Linear Momentum and Impulse

Linear Momentum

- Linear momentum (p) is a vector
- p is parallel to v
- Unit: kg·m/s
- The net momentum of a collection of objects is the vector sum of the momentum of each object

$$\vec{p} = m\vec{v}$$

$$\vec{p} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots$$
$$= m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \ldots$$

Momentum problem

A bullet of mass 0.005 kg moving at a speed of 100 m/s lodges within a 1-kg block of wood resting on a frictionless surface and attached to a horizontal spring of k=50N/m.

a) What is the velocity of the block the instant after the bullet strikes it?

$$m_1 v_1 + m_2 v_2 = m_1 v_1^{'} + m_2 v_2^{'}$$

$$m_1 v_1 + 0 = (m_1 + m_2) v^{'}$$

$$v^{'} = \frac{m_1 v_1}{m_1 + m_2}$$

$$= \frac{(0.005kg)(100m/s)}{0.005kg + 1.00kg}$$

$$= 0.50m/s$$

Momentum problem

A bullet of mass 0.005 kg moving at a speed of 100 m/s lodges within a 1-kg block of wood resting on a frictionless surface and attached to a horizontal spring of k=50N/m.

b) What is the maximum compression of the spring?

$$E_T = E_T$$

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}kx^2$$

$$x = \sqrt{\frac{mv^2}{k}}$$

$$= \sqrt{\frac{(1.005kg)(0.5m/s)^2}{50N/m}}$$

$$= 0.07m$$

Force and momentum

The time rate of change of the momentum of a particle is equal to the net force acting on the particle.

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

"Let the time rate of change of momentum be with you."

Force and momentum

- In general, force and momentum are related as:

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$$

- When the mass is constant, this becomes the familiar equation:

$$\vec{F}_{net} = m\frac{d\vec{v}}{dt} = m\vec{a}$$

Non-constant mass

- Mass changes for rockets, bags of sand (with holes in them), and other special cases.

See below for a changing mass:

http://eepybird.com/dcm1.html

Changing mass problem

A rocket whose initial mass is 850kg consumes fuel at the rate of 2.3kg/s. The speed of the exhaust gases relative to the rocket engine is 2800m/s.

What thrust does the rocket engine provide?

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$$

$$= 0 + \vec{v}\frac{dm}{dt}$$

$$= (2800m/s)(2.3kg/s)$$

$$= 6440N$$

Impulse

Impulse (J) is a change in momentum.

Start with Newton's second Law:

$$\vec{F} = \frac{d\vec{p}}{dt}$$

Which can be written as:

$$d\vec{p} = \vec{F}(t)dt$$

Integrate with respect to time to find the change in momentum:

$$\vec{J} = \int_{\vec{p}_i}^{\vec{p}_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}(t)dt$$

Impulse

A 2.0 kg toy car travels at 0.50m/s East before a turn and at 0.40m/s South after the turn.

What is the impulse (change in momentum) of the car due to the turn?

$$
\begin{aligned}
\vec{J} &= \Delta\vec{p} = \vec{p}_f - \vec{p}_i \\
&= m\vec{v}_f - m\vec{v}_i \\
&= m(\vec{v}_f - \vec{v}_i) \\
&= (2.0kg)\left[(-0.40m/s)\hat{j} - (0.50m/s)\hat{i}\right] \\
&= -(1.0\hat{i} + 0.8\hat{j})kg \cdot m/s
\end{aligned}
$$

Conservation of Linear Momentum

The linear momentum (p) of a system is conserved (does not change) unless the system experiences an external force.

Problem

An 80-kg lumberjack stands at one end of a floating 400-kg log that is at rest relative to the shore of a lake.

If the lumberjack jogs to the other end of the log at 2m/s relative to the shore, what happens to the log while he is moving?

Elastic collision

In an elastic collision objects typically bounce, and ***NO*** energy is lost

- Momentum is conserved

$$\vec{p}_{before} = \vec{p}_{after}$$

- Kinetic energy is conserved

$$K_{before} = K_{after}$$

Inelastic collision

In a totally inelastic collision, objects stick together.

- Momentum is conserved

$$\vec{p}_{before} = \vec{p}_{after}$$

- Energy is lost to sound, sparks, mechanical deformation, etc.

$$K_{before} \neq K_{after}$$

Example of Inelastic collision:

Would you be safer in an old (heavy) 1959 Chevrolet Bel Air or in a newer (lighter) 2009 Chevy Malibu?

http://www.youtube.com/watch?v=fPF4fBGNK0U

Collision review

- Momentum is conserved in all collisions
- Elastic collisions: no deformation occurs
  - Kinetic energy is also conserved
- Inelastic collisions: deformation occurs
  - Kinetic energy is “lost”
- Perfectly inelastic collisions
  - Objects stick together; kinetic energy is “lost”
- Explosions
  - Reverse of perfectly inelastic collisions; kinetic energy is “gained”

Collision problem

Two skaters collide and embrace, in a completely inelastic collision. Dean, of mass 83 kg, is initially moving east with speed 6.2 m/s. Torvill, of mass 55 kg, is initially traveling north with speed 7.8 m/s.

What are the skaters' speed and direction after the collision?

$$v = 4.86 m/s \approx 4.9 m/s$$

$$\theta = 39.8^\circ \approx 40^\circ$$