) Conservation of momentum:

$$0.5\text{ kg}\left(2\hat{\mathbf{i}}-3\hat{\mathbf{j}}+1\hat{\mathbf{k}}\right)\text{ m/s}+1.5\text{ kg}\left(-1\hat{\mathbf{i}}+2\hat{\mathbf{j}}-3\hat{\mathbf{k}}\right)\text{ m/s}$$

$$=0.5\text{ kg}\left(-1\hat{\mathbf{i}}+3\hat{\mathbf{j}}-8\hat{\mathbf{k}}\right)\text{ m/s}+1.5\text{ kg }\mathbf{v}_{2f}$$

$$\mathbf{v}_{2f}=\frac{\left(-0.5\hat{\mathbf{i}}+1.5\hat{\mathbf{j}}-4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}+\left(0.5\hat{\mathbf{i}}-1.5\hat{\mathbf{j}}+4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}}{1.5\text{ kg}}=\boxed{0}$$

The original kinetic energy is

$$\frac{1}{2}0.5\text{ kg}\left(2^2+3^2+1^2\right)\text{ m}^2/\text{s}^2+\frac{1}{2}1.5\text{ kg}\left(1^2+2^2+3^2\right)\text{ m}^2/\text{s}^2=14.0\text{ J}$$

The final kinetic energy is $\frac{1}{2}0.5\text{ kg}\left(1^2+3^2+8^2\right)\text{ m}^2/\text{s}^2+0=18.5\text{ J}$ different from the original energy so the collision is $\boxed{\text{inelastic}}$.

(b) We follow the same steps as in part (a):

$$\left(-0.5\hat{\mathbf{i}}+1.5\hat{\mathbf{j}}-4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}=0.5\text{ kg}\left(-0.25\hat{\mathbf{i}}+0.75\hat{\mathbf{j}}-2\hat{\mathbf{k}}\right)\text{ m/s}+1.5\text{ kg }\mathbf{v}_{2f}$$

$$\mathbf{v}_{2f}=\frac{\left(-0.5\hat{\mathbf{i}}+1.5\hat{\mathbf{j}}-4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}+\left(0.125\hat{\mathbf{i}}-0.375\hat{\mathbf{j}}+1\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}}{1.5\text{ kg}}$$

$$=\boxed{\left(-0.250\hat{\mathbf{i}}+0.750\hat{\mathbf{j}}-2.00\hat{\mathbf{k}}\right)\text{ m/s}}$$

We see $\mathbf{v}_{2f}=\mathbf{v}_{1f}$, so the collision is $\boxed{\text{perfectly inelastic}}$.

(c) Conservation of momentum:

$$\left(-0.5\hat{\mathbf{i}}+1.5\hat{\mathbf{j}}-4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}=0.5\text{ kg}\left(-1\hat{\mathbf{i}}+3\hat{\mathbf{j}}+a\hat{\mathbf{k}}\right)\text{ m/s}+1.5\text{ kg }\mathbf{v}_{2f}$$

$$\mathbf{v}_{2f}=\frac{\left(-0.5\hat{\mathbf{i}}+1.5\hat{\mathbf{j}}-4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}+\left(0.5\hat{\mathbf{i}}-1.5\hat{\mathbf{j}}-0.5a\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}}{1.5\text{ kg}}$$

$$=\boxed{(-2.67-0.333a)\hat{\mathbf{k}}\text{ m/s}}$$

Conservation of energy:

$$14.0\text{ J}=\frac{1}{2}0.5\text{ kg}\left(1^2+3^2+a^2\right)\text{ m}^2/\text{s}^2+\frac{1}{2}1.5\text{ kg}(2.67+0.333a)^2\text{ m}^2/\text{s}^2$$

$$=2.5\text{ J}+0.25a^2+5.33\text{ J}+1.33a+0.083\,3a^2$$

$$0=0.333a^2+1.33a-6.167$$

$$a=\frac{-1.33\pm\sqrt{1.33^2-4(0.333)(-6.167)}}{0.667}$$

$a=2.74$ or -6.74. Either value is possible.

$$\therefore\ \boxed{a=2.74},\quad \mathbf{v}_{2f}=\left(-2.67-0.333(2.74)\right)\hat{\mathbf{k}}\text{ m/s}=\boxed{-3.58\hat{\mathbf{k}}\text{ m/s}}$$

$$\therefore\ \boxed{a=-6.74},\quad \mathbf{v}_{2f}=\left(-2.67-0.333(-6.74)\right)\hat{\mathbf{k}}\text{ m/s}=\boxed{-0.419\hat{\mathbf{k}}\text{ m/s}}$$

P9.60 (a) The initial momentum of the system is zero, which remains constant throughout the motion. Therefore, when m_1 leaves the wedge, we must have

$$m_2 v_{\text{wedge}} + m_1 v_{\text{block}} = 0$$

or $$(3.00\text{ kg})v_{\text{wedge}} + (0.500\text{ kg})(+4.00\text{ m/s}) = 0$$

so $$v_{\text{wedge}} = \boxed{-0.667\text{ m/s}}$$

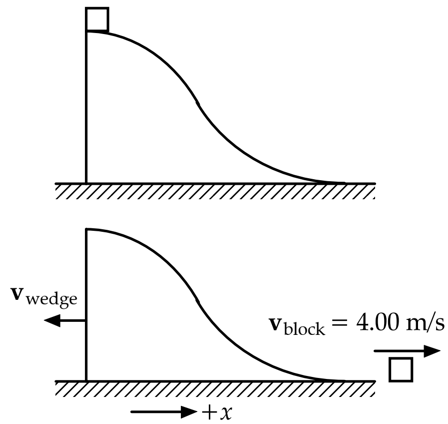

FIG. P9.60

(b) Using conservation of energy for the block-wedge-Earth system as the block slides down the smooth (frictionless) wedge, we have

$$\left[K_{\text{block}} + U_{\text{system}}\right]_i + \left[K_{\text{wedge}}\right]_i = \left[K_{\text{block}} + U_{\text{system}}\right]_f + \left[K_{\text{wedge}}\right]_f$$

or $$\left[0 + m_1 g h\right] + 0 = \left[\frac{1}{2}m_1(4.00)^2 + 0\right] + \frac{1}{2}m_2(-0.667)^2$$ which gives $\boxed{h = 0.952\text{ m}}$.

***P9.61** (a) Conservation of the x component of momentum for the cart-bucket-water system:

$$mv_i + 0 = (m + \rho V)v \qquad \boxed{v_i = \frac{m + \rho V}{m}v}$$

(b) Raindrops with zero x-component of momentum stop in the bucket and slow its horizontal motion. When they drip out, they carry with them horizontal momentum. Thus the cart slows with constant acceleration.

P9.62 Consider the motion of the firefighter during the three intervals:

(1) before, (2) during, and (3) after collision with the platform.

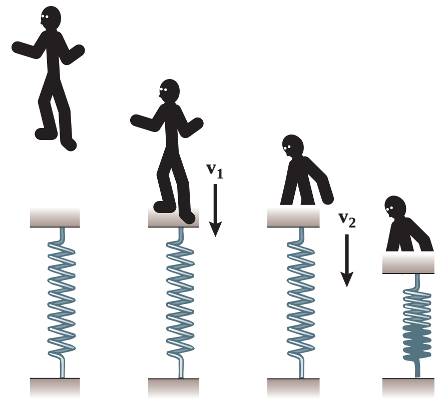

FIG. P9.62

(a) While falling a height of 4.00 m, his speed changes from $v_i = 0$ to v_1 as found from

$$\Delta E = \left(K_f + U_f\right) - \left(K_i - U_i\right), \text{ or}$$
$$K_f = \Delta E - U_f + K_i + U_i$$

When the initial position of the platform is taken as the zero level of gravitational potential, we have

$$\frac{1}{2}mv_1^2 = fh\cos(180°) - 0 + 0 + mgh$$

Solving for v_1 gives

$$v_1 = \sqrt{\frac{2(-fh + mgh)}{m}} = \sqrt{\frac{2(-300(4.00) + 75.0(9.80)4.00)}{75.0}} = \boxed{6.81\ \text{m/s}}$$

(b) During the inelastic collision, momentum is conserved; and if v_2 is the speed of the firefighter and platform just after collision, we have $mv_1 = (m+M)v_2$ or

$$v_2 = \frac{m_1 v_1}{m + M} = \frac{75.0(6.81)}{75.0 + 20.0} = 5.38\ \text{m/s}$$

Following the collision and again solving for the work done by non-conservative forces, using the distances as labeled in the figure, we have (with the zero level of gravitational potential at the initial position of the platform):

$$\Delta E = K_f + U_{fg} + U_{fs} - K_i - U_{ig} - U_{is}, \text{ or}$$
$$-fs = 0 + (m+M)g(-s) + \frac{1}{2}ks^2 - \frac{1}{2}(m+M)v^2 - 0 - 0$$

This results in a quadratic equation in s:

$$2\,000s^2 - (931)s + 300s - 1\,375 = 0 \text{ or } \boxed{s = 1.00\ \text{m}}$$

***P9.63** (a) Each object swings down according to

$$mgR = \frac{1}{2}mv_1^2 \qquad MgR = \frac{1}{2}Mv_1^2 \qquad v_1 = \sqrt{2gR}$$

The collision: $-mv_1 + Mv_1 = +(m+M)v_2$

$$v_2 = \frac{M-m}{M+m}v_1$$

Swinging up: $\frac{1}{2}(M+m)v_2^2 = (M+m)gR(1-\cos 35°)$

$$v_2 = \sqrt{2gR(1-\cos 35°)}$$

$$\sqrt{2gR(1-\cos 35°)}(M+m) = (M-m)\sqrt{2gR}$$

$$0.425M + 0.425m = M - m$$

$$1.425m = 0.575M$$

$$\boxed{\frac{m}{M} = 0.403}$$

(b) No change is required if the force is different. The nature of the forces within the system of colliding objects does not affect the total momentum of the system. With strong magnetic attraction, the heavier object will be moving somewhat faster and the lighter object faster still. Their extra kinetic energy will all be immediately converted into extra internal energy when the objects latch together. Momentum conservation guarantees that none of the extra kinetic energy remains after the objects join to make them swing higher.

P9.64 (a) Use conservation of the horizontal component of momentum for the system of the shell, the cannon, and the carriage, from just before to just after the cannon firing.

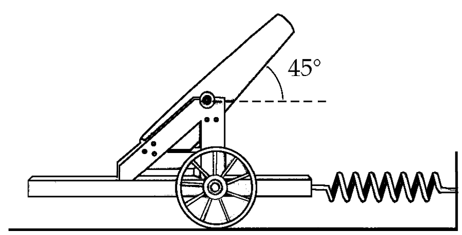

FIG. P9.64

$p_{xf} = p_{xi}$: $\quad m_{\text{shell}}v_{\text{shell}}\cos 45.0° + m_{\text{cannon}}v_{\text{recoil}} = 0$

$$(200)(125)\cos 45.0° + (5\,000)v_{\text{recoil}} = 0$$

or $\quad v_{\text{recoil}} = \boxed{-3.54 \text{ m/s}}$

(b) Use conservation of energy for the system of the cannon, the carriage, and the spring from right after the cannon is fired to the instant when the cannon comes to rest.

$K_f + U_{gf} + U_{sf} = K_i + U_{gi} + U_{si}$: $\quad 0 + 0 + \frac{1}{2}kx_{\max}^2 = \frac{1}{2}mv_{\text{recoil}}^2 + 0 + 0$

$$x_{\max} = \sqrt{\frac{mv_{\text{recoil}}^2}{k}} = \sqrt{\frac{(5\,000)(-3.54)^2}{2.00\times 10^4}} \text{ m} = \boxed{1.77 \text{ m}}$$

(c) $|F_{s,\max}| = kx_{\max}$ $\quad |F_{s,\max}| = (2.00\times 10^4 \text{ N/m})(1.77 \text{ m}) = \boxed{3.54\times 10^4 \text{ N}}$

(d) No. The rail exerts a vertical external force (the normal force) on the cannon and prevents it from recoiling vertically. Momentum is not conserved in the vertical direction. The spring does not have time to stretch during the cannon firing. Thus, no external horizontal force is exerted on the system (cannon, carriage, and shell) from just before to just after firing. Momentum of this system is conserved in the horizontal direction during this interval.

P9.65 (a) Utilizing conservation of momentum,

$$m_1 v_{1A} = (m_1 + m_2) v_B$$

$$v_{1A} = \frac{m_1 + m_2}{m_1}\sqrt{2gh}$$

$$v_{1A} \cong \boxed{6.29 \text{ m/s}}$$

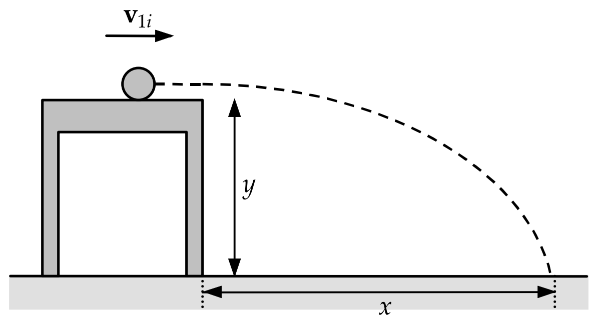

FIG. P9.65

(b) Utilizing the two equations,

$$\frac{1}{2}gt^2 = y \text{ and } x = v_{1A}t$$

we combine them to find

$$v_{1A} = \frac{x}{\sqrt{\frac{2y}{g}}}$$

From the data, $v_{1A} = \boxed{6.16 \text{ m/s}}$

Most of the 2% difference between the values for speed is accounted for by the uncertainty in the data, estimated as $\frac{0.01}{8.68} + \frac{0.1}{68.8} + \frac{1}{263} + \frac{1}{257} + \frac{0.1}{85.3} = 1.1\%$.

***P9.66** The ice cubes leave the track with speed determined by $mgy_i = \frac{1}{2}mv^2$;

$$v = \sqrt{2\left(9.8 \text{ m/s}^2\right)1.5 \text{ m}} = 5.42 \text{ m/s}.$$

Its speed at the apex of its trajectory is $5.42 \text{ m/s} \cos 40° = 4.15 \text{ m/s}$. For its collision with the wall we have

$$mv_i + F\Delta t = mv_f$$

$$0.005 \text{ kg } 4.15 \text{ m/s} + F\Delta t = 0.005 \text{ kg}\left(-\frac{1}{2}4.15 \text{ m/s}\right)$$

$$F\Delta t = -3.12 \times 10^{-2} \text{ kg} \cdot \text{m/s}$$

The impulse exerted by the cube on the wall is to the right, $+3.12 \times 10^{-2} \text{ kg} \cdot \text{m/s}$. Here F could refer to a large force over a short contact time. It can also refer to the average force if we interpret Δt as $\frac{1}{10}$ s, the time between one cube's tap and the next's.

$$F_{\text{av}} = \frac{3.12 \times 10^{-2} \text{ kg} \cdot \text{m/s}}{0.1 \text{ s}} = \boxed{0.312 \text{ N to the right}}$$

P9.67 (a) Find the speed when the bullet emerges from the block by using momentum conservation:

$$mv_i = MV_i + mv$$

The block moves a distance of 5.00 cm. Assume for an approximation that the block quickly reaches its maximum velocity, V_i, and the bullet kept going with a constant velocity, v. The block then compresses the spring and stops.

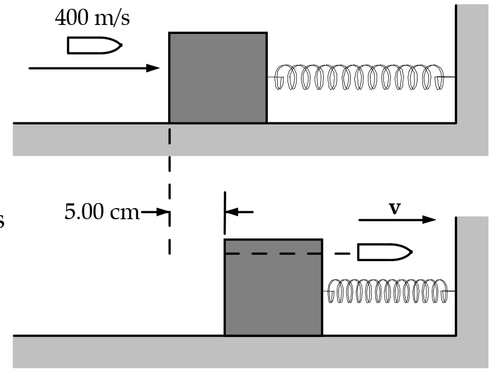

FIG. P9.67

$$\frac{1}{2}MV_i^2 = \frac{1}{2}kx^2$$

$$V_i = \sqrt{\frac{(900\ \text{N/m})(5.00\times10^{-2}\ \text{m})^2}{1.00\ \text{kg}}} = 1.50\ \text{m/s}$$

$$v = \frac{mv_i - MV_i}{m} = \frac{(5.00\times10^{-3}\ \text{kg})(400\ \text{m/s}) - (1.00\ \text{kg})(1.50\ \text{m/s})}{5.00\times10^{-3}\ \text{kg}}$$

$$v = \boxed{100\ \text{m/s}}$$

(b) $$\Delta E = \Delta K + \Delta U = \frac{1}{2}(5.00\times10^{-3}\ \text{kg})(100\ \text{m/s})^2 - \frac{1}{2}(5.00\times10^{-3}\ \text{kg})(400\ \text{m/s})^2 + \frac{1}{2}(900\ \text{N/m})(5.00\times10^{-2}\ \text{m})^2$$

$\Delta E = -374$ J, or there is an energy loss of $\boxed{374\ \text{J}}$.

***P9.68** The orbital speed of the Earth is

$$v_E = \frac{2\pi r}{T} = \frac{2\pi 1.496\times10^{11}\ \text{m}}{3.156\times10^{7}\ \text{s}} = 2.98\times10^{4}\ \text{m/s}$$

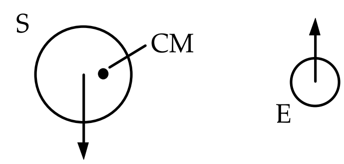

FIG. P9.68

In six months the Earth reverses its direction, to undergo momentum change

$$m_E|\Delta \mathbf{v}_E| = 2m_E v_E = 2(5.98\times10^{24}\ \text{kg})(2.98\times10^{4}\ \text{m/s}) = 3.56\times10^{25}\ \text{kg}\cdot\text{m/s}.$$

Relative to the center of mass, the sun always has momentum of the same magnitude in the opposite direction. Its 6-month momentum change is the same size, $m_S|\Delta \mathbf{v}_S| = 3.56\times10^{25}\ \text{kg}\cdot\text{m/s}$.

Then $|\Delta \mathbf{v}_S| = \frac{3.56\times10^{25}\ \text{kg}\cdot\text{m/s}}{1.991\times10^{30}\ \text{kg}} = \boxed{0.179\ \text{m/s}}$.

P9.69 (a) $\mathbf{p}_i + \mathbf{F}t = \mathbf{p}_f$: $(3.00 \text{ kg})(7.00 \text{ m/s})\hat{\mathbf{j}} + (12.0 \text{ N}\hat{\mathbf{i}})(5.00 \text{ s}) = (3.00 \text{ kg})\mathbf{v}_f$

$$\mathbf{v}_f = \boxed{(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}}) \text{ m/s}}$$

(b) $\mathbf{a} = \dfrac{\mathbf{v}_f - \mathbf{v}_i}{t}$: $\mathbf{a} = \dfrac{(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}} - 7.00\hat{\mathbf{j}}) \text{ m/s}}{5.00 \text{ s}} = \boxed{4.00\hat{\mathbf{i}} \text{ m/s}^2}$

(c) $\mathbf{a} = \dfrac{\sum \mathbf{F}}{m}$: $\mathbf{a} = \dfrac{12.0 \text{ N}\hat{\mathbf{i}}}{3.00 \text{ kg}} = \boxed{4.00\hat{\mathbf{i}} \text{ m/s}^2}$

(d) $\Delta\mathbf{r} = \mathbf{v}_i t + \dfrac{1}{2}\mathbf{a}t^2$: $\Delta\mathbf{r} = (7.00 \text{ m/s}\,\hat{\mathbf{j}})(5.00 \text{ s}) + \dfrac{1}{2}(4.00 \text{ m/s}^2\,\hat{\mathbf{i}})(5.00 \text{ s})^2$

$$\Delta\mathbf{r} = \boxed{(50.0\hat{\mathbf{i}} + 35.0\hat{\mathbf{j}}) \text{ m}}$$

(e) $W = \mathbf{F}\cdot\Delta\mathbf{r}$: $W = (12.0 \text{ N}\hat{\mathbf{i}})\cdot(50.0 \text{ m}\hat{\mathbf{i}} + 35.0 \text{ m}\hat{\mathbf{j}}) = \boxed{600 \text{ J}}$

(f) $\dfrac{1}{2}mv_f^2 = \dfrac{1}{2}(3.00 \text{ kg})(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}})\cdot(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}}) \text{ m}^2/\text{s}^2$

$$\frac{1}{2}mv_f^2 = (1.50 \text{ kg})(449 \text{ m}^2/\text{s}^2) = \boxed{674 \text{ J}}$$

(g) $\dfrac{1}{2}mv_i^2 + W = \dfrac{1}{2}(3.00 \text{ kg})(7.00 \text{ m/s})^2 + 600 \text{ J} = \boxed{674 \text{ J}}$

P9.70 We find the mass from $M = 360 \text{ kg} - (2.50 \text{ kg/s})t$.

We find the acceleration from $a = \dfrac{\text{Thrust}}{M} = \dfrac{v_e|dM/dt|}{M} = \dfrac{(1\,500 \text{ m/s})(2.50 \text{ kg/s})}{M} = \dfrac{3\,750 \text{ N}}{M}$

We find the velocity and position according to Euler,

from $v_{\text{new}} = v_{\text{old}} + a(\Delta t)$

and $x_{\text{new}} = x_{\text{old}} + v(\Delta t)$

If we take $\Delta t = 0.132$ s, a portion of the output looks like this:

Time t(s)	Total mass (kg)	Acceleration $a(\text{m/s}^2)$	Speed, v (m/s)	Position x(m)
0.000	360.00	10.4167	0.0000	0.0000
0.132	359.67	10.4262	1.3750	0.1815
0.264	359.34	10.4358	2.7513	0.54467
...				
65.868	195.330	19.1983	916.54	27191
66.000	195.000	19.2308	919.08	27312
66.132	194.670	19.2634	921.61	27433
...				
131.736	30.660	122.3092	3687.3	152382
131.868	30.330	123.6400	3703.5	152871
132.000	30.000	125.0000	3719.8	153362

(a) The final speed is $v_f = \boxed{3.7 \text{ km/s}}$

(b) The rocket travels $\boxed{153 \text{ km}}$

P9.71 The force exerted by the table is equal to the change in momentum of each of the links in the chain.

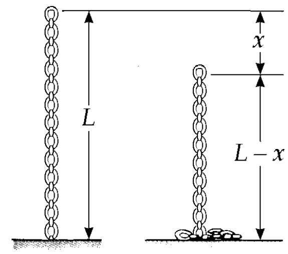

FIG. P9.71

By the calculus chain rule of derivatives,

$$F_1 = \frac{dp}{dt} = \frac{d(mv)}{dt} = v\frac{dm}{dt} + m\frac{dv}{dt}.$$

We choose to account for the change in momentum of each link by having it pass from our area of interest just before it hits the table, so that

$$v\frac{dm}{dt} \neq 0 \text{ and } m\frac{dv}{dt} = 0.$$

Since the mass per unit length is uniform, we can express each link of length dx as having a mass dm:

$$dm = \frac{M}{L}dx.$$

The magnitude of the force on the falling chain is the force that will be necessary to stop each of the elements dm.

$$F_1 = v\frac{dm}{dt} = v\left(\frac{M}{L}\right)\frac{dx}{dt} = \left(\frac{M}{L}\right)v^2$$

After falling a distance x, the square of the velocity of each link $v^2 = 2gx$ (from kinematics), hence

$$F_1 = \frac{2Mgx}{L}.$$

The links already on the table have a total length x, and their weight is supported by a force F_2:

$$F_2 = \frac{Mgx}{L}.$$

Hence, the *total* force on the chain is

$$F_{\text{total}} = F_1 + F_2 = \boxed{\frac{3Mgx}{L}}.$$

That is, *the total force is three times the weight of the chain on the table at that instant.*

P9.72 A picture one second later differs by showing five extra kilograms of sand moving on the belt.

(a) $\frac{\Delta p_x}{\Delta t} = \frac{(5.00\text{ kg})(0.750\text{ m/s})}{1.00\text{ s}} = \boxed{3.75\text{ N}}$

(b) The only horizontal force on the sand is belt friction,

so from $p_{xi} + f\Delta t = p_{xf}$ this is $f = \frac{\Delta p_x}{\Delta t} = \boxed{3.75\text{ N}}$

(c) The belt is in equilibrium:

$\sum F_x = ma_x: \quad +F_{\text{ext}} - f = 0$ and $F_{\text{ext}} = \boxed{3.75\text{ N}}$

(d) $W = F\Delta r\cos\theta = 3.75\text{ N}(0.750\text{ m})\cos 0° = \boxed{2.81\text{ J}}$

(e) $\frac{1}{2}(\Delta m)v^2 = \frac{1}{2}5.00\text{ kg}(0.750\text{ m/s})^2 = \boxed{1.41\text{ J}}$

(f) $\boxed{\text{Friction between sand and belt converts half of the input work into extra internal energy.}}$

***P9.73** $x_{\text{CM}} = \frac{\sum m_i x_i}{\sum m_i} = \frac{m_1\left(R + \frac{\ell}{2}\right) + m_2(0)}{m_1 + m_2} = \boxed{\frac{m_1\left(R + \frac{\ell}{2}\right)}{m_1 + m_2}}$

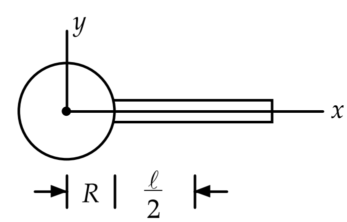

FIG. P9.73

ANSWERS TO EVEN PROBLEMS

P9.2 (a) 0; (b) $1.06\text{ kg}\cdot\text{m/s}$; upward

P9.4 (a) 6.00 m/s to the left; (b) 8.40 J

P9.6 The force is 6.44 kN

P9.8 $1.39\text{ kg}\cdot\text{m/s}$ upward

P9.10 (a) $5.40\text{ N}\cdot\text{s}$ toward the net; (b) –27.0 J

P9.12 $\sim 10^3\text{ N}$ upward

P9.14 (a) and (c) see the solution; (b) small; (d) large; (e) no difference

P9.16 1.67 m/s

P9.18 (a) 2.50 m/s; (b) $3.75\times 10^4\text{ J}$

P9.20 0.556 m

P9.22 1.78 kN on the truck driver; 8.89 kN in the opposite direction on the car driver

P9.24 $v = \frac{4M}{m}\sqrt{g\ell}$

P9.26 7.94 cm

P9.28 (a) 2.88 m/s at 32.3°; (b) 783 J becomes internal energy

P9.30 $v_Y = v_i\sin\theta$; $v_O = v_i\cos\theta$

P9.32 No; his speed was 41.5 mi/h

P9.34 (a) $v = \frac{v_i}{\sqrt{2}}$; (b) 45.0° and –45.0°

P9.36 (a) $\sqrt{2}v_i$; $\sqrt{\frac{2}{3}}v_i$; (b) 35.3°

P9.38 $(0,\ 1.00\text{ m})$

P9.40 4.67×10^{6} m from the Earth's center

P9.42 (a) see the solution; (b) 3.57×10^{8} J

P9.44 $0.063\,5L$

P9.46 (a) see the solution;
(b) $(-2.00\text{ m},\ -1.00\text{ m})$;
(c) $\left(3.00\hat{\mathbf{i}}-1.00\hat{\mathbf{j}}\right)$ m/s;
(d) $\left(15.0\hat{\mathbf{i}}-5.00\hat{\mathbf{j}}\right)$ kg·m/s

P9.48 (a) $-0.780\hat{\mathbf{i}}$ m/s; $1.12\hat{\mathbf{i}}$ m/s; (b) $0.360\hat{\mathbf{i}}$ m/s

P9.50 (a) 787 m/s; (b) 138 m/s

P9.52 see the solution

P9.54 (a) $\frac{m_1\mathbf{v}_1+m_2\mathbf{v}_2}{m_1+m_2}$;
(b) $(v_1-v_2)\sqrt{\frac{m_1m_2}{k(m_1+m_2)}}$;
(c) $\mathbf{v}_{1f}=\left(\frac{m_1-m_2}{m_1+m_2}\right)\mathbf{v}_1+\left(\frac{2m_2}{m_1+m_2}\right)\mathbf{v}_2$;
$\mathbf{v}_{2f}=\left(\frac{2m_1}{m_1+m_2}\right)\mathbf{v}_1+\left(\frac{m_2-m_1}{m_1+m_2}\right)\mathbf{v}_2$

P9.56 291 N

P9.58 $\left(\frac{M+m}{m}\right)\sqrt{\frac{gd^2}{2h}}$

P9.60 (a) -0.667 m/s; (b) 0.952 m

P9.62 (a) 6.81 m/s; (b) 1.00 m

P9.64 (a) -3.54 m/s; (b) 1.77 m; (c) 35.4 kN;
(d) No. The rails exert a vertical force to change the momentum

P9.66 0.312 N to the right

P9.68 0.179 m/s

P9.70 (a) 3.7 km/s; (b) 153 km

P9.72 (a) 3.75 N to the right; (b) 3.75 N to the right; (c) 3.75 N; (d) 2.81 J; (e) 1.41 J;
(f) Friction between sand and belt converts half of the input work into extra internal energy.