LP03

Chapter 5

Prime Numbers

A prime number is a natural number greater that 1 that has only itself and 1 as factors.

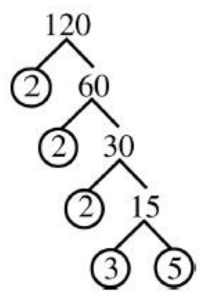
$$2, 3, 5, 7, 11, 13, 17, 19, 23, 29, \ldots$$

Question 1

Find the prime factorization of 120.

Solution

You must write 120 as a product using only prime numbers.
So, start dividing it by 2, 2, 2, 3, and then 5.



The prime factorization of 120 is:

$$2^3 \cdot 3 \cdot 5$$

Finding the Least Common Multiple Using Prime Factorizations

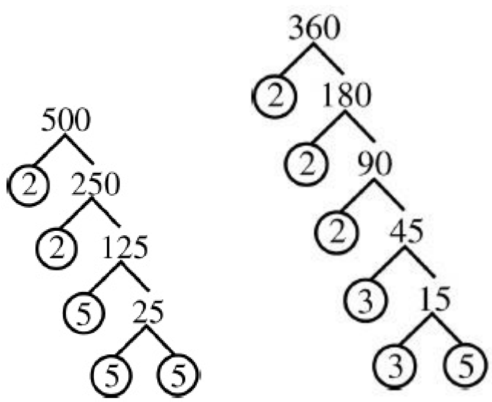
To find the least common multiple of two or more numbers,

1. Write the prime factorization of each number.
2. Select every prime factor that occurs, raised to the greatest power to which it occurs, in these factorizations.
3. Form the product of the numbers from step 2. The least common multiple is the product of these factors.

Question 2

Find the Least Common Multiple of 500 and 360.

Solution



$$500 = 2^2 \cdot 5^3$$

$$360 = 2^3 \cdot 3^2 \cdot 5$$

Now choose the numbers that you see in any factorizations with their largest exponents (I see the numbers 2^3, 3^2, and 5^3) and multiply them:

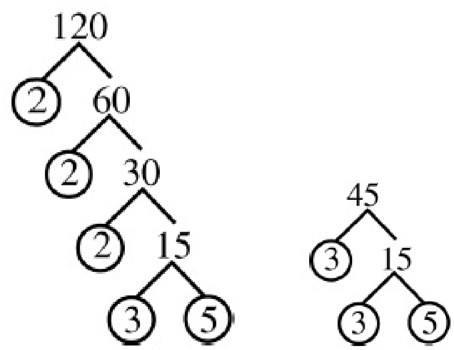
$$\begin{aligned} LCM &= 2^3 \cdot 3^2 \cdot 5^3 \\ &= 9000 \end{aligned}$$

Question 3

A relief worker needs to divide 120 bottles of water and 45 cans of food into groups that each contain the same number of items. Also, each group must contain the same type of item (bottled water or canned food). What is the largest number of relief supplies that can be put in each group?

Solution

The problem is about finding the Greatest Common Divisor.



$$120 = 2^3 \cdot 3 \cdot 5$$

$$45 = 3^2 \cdot 5$$

Now choose only the numbers that you see in both factorizations with their smallest exponents (I see 3 and 5 in both factorizations) and multiply:

$$GCD = 3 \cdot 5$$
$$= 15$$

So 15 is the largest number that goes into both 120 and 45.

15 is the largest number of supplies that can be put in each group.

Question 4

Express the following number as a decimal.

$$\frac{1}{2}$$

Solution

You can use long division or the calculator to divide $1 \div 2 = 0.5$

Question 5

The list of ingredients for chocolate brownies given will make 16 brownies.

$\frac{1}{3}$ cup butter
3 ounces chocolate
$\frac{3}{4}$ cups sugar
$1\frac{1}{2}$ teaspoons vanilla
2 eggs
$\frac{2}{3}$ cup flour

How much of each ingredient is needed to make 12 brownies?

Solution
So, you need only 12 brownies out of 16. You will have to multiply each of these quantities by the fraction $\frac{12}{16}$.
Also, it would be easier, if you reduce it first:

$$\frac{12}{16} = \frac{12 \div 4}{16 \div 4} = \frac{3}{4}$$

Now start multiplying each quantity by $\frac{3}{4}$.

$\frac{1}{3} \cdot \frac{3}{4} = \frac{1}{4}$	Cup butter
$3 \cdot \frac{3}{4} = \frac{3}{1} \cdot \frac{3}{4} = \frac{9}{4} = 2\frac{1}{4}$	Ounces chocolate
$\frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}$	Cups sugar
$1\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{2} \cdot \frac{3}{4} = \frac{9}{8} = 1\frac{1}{8}$	Teaspoons vanilla
$2 \cdot \frac{3}{4} = \frac{2}{1} \cdot \frac{3}{4} = \frac{6}{4} = \frac{3}{2} = 1\frac{1}{2}$	Eggs
$\frac{2}{3} \cdot \frac{3}{4} = \frac{1}{2}$	Cup flour

Question 6

If you walk $\frac{5}{6}$ mile and then jog $\frac{1}{5}$mile, what is the total distance covered? How much farther did you walk than jog?

Solution

Find the total distance:

$$\frac{5}{6}+\frac{1}{5}$$

$$=\frac{5\cdot 5}{6\cdot 5}+\frac{1\cdot 6}{5\cdot 6}$$

$$=\frac{25}{30}+\frac{6}{30}$$

$$=\frac{31}{30}$$

$$=1\frac{1}{30}$$

Now find the difference:

$$\frac{5}{6}-\frac{1}{5}$$

$$=\frac{5\cdot 5}{6\cdot 5}-\frac{1\cdot 6}{5\cdot 6}$$

$$=\frac{25}{30}-\frac{6}{30}$$

$$=\frac{19}{30}$$

A perfect square is a number that is a square of a rational number.

Examples of Perfect Squares

$1^2 = 1$	$11^2 = 121$
$2^2 = 4$	$12^2 = 144$
$3^2 = 9$	$13^2 = 169$
$4^2 = 16$	$14^2 = 196$
$5^2 = 25$	$15^2 = 225$
$6^2 = 36$	$16^2 = 256$
$7^2 = 49$	$17^2 = 289$
$8^2 = 64$	$18^2 = 324$
$9^2 = 81$	$19^2 = 361$
$10^2 = 100$	$20^2 = 400$

Question 7

Evaluate the expression.

$$\sqrt{169}$$

Solution

$$\sqrt{169} = 13$$

Question 8

Simplify:

$$(-4x^5y^{-7})(5x^{-1}y)$$

Solution

$$(-4x^5y^{-7})(5x^{-1}y)$$

$$= -20x^{5+(-1)}y^{-7+1}$$

$$= -20x^4y^{-6}$$

$$= -\frac{20x^4}{y^6}$$

Question 9

Write the number in decimal notation without using positive exponents.

$$3.85 \times 10^{-2}$$

Solution

Move the decimal point two places to the left.

$$3.85 \times 10^{-2} = 0.0385$$

Question 10

Express the number in scientific notation.

$$0.00000293$$

Solution

Move the decimal point 6 places to the right.

$$0.00000293 = 2.93 \times 10^{-6}$$

Question 11

Write the first six terms of the arithmetic sequence with the first term, a_1, and common difference, d.

$$a_1 = 200, \;\; d = 20$$

Solution

$$a_1 = 200$$

$$a_2 = 200 + 20 = 220$$

$$a_3 = 220 + 20 = 240$$

$$a_4 = 240 + 20 = 260$$

$$a_5 = 260 + 20 = 280$$

$$a_6 = 280 + 20 = 300$$

Question 12

Write the first six terms of the geometric sequence with the first term, a_1, and common ratio, r.

$$a_1 = -3, \;\; r = -2$$

Solution

$$a_1 = -3$$

$$a_2 = -3 \cdot (-2) = 6$$

$$a_3 = 6 \cdot (-2) = -12$$

$$a_4 = -12 \cdot (-2) = 24$$

$$a_5 = 24 \cdot (-2) = -48$$

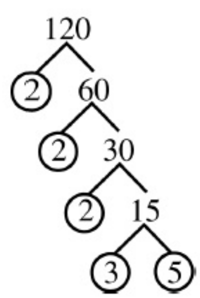
$$a_6 = -48 \cdot (-2) = 96$$

Question 13

Find the prime factorization of 120 of the composite number.

Solution

You must write 120 as a product using only prime numbers.
So, start dividing it by 2, 2, 2, 3, and then 5.



The prime factorization of 120 is:

$$2^3 \cdot 3 \cdot 5$$

Finding the Greatest Common Divisor Using Prime Factorizations

To find the greatest common divisor of two or more numbers,

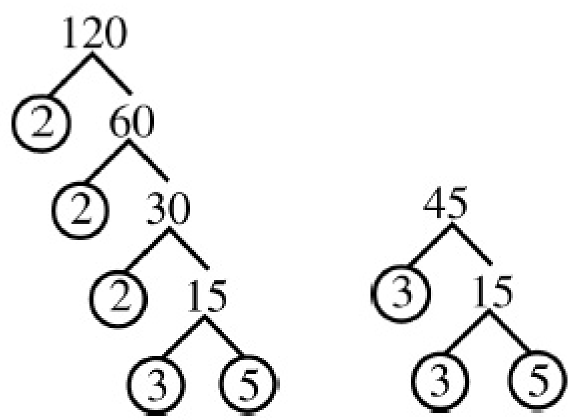
1. Write the prime factorization of each number.
2. Select each prime factor with the smallest exponent that is common to each of the prime factorizations.
3. Form the product of the numbers from step 2. The greatest common divisor is the product of these factors.

Question 14

Find the Greatest Common Divisor of 120 and 45.

Solution

Find the prime factorization for each number.



$$120 = 2^3 \cdot 3 \cdot 5$$

$$45 = 3^2 \cdot 5$$

Now choose only the numbers that you see in both factorizations with their smallest exponents (I see 3 and 5 in both factorizations) and multiply them:

$$\begin{aligned} GCD &= 3 \cdot 5 \\ &= 15 \end{aligned}$$

So 15 is the largest number that goes into both 120 and 45.

Question 15

A store owner wishes to stack books into equal piles, each pile containing only one title. There are 39 books of one title and 91 books of another title in the shipment. What is the largest number of books that can be stacked in each pile?

Solution

Find the prime factorization of each number.

$$39 = 3 \cdot 13$$

$$91 = 7 \cdot 13$$

The largest number that goes into both 39 and 91 is 13.

So, the largest number of books that can be stacked in each pile is 13 books.

Question 16

Insert < or > in the area between the integers to make the statement true.

$$-23 \qquad -15$$

Solution

$$-23 < -15$$

because on the number line, -23 is to the left of -15.

Absolute Value

The absolute value of an integer a, denoted by $|a|$, is the distance from 0 to a on the number line. Because absolute value describes a distance, it is never negative.

Examples:

$$|-4| = 4$$
$$|4| = 4$$
$$-|-4| = -4$$
$$-|4| = -4$$

Question 17

Find the absolute value.

$$|-10|$$

Solution

$$|-10| = 10$$

Rules for Addition of Integers

Rule	Example
If the integers have the same sign, 1. Add their absolute values. 2. The sign of the sum is the same as the sign of the two numbers.	$3 + 5 = 8$ $-3 + (-5) = -8$
If the integers have different signs, 1. Subtract the smaller absolute value from the larger absolute value. 2. The sign of the sum is the same as the sign of the number with the larger absolute value.	$-12 + 5 = -7$ $12 + (-5) = 7$

Rules for Multiplying Integers

Rule	**Examples**
The product of two integers with different signs is found by multiplying their absolute values. The product is negative.	$7(-5) = -35$ $-7(5) = -35$
The product of two integers with the same sign is found by multiplying their absolute values. The product is positive.	$4 \cdot 5 = 20$ $(-4)(-5) = 20$
The product of 0 and any integer is 0.	$7(0) = 0$
If no number is 0, a product with an odd number of negative factors is found by multiplying absolute values. The product is negative.	$(-2)(-4)(-5) = -40$
If no number is 0, a product with an even number of negative factors is found by multiplying absolute values. The product is positive.	$-2(4)(-5) = 40$

Rules for Dividing Integers

Rule	Examples
The quotient of two integers with different signs is found by dividing their absolute values. The quotient is negative.	$\frac{80}{-4} = -20$ $\frac{-15}{5} = -3$
The quotient of two integers with the same sign is found by dividing their absolute values. The quotient is positive.	$\frac{27}{9} = 3$ $\frac{-45}{-3} = 15$
Zero divided by any nonzero integer is zero.	$\frac{0}{-5} = 0$ (because $-5(0) = 0$)
Division by 0 is undefined.	$\frac{7}{0}$ *is undefined* (because 0 cannot be multiplied by an integer to obtain 7)

Question 18

Add.

$$1 + (-7)$$

Solution

$$1 + (-7) = -6$$

Question 19

Subtract.

$$-12 - 15$$

Solution

$$-12 - 15 = -27$$

Question 20

Evaluate the exponential expression.

$$-5^2$$

Solution

$$-5^2 = -5 \cdot 5 = -25$$

Question 21

Evaluate the exponential expression.

$$(-5)^2$$

Solution

$$(-5)^2 = (-5)(-5) = 25$$

Order of Operations

1. Perform all operations within grouping symbols.
2. Evaluate all exponential expressions.
3. Do all multiplications and divisions in the order in which they occur, working from left to right.
4. Finally, do all additions and subtractions in the order in which they occur, working from left to right.

Question 22

Use the order of operations to find the value of the expression.

$$9 - 4(8 - 6) - 10$$

Solution

$$9 - 4(8 - 6) - 10$$

$$= 9 - 4(2) - 10$$

$$= 9 - 8 - 10$$

$$= -9$$

Question 23

Use the order of operations to find the value of the expression.

$$4(-5)^2 - 5(-3)^2$$

Solution

$$4(-5)^2 - 5(-3)^2$$

$$= 4(25) - 5(9)$$

$$= 100 - 45$$

$$= 55$$

Question 24

Reduce the rational number to its lowest terms.

$$\frac{12}{18}$$

Solution

Divide the numerator and the denominator by 6.

$$\frac{12 \div 6}{18 \div 6} = \frac{2}{3}$$

Converting a Positive Mixed Number to an Improper Fraction

1. Multiply the denominator of the rational number by the integer and add the numerator to this product.
2. Place the sum in step 1 over the denominator in the mixed number.

Question 25

Convert the mixed number to an improper fraction.

$$4\frac{3}{5}$$

Solution

$$4\frac{3}{5} = \frac{4 \cdot 5 + 3}{5} = \frac{23}{5}$$

Question 26

Express the rational number as a decimal.

$$\frac{3}{8}$$

Solution

You can use long division or you can use a calculator.

$$3 \div 8 = 0.375$$

Question 27

Perform the indicated operations. Where possible, reduce the answer to the lowest terms.

$$\left(-\frac{2}{9}\right)\left(-\frac{7}{8}\right)$$

Solution

Simplify first, by dividing 2 and 8 by 2. Then multiply the numerators together, and the denominators together.

$$\left(-\frac{2}{9}\right)\left(-\frac{7}{8}\right) = \left(-\frac{1}{9}\right)\left(-\frac{7}{4}\right)$$

$$= \frac{7}{36}$$

Dividing Rational Numbers

The quotient of two rational numbers is the product of the first number and the reciprocal of the second number.

If $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers and $\frac{c}{d}$ is not 0, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}$

Question 28

Perform the indicated operations. Where possible, reduce the answer to the lowest terms.

$$-\frac{3}{8} \div \frac{4}{9}$$

Solution

$$-\frac{3}{8} \div \frac{4}{9}$$

$$= -\frac{3}{8} \cdot \frac{9}{4}$$

$$= -\frac{27}{32}$$

Question 29

Find the indicated term for the arithmetic sequence with first term, a_1, and common difference, d.

Find a_6, when $a_1 = 13, \ \ d = 4$.

Solution

Use formula:

$$a_n = a_1 + (n-1)d$$

$$a_6 = 13 + (6-1) \cdot 4$$

$$= 13 + 5 \cdot 4$$

$$= 13 + 20$$

$$= 33$$

Question 30

Find the indicated term for the geometric sequence with the first term, a_1, and the common ratio, r.

Find a_6, when $a_1 = 18, \ \ r = -2$.

Solution

Use the formula

$$a_n = a_1 r^{n-1}$$

$$a_6 = 18 \cdot (-2)^{6-1}$$

$$= 18 \cdot (-2)^5$$

$$= 18 \cdot (-32)$$

$$= -576$$