Log Gaussian Cox Processes

Chi Group Meeting

February 23, 2016

Outline

- Typical motivating application
- Introduction to LGCP model
- Brief overview of inference
- Applications in my work

…just getting started with this…

Motivation

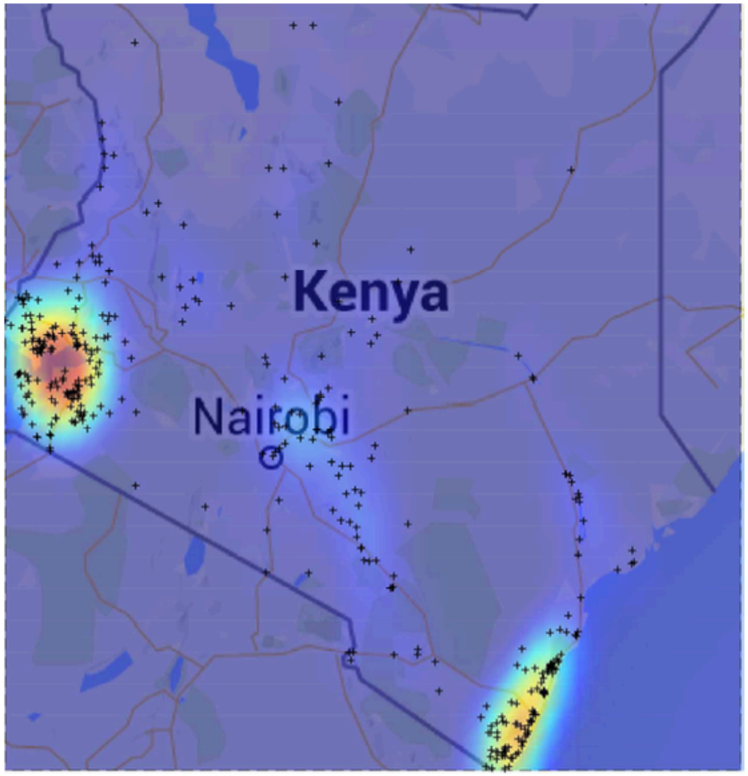

Image: Lloyd, et al. 2015. ICML.

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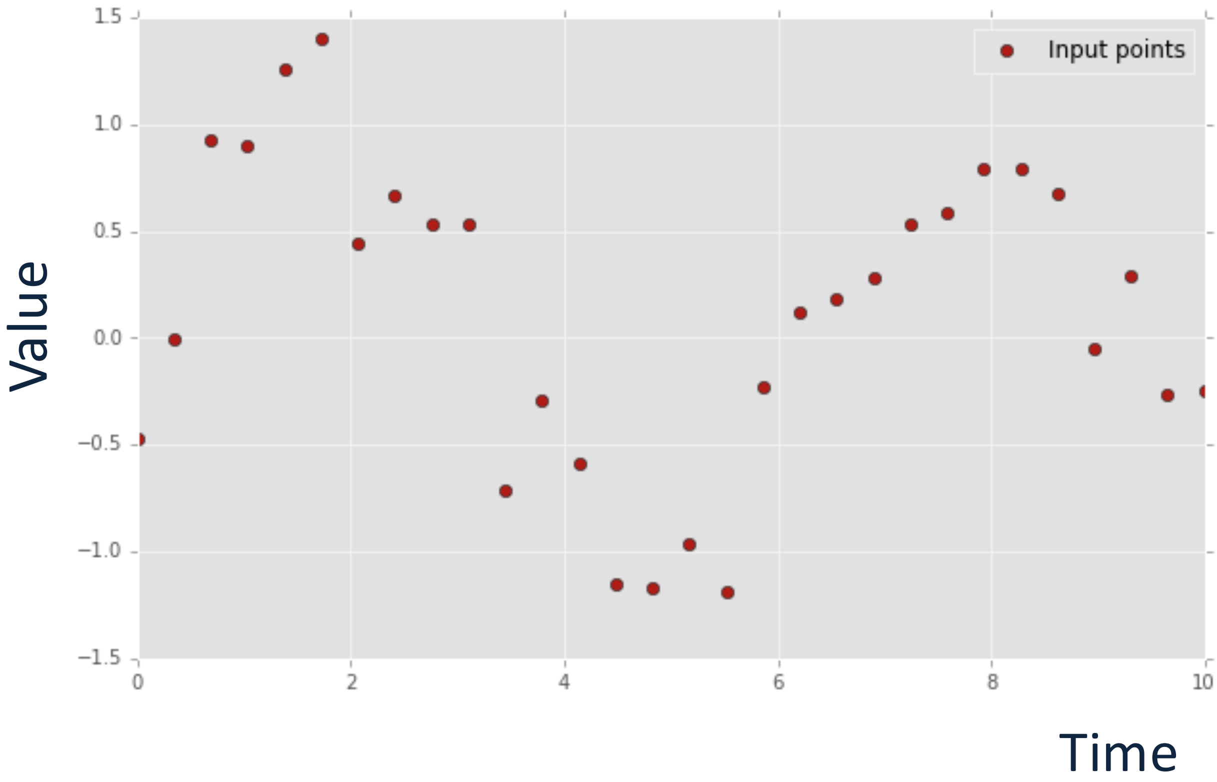
What are Gaussian processes?



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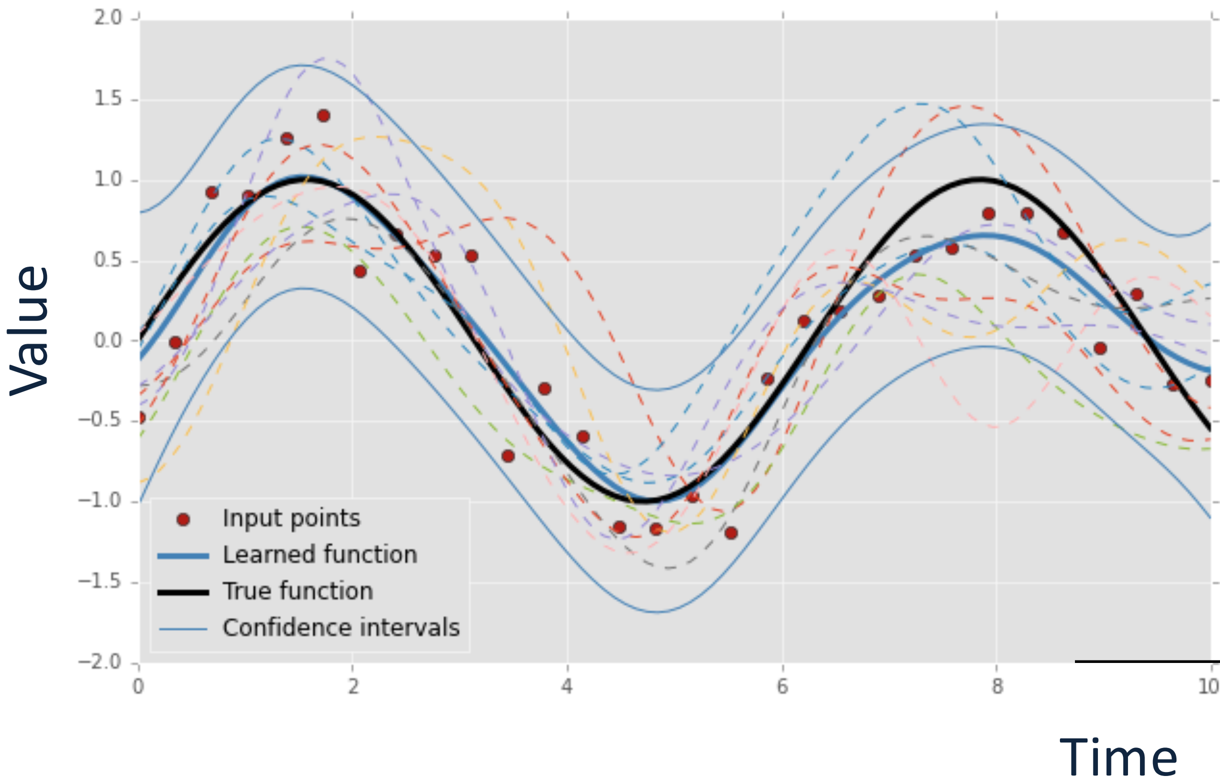


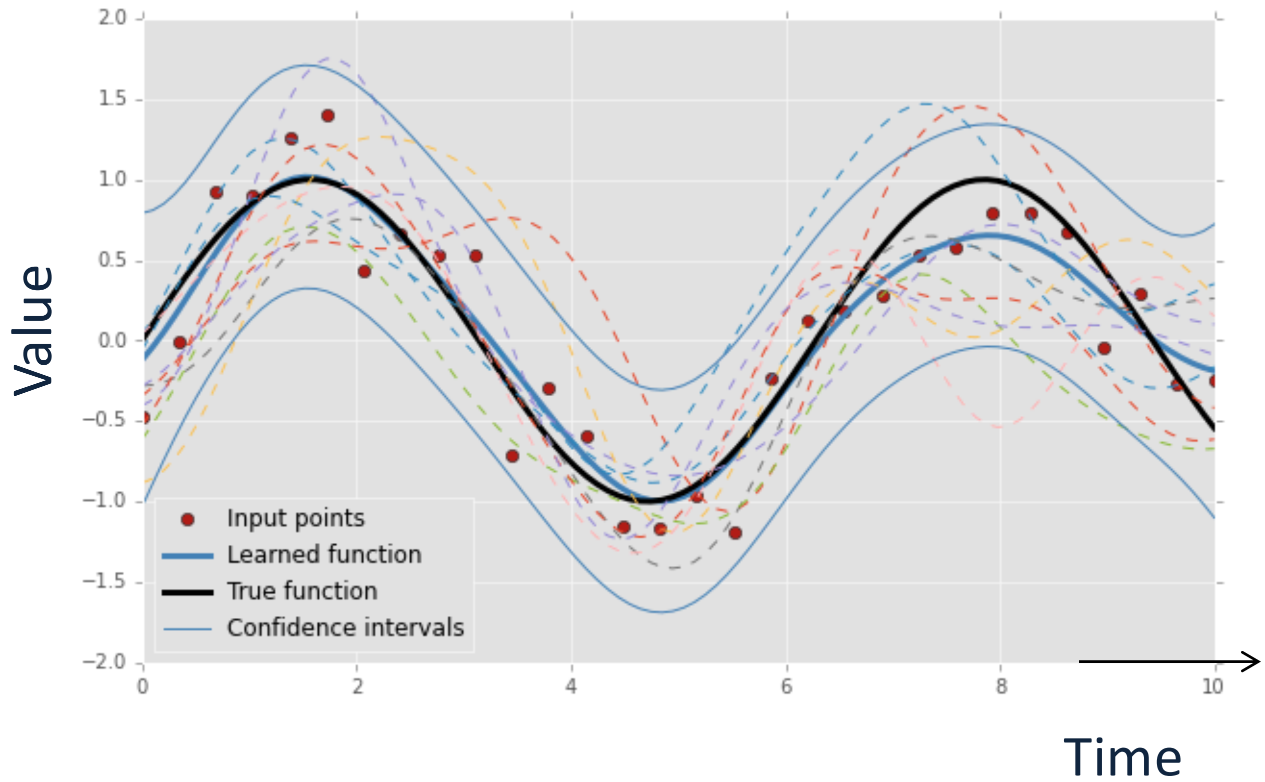
Posterior = $\frac{\text{Likelihood * prior}}{\text{Marginal likelihood}}$

→ Infinite dimensional Gaussian

likelihood prior

$$p(\mathbf{y}|\mathbf{f}, X)p(\mathbf{f}|X)$$

What are Gaussian processes?



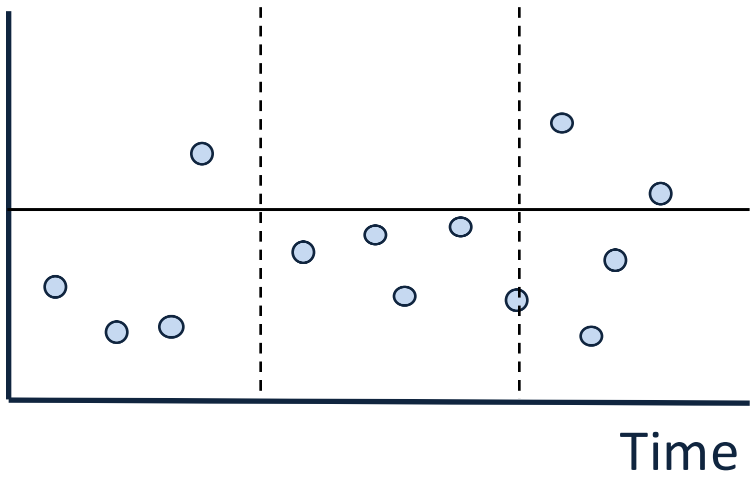
Infinite dimensional Gaussian

$\mathbf{y}|\mathbf{f} \sim \mathcal{N}(\mathbf{f}, \sigma_n^2 I)$ likelihood

prior $\mathbf{f}|X \sim \mathcal{N}(\mathbf{0}, K)$

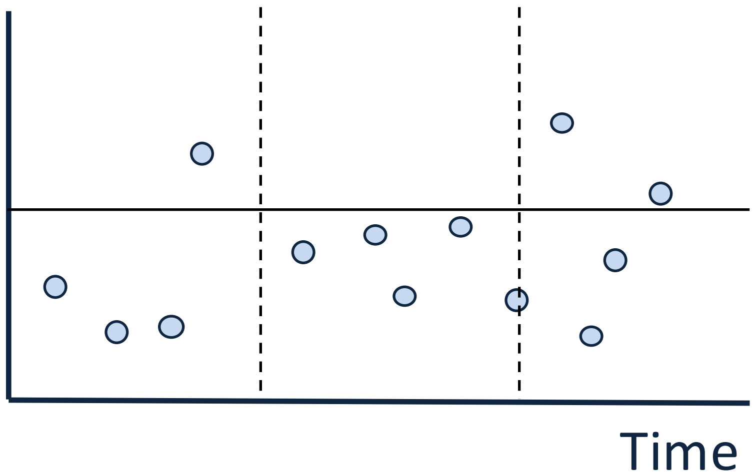
$$p(\mathbf{y}|\mathbf{f}, X)p(\mathbf{f}|X)$$

What is a Poisson process?

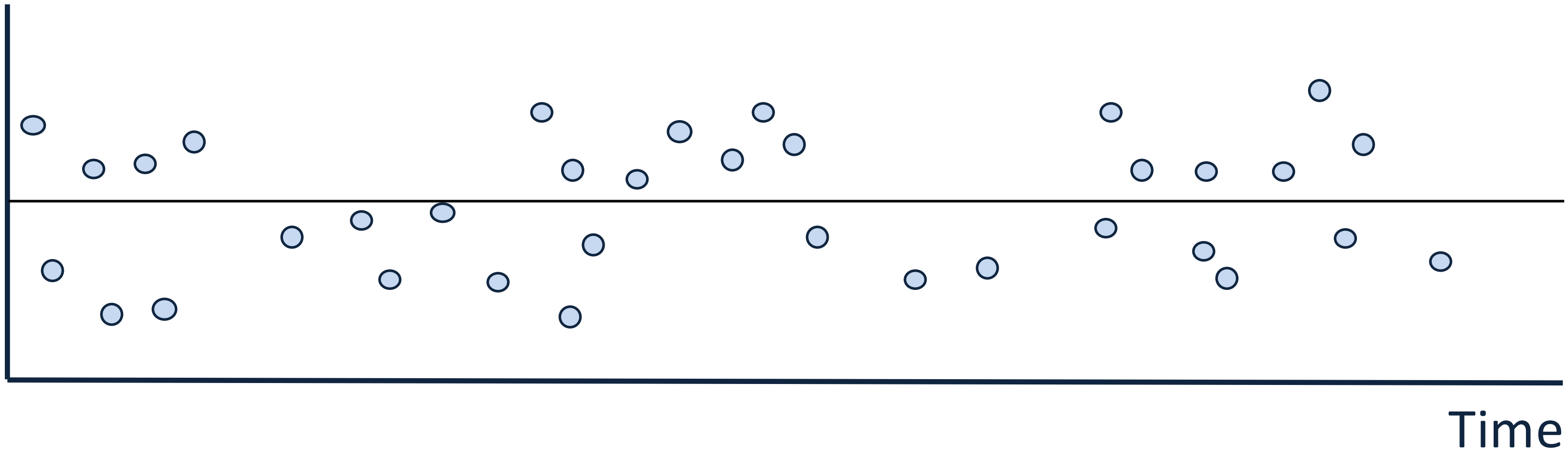


Rate of point appearance: λ(t)

What is a Poisson process?



Rate of point appearance: $\lambda(t)$

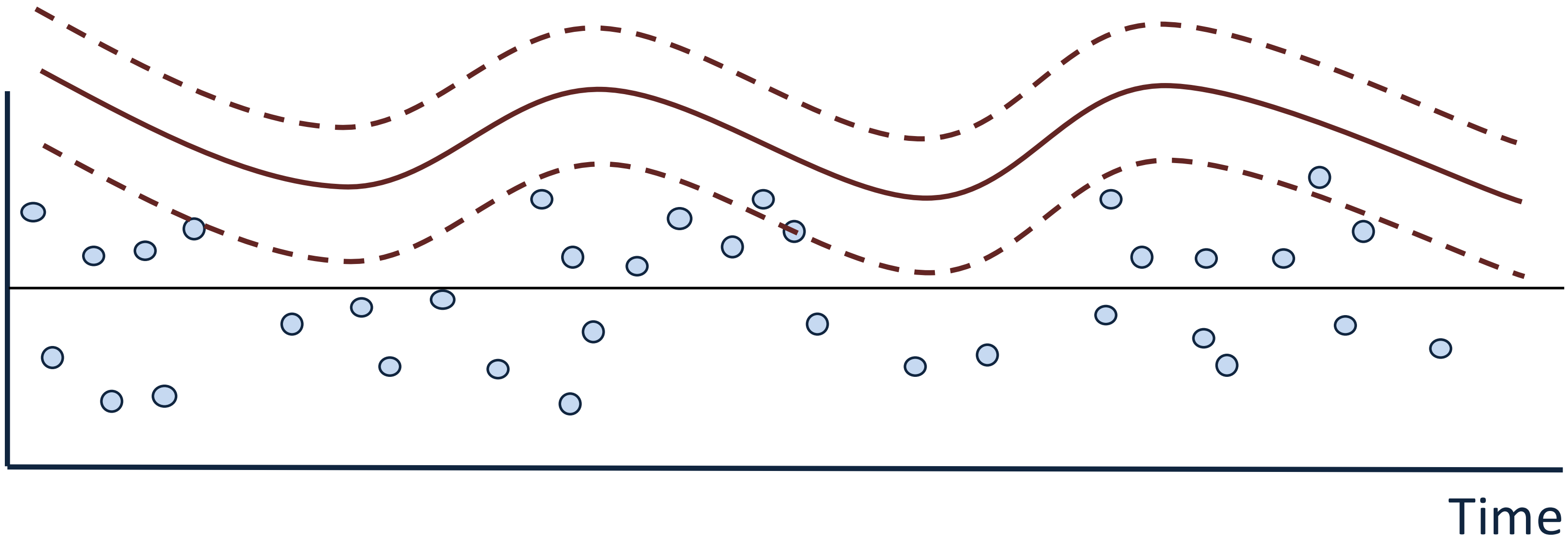


What is a Log Gaussian Cox Process?

… doubly stochastic Poisson process

… Gaussian process modulated Poisson process

… sigmoidal Gaussian Cox process



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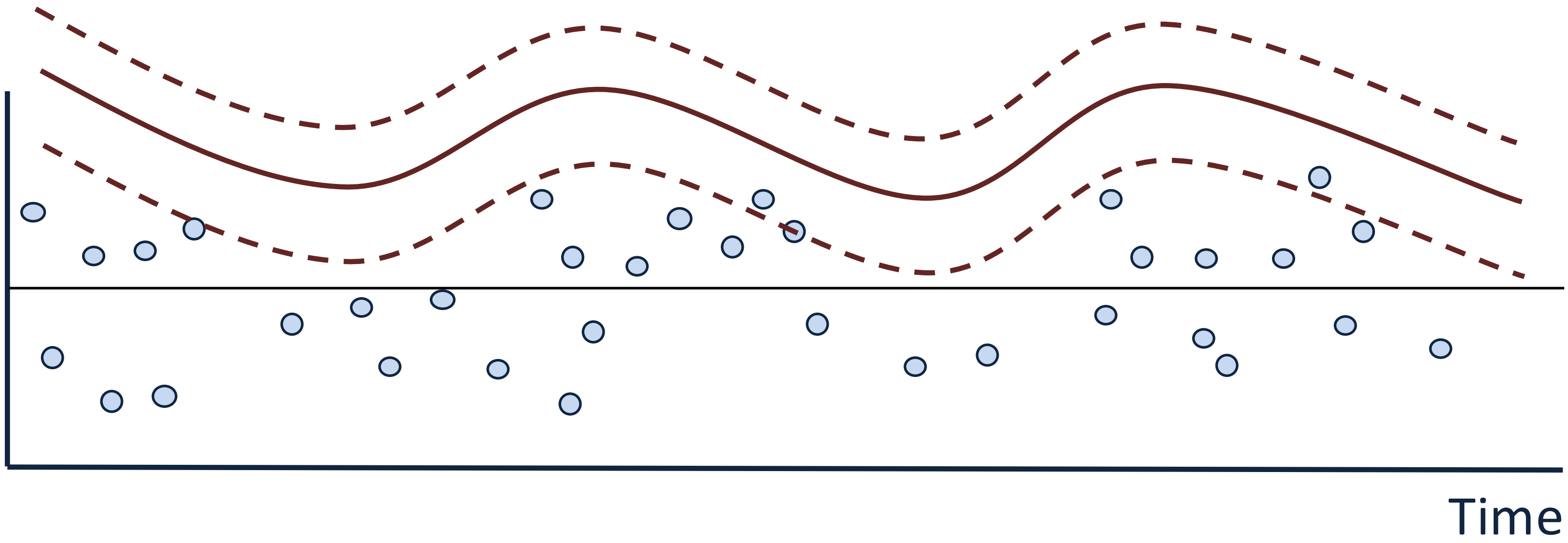
Cox process = inhomogeneous Poisson process with stochastic intensity



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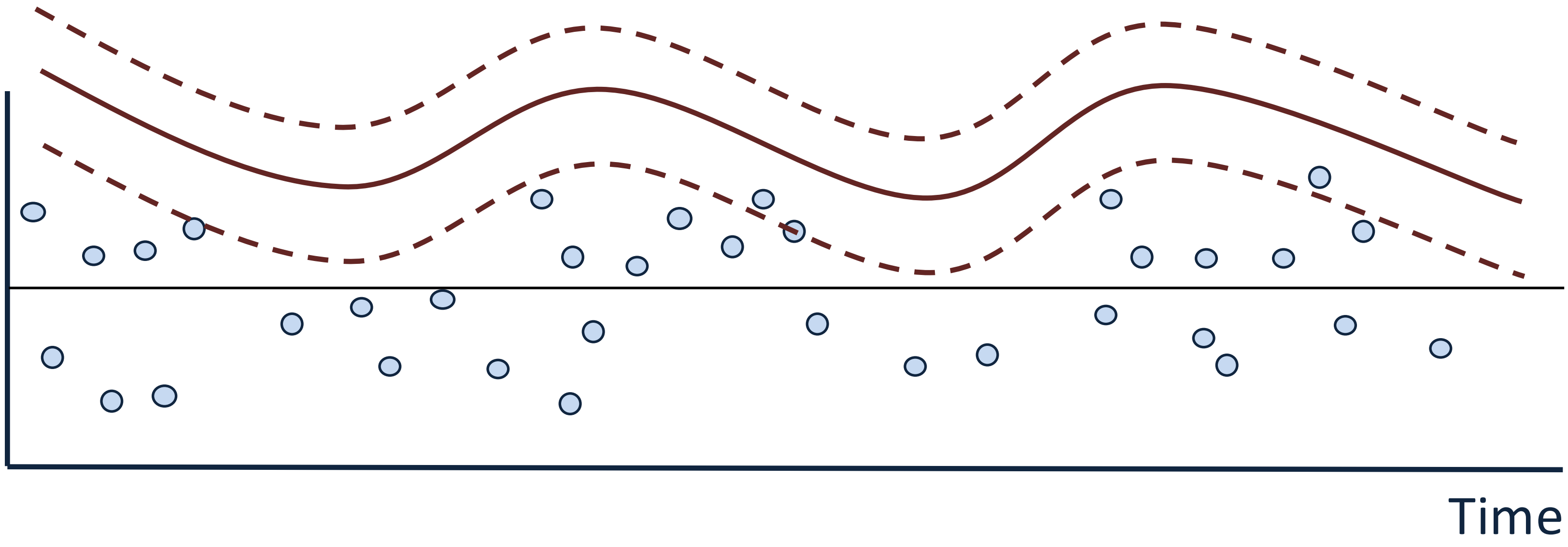
Cox process = inhomogeneous Poisson process with stochastic intensity

$$\mathbf{y}|\mathbf{f} \sim \mathcal{N}(\mathbf{f}, \sigma_n^2 I)$$

$$y_i|f(s_i) \sim \text{Poisson}\left(\exp[f(s_i)]\right)$$



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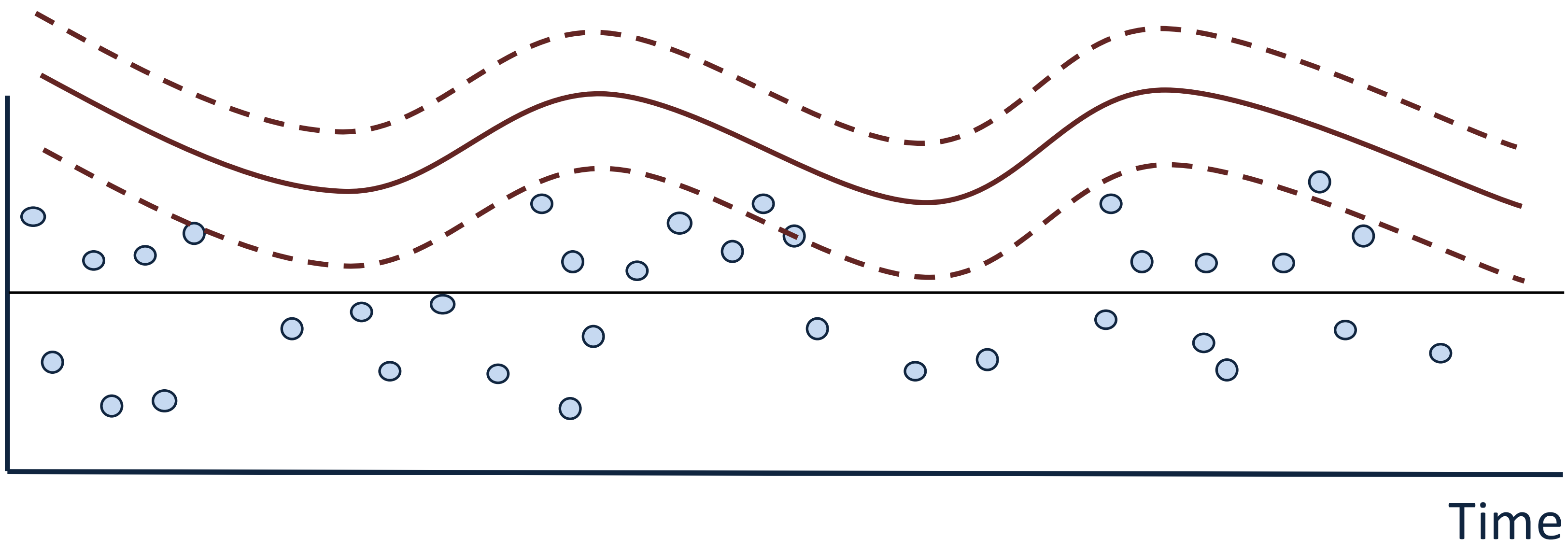
$$y_i|f(s_i) \sim \text{Poisson}\left(\exp[f(s_i)]\right)$$

Always need a positive intensity, so…

- Take exponential
- Sigmoid
- Square



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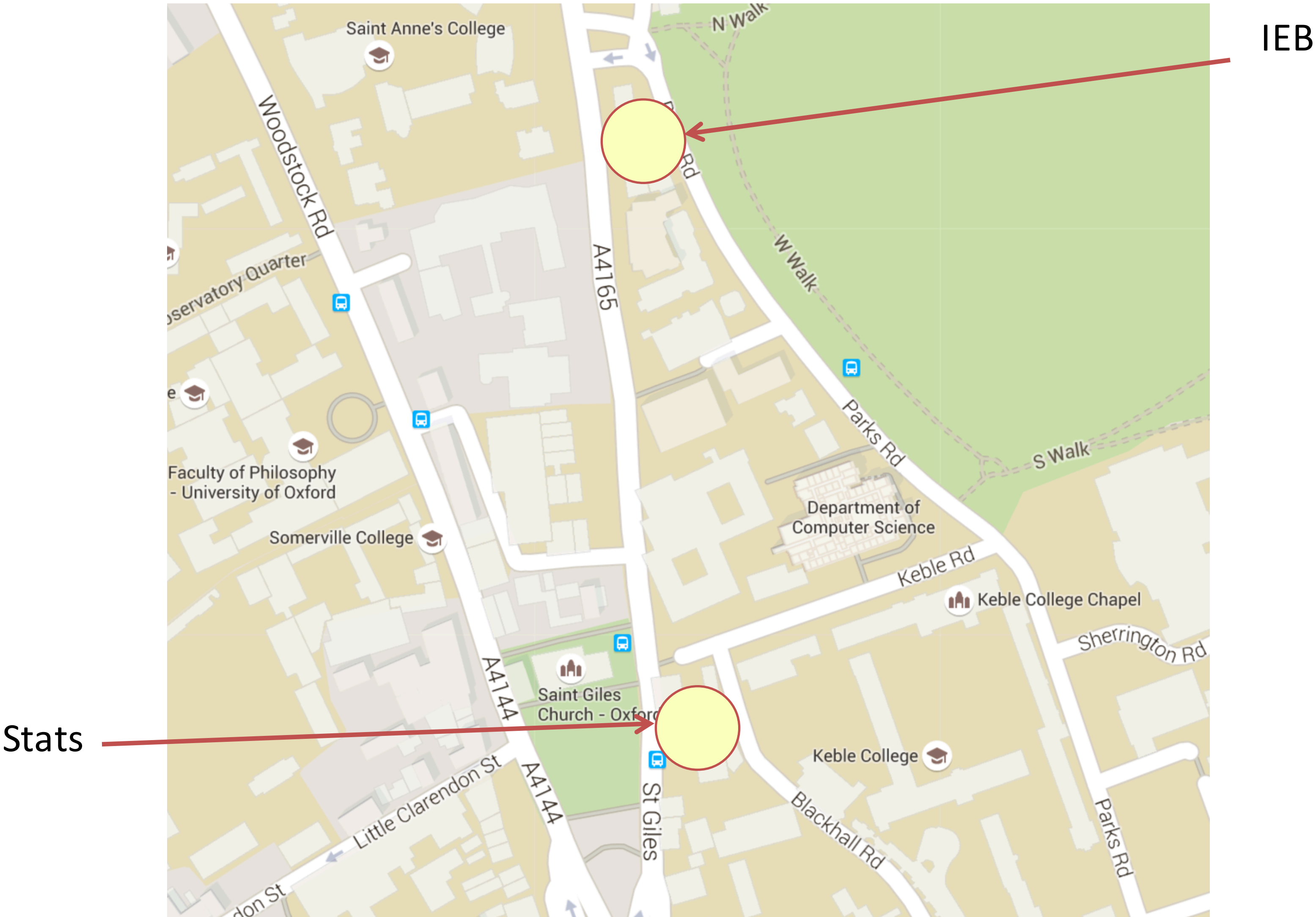
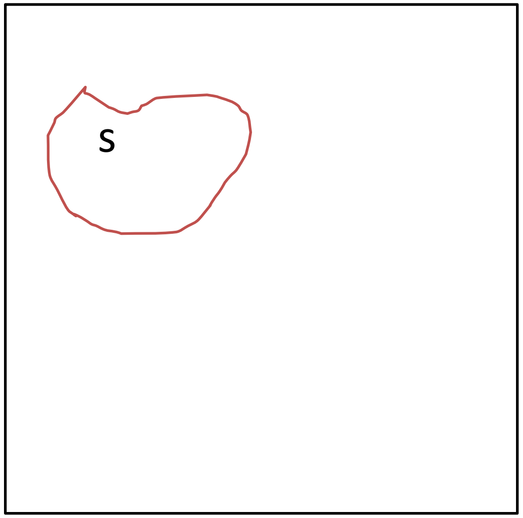
How can I do inference with this sort of model?



More model specifications...

Number of points inside a given space-time region:

$$y_S | \lambda(s) \sim \text{Poisson}\left(\int_{s \in S} \lambda(s)\, ds\right)$$



[Flaxman, et al. 2015. "Fast Kronecker inference in Gaussian processes with non-Gaussian likelihoods." ICML.]

More model specifications…

Number of points inside a given space-time region:

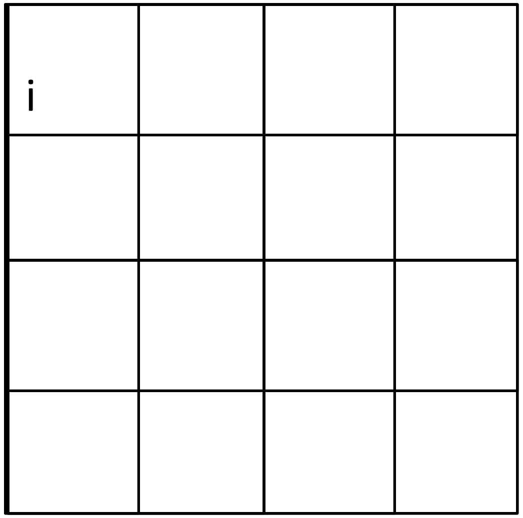
$$y_S|\lambda(s) \sim \text{Poisson}\left(\int_{s\in S} \lambda(s)\, ds\right)$$

Simplify by introducing a spatial grid,

y_i = count of points inside grid cell i

$$y_i|f(s_i) \sim \text{Poisson}\left(\exp[f(s_i)]\right)$$

$$f(s) \sim \mathcal{GP}(\mu(s), k_\theta(\cdot,\cdot))$$



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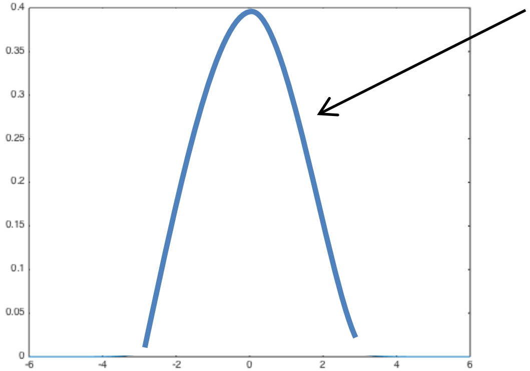
How can I do inference with this sort of model?

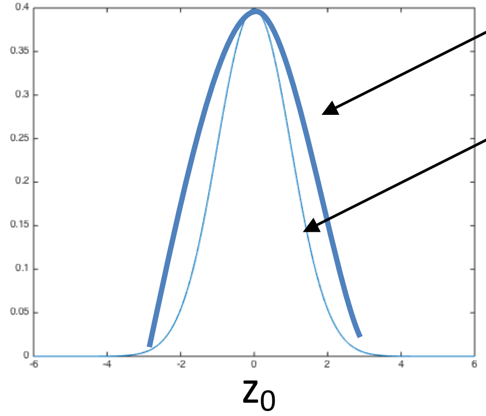
Laplace approximation

Posterior distribution, $p(\boldsymbol{f}|\boldsymbol{y}, X)$.



How can I do inference with this sort of model?

Laplace approximation



Posterior distribution, $p(\boldsymbol{f}|\boldsymbol{y}, X)$.

Gaussian approximation

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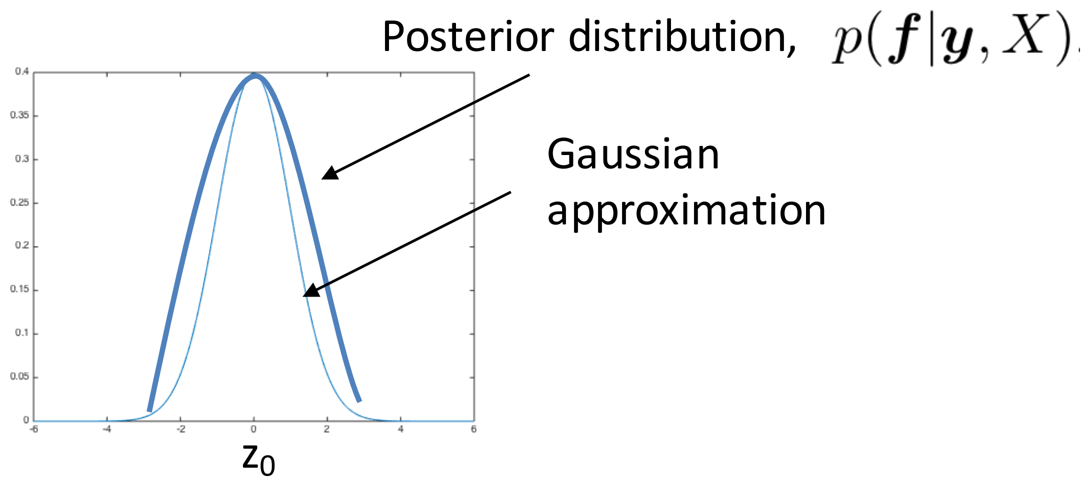
Laplace approximation



If assume a Normal centered at x_0,

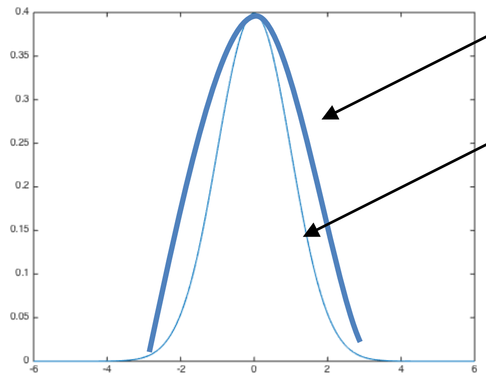
and take Taylor series expansion around x_0,

math works out to show that Gaussian approximation of distribution is:

posterior = $\mathcal{N}(\mathbf{z}|\mathbf{z}_0, \mathbf{A}^{-1})$

How can I do inference with this sort of model?

Laplace approximation



z_0

Posterior distribution, $p(\boldsymbol{f}|\boldsymbol{y}, X)$

Gaussian approximation

Need:

- Find maximum of posterior
- Hessian (-A) at maximum

If assume a Normal centered at x_0,

and take Taylor series expansion around x_0,

math works out to show that Gaussian approximation of distribution is:

posterior = $\mathcal{N}(\mathbf{z}|\mathbf{z}_0, \mathbf{A}^{-1})$

How can I do inference with this sort of model?

Laplace approximation + Kronecker methods

Can decompose GP kernel as a product of covariance matrices (because on grid)

$$K = K_1 \otimes \cdots \otimes K_D.$$

Need to do lots of inversions and log-determinants when doing GP regression

↓

Kronecker methods can speed this up quite a bit

How can I do inference with this sort of model?

Variational Bayes

No grid required, but do need inducing points
(...which can best be set on a rectangular grid)

[Lloyd, et al. 2015. "Variational inference for Gaussian process modulated Poisson processes" ICML.]

Sampling

Metropolis Hastings x 2
Hamiltonian Monte Carlo x 2

[Adams, et al. 2009. "Tractable nonparametric Bayesian inference in Poisson processes with Gaussian process intensities." ICML.]

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References of interest

	Transformation	Inference	Notes
Adams, et al. 2009. "Tractable nonparametric Bayesian inference in Poisson processes with Gaussian process intensities." ICML.	Sigmoid function	Multiple sampling schemes	
Flaxman, et al. 2015. "Fast Kronecker inference in Gaussian processes with non-Gaussian likelihoods." ICML.	Exponential	Laplace approximation	Implemented in GPML: http://www.cs.cmu.edu/~andrewgw/pattern/
Gunter, et al. 2014. "Efficient Bayesian nonarametric modelling of structured point processes." UAI.	Sigmoid function	Many sampling schemes	Multiple realizations from latent LCGP
Lloyd, et al. 2015. "Variational inference for Gaussian process modulated Poisson processes" ICML.	Square	Variational Bayes	

That’s all…