

WORCESTER POLYTECHNIC INSTITUTE

Longevity Insurance Pricing Development

A Major Qualifying Project Report for A requirement of the Degree of Bachelor of Science from
WORCESTER POLYTECHNIC INSTITUTE

Written By:

Rachael Fahey

Kimberly Maciejczyk

Craig Melillo

Daniel Roberge

Project Center: Worcester Polytechnic Institute

Term: A12-B12

Sponsoring Agency: N/A

Project Advisor: Jon Abraham

Abstract

This Major Qualifying Project (MQP) developed and priced a longevity insurance product for men and women between the ages 50 and 75 planning for retirement. Using current mortality rates and analyzing trends of future interest rates, simulations of random bond defaults and random deaths were performed to determine the costs and expected profitability of a longevity insurance product.

Acknowledgments

We would like to thank Professor Jon Abraham for providing us with guidance and support throughout this project to ensure that we considered all appropriate factors in pricing a longevity insurance product. His insight and recommendations were essential to the completion of our project, and we truly appreciate all of his help and support.

Thank you, Professor Abraham,

Rachael Fahey, Kim Maciejczyk, Dan Roberge and Craig Melillo

Table of Contents

Abstract.....	1
Acknowledgments	2
Table of Contents.....	3
Table of Tables.....	5
Table of Figures.....	5
1. Introduction	6
1.1 Longevity Insurance	6
1.2 Planning for Retirement.....	7
1.3 Existence of Longevity Insurance in the Market Today	9
2. Target Market	10
2.1 Defining a Need in the Market.....	10
2.2 Psychology of Potential Consumers.....	11
3. Investing the Premiums	12
3.1 Choosing an Investment Outlet	12
3.1.1 Investing in the Stock Market	12
3.1.2 Investing in Mutual Funds.....	13
3.1.3 Investing in Bonds	13
3.2 Selecting the Right Bond Type	14
3.2.1 Coupon Bonds versus Zero-Coupon Bonds	14
3.2.2 Municipal Bonds versus Corporate Bonds	15
3.2.3 Risk Associated with Bonds.....	15
4. Pricing and Modeling	18
4.1 Pricing Longevity Insurance	18
4.1.1 Determining the Pure Premium	18
4.1.2 Determining Total Costs Associated with Longevity Insurance	19
4.2 Modeling.....	23
4.2.1 Death Model	23
4.2.2 Bond Default Model.....	24
4.2.3 Investment Model.....	25
5. Conclusion.....	28
5.1 Profitability of the Product	28

Works Cited 31

Table of Tables

Table 1 – Moody’s Investor Services Bond Ratings.....	16
Table 2 – Total Direct Costs: Fixed and Variable Costs	21
Table 3 – Expected Costs of Single Net Premiums with and without Costs.....	22

Table of Figures

Figure 1 – Projected Spot Rate Yield Curves	17
Figure 2 – Model Simulation Data	29

1. Introduction

Over the past century, life expectancy in America has been on the rise due to various improvements in medicine and healthier lifestyles. According to an article by Stanley and Barney, among individuals who have reached age 65, about 20 percent of men and 30 percent of women are expected to reach age 90. For a 65 year old couple, there is a 50 percent chance that at least one individual will live to 92ⁱ. In another article by Kahalaf-Allah, Haberman, and Verall, mortality rates have been improving steadily in countries that are more developed, such as the United States and the United Kingdom. The authors go on to suggest that, while there may be some limit to human longevity, currently there is no evidence to suggest mortality improvements are decreasing to approach some lower limitⁱⁱ.

1.1 Longevity Insurance

As life expectancies improve, there is increasing concern among many people about outliving their wealth during retirement. This concern has more people looking into the annuity market and considering different products to mitigate this risk, such as longevity insurance. Longevity insurance is a deferred life annuity product that an individual can buy as part of their retirement planning. Once the individual reaches a certain commencement age, the annuity will then provide payments each year until the person dies. This way the individual can plan ahead and make sure their current wealth will sustain them until the annuity reaches maturity and they start receiving the annuity payments. The difference between this product and other annuity products that are available is that this annuity is usually deferred by 30-35 years.

Because this product is deferred for such a long time, this poses a type of risk known as longevity risk for insurers in the annuity market.

For insurers, mortality and longevity risk have been steadily on the rise. Mortality risk is the risk that future trends in mortalities differ from the current mortality assumptions in place for those companies selling annuitiesⁱⁱⁱ. Longevity risk consists of both specific and aggregate longevity risk. Specific longevity risk is the possibility that individuals will live well beyond their expected life time. Specific risk balances out over the long run since other individuals can be expected to die at younger ages; however, aggregate risk is what most insurers are worried about. Aggregate risk is the possibility that unanticipated changes in lifestyles or advances in medicine significantly improve the average lifespan^{iv}.

This risk, along with mortality improvements and low interest rates, caused by the stock market drop in 2002, has made investing in annuities extremely difficult. To deal with these issues, insurers have had to either raise the price of annuities, making them unaffordable or unattractive for many people, or have begun reducing the payments made to the insured.

1.2 Planning for Retirement

When selling longevity insurance, insurers must consider the issues for the people mulling retirement regarding their asset allocation. The traditional approach of investing in stocks, bonds or mutual funds is inefficient when trying to mitigate longevity risk for potential retirees.^v

While purchasing lifetime annuities seems like the most logical answer to the question: “how to hedge against longevity risk?” there are many that argue the premiums paid toward purchasing longevity insurance are worth more than the benefits received. This presents

another problem because lump sum fixed annuities have no liquidity, making them unattractive for people unwilling to commit to long term investments. Fixed lifetime annuities may also decline in real value over time due to rises in inflation.

When trying to attract buyers for longevity insurance, there are three potential customer profile types to choose from: people planning to leave an inheritance, people planning to consume all their wealth within their lifetime, and people with mixed consumption and inheritance motives^{vi}.

Men and women who tend to leave the majority of their assets as an inheritance would be reluctant to place their assets into annuities. They would be more inclined to invest in stocks and bonds; however, their asset allocation to stocks would decrease if they were inherently sensitive to risk. Because returns on annuities are more reliable than stocks and can be more profitable than bonds,^{vii} people planning to use their retirement funds entirely during their lifetime would theoretically invest solely in annuities. They would essentially be called the “die broke crowd” because they wish to maximize their income and spending while alive. People in this category would want to balance between investing in immediate fixed annuities and immediate variable annuities.

Most common though are the people who fall in the category of mixing their retirement plans. Investments made with the intentions to be left as an inheritance would use a mixture of investment products, such as bonds, stocks, mutual funds, etc. While investments made with the intent to be used up would probably be invested in fixed and variable annuities. These factors must be considered when establishing a target consumer for a longevity insurance product. In this paper, the target consumer’s profile will help determine whether the target

market is large enough to produce enough profit to warrant the longevity insurance product implementation.

1.3 Existence of Longevity Insurance in the Market Today

As longevity insurance products are annuities with deferment periods of at least 30 plus years, their existence in the current market is scarce. Genworth Financial Inc. and Symetra Financial used to issue longevity insurance, but have ceased to sell them. Genworth Financial does sell deferred annuities, but clients will only be paid ten to twenty years down the road^{viii}. Symetra offers a similar product as Genworth, but the annuities may only be deferred for 3, 5 or 7 years^{ix}.

Companies that offer products similar to traditional longevity insurance are The Hartford, New York Life Insurance Co. and MetLife. The Hartford sells guaranteed fixed income with an option that starts immediately or delays the income to when the client will need it. Clients can also purchase survivor benefits for an additional fee. New York Life offers late-in-life protection optional feature to its immediate Lifetime Income Annuity, called “changing needs”. Clients who purchased this would be able to collect immediately, but they would be able to increase or decrease their future payment as they wish. While the payout is less, MetLife offers a version of its Income Guarantee product that allows policyholders to collect when they need it and offers a death benefit as well as a liquidity option if needed. MetLife also offers a deferred income annuity product which is most similar to traditional longevity insurance^x.

In this paper, an appropriate price for this longevity insurance product is formulated, taking into account the current interest rates and various costs associated with creating a new

product. A model was then created to project different scenarios to see if the product would be profitable for an insurance company to sell.

2. Target Market

The longevity insurance product discussed in this paper has a very specific target market for potential consumers. Because the purpose of the product is to provide a structured plan for people with uncertainty in their retirement funds, the target market is directed toward adults between the ages of 50 to 75 seeking financial stability in their later years of life.

2.1 Defining a Need in the Market

A common problem that many adults face when planning for their retirement is how to evenly distribute their funds from the beginning of their retirement until death. While the start date for retirement can easily be determined, the time of death is unpredictable. So, for an individual who accumulates one lump sum when he or she retires (as opposed to annual pension funds), he or she must decide the most efficient method of allotting their wealth on a yearly basis. Using a maturity date of 85, our deferred annuity product will pay this individual annual payments starting at age 85 and continuing until death. This allows the purchaser to effectively plan exactly how he or she is going to spend their retirement savings on a scheduled basis for the rest of their life. Their accumulated wealth can be targeted to the years between retirement and age 85, at which point the annuity payments take over.

2.2 Psychology of Potential Consumers

When looking at the psychology of individual potential buyers, it is important to focus on relieving their concerns over purchasing an annuity that may be deferred for 35 years. Ideally this product would be marketed towards anyone looking to plan for retirement; however, because of the high cost of this product, it is likely to expect that only adults with an already developed retirement fund in place would be able to afford this. The high cost upfront can also lead prospective customers to be skeptical of whether or not they would truly receive enough value back for their investment. Because the annuity does not start benefits until age 85, there is the chance that the individual may never receive any payments and lose a rather large investment. By nature, people can be apprehensive to commit to long term investments because they do not want to wait long periods of time to receive their return on investment, even if it is guaranteed. In response to these claims, this product will theoretically provide a low risk of loss, as opposed to investing in stocks which are unstable and unpredictable. Also, the buyer knows exactly what to earn on their investment.

Considering these concerns, this product will still be marketed toward those individuals who are planning on retiring and between the ages of 50 and 75. Financial stability and financial certainty are the two biggest reasons an individual would choose to purchase this product. This longevity insurance product will also allow people to make precise plans for their future and it will help eliminate the anxiety that would surround budgeting future spending.

3. Investing the Premiums

To obtain enough capital to cover the benefits for the buyers of the longevity insurance, the single net premiums need to be invested in blocks of low risk, high yield securities. The goal of investing these premiums is to redeem 100% of the initial capital invested plus interest within a specific time frame. Because not every longevity insurance issued will reach maturity at the same time, these securities need to be structured so they will pay out to the buyers in accordance with their longevity insurances' maturity dates. The biggest obstacle encountered when investing is finding balance between minimizing the probability of default without sacrificing high returns.

3.1 Choosing an Investment Outlet

In this MQP, different types of investment options were evaluated based on volatility and profitability. The different types of investment opportunities were narrowed down to stocks, mutual funds, and bonds.

3.1.1 Investing in the Stock Market

The stock market is a market where businesses issue and purchase shares of their company. The share prices are based on company performance of the past present and future. Additionally, it provides companies with access to capital and investors to claim partial ownership of companies.^{xi} Investments in the stock market are perceived as risky because stock prices fluctuate up and down daily with unlimited range. For this reason, it was found that the stock market's unpredictability would not make for a reliable investment option.

3.1.2 Investing in Mutual Funds

The second option evaluated was mutual funds. A mutual fund is an investment outlet managed by one or more people where money is taken in from investors and reinvested in different opportunities, like the stock market or bonds. The combined holdings of stocks, bonds and other assets are known as a portfolio. Each investor in the fund owns shares based on the sized of portfolio, which represent a part of the total holdings.^{xii} Since this option is dependent on variables such as the stock market, it very little predictability; therefore it is not a viable option for investment in this MQP.

3.1.3 Investing in Bonds

The third option evaluated was bonds. A bond is a structured investment tool where one party purchases a premium from another and in return receives a set number of interest payments called coupons over a certain time frame, plus the initial premium at the end of the time frame. The coupon amounts are determined by the interest rate and by the size of the initial premium agreed upon between the two parties. With bonds, the probability of default can be reduced by examining bond ratings. Financial agencies, such as the Moody's Investors Service, offer bond ratings that specify the riskiness of a bond. Because bonds have structured coupon rates, designated maturity dates and default ratings it was decided that they were the best option for investment for this MQP.

3.2 Selecting the Right Bond Type

Bond types vary due to different factors based on payment structure, the rate of return, the issuer of the bond, and risk. Choosing the right type from the different combination of factors will provide the best investment strategy for this MQP

3.2.1 Coupon Bonds versus Zero-Coupon Bonds

The two payment structures considered for this MQP were zero coupon bonds and coupon bonds. A zero coupon bond is structured so all coupons are delayed to the end of the lifetime of the bond. It is unique because rather than seeing coupons over the life of the bond they are paid out at the maturity date along with the premium. An issue with this payment method is that if the bond defaults at any time before the maturity date the buyer will not see receive any coupons over the life of the bond. This concern made zero coupon bonds unattractive for this MQP.

Coupon bonds yield a coupon (whose amount is dependent on the interest rate) occurring at the end of each year starting from when the bond was purchased and continuing until the bond reaches maturity. For example, if a \$1,000 ten year bond was purchased with a 10% interest rate; a coupon of \$100 would be received every year for ten years. On the tenth year, both the \$100 coupon and \$1,000 premium would be paid back. This payment structure was found to be less risky than the zero-coupon bond because if default occurred any time after the first year, the owner of the bond would still receive some coupons. For this reason, only coupon bonds were used in this MQP.

3.2.2 Municipal Bonds versus Corporate Bonds

The next bond characteristic examined was the issuer of the bond. The issuer of a bond can be classified into two groups; municipal bonds and corporate bonds. Municipal bonds are issued by towns and cities to fund public works projects. While municipal bonds have very low probabilities of default, there are not enough of them in circulation to provide the amount of capital needed to cover the all of the cash flows occurring in this MQP.

Corporate bonds are issued by companies to provide capital to carry out projects and other day to day operations. Corporate bonds are less reliable than municipal bonds as the institution of a state or town is more likely to remain intact as opposed to a private business, which always has the chance of dissolving. However, bond rating agencies like Moody's allow buyers to differentiate between the stable and unstable corporate bonds. Because corporate bonds exist in high abundance combined with their ability to be rated based on their probability of default, it was decided that only corporate bonds would be used in this MQP.

3.2.3 Risk Associated with Bonds

As previously mentioned, bond rating agencies exist to determine the reliability of corporate bonds. The bonds used in this MQP were chosen based on the ratings provided by Moody's Investors Service^{xiii}. Moody's Investors Service examines a company's performance, finances and history with other investors to determine how credible the company is in repaying its debts. The rating system ranges from grades of Aaa, which means a bond has a relatively low probability of default but also has a relatively low interest rate^{xiv}, to C, which means a bond has

a relatively high probability of default as well as a relatively high interest rate^{xv}. These ratings were the key determinant in selecting the type of corporate bonds to purchase.

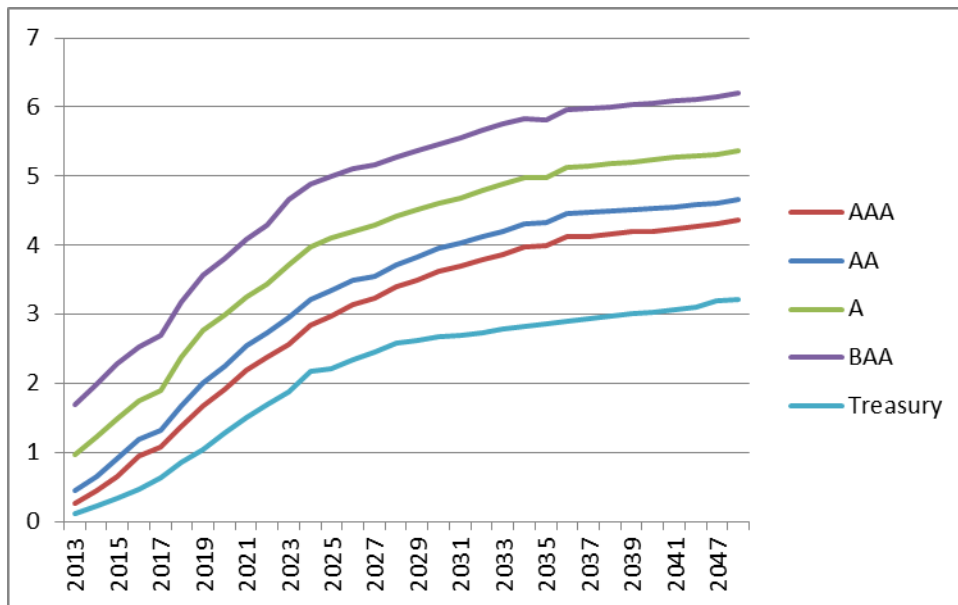
In Table 1, the bond rating along with the corresponding probabilities of default found using Moody's Investors Service^{xvi} is shown.

Table 1 – Moody's Investor Services Bond Ratings

Moody's Bond Rating	Probability of Default
Treasury	0
Aaa	0.52
Aa	0.58
A	1.29
Ba	4.64
Ba	19.12
B	43.34
Caa-C	69.18

Using these ratings a yield curve was developed for each bond type projecting the trend of expected interest rates from year to year.^{xvii} The projected yield curves are shown in Figure 1.

Figure 1 – Projected Spot Rate Yield Curves



The expected interest rates shown in Figure 1 represent the expected rate of return that would be used for different bonds purchased during a specific year.^{xviii} After the potential risks and rewards of different bonds were assessed, it was concluded that the only bonds to be used for investing in this MQP would be Aaa, Aa and A. The bonds purchased with these ratings would vary in their maturity dates depending on when funds would be needed for the payment of benefits.

4. Pricing and Modeling

In this chapter, the face value of the longevity insurance and the expected cash flows to occur over the life of the longevity insurance product were determined. These figures were found using mortality rates from the GAM 94 table, costs associated with developing insurance products, and random models simulating the insurance payouts based on the random occurrence of death.

4.1 Pricing Longevity Insurance

In order to come up with an appropriate price for this longevity insurance product, two factors associated with the premium were examined: the pure premium, which takes into account the payments' present value made to the annuitant and the probability of having to make payments for people at different ages, and additional costs that are needed to create and maintain the product.

4.1.1 Determining the Pure Premium

To calculate the price's pure premium, it was assumed that an individual would receive \$65,000 payments commencing when the individual reached age 85, and they would last until the annuitant's time of death. These cash flows were then discounted back to time zero, the time when the individual purchased the annuity, using the current and future projected spot rates.

To find the probabilities of death for an individual of a certain age, the calculations performed used the 1994 Group Annuity Mortality table, or GAM-94. The GAM-94 table was

created by building upon the data used to develop the GAM-83 table. Scholars found, between the years 1986-1990, there was a steady decline between actual mortality experiences compared to expected mortalities calculated for GAM-83. This trend was prevalent within the male population. The GAM-94 is mainly used for insurance reserves and includes a 7% margin of error built in. This extra margin takes into account any differences in the work type, income level, or geography that can be experienced in a large annuitants group^{xix}.

Using this table, the mortality aspect was included in the annuity calculation. Once the payments' present value was determined, the present value was multiplied by the probability that an individual lives long enough to receive a benefit and then die within the next year. The sum of these expected present value payments was calculated, providing an expected payout for individuals between the ages 50 to 75. It is important to note, this value only takes into account the amount paid to the annuitant; it does not take into account the additional expenses or profits that are associated with this product's upkeep and the business as a whole.

4.1.2 Determining Total Costs Associated with Longevity Insurance

The total expenses associated with the development and application of an insurance product can be divided into two different categories: direct costs and indirect costs. Direct costs are the expenses associated with the direct material used in creating the insurance product. This includes salaries designated for research and development, marketing, sales etc. It was found that the positions within an insurance company that accounted for the direct costs were: actuarial consultant, strategy and planning consultant, loss prevention consultant, marketing

and communications consultant, administrative services, accounting consultant, investment consultant, underwriting consultant and sales agent.

The estimated costs for each role were based on current average salaries for each position and also the length of time for which each person would be needed during the development process. However, the sales commission for the sales agent was dependent on the amount of the single net premium. On average, life insurance agents make 6% on products they sell that fall under accounts receivable for longevity insurance. Commission for insurance agents varies based on the company employing them as well as the type of insurance being sold. Typically, insurance agents collect higher commissions on whole-life insurance policies as opposed to term insurance policies. The Wall Street Journal reported that the average commission rate for an insurance agent on an annuity today is 6.3% of the principal payment.^{xx} Since this longevity insurance is a type of life annuity, it will be expected that the commission rate will be greater than 6.3%.

In addition, the cost of the insurance agent to the insurer is contingent on the payment method of the principle payment. With a single net premium, the commission is likely to be paid in a lump sum, as opposed to a payment structured annually. For the longevity insurance product being developed in this MQP a 7.0% commission rate will be assumed. However in addition to the sales agent, an extra 3.0% can be added on to account for upper management positions within the sales team, resulting in 10.0% of premiums being allotted to sales and marketing. Table 2 outlines the total direct costs for the development of the longevity insurance product.

Table 2 – Total Direct Costs: Fixed and Variable Costs

Direct Fixed Costs ^{xxi}		Direct Variable Costs ^{xxii}	
Cost of Actuaries	\$61,937.00	Sales Commission	(7+3) = 10%
Strategy and Planning Consultant	\$29,086.67		
Loss Prevention Consultant	\$22,768.67		
Marketing and Communications Consultant	\$20,832.33		
Administrative Services	\$40,285.00		
Finance and Accounting	\$47,500.00		
Investment Consultant	\$52,500.00		
Underwriting Consultant	\$53,000.00		
Total Direct Fixed Costs: \$327,909.67		Total Variable Costs: 10% of revenue	

The indirect costs for this project are all the costs related to the day to day activities within an insurance company (not associated with salaries). These costs are called overhead costs and were determined using an overhead rate. Based on a series of current national surveys for firms of all sizes, the overhead rate for this project was assumed to be 2.45^{xxiii} (before any type of tax or bonus is paid out). This ratio compares the indirect expenses to direct expenses. The combination of indirect and direct expenses was \$1,131,288.36. Using these costs and the time value of money for benefits of \$65,000.00 potentially paid for thirty five years, single net premiums were calculated for men and women, ages 50 to 75, purchasing

longevity insurance at time zero. The cost of each longevity insurance product is shown in Table

3.

Table 3 – Expected Costs of Single Net Premiums with and without Costs

Expected Cost of Single Net Premium				
Age	Male		Female	
	Without Costs	With Costs	Without Costs	With Costs
50	\$ 45,937.75	\$ 136,207.80	\$ 58,874.73	\$ 151,150.01
51	\$ 47,902.57	\$ 138,477.16	\$ 61,630.92	\$ 154,333.41
52	\$ 49,943.95	\$ 140,834.96	\$ 64,508.23	\$ 157,656.70
53	\$ 52,066.80	\$ 143,286.85	\$ 67,515.43	\$ 161,130.02
54	\$ 54,278.24	\$ 145,841.06	\$ 70,660.03	\$ 164,762.03
55	\$ 56,583.95	\$ 148,504.16	\$ 73,949.92	\$ 168,561.85
56	\$ 59,054.54	\$ 151,357.69	\$ 77,466.50	\$ 172,623.50
57	\$ 61,705.45	\$ 154,419.49	\$ 81,231.38	\$ 176,971.94
58	\$ 64,514.05	\$ 157,663.42	\$ 85,217.85	\$ 181,576.31
59	\$ 67,513.81	\$ 161,128.15	\$ 89,463.53	\$ 186,480.07
60	\$ 70,717.52	\$ 164,828.43	\$ 93,992.77	\$ 191,711.34
61	\$ 74,089.67	\$ 168,723.26	\$ 98,765.57	\$ 197,223.93
62	\$ 78,009.41	\$ 173,250.56	\$ 104,224.00	\$ 203,528.41
63	\$ 82,099.96	\$ 177,975.15	\$ 109,933.72	\$ 210,123.14
64	\$ 86,690.88	\$ 183,277.66	\$ 116,269.55	\$ 217,441.02
65	\$ 91,689.62	\$ 189,051.21	\$ 123,137.53	\$ 225,373.54
66	\$ 97,187.29	\$ 195,401.01	\$ 130,645.03	\$ 234,044.70
67	\$ 103,171.17	\$ 202,312.40	\$ 138,742.19	\$ 243,396.92
68	\$ 109,741.76	\$ 209,901.43	\$ 147,561.76	\$ 253,583.53
69	\$ 116,841.20	\$ 218,101.28	\$ 157,050.97	\$ 264,543.57
70	\$ 124,674.05	\$ 227,148.23	\$ 167,397.62	\$ 276,493.95
71	\$ 133,020.00	\$ 236,787.80	\$ 178,367.50	\$ 289,164.16
72	\$ 142,124.09	\$ 247,303.02	\$ 190,146.52	\$ 302,768.92
73	\$ 152,042.57	\$ 258,758.86	\$ 202,829.55	\$ 317,417.82
74	\$ 163,222.19	\$ 271,671.32	\$ 216,879.42	\$ 333,645.43
75	\$ 175,571.37	\$ 285,934.63	\$ 232,228.95	\$ 351,374.13

4.2 Modeling

There were three different models used for the development of the longevity insurance model. A death model was used to simulate the random occurrence of death for men and women between the ages of 50 and 75 who purchased the product. A Bond Default Model was created to simulate the default rates of corporate coupon bonds differing in ratings of Aaa, Aa, and A. These bonds also varied in their time until maturity from ten year bonds to forty year bonds. After both models were completed, they were combined to generate the total expected cash flows over a seventy year period for the collection of premiums, the investment of premiums in bonds, the fixed costs for each year and the benefits paid out to each annuitant.

4.2.1 Death Model

In order to simulate how many individuals within a group of annuitants would receive benefits and how long we can come to expect to keep paying individuals, a model was created that simulated the life spans of the group of annuitants. Using the Excel function, rand(), and mortality rates provided by the GAM 1994 Table, a random number was generated. If the random number generated was less than or equal to the mortality rate for a single individual in one year then the individual would die within that year, however, if the random number was greater than that mortality rate, the individual survived. This concept was reapplied to each surviving annuitant to see if the individual survived to the next year, and so on, until the annuitant reached the upper limit age of 120. The number of annuitants from age 50 to 75 expected to purchase the longevity insurance product was specified, and the process was

applied to 1000 men and women combined. To account for any random occurrences of unusually high or low death rates, this procedure was performed 100 times. In addition to producing the number of people who survived from year to year, this model also produced the age of each person for each year that they survived. Since the death model yielded the number of people that survived from year to year, along with their age, it also provided the expected total number of benefits that each annuitant would expect to receive.

4.2.2 Bond Default Model

The bond default model for the premiums was able to employ the same concept used in the death model because the default of a single bond follows the same pattern as the loss of a single life. Each bond had a probability of defaulting each year just like each person had a probability of dying. The Excel function rand() was used to generate a random number and if the number was less than the probability of default, the bond would fail and the expected coupons would not be seen. Vice versa, if the random number generated was greater than the probability of default, the bond would survive and continue to the next year paying out coupons each year until it reached maturity. Upon reaching maturity it paid out the face value as well as the coupon.

When the bonds were purchased, 40% of the assets were used to by AAA rated bonds which had a 0.52% default rate, 30% were used to by AA rated bonds which had a 0.58% default rate and the final 30% was used to purchase A rated bonds which had a 1.29% chance of default. It is important to note, as mentioned earlier in Section 3.2.3 on bond ratings, a higher probability of default indicates a higher rate of return.

The bond default model had an extra layer in comparison to the death model though. Because coupons were collected by the insurer each year, the coupons were eligible for reinvestment. Since the expected benefits had the potential to be extremely high, it was necessary to hedge against this risk by continuing to purchase bonds well after the first year. At the end of each year when the coupons were collected or the face value of a bond was redeemed, costs were extracted as well as profits for the insurers. The remaining funds were reinvested in Aaa, Aa and A rated bonds, with 40% of the funds going towards Aaa, 30% of the funds going towards Aa bonds and the remaining 30% going towards A bonds. This was repeated at the end of each year during the trials of the Bond Default Model.

4.2.3 Investment Model

The investment model created to simulate the cash flows in the application of the longevity insurance product combined the death and the bond default models, as well other aspects that directly affected the cash flows. After the number of men and women at different ages purchasing the product was determined, those numbers were multiplied by the present value of the single net premiums for the longevity insurances. Assuming 1000 individuals purchased this product, with 60% of this population being male and 40% female, it was determined that the initial revenue collected from a one-time sale of this product was \$165,442,306.43.

In the first year of investment, it was determined that only blocks of ten forty-year \$1,000.00 bonds would be purchased. However, these ten \$1,000.00 bonds behaved identically to one \$10,000.00 bond, so for the purpose of the model each block of bonds purchased is considered to be one bond.

As each year passed, the lives of each annuitant followed the same death model as the one previously described in Section 4.2.1. For this model though, with the loss of each life meant a series of benefits that would not be paid out upon the annuitant reaching age 85. Similarly, the life of each bond followed the same default model as the model described in Section 4.2.2. So for each year, every bond that defaulted meant the loss of a series of coupons as well as a \$10,000.00 premium. In addition to the lost capital from defaulted bonds, at the end of each year after all the coupons were collected, an amount of \$1,131,288.36 was taken out to account for total fixed costs. Once all the yearly cash flows were subtracted, the remaining funds were reinvested in ten year \$10,000.00 Aaa, Aa and A rated bonds using the same proportion of bond ratings referenced in Section 4.2.2. These practices were carried out at the end of each year barring two events; paying out benefits and negative cash flows.

After ten years of investing and reinvesting, when the first round of annuitants who survived to age 85 were finally eligible to receive the \$65,000.00 benefits needed to be accounted for in yearly cash flows. At the end of each year starting ten years after the initial purchase of longevity insurances, \$65,000.00 was subtracted for every man and woman who survived to age 85, and for every man and woman who survived another year after age 85. This cash flow was taken out before any fixed costs or reinvestments were accounted for.

Because numerous trials of this model were performed, it produced scenarios displaying combinations of high default rates and low mortality rates; this led to the problem of negative cash flows. To account for this, whenever negative cash flows were observed, bonds were sold back for the net present value of their remaining coupons plus the net present value of their

redemption amount. This decreased the value of future assets, but it helped balance the cash flows so benefits and fixed costs could be paid.

5. Conclusion

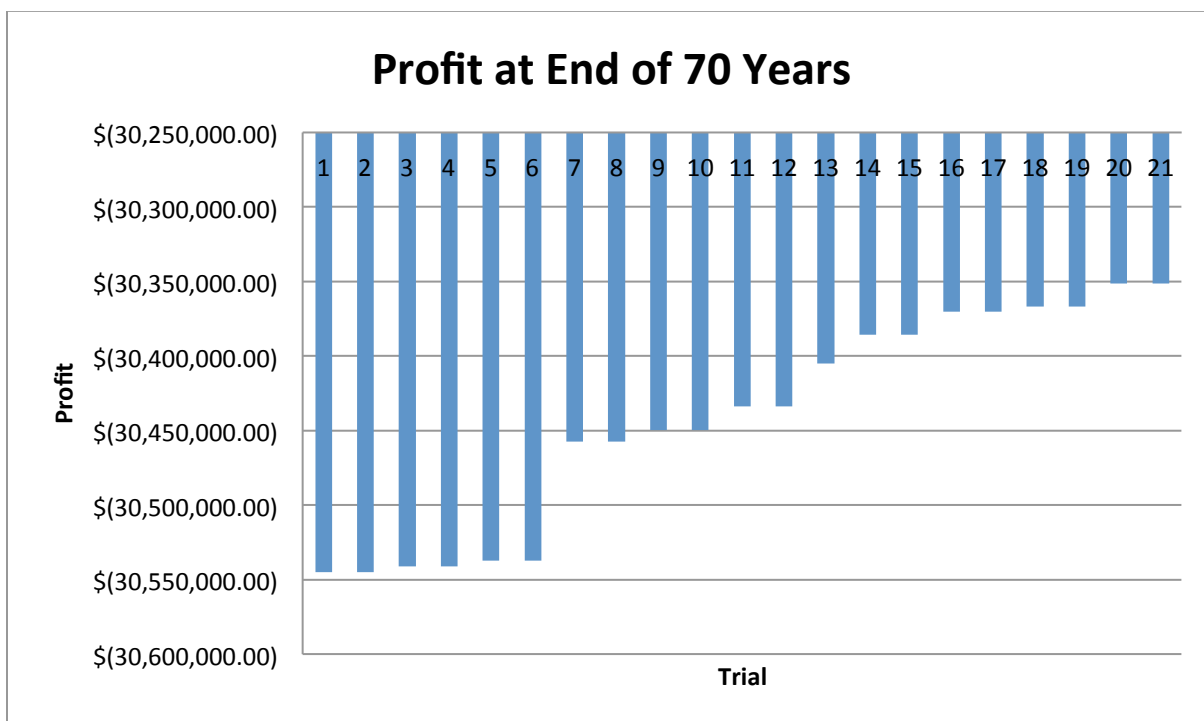
After different scenarios were modeled, the final profit recorded at time 70 was observed to see if the longevity insurance would be a profitable product to sell. The following chapter describes the data observed along with the factors that helped determine the outcome.

5.1 Profitability of the Product

In the vast majority of the model's trials, positive profits were rarely seen. Because of low interest rates on the bonds being purchased in the reinvestment portion of the project, almost all of the cash flows coming in needed to be paid out for yearly costs and for the benefits being paid out. This left little to no money left to be collected as profit. When trying to combat this problem, it was revealed that it could not be solved by reinvesting in more profitable bonds because their high default rates led to even worse losses.

From these observations, the observed profit averaged a loss of about -\$400 million in each trial. Simulations were then ran by changing the interest rates of the bonds to be 5%, 6%, and 7% for Aaa, Aa, and A bonds, respectively. With this, we also assumed a higher premium for the bond: Instead of bonds being worth \$10,000 at maturity, we assumed the bonds were each worth \$100,000. This gave us a much higher coupon payment, making reinvestment possible and allowing us to cover some of our expenses. This, however, proved to also be unprofitable, observing losses around -\$30 million in all 21 trials.

Figure 2



It was determined that the longevity insurance product developed in this project would not be a profitable product to sell. In today's economy, interest rates are still much too low to support a product of this type. Increasing the price of the single net premium to increase profits would also not be a feasible solution to attaining higher profits because the higher prices needed would become much too unattractive for consumers. Another idea would be to lower the amount received each year. While this may be a good idea, consumers may still be unwilling to purchase a product of this type, when they can just save the money themselves.

In order to have a profitable product in the future, we would have to see a brand new market. Interest rates will have to be much higher than they are today. There must also be lower risk for bonds of a lower grade, meaning that investing in bonds with a higher rate and higher probability of default will not default very often. Mortality assumptions, unfortunately,

must also rise. If there is a higher probability that someone will not live longer than 85, the payouts promised by the insurer will not be as great, thus leading to more profit.

Even though the results from this model provided accurate data, there were obstacles within the pricing process that prevented more precise results from being achieved. If a more user friendly programming language than visual basic had been used to write the functions in this project, it would have allowed for more pricing factors to have been considered. Because of difficulties with the programming language all of the bonds used in this project were assumed to be forty year bonds. Had a more strategic reinvestment strategy been used, greater profits might have been seen. Also, because of the slow run time with the model, there was a limit to the amount of trials that could be run using the model created in Microsoft Excel. Had a program with a faster run time been used, then more trials could have given insight into what needed to be done to achieve positive profits.

Works Cited

ⁱ "Retirement Standard Risk Review." Morgan Stanley Smith Barney LLC, 2011. Web. 2 Nov. 2012.
<<http://www.topretirements.com/blog/wp-content/uploads/2012/09/Risk-Review.pdf>>.

ⁱⁱ Khalaf-Allah, M., S. Haberman, and R. Verrall. "Measuring the Effect of Mortality Improvements on the Cost of Annuities." *ScienceDirect.com*. Web. 12 Oct. 2012.
<<http://www.sciencedirect.com/science/article/pii/S0167668706000497>>.

ⁱⁱⁱ Bauer, Daniel. "Pricing Longevity Bonds Using Implied Survival Probabilities." DFG Research Training Group, Apr. 2006. Web. 23 Oct. 2012. <<http://www.ifa-ulm.de/downloads/ImpliedSurv.pdf>>.

^{iv} Blake, David, Tom Boardman, and Andrew Cairns. "The Case for Longevity Bonds." Center for Retirement Research at Boston College, June 2011. Web. 30 Oct. 2012. <http://www.pensions-institute.org/commentaries/The_case_for_Longevity_bonds_June2010.pdf>.

^v Chen, Peng, and Moshe A. Milevsky. "Merging Asset Allocation and Longevity Insurance: An Optimal Perspective on Payout Annuities." 20 Feb. 2003. Web. 20 Nov. 2012.
<<http://corporate.morningstar.com/ib/documents/MethodologyDocuments/IBBAssociates/LongevityInsurance.pdf>>.

- ^{vi}Chen, Peng, and Moshe A. Milevsky. "Merging Asset Allocation and Longevity Insurance: An Optimal Perspective on Payout Annuities." 20 Feb. 2003. Web. 20 Nov. 2012.
<<http://corporate.morningstar.com/ib/documents/MethodologyDocuments/IBBAssociates/LongevityInsurance.pdf>>.
- ^{vii}Chen, Peng, and Moshe A. Milevsky. "Merging Asset Allocation and Longevity Insurance: An Optimal Perspective on Payout Annuities." 20 Feb. 2003. Web. 20 Nov. 2012.
<<http://corporate.morningstar.com/ib/documents/MethodologyDocuments/IBBAssociates/LongevityInsurance.pdf>>.
- ^{viii} "Annuities Overview." *Annuities Retirement Planning*. 15 Dec. 2011. Web. 12 Nov. 2012.
<http://www.genworth.com/content/products/retirement_solutions/Annuities.html>.
- ^{ix} Chen, Peng, and Moshe A. Milevsky. "Merging Asset Allocation and Longevity Insurance: An Optimal Perspective on Payout Annuities." 20 Feb. 2003. Web. 20 Nov. 2012.
<<http://corporate.morningstar.com/ib/documents/MethodologyDocuments/IBBAssociates/LongevityInsurance.pdf>>.
- ^x "Is Longevity Insurance Right for You?" *Bankrate.com*. 13 June 2011. Web. 10 Oct. 2012.
<<http://www.bankrate.com/finance/retirement/is-longevity-insurance-right-for-you-2.aspx>>.

^{xi} "Stock Market." *Investopedia*. Web. 1 Dec. 2012.

<<http://www.investopedia.com/terms/s/stockmarket.asp>>.

^{xii} "Mutual Fund." *Investopedia*. Web. 1 Dec. 2012. <<http://www.sec.gov/investor/tools/mfcc/mutual-fund-help.htm>>.

^{xiii} "Moody's Rating Symbols & Definitions." Moody's Investors Service, June 2009. Web. 13 Nov. 2012.

<<http://www.moody.com/sites/products/AboutMoodyRatingsAttachments/MoodyRatingsSymbolsand%20Definitions.pdf>>.

^{xiv} "Moody's Rating Symbols & Definitions." Moody's Investors Service, June 2009. Web. 13 Nov. 2012.

<<http://www.moody.com/sites/products/AboutMoodyRatingsAttachments/MoodyRatingsSymbolsand%20Definitions.pdf>>.

^{xv} "Moody's Rating Symbols & Definitions." Moody's Investors Service, June 2009. Web. 13 Nov. 2012.

<<http://www.moody.com/sites/products/AboutMoodyRatingsAttachments/MoodyRatingsSymbolsand%20Definitions.pdf>>.

^{xvi} "Bond Default Probabilities." *Monevator RSS*. 9 Apr. 2010. Web. 11 Oct. 2012.

<<http://monevator.com/bond-default-rating-probability/>>.

^{xvii} "Yield Curve." *Investopedia*. Web. 12 Dec. 2012.

<<http://www.investopedia.com/terms/y/yieldcurve.asp>>.

^{xviii} "Spot Rate." *Investopedia*. Web. 12 Oct. 2012.

<http://www.investopedia.com/terms/s/spot_rate.asp>.

^{xix} Kessler, Emily. "Pension Tables Should Reflect Improving Mortality." *American Academy of Actuaries*.

2005. Web. 3 Oct. 2012. <http://www.contingencies.org/novdec05/workshop_1105.asp>.

^{xx} Scism, Leslie. "Insurance Fees Revealed." *Wall Street Journal*, 30 Mar. 2012. Web. 21 Nov. 2012.

<<http://online.wsj.com/article/SB10001424052702304177104577305930202770336.html>>.

^{xxi} "Glassdoor - an inside Look at Jobs and Companies." *Glassdoor*. Web. 12 Nov. 2012.

<<http://www.glassdoor.com/index.htm>>.

^{xxii} Scism, Leslie. "Insurance Fees Revealed." *Wall Street Journal*, 30 Mar. 2012. Web. 21 Nov. 2012.

<<http://online.wsj.com/article/SB10001424052702304177104577305930202770336.html>>.

^{xxiii} "Overhead Defined." Web. 29 Nov. 2012. <http://www.wahby.com/articles/overhead_defined.htm>.

Appendix

The content in the appendix is the code used to write the excel functions and macros that carried the simulations of the longevity insurance.

Random Death Model

Sub KillEmOff()

'Runs for 8s

Application.ScreenUpdating = False

Sheets("Projected Lives").Select

Range("F3").Select

'MALE SECTION

Dim A, b, c, d, e, f, h, i, j, k, l, m, itnum, sum_m(71), sum_f(71), purnum_m(26), purnum_f(26) As Integer

'Initialize sum array to zero

For A = 0 To 70

 sum_m(A) = 0

Next

'Reads number of people to purchase annuity at each age

For b = 0 To 25

 purnum_m(b) = Selection.Value

 Selection.Offset(1, 0).Select

Next

'Starts cursor at beginning of table

Range("C33").Select

'Moves from age to age

For c = 0 To 25

 itnum = purnum_m(c) 'Sets number of iterations to the number of purchasers

 'Runs specified number of trials

```

For d = 1 To itnum
  'Adds individual if alive at time k
  For e = 0 To 70
    sum_m(e) = sum_m(e) + Selection.Value 'Sums number alive
    Selection.Offset(0, 1).Select
  Next
  Selection.Offset(0, -71).Select
  Calculate 'Recalculates spreadsheet
Next
Selection.Offset(1, 0).Select
'Produces output on line below
For f = 0 To 70
  Selection.Value = sum_m(f)
  Selection.Offset(0, 1).Select
Next
'Reinitialize sum array to zero
For A = 0 To 70
  sum_m(A) = 0
Next
Selection.Offset(1, -71).Select
Next
'-----
'FEMALE SECTION

Range("CD3").Select

'Initialize sum array to zero
For h = 0 To 70
  sum_f(h) = 0
Next

'Reads number of people to purchase annuity at each age
For i = 0 To 25
  purnum_f(i) = Selection.Value
  Selection.Offset(1, 0).Select
Next

'Starts cursor at beginning of table
Range("CA33").Select

```

```

'Moves from age to age
For j = 0 To 25
    itnum = purnum_f(j) 'Sets number of iterations to the number of purchasers
    'Runs specified number of trials
    For k = 1 To itnum
        'Adds individual if alive at time k
        For l = 0 To 70
            sum_f(l) = sum_f(l) + Selection.Value 'Sums number alive
            Selection.Offset(0, 1).Select
        Next
    Next

```

Bond Default Function

```

Public Function default_model(bond_grade As String, _
    New_bonds As Double, _
    bond_number As Double, _
    cumulative_bond As Double, _
    Optional is_first_column As Boolean = False, _
    Optional AAA_prob As Double = 0.0052, _
    Optional AA_prob As Double = 0.0058, _
    Optional A_prob As Double = 0.0129)

```

```

Application.Volatile

```

```

If is_first_column = True Then
    If New_bonds >= bond_number Then
        default_model = 1
    Else
        default_model = 0
    End If
Exit Function
End If

```

```

Dim bond_prob As Double

```

```

If bond_grade = "AAA" Then
    bond_prob = AAA_prob
ElseIf bond_grade = "AA" Then
    bond_prob = AA_prob
ElseIf bond_grade = "A" Then

```

```

    bond_prob = A_prob
End If

```

```

If Application.WorksheetFunction.sum(Range(Cells(Application.Caller.Row, 3),
Application.Caller.Offset(0, -1))) > 40 Then
    default_model = 0

```

```

ElseIf Application.WorksheetFunction.sum(Range(Cells(Application.Caller.Row, 3),
Application.Caller.Offset(0, -1))) >= 1 And Application.Caller.Offset(0, -1).Value = 0 Then
    default_model = 0

```

```

ElseIf Application.Caller.Offset(0, -1).Value = 1 Then
    If Rnd() >= bond_prob Then
        default_model = 1
    Else
        default_model = 0
    End If

```

```

ElseIf bond_number - 1 < cumulative_bond Then
    default_model = 1

```

```

Else
    default_model = 0

```

```

End If

```

Bond Grade Function

```

Public Function get_bond_grade(Existing_Bonds As Range, _
Optional AAA_Percent As Double = 0.4, _
Optional AA_Percent As Double = 0.3, _
Optional A_Percent As Double = 0.3)

```

```

Application.Volatile

```

```

Dim MyCounter_all As Double, MyCounter_AAA As Double, MyCounter_AA As Double, MyCounter_A As
Double

```

```

MyCounter_all = 0
MyCounter_AAA = 0

```

```
MyCounter_AA = 0
```

```
MyCounter_A = 0
```

```
For Each mycell In Existing_Bonds
```

```
    MyCounter_all = MyCounter_all + 1
```

```
    If mycell.Value = "AAA" Then MyCounter_AAA = MyCounter_AAA + 1
```

```
    If mycell.Value = "AA" Then MyCounter_AA = MyCounter_AA + 1
```

```
    If mycell.Value = "A" Then MyCounter_A = MyCounter_A + 1
```

```
Next
```

```
If MyCounter_AAA / MyCounter_all < AAA_Percent Then
```

```
    get_bond_grade = "AAA"
```

```
Elseif MyCounter_AA / MyCounter_all < AA_Percent Then
```

```
    get_bond_grade = "AA"
```

```
Else
```

```
    get_bond_grade = "A"
```

```
End If
```

```
End Function
```

Get Money Function

```
Public Function get_money(ben_needed As Double, _
```

```
rev As Double, _
```

```
cash_back As Range, _
```

```
is_alive As Double, _
```

```
pmt As Double, _
```

```
bond_return As Double)
```

```
If ben_needed < rev Then
```

```
    get_money = is_alive * pmt
```

```
Exit Function
```

```
Elseif ben_needed >= rev And is_alive = 1 Then
```

```
    If Application.WorksheetFunction.sum(cash_back) <= ben_needed Then
```

```
        get_money = bond_return
```



```
Else
  get_money = is_alive * pmt
```

```
End If
```

```
Else
  get_money = 0
```

```
End If
```

```
End Function
```

Coupon Value Function

```
Public Function payout(bond_grade As String, _
  alive As Range, _
  alive_before As Range, _
  pmt As Range, _
  pmt_before As Range, _
  Optional AAA_coup As Double = 325.11, _
  Optional AA_coup As Double = 356.45, _
  Optional A_coup As Double = 424.88)
```

```
  Dim bond_coup As Double
```

```
  If bond_grade = "AAA" Then
    bond_coup = AAA_coup
  ElseIf bond_grade = "AA" Then
    bond_coup = AA_coup
  Else
    bond_coup = A_coup
  End If
```

```
  If alive = 1 And Application.WorksheetFunction.sum(alive_before) = 0 Then
    payout = -10000
```

```
  ElseIf Application.WorksheetFunction.Min(pmt_before) = -10000 And
    Application.WorksheetFunction.CountIf(pmt_before, bond_coup) < 39 Then
    payout = bond_coup
```

```

Elseif Application.WorksheetFunction.Min(pmt_before) = -10000 And
Application.WorksheetFunction.CountIf(pmt_before, bond_coup) = 39 And pmt <> (10000 + bond_coup)
And pmt <> 0 Then
    payout = (10000 + bond_coup)

Else
    payout = 0

End If

End Function

```

Net Present Value Function

```

Public Function pv_calc(bond As String, _
year As Integer, _
pmt_series As Range, _
Optional AAA_coup As Double = 325.11, _
Optional AA_coup As Double = 356.45, _
Optional A_coup As Double = 424.88)

Dim i, j As Integer
Dim time As String
Dim coup, interest, pv_sum As Double

time = bond & year

If bond = "AAA" Then
    coup = AAA_coup
Elseif bond_grade = "AA" Then
    coup = AA_coup
Else
    coup = A_coup
End If

j = pv_sum = 0

If year <= 40 Then
    interest = Application.WorksheetFunction.VLookup(time, Range("ET1019:EU1141"), 2, False)

```

```
Else
    interest = Application.WorksheetFunction.VLookup((bond & 40), Range("ET1019:EU1141"), 2, False)
End If

count = 39 - Application.WorksheetFunction.CountIf(pmt_series, coup)

For i = 1 To count
    pv_sum = pv_sum + (coup / ((1 + interest) ^ i))
    j = j + 1
Next
pv_sum = pv_sum + (10000 / ((1 + interest) ^ (i - 1)))

pv_calc = pv_sum

End Function
```