

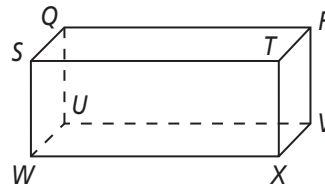
3-1 Practice

Lines and Angles

Form G

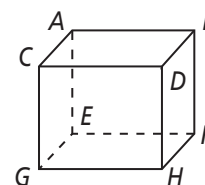
Use the diagram to name each of the following.

- a pair of parallel planes **one of the following pairs:**
QRTS, UVXW; QUWS, RVXT; STXW, QRVU
- all lines that are parallel to \overleftrightarrow{RV}
 $\overleftrightarrow{TX}, \overleftrightarrow{QU}, \overleftrightarrow{SW}$
- four lines that are skew to \overleftrightarrow{WX}
Answers may vary. Sample: *$\overleftrightarrow{TR}, \overleftrightarrow{QS}, \overleftrightarrow{RV}, \overleftrightarrow{QU}$*
- all lines that are parallel to plane $QUVR$
Answers may vary. Sample: *$\overleftrightarrow{ST}, \overleftrightarrow{TX}, \overleftrightarrow{WX}, \overleftrightarrow{SW}$*
- a plane parallel to plane $QUWS$
RVXT



In Exercises 6–11, describe the statement as *true* or *false*. If false, explain.

- \overleftrightarrow{AE} and \overleftrightarrow{EF} are skew lines.
False; \overleftrightarrow{AE} and \overleftrightarrow{EF} intersect.
 - $\overleftrightarrow{GH} \parallel \overleftrightarrow{EF}$ **true**
 - plane $DBF \parallel$ plane ABD
False; the planes intersect.
 - $\overleftrightarrow{DB} \parallel \overleftrightarrow{AE}$
False; the lines are skew because they are noncoplanar.
 - plane $EFH \parallel$ plane ABD **true**
 - \overleftrightarrow{FH} and \overleftrightarrow{CD} are skew lines. **true**
- You are driving over a bridge that runs east to west. Below the bridge, a highway runs north to south. Are the bridge and the highway *parallel*, *skew*, or *neither*? Explain.
Skew; because the bridge is above the highway and they run in different directions, they are noncoplanar and cannot intersect.
 - Open-Ended** List parts of your classroom that fit each description below.
 - parallel to the top of a window
Sample: bottom of the window
 - skew with one side of the door
Sample: top of the chalkboard
 - parallel to the plane of the floor
Sample: plane of the ceiling
 - Reasoning** Your friend says that the sides of a ladder and the rungs of a ladder are skew. Is this true? Explain.
No; the rungs of a ladder and the sides of a ladder intersect. Skew lines do not intersect.
 - Visualization** If two planes are parallel, must all lines within those planes be parallel? Explain.
Answers may vary. Sample: No; even if the planes are parallel, the lines could be skew. It depends upon the direction of the lines.



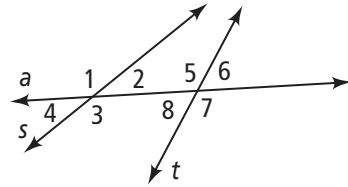
3-1 Practice (continued)

Lines and Angles

Form G

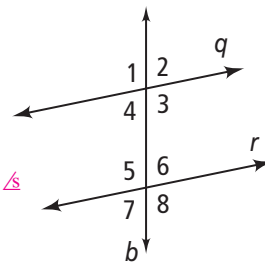
Identify all pairs of each type of angle in the diagram below right.

- 16. corresponding angles
 $\angle 1$ and $\angle 5$; $\angle 2$ and $\angle 6$; $\angle 4$ and $\angle 8$; $\angle 3$ and $\angle 7$
- 17. same-side interior angles
 $\angle 2$ and $\angle 5$; $\angle 3$ and $\angle 8$
- 18. alternate interior angles
 $\angle 3$ and $\angle 5$; $\angle 2$ and $\angle 8$
- 19. alternate exterior angles
 $\angle 1$ and $\angle 7$; $\angle 4$ and $\angle 6$



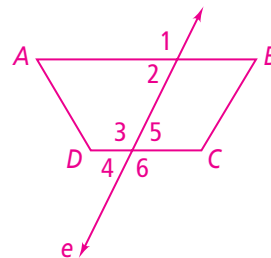
Decide whether the angles are *alternate interior angles*, *same-side interior angles*, *corresponding angles*, or *alternate exterior angles*.

- 20. $\angle 2$ and $\angle 7$ **alt. ext.** \triangle
- 21. $\angle 5$ and $\angle 4$ **same-side int.** \triangle
- 22. $\angle 8$ and $\angle 3$ **corr.** \triangle
- 23. $\angle 6$ and $\angle 4$ **alt. int.** \triangle
- 24. $\angle 1$ and $\angle 5$ **corr.** \triangle



25. **Draw a Diagram** Line e intersects trapezoid $ABCD$. Sketch a diagram that meets the following conditions.

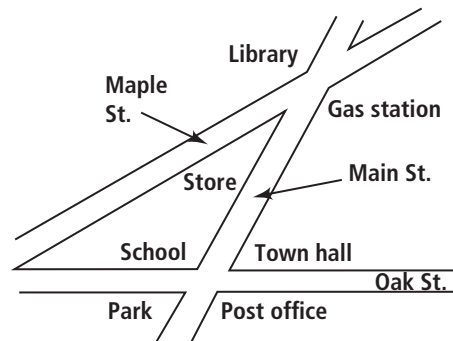
- a. \overleftrightarrow{AB} and \overleftrightarrow{DC} are parallel. **Answers may vary. Sample:**
- b. $\angle 1$ and $\angle 6$ are alternate exterior angles.
- c. $\angle 2$ and $\angle 3$ are same-side interior angles.
- d. $\angle 4$ and $\angle 5$ are each supplementary to $\angle 3$.



26. **Writing** Describe three real-world objects that represent two lines intersected by a transversal. **Answers may vary. Samples:** The sides of window panes are parallel lines intersected by the transversal of the center strip. Train track ties are transversals intersecting the parallel rails. In a bridge framework, the crosspieces intersect parallel and non-parallel lines.

27. The map at the right shows the intersection of Maple Street and Oak Street by Main Street. Name the angle pairs represented by the locations listed below.

- a. town hall and gas station **same-side interior**
- b. school and library **corresponding**
- c. library and post office **alternate exterior**
- d. school and gas station **alternate interior**



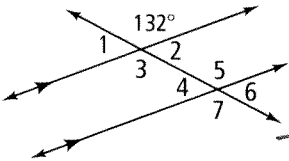
3-2

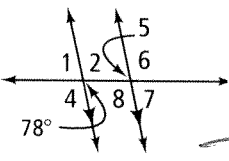
Practice

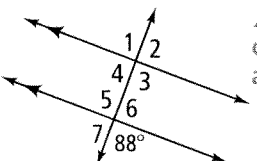
Form G

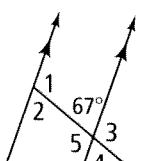
Properties of Parallel Lines

Identify all the numbered angles that are congruent to the given angle. Justify your answers.

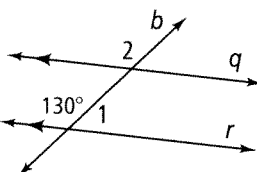
1.  $\angle 3$; vert. \triangle are \cong ; $\angle 5$; corresp. \triangle are \cong ; $\angle 7$; alt. ext. \triangle are \cong .

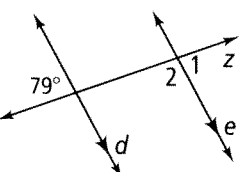
2.  $\angle 1$; vert. \triangle are \cong ; $\angle 5$; alt. int. \triangle are \cong ; $\angle 7$; corresp. \triangle are \cong .

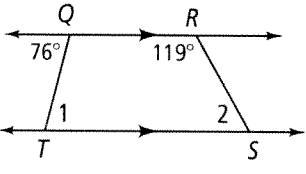
3.  $\angle 5$; vert. \triangle are \cong ; $\angle 3$; corresp. \triangle are \cong ; $\angle 1$; alt. ext. \triangle are \cong .

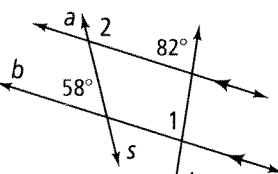
4.  $\angle 4$; vert. \triangle are \cong ; $\angle 2$; alt. int. \triangle are \cong .

Find $m\angle 1$ and $m\angle 2$. Justify each answer.

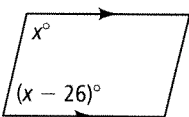
5.  $m\angle 1 = 50$; \triangle that form a linear pair are suppl.; $m\angle 2 = 130$; corresp. \triangle are \cong .

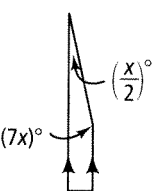
6.  $m\angle 1 = 79$; alt. ext. \triangle are \cong ; $m\angle 2 = 101$; \triangle that form a linear pair are suppl.

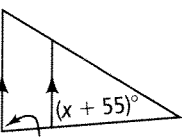
7.  $m\angle 1 = 76$; alt. int. \triangle are \cong ; $m\angle 2 = 61$; same-side int. \triangle are suppl.

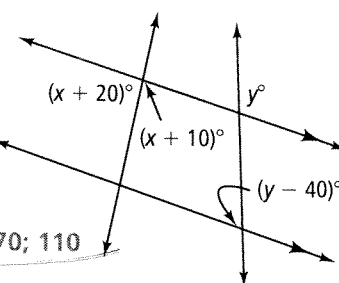
8.  $m\angle 1 = 82$; corresp. \triangle are \cong ; $m\angle 2 = 122$; the 58° \angle and the \angle below $\angle 2$ are alt. int. \triangle and are \cong . Because $\angle 2$ and the \angle below it form a linear pair, they are suppl.

Algebra Find the value of x and y . Then find the measure of each labeled angle.

9.  $103; 77; 103^\circ$

10.  $24; 12; 168$

11.  $30; 85; 85$

12.  $75; 95; 85; 70; 110$

3-2 Practice (continued)

Properties of Parallel Lines

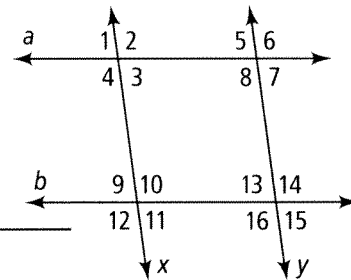
Form G

13. Write a two-column proof.

Given: $a \parallel b, x \parallel y$

Prove: $\angle 4$ is supplementary to $\angle 15$.

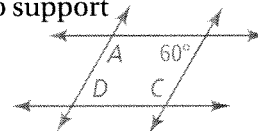
Answers may vary. Sample:



Statements	Reasons
1) $x \parallel y; a \parallel b$	1) Given
2) $\angle 15 \cong \angle 9$	2) Alt. ext. angles are \cong .
3) $m\angle 15 = m\angle 9$	3) Definition of congruent
4) $\angle 9$ and $\angle 4$ are suppl.	4) Same-side int. \angle s are suppl.
5) $m\angle 9 + m\angle 4 = 180$	5) Def. of suppl. \angle s
6) $m\angle 15 + m\angle 4 = 180$	6) Substitution property
7) $\angle 15$ and $\angle 4$ are suppl.	7) Def. of suppl. \angle s

14. **Visualization** One pair of parallel lines intersect a second pair of parallel lines. One of the angles of intersection has a measure of 60. How can you determine the measure of the four interior angles? Draw a sketch to support

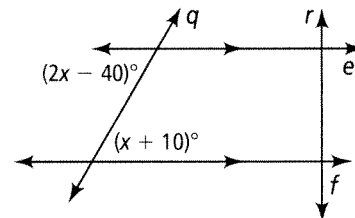
your answer. Answers may vary. Sample: If the measure of the given angle is 60, then $m\angle A$ and $m\angle C$ are both 120 because same-side interior angles are supplementary. Because $\angle C$ and $\angle D$ are also supplementary, $m\angle D$ is 60.



15. **Error Analysis** Which solution for the figure at the right is incorrect? Explain.

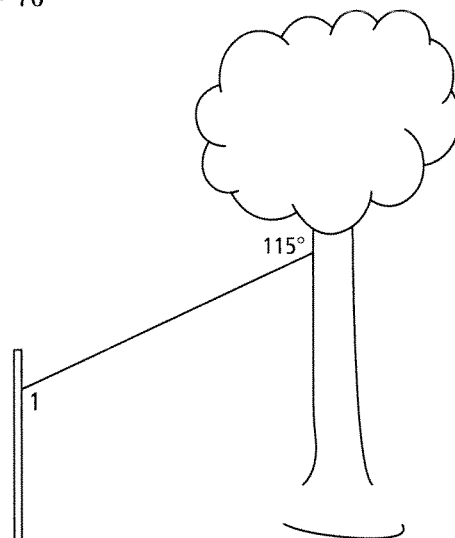
$$\begin{array}{l}
 2x - 40 = x + 10 \qquad 2x - 40 + (x + 10) = 180 \\
 x - 40 = 10 \qquad \qquad \qquad 3x - 30 = 180 \\
 x = 50 \qquad \qquad \qquad 3x = 210 \\
 \qquad \qquad \qquad \qquad \qquad \qquad x = 70
 \end{array}$$

Second solution; the angles are alternate interior angles, which means they are congruent.



16. A zip line consists of a pulley attached to a cable that is strung at an angle between two objects. In the zip line at the right, one end of the cable is attached to a tree. The other end is attached to a post parallel to the tree. What is the measure of $\angle 1$? What type of angle pair do $\angle 1$ and the given angle represent?

115°; alternate interior angles



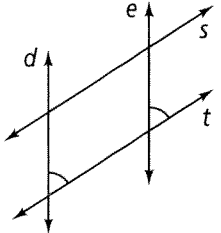
3-3

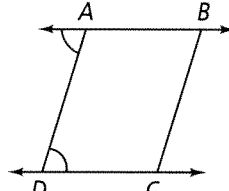
Practice

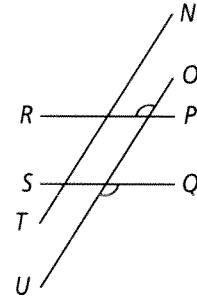
Form G

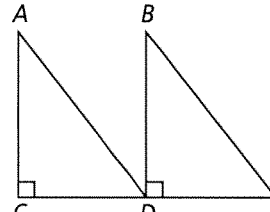
Proving Lines Parallel

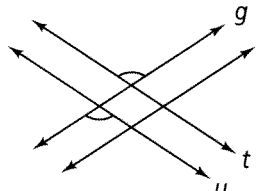
Which lines or segments are parallel? Justify your answer.

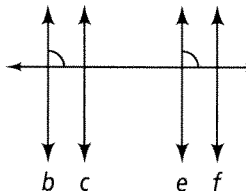
1. 
 $d \parallel e$; corr. angles

2. 
 $\overline{AB} \parallel \overline{DC}$; alt. int. angles

3. 
 $\overline{RP} \parallel \overline{SQ}$; alt. ext. \angle s

4. 
 $\overline{AC} \parallel \overline{BD}$; corr. angles

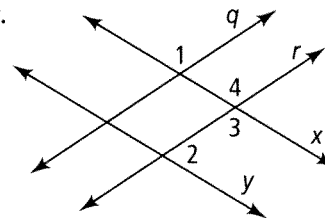
5. 
 $t \parallel u$; alt. ext. angles

6. 
 $b \parallel c$; corr. angles

7. **Developing Proof** Complete the flow proof below.

Given: $\angle 1$ and $\angle 2$ are supplementary; $x \parallel y$

Prove: $q \parallel r$

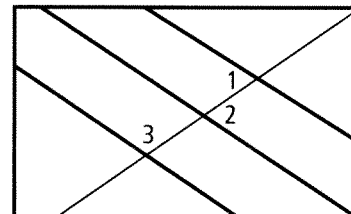


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    graph TD
      A["x || y  
Given"] --> B["∠2 and ∠3  
are suppl."]
      A --> C["∠1 and ∠2  
are suppl.  
Given"]
      B --> D["∠1 ≅ ∠3  
∠s suppl. to  
the same  
∠ are ≅."]
      C --> D
      D --> E["∠3 ≅ ∠4  
Vert. ∠s are ≅."]
      D --> F["∠1 ≅ ∠4  
Transitive  
Property of ≅"]
      E --> F
      F --> G["q || r  
If corresp.  
∠s are ≅,  
lines are ||."]
  
```

8. The art club is designing a new flag for the marching band. In the diagram, $m\angle 1 = 45$, $m\angle 2 = 45$, and $m\angle 3 = 145$. Does the flag contain three parallel lines? Explain.

The top two lines are parallel because $\angle 1 \cong \angle 2$ and they are alt. int. \angle s. The angle vertical to $\angle 2$ is suppl. to $\angle 3$. Because $45 + 145 \neq 180$, the bottom line is not parallel to the top two.

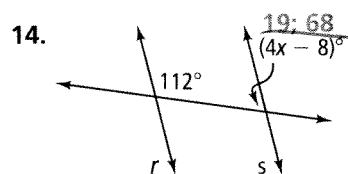
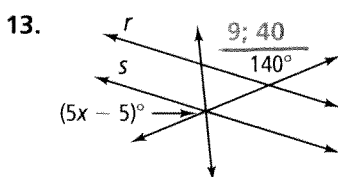
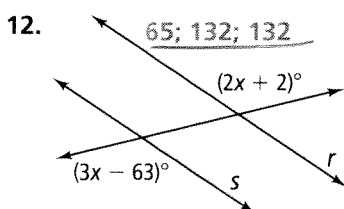
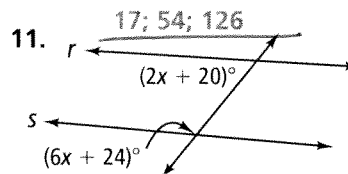
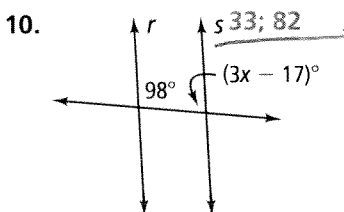
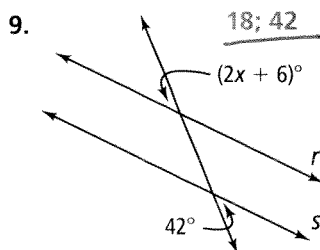


3-3 Practice (continued)

Form G

Proving Lines Parallel

Algebra Determine the value of x for which $r \parallel s$. Then find the measure of each labeled angle.



Developing Proof Use the given information to determine which lines, if any, are parallel. Justify each conclusion with a theorem or postulate.

15. $\angle 11$ is supplementary to $\angle 10$.
 $t \parallel u$; same-side int. \angle s are suppl.

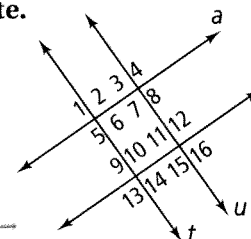
17. $\angle 13$ is supplementary to $\angle 14$.
no lines; linear pair

19. $\angle 12$ is supplementary to $\angle 3$.
 $a \parallel b$; $\angle 12$ and $\angle 16$ are linear pair; alt. ext. \angle s are \cong .

16. $\angle 6 \cong \angle 9$
 $a \parallel b$; alt. int. \angle s are \cong .

18. $\angle 13 \cong \angle 15$
 $t \parallel u$; corr. \angle s are \cong .

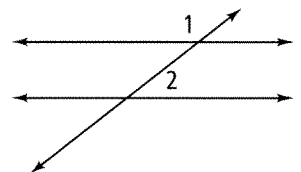
20. $\angle 2 \cong \angle 13$
 $a \parallel b$; alt. ext. \angle s are \cong .



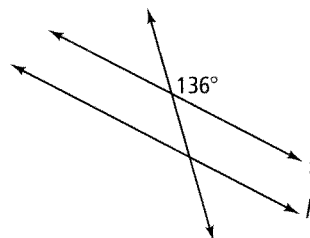
Algebra Determine the value of x for which $j \parallel k$. Then find $m\angle 1$ and $m\angle 2$.

21. $m\angle 1 = 7x + 14$, $m\angle 2 = 2x + 4$ 18; 140; 40

22. $m\angle 1 = 4x - 5$, $m\angle 2 = x + 20$ 33; 127; 53



23. **Open-Ended** Choose a value for x and write an expression for one of the angles in terms of x that will prove that g and h are parallel. Check students' work.



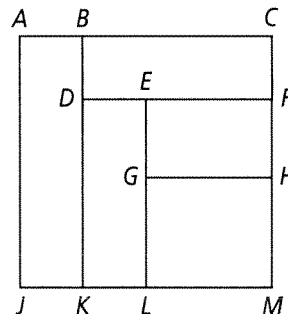
3-4

Practice

Form G

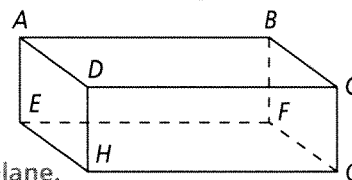
Parallel and Perpendicular Lines

- Suppose you are laying tiles. You place several different rectangles together to form a larger rectangle.
 - \overline{BC} is parallel to \overline{DF} , \overline{DF} is parallel to \overline{GH} . What is the relationship between \overline{BC} and \overline{GH} ? Explain.
 - \overline{BK} is parallel to \overline{EL} . \overline{GH} is perpendicular to \overline{BK} .



What is the relationship between \overline{GH} and \overline{EL} ? $\overline{GH} \perp \overline{EL}$.
 If a line is \perp to one of two parallel lines, it is also \perp to the other.

- Error Analysis** A student says that according to Theorem 3-9, \overrightarrow{AB} and \overrightarrow{BC} must be parallel because they are both perpendicular to \overrightarrow{BF} . Explain the student's error.



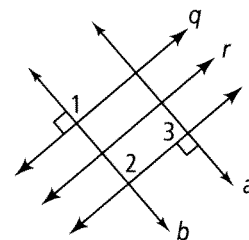
Theorem 3-9 states that the lines must all be in the same plane. \overrightarrow{AB} and \overrightarrow{BC} are \perp to \overrightarrow{BF} in different planes. \overrightarrow{AB} and \overrightarrow{BC} are \perp .

- Developing Proof** Copy and complete this paragraph proof.

Given: $q \parallel r$, $r \parallel s$, $b \perp q$, and $a \perp s$

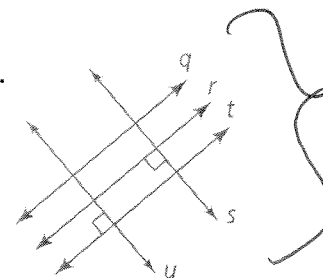
Prove: $a \parallel b$

Proof: Because it is given that $q \parallel r$ and $r \parallel s$, then $q \parallel s$ by the Trans. Prop. of \parallel Lines. This means that $\angle 1 \cong \angle 2$ because they are corresponding angles. Because $b \perp q$, $m\angle 1 = 90$. So, $m\angle 2 = 90$. This means $s \perp b$, by definition of perpendicular lines. It is given that $a \perp s$, so $a \parallel b$ by Theorem 3-9.



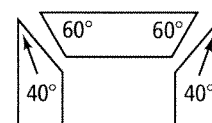
- Open-Ended** Draw a diagram that meets the criteria listed below. Then describe how all the lines are related to each other.

- $q \parallel r$
 - $r \perp s$
 - $t \parallel q$
 - $u \perp t$
- $q \parallel r \parallel t$; $s \perp q, r$, and t ; $u \perp q, r$, and t ; $s \parallel u$



- A puppeteer cuts the pieces shown at the right to frame the stage of a puppet theater. Will the sides of the pieces on the left and right be parallel?

No; the sum of the angles at the corners will be 100° . For the sides on the left and right to be parallel, they must form 90° angles.



In Exercises 6 and 7, a , b , c , and d are distinct lines in the same plane. For each combination of relationships, tell how a and c relate. Justify your answer.

6. $a \perp b$; $b \perp c$ $a \parallel c$; Theorem 3-9

7. $a \perp b$; $b \parallel c$ $a \perp c$; Theorem 3-10

3-4

Practice (continued)

Form G

Parallel and Perpendicular Lines

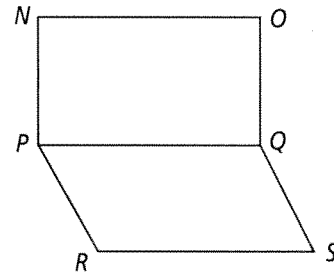
8. Write a paragraph proof.

Given: $\overleftrightarrow{NP} \perp \overleftrightarrow{NO}$; $\overleftrightarrow{NP} \perp \overleftrightarrow{PQ}$

$\angle PQS$ and $\angle QSR$ are supplementary.

Prove: $\overleftrightarrow{NO} \parallel \overleftrightarrow{RS}$

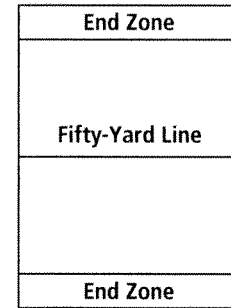
Since it is given that $\overleftrightarrow{NP} \perp \overleftrightarrow{NO}$ and $\overleftrightarrow{NP} \perp \overleftrightarrow{PQ}$, then $\overleftrightarrow{NO} \parallel \overleftrightarrow{PQ}$ by Theorem 3-9. Since it is given that $\angle PQS$ and $\angle QSR$ are supplementary, then $\overleftrightarrow{RS} \parallel \overleftrightarrow{PQ}$ by Theorem 3-6. So, $\overleftrightarrow{NO} \parallel \overleftrightarrow{RS}$ by Theorem 3-8.



9. The recreation department is setting up the football field.

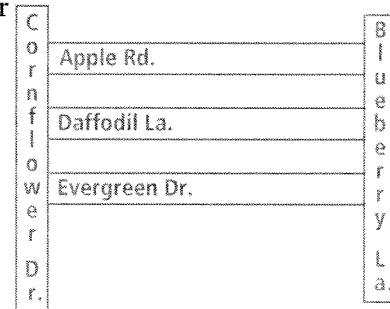
They check to make sure that the 50-yd line and the end zone lines are perpendicular to the right sideline. Does this mean both sidelines are parallel? Explain.

No; the 50-yd line and the end zone lines need to be perpendicular to the left sideline as well for the sidelines to be parallel.



10. **Draw a Diagram** Apple Road is perpendicular to Blueberry Lane. Blueberry Lane is parallel to Cornflower Drive. Cornflower Drive is perpendicular to Daffodil Lane. Daffodil Lane is parallel to Evergreen Drive. Draw a diagram to explain how each street is related to every other street. What can you conclude about Apple Road and Evergreen Drive? Explain.

They are parallel; the Perp. Transversal Thm. shows they are both perpendicular to Cornflower Drive and Theorem 3-9 proves they are parallel to each other.



11. **Compare and Contrast** How is the Transitive Property of Parallel Lines similar to the Transitive Property of Congruence? How are they different?

They are the same because they both compare two items to another: If two lines are parallel to the same line, they are parallel to each other, or, if two angles are congruent to the same angle they are congruent to each other. They are different in that the Transitive Property of Parallel Lines can be applied only to lines, not to angles or other figures.

12. **Writing** How is Theorem 3-9 related to the postulates and theorems you learned in Lesson 3-3?

Both Theorem 3-9 and the theorems and postulates in Lesson 3-3 can prove lines are parallel.

The following statements describe a set of railroad tracks. Based only on the statement, make a conclusion about the rails or the railroad ties. Explain.

13. The railroad ties are each perpendicular to one rail.

The railroad ties are all parallel to each other if they are in the same plane.

14. The rails are parallel. One railroad tie is perpendicular to one rail.

The railroad tie is perpendicular to both rails if it is in the same plane as both rails.

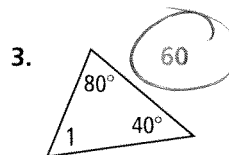
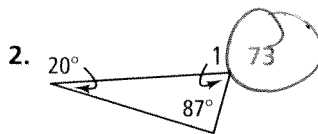
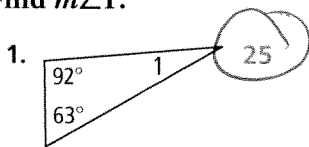
3-5

Practice

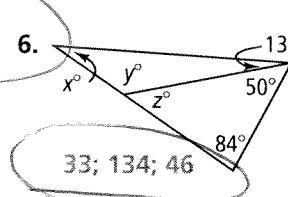
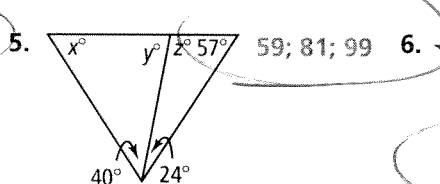
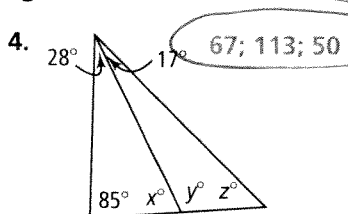
Form G

Parallel Lines and Triangles

Find $m\angle 1$.



Algebra Find the value of each variable.



7. Use the diagram at the right to answer the questions.

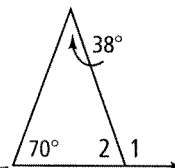
a. Which angle is an exterior angle? $\angle 1$

b. What are its remote interior angles?

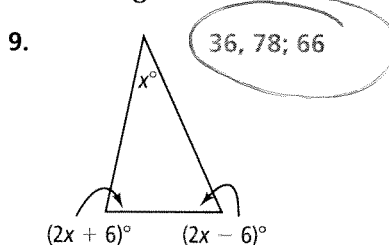
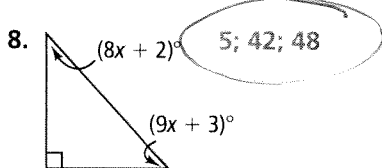
$\angle 1$ has the remote interior angle measurements of 70 and 38.

c. Find $m\angle 1$ and $m\angle 2$.

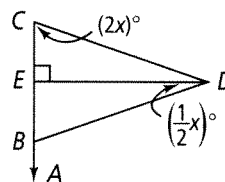
$m\angle 1 = 108$; $m\angle 2 = 72$



Find the value of the variables and the measures of the angles.



10. In the figure at the right, $\overline{ED} \perp \overline{CB}$ and \overline{ED} bisects $\angle CDB$. Find $m\angle DBA$. 108



11. Reasoning What is the measure of each angle in an isosceles right triangle? Explain. 45, 45, 90; Sample: because the triangle is isosceles, two sides are equal measure. So two angles are equal measure and one angle is 90°, since the triangle is right and 45 + 45 + 90 = 180.

12. The ratio of the angle measures of the acute angles in a right triangle is 2 : 3. Find the measures of the acute angles. 36 and 54

3-5 Practice (continued)

Parallel Lines and Triangles

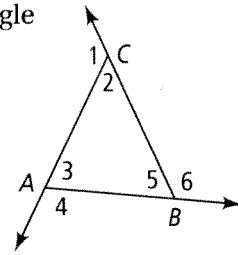
Form G

13. Paragraph Proof Prove that the sum of the exterior angles of a triangle is always 360° .

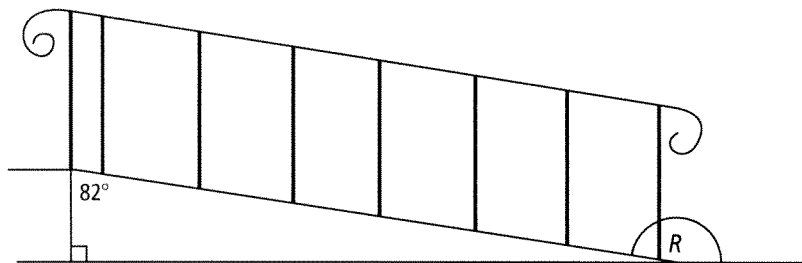
Given: $\triangle ABC$

Prove: $m\angle 1 + m\angle 4 + m\angle 6 = 360$

Sample: The sum of the interior angles of a triangle is 180, so $m\angle 2 + m\angle 3 + m\angle 5 = 180$. Because $\angle 1$ and $\angle 2$, $\angle 3$ and $\angle 4$, $\angle 5$ and $\angle 6$ are linear pairs, the sum of the measures of each pair is 180. So, $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 + m\angle 6 = 540$. Using the Substitution Property of Equality, $m\angle 1 + m\angle 4 + m\angle 6 + 180 = 540$. Then, by the Subtraction Property of Equality, $m\angle 1 + m\angle 4 + m\angle 6 = 360$.



14. A ramp built for wheelchairs is shown below.



a. Find the measures of the remote interior angles for $\angle R$. **90; 82**

b. Find $m\angle R$. **172**

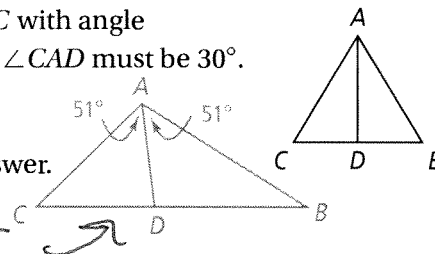
15. Error Analysis A student draws a diagram of $\triangle ABC$ with angle bisector \overline{AD} as shown, and says that the measure of $\angle CAD$ must be 30° .

a. What is the student's error in reasoning?

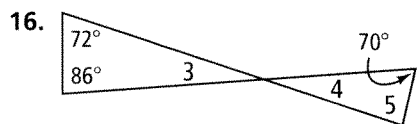
The triangle might not be an equilateral triangle.

b. Draw a diagram that would show an alternate answer.

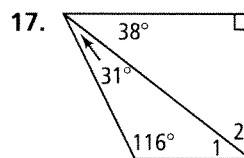
Answers may vary. Sample shown at the right.



Find each missing angle measure.



$m\angle 3 = 22; m\angle 4 = 22; m\angle 5 = 88$



$m\angle 1 = 33; m\angle 2 = 52$

18. Reasoning Two angles of a triangle measure 53 and 39. What is the measure of the largest exterior angle of the triangle? Explain.

141; answers may vary. Sample: If two angles are 53 and 39, the third angle will be 88. The measure of the largest exterior angle will be the sum of the two larger angles, and $53 + 88 = 141$.