CALCULUS KINEMATICS

CALCULUS KINEMATICS IN SCALAR FORM

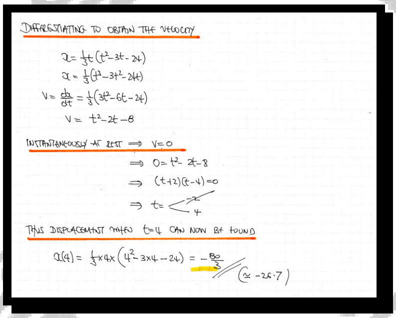
Question 1 (**)

A particle P is moving on the x axis and its displacement from the origin, x m, t seconds after a given instant, is given by

$$x = \frac{1}{3}t\left(t^2 - 3t - 24\right),\ t \geq 0.$$

Determine the displacement of P when it is instantaneously at rest.

$\boxed{}$, $\boxed{x = -26\frac{2}{3}\text{ m}}$



Question 2 (**)

A particle P is moving on the x axis and its acceleration a ms^{-2}, t seconds after a given instant, is given by

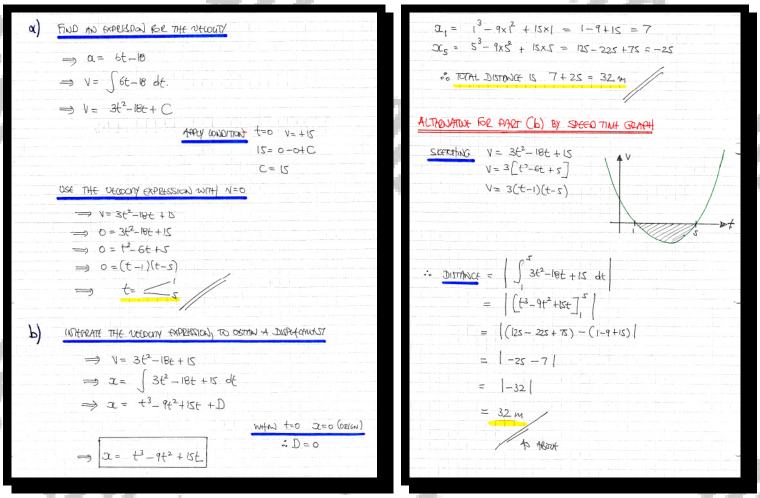
$$a = 6t - 18, \ t \geq 0.$$

The particle is initially at the origin O, moving with a speed of 15 ms^{-1} in the positive x direction.

a) Determine the times when P is instantaneously at rest.

b) Find the distance between the points, at which P is instantaneously at rest.

$\boxed{}$, $\boxed{t = 1, \quad t = 5}$, $\boxed{d = 32\,\text{m}}$


a) FIND AN EXPRESSION FOR THE VELOCITY
$\Rightarrow a = 6t - 18$
$\Rightarrow v = \int 6t - 18 \, dt$
$\Rightarrow v = 3t^2 - 18t + C$
APPLY CONDITION: $t = 0$, $v = +15$
$15 = 0 - 0 + C$
$C = 15$
USE THE VELOCITY EXPRESSION WITH $v = 0$
$\Rightarrow v = 3t^2 - 18t + 15$
$\Rightarrow 0 = 3t^2 - 18t + 15$
$\Rightarrow 0 = t^2 - 6t + 5$
$\Rightarrow 0 = (t-1)(t-5)$
$\Rightarrow t = 1, \ 5$
b) INTEGRATE THE VELOCITY EXPRESSION TO OBTAIN A DISPLACEMENT
$\Rightarrow v = 3t^2 - 18t + 15$
$\Rightarrow x = \int 3t^2 - 18t + 15 \, dt$
$\Rightarrow x = t^3 - 9t^2 + 15t + D$
WHEN $t = 0$, $x = 0$ (ORIGIN)
$\therefore D = 0$
$\Rightarrow x = t^3 - 9t^2 + 15t$
$x_1 = 1^3 - 9 \times 1^2 + 15 \times 1 = 1 - 9 + 15 = 7$
$x_5 = 5^3 - 9 \times 5^2 + 15 \times 5 = 125 - 225 + 75 = -25$
$\therefore$ TOTAL DISTANCE IS $7 + 25 = 32$ m
ALTERNATIVE FOR PART (b) BY SPEED TIME GRAPH
SKETCHING $v = 3t^2 - 18t + 15$
$v = 3[t^2 - 6t + 5]$
$v = 3(t-1)(t-5)$
$\therefore$ DISTANCE $= \left| \int_1^5 3t^2 - 18t + 15 \, dt \right|$
$= \left| \left[t^3 - 9t^2 + 15t \right]_1^5 \right|$
$= |(125 - 225 + 75) - (1 - 9 + 15)|$
$= |-25 - 7|$
$= |-32|$
$= 32$ m
AS ABOVE

Question 3 (**)

A particle P is moving on the x axis and its velocity v ms^{-1}, t seconds after a given instant, is given by

$$v = t^2 - 4t - 12, \ t \geq 0.$$

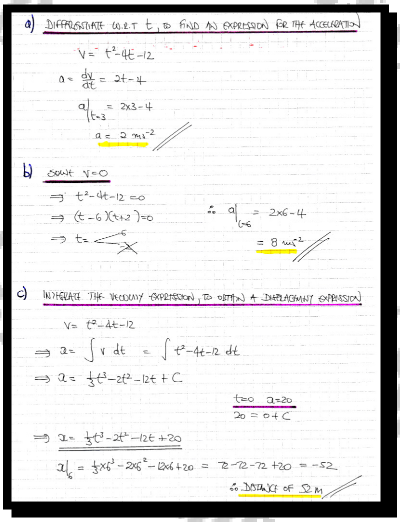
When $t = 0$, its displacement x from the origin O is 20 m.

a) Find the acceleration of P when $t = 3$.

b) Find the acceleration of P, when P is instantaneously at rest.

c) Determine the distance of P from O, when P is instantaneously at rest.

$\boxed{}$, $\boxed{a = 2 \text{ ms}^{-2}}$, $\boxed{a = 8 \text{ ms}^{-2}}$, $\boxed{d = 52 \text{ m}}$


a) DIFFERENTIATE W.R.T t, TO FIND AN EXPRESSION FOR THE ACCELERATION

$v = t^2 - 4t - 12$

$a = \frac{dv}{dt} = 2t - 4$

$a|_{t=3} = 2 \times 3 - 4$

$a = 2 \text{ ms}^{-2}$

b) SOLVE $v = 0$

$\Rightarrow t^2 - 4t - 12 = 0$

$\Rightarrow (t-6)(t+2) = 0$

$\Rightarrow t = 6$ ~~-2~~

$\therefore a|_{t=6} = 2 \times 6 - 4 = 8 \text{ ms}^{-2}$

c) INTEGRATE THE VELOCITY EXPRESSION, TO OBTAIN A DISPLACEMENT EXPRESSION

$v = t^2 - 4t - 12$

$\Rightarrow x = \int v \, dt = \int t^2 - 4t - 12 \, dt$

$\Rightarrow x = \frac{1}{3}t^3 - 2t^2 - 12t + C$

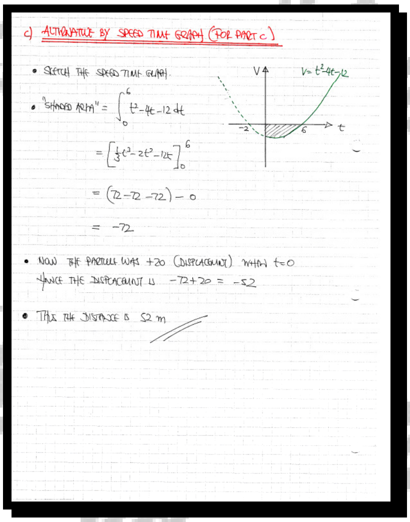
$t = 0 \quad x = 20$

$20 = 0 + C$

$\Rightarrow x = \frac{1}{3}t^3 - 2t^2 - 12t + 20$

$x|_6 = \frac{1}{3} \times 6^3 - 2 \times 6^2 - 12 \times 6 + 20 = 72 - 72 - 72 + 20 = -52$

$\therefore$ DISTANCE OF 52 m


c) ALTERNATIVE BY SPEED TIME GRAPH (FOR PART c)

- SKETCH THE SPEED TIME GRAPH.
- "SHADED AREA" $= \int_0^6 t^2 - 4t - 12 \, dt$

$= \left[\frac{1}{3}t^3 - 2t^2 - 12t\right]_0^6$

$= (72 - 72 - 72) - 0$

$= -72$

- NOW THE PARTICLE WAS +20 (DISPLACEMENT) WHEN $t = 0$. HENCE THE DISPLACEMENT IS $-72 + 20 = -52$
- THUS THE DISTANCE IS 52 m

Question 4 (**+)

A particle P is moving on the x axis and its velocity v ms^{-1}, t s after a given instant, is given by

$$v = t^2(3-t), \ t \geq 0.$$

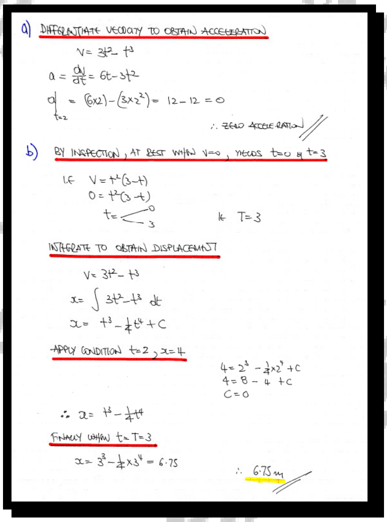
When $t=2$, P is observed to be 4 m from the origin O, in the positive x direction.

a) Find the acceleration of P when $t=2$.

The particle is at instantaneous rest initially, and when $t=T$.

b) Determine the distance of P from O when $t=T$.

$\boxed{}$, $\boxed{a=0}$, $\boxed{d=6.75\,\text{m}}$



a) DIFFERENTIATE VELOCITY TO OBTAIN ACCELERATION

$v = 3t^2 - t^3$

$a = \frac{dv}{dt} = 6t - 3t^2$

$a\big|_{t=2} = (6\times2) - (3\times2^2) = 12 - 12 = 0$

$\therefore$ ZERO ACCELERATION

b) BY INSPECTION, AT REST WHEN $v=0$, YIELDS $t=0$ & $t=3$

I.E $v = t^2(3-t)$

$0 = t^2(3-t)$

$t = 0, 3$ i.e. $T=3$

INTEGRATE TO OBTAIN DISPLACEMENT

$v = 3t^2 - t^3$

$x = \int 3t^2 - t^3 \, dt$

$x = t^3 - \frac{1}{4}t^4 + C$

APPLY CONDITION $t=2$, $x=4$

$4 = 2^3 - \frac{1}{4}\times2^4 + C$

$4 = 8 - 4 + C$

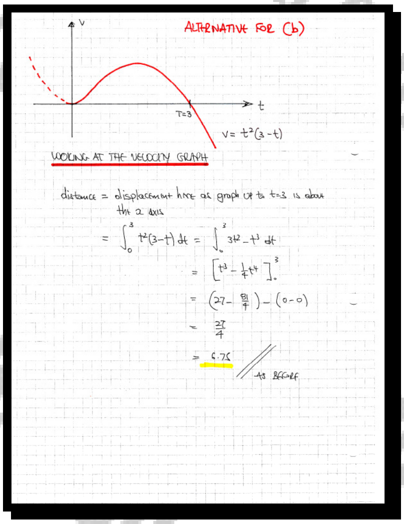
$C = 0$

$\therefore x = t^3 - \frac{1}{4}t^4$

FINALLY WHEN $t=T=3$

$x = 3^3 - \frac{1}{4}\times3^4 = 6.75$

$\therefore$ 6.75 m



ALTERNATIVE FOR (b)

LOOKING AT THE VELOCITY GRAPH

distance = displacement here as graph up to $t=3$ is above the x axis

$= \int_0^3 t^2(3-t)\,dt = \int_0^3 3t^2 - t^3 \, dt$

$= \left[t^3 - \frac{1}{4}t^4\right]_0^3$

$= \left(27 - \frac{81}{4}\right) - (0-0)$

$= \frac{27}{4}$

$= 6.75$ AS BEFORE

Question 5 (***)

A particle P is moving on the x axis and its acceleration a ms^{-2}, t seconds after a given instant, is given by

$$a = 8 - 2t, \quad t \geq 0.$$

Initially, P is on the positive x axis 84 m away from the origin O, and is moving towards O with a speed of 7 ms^{-1}.

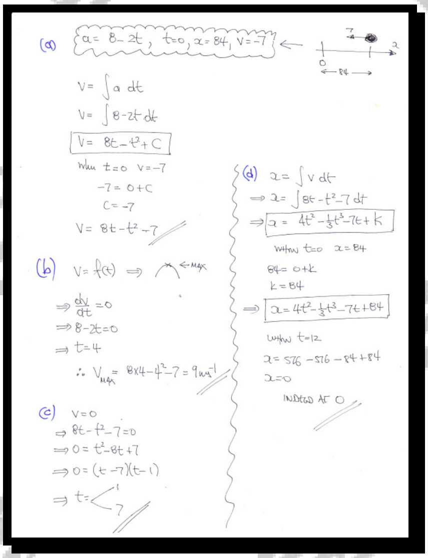
a) Find an expression for the velocity of P.

b) Calculate the maximum velocity of P.

c) Determine the times when P is instantaneously at rest.

d) Show that when $t = 12$, P is passing through O.

$\boxed{v = -t^2 + 8t - 7}$, $\boxed{v_{max} = 9 \text{ ms}^{-1}}$, $\boxed{t = 1, 7}$


(a) $a = 8 - 2t$, $t=0$, $x = 84$, $v = -7$

$v = \int a \, dt$

$v = \int 8 - 2t \, dt$

$v = 8t - t^2 + C$

when $t = 0$ $v = -7$

$-7 = 0 + C$

$C = -7$

$v = 8t - t^2 - 7$

(b) $v = f(t) \Rightarrow$ MAX

$\Rightarrow \frac{dv}{dt} = 0$

$\Rightarrow 8 - 2t = 0$

$\Rightarrow t = 4$

$\therefore V_{MAX} = 8 \times 4 - 4^2 - 7 = 9 \text{ ms}^{-1}$

(c) $v = 0$

$\Rightarrow 8t - t^2 - 7 = 0$

$\Rightarrow 0 = t^2 - 8t + 7$

$\Rightarrow 0 = (t-7)(t-1)$

$\Rightarrow t = 1, 7$

(d) $x = \int v \, dt$

$\Rightarrow x = \int 8t - t^2 - 7 \, dt$

$\Rightarrow x = 4t^2 - \frac{1}{3}t^3 - 7t + k$

when $t = 0$ $x = 84$

$84 = 0 + k$

$k = 84$

$\Rightarrow x = 4t^2 - \frac{1}{3}t^3 - 7t + 84$

when $t = 12$

$x = 576 - 576 - 84 + 84$

$x = 0$

INDEED AT O

Question 6 (***)

A particle is moving in a straight line.

At time t s, the particle has displacement x m from a fixed origin O and is moving with velocity v ms^{-1}.

When $t = 1$, $x = -5$ and $v = 1$.

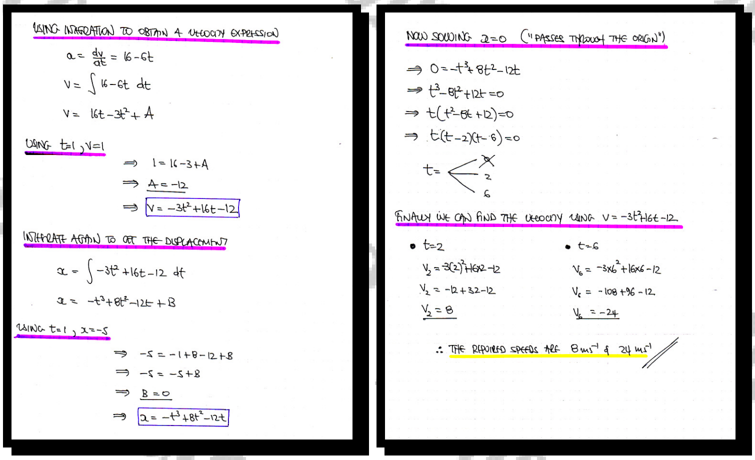
The acceleration a of the particle is given by

$$a = (16 - 6t) \text{ ms}^{-2}, \ t \geq 0.$$

The particle passes through O with speed U when $t = T$, $T > 0$.

Find the possible values of U.

$\boxed{}$, $\boxed{U = 8, \ 24}$



USING INTEGRATION TO OBTAIN A VELOCITY EXPRESSION

$a = \frac{dv}{dt} = 16 - 6t$

$v = \int 16 - 6t \, dt$

$v = 16t - 3t^2 + A$

USING $t=1$, $v=1$

$\Rightarrow 1 = 16 - 3 + A$

$\Rightarrow A = -12$

$\Rightarrow \boxed{v = -3t^2 + 16t - 12}$

INTEGRATE AGAIN TO GET THE DISPLACEMENT

$x = \int -3t^2 + 16t - 12 \, dt$

$x = -t^3 + 8t^2 - 12t + B$

USING $t=1$, $x=-5$

$\Rightarrow -5 = -1 + 8 - 12 + B$

$\Rightarrow -5 = -5 + B$

$\Rightarrow B = 0$

$\Rightarrow \boxed{x = -t^3 + 8t^2 - 12t}$

NOW SOLVING $x=0$ ("PASSES THROUGH THE ORIGIN")

$\Rightarrow 0 = -t^3 + 8t^2 - 12t$

$\Rightarrow t^3 - 8t^2 + 12t = 0$

$\Rightarrow t(t^2 - 8t + 12) = 0$

$\Rightarrow t(t-2)(t-6) = 0$

$t = \cancel{0}, 2, 6$

FINALLY WE CAN FIND THE VELOCITY USING $v = -3t^2 + 16t - 12$

• $t=2$

$v_2 = -3(2)^2 + 16\times2 - 12$

$v_2 = -12 + 32 - 12$

$v_2 = 8$

• $t=6$

$v_6 = -3\times6^2 + 16\times6 - 12$

$v_6 = -108 + 96 - 12$

$v_6 = -24$

∴ THE REQUIRED SPEEDS ARE 8 ms^{-1} & 24 ms^{-1}

Question 7 (***+)

A particle P is moving on the x axis and its displacement from the origin, x m, t seconds after a given instant, is given by

$$x = 2t^3 - 3t^2 + At + B, \quad t \geq 0,$$

where A and B are constants.

a) Find the value of t when the acceleration of P is zero.

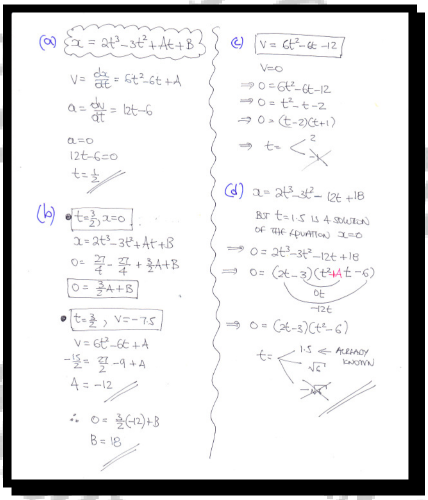
When $t = 1.5$ s, P is passing through the origin O, and is moving in the negative x direction with speed 7.5 ms^{-1}.

b) Determine the value of A and the value of B.

c) Determine the time when P is instantaneously at rest.

d) Calculate as an exact surd the value of t, when P is passing through O again.

$\boxed{t = \frac{1}{2}}$, $\boxed{A = -12}$, $\boxed{B = 18}$, $\boxed{t = 2}$, $\boxed{t = \sqrt{6}}$



Question 8 (***+)

A car is travelling on a straight horizontal road with constant velocity of 37.5 ms^{-1}.

The driver applies the brakes and the car decelerates at $(9.25 - t) \text{ ms}^{-2}$, where t s is the time since the instant when the brakes where first applied.

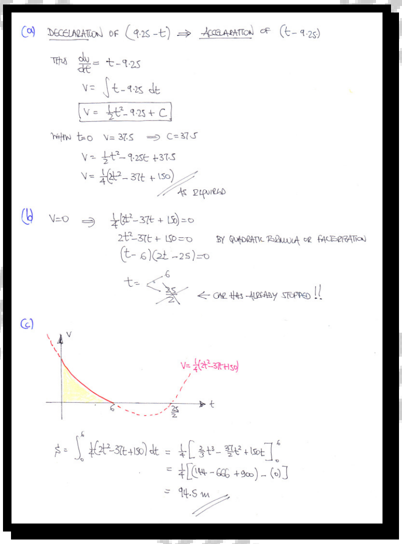
a) Show that while the car is decelerating its velocity is given by

$$\frac{1}{4}\left(2t^2 - 37t + 150\right) \text{ ms}^{-1}.$$

b) Hence find the time taken to bring the car to rest.

c) Determine the distance covered while the car was decelerating.

$\boxed{t = 6 \text{ s}}$, $\boxed{d = 94.5 \text{ m}}$



Question 9 (***+)

A particle P is moving on the x axis and its velocity v ms^{-1} in the positive x direction, t seconds after a given instant, is given by

$$v = t^2 - 2t - 24 \, , \; t \geq 0 \, .$$

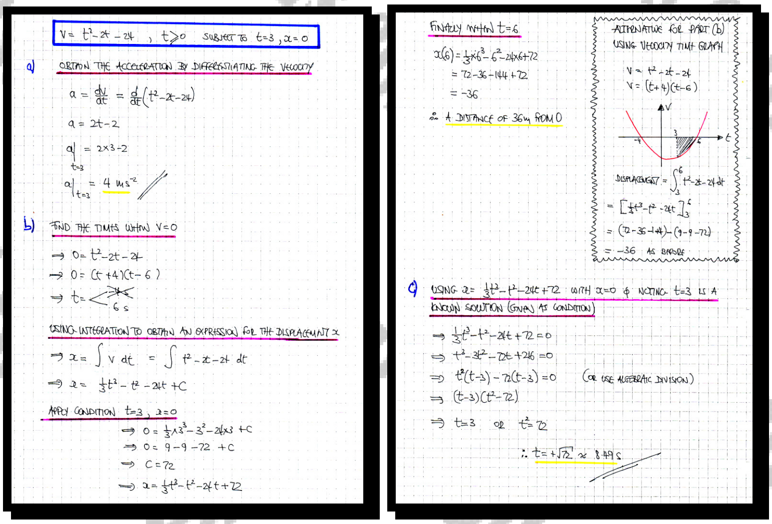
When $t = 3$, P is observed passing through the origin.

a) Find the acceleration of P when $t = 3$.

b) Determine the distance of P from O when it is instantaneously at rest.

c) Find the time at which P is passing through O again.

[ ] , $a = 4 \text{ ms}^{-2}$, $d = 36 \text{ m}$, $t = \sqrt{72} \approx 8.49$



Question 10 (***+)

A particle P is moving on the x axis and its velocity v ms^{-1} in the positive x direction, t seconds after a given instant, is given by

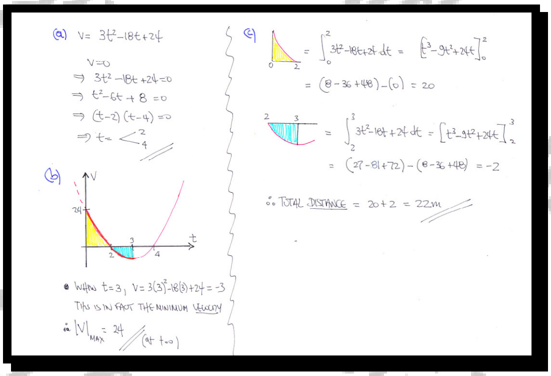
$$v = 3t^2 - 18t + 24, \; t \geq 0.$$

a) Find the times when P is instantaneously at rest.

b) Determine the greatest speed of P in the interval $0 \leq t \leq 3$.

c) Calculate the total distance covered by P in the interval $0 \leq t \leq 3$.

$\boxed{t = 2, 4}$, $\boxed{|v|_{\max} = 24 \text{ ms}^{-1}}$, $\boxed{d = 22 \text{ m}}$



Question 11 (***+)

A particle P is moving on a straight line.

At time t seconds, the distance of P from a fixed origin O is x metres and its acceleration is

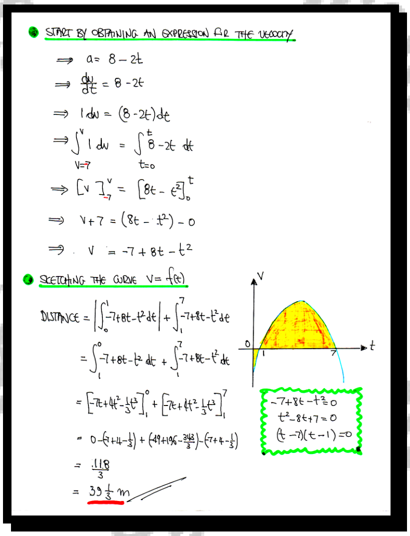
$$(8-2t) \text{ ms}^{-1}$$

in the direction of x increasing.

It is further given that when $t=0$, P was moving towards O with speed 7 ms^{-1}.

Determine the total distance covered by P in the first 7 seconds.

$d = 39\frac{1}{3}$ m



Question 12 (***+)

A particle is moving in a straight line in an electromagnetic field.

Its velocity, v ms^{-1}, at time t s, $t \geq 0$, is given by

$$v = t^2 + kt + 3.2,$$

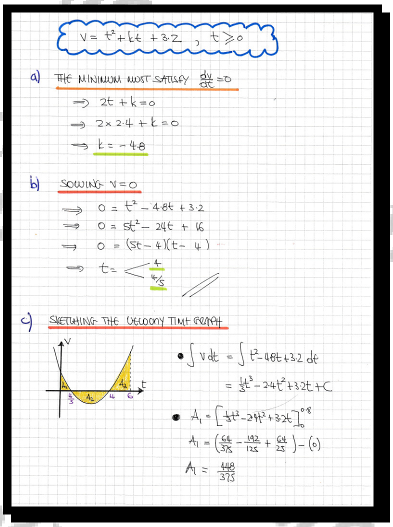
where k is a non zero constant.

a) Given that the particle achieves its minimum velocity when $t = 2.4$ s, show that $k = -4.8$.

b) Determine the values of t when the particle is instantaneously at rest.

c) Calculate the total distance covered by the particle for $0 \leq t \leq 6$.

[ ] , $\boxed{t = 0.8, \quad t = 4}$, $\boxed{d = \frac{5896}{375} \approx 15.72 \text{ m}}$



$v = t^2 + kt + 3.2, \quad t \geq 0$

a) THE MINIMUM MUST SATISFY $\frac{dv}{dt} = 0$

$\Rightarrow 2t + k = 0$

$\Rightarrow 2 \times 2.4 + k = 0$

$\Rightarrow k = -4.8$

b) SOLVING $v = 0$

$\Rightarrow 0 = t^2 - 4.8t + 3.2$

$\Rightarrow 0 = 5t^2 - 24t + 16$

$\Rightarrow 0 = (5t - 4)(t - 4)$

$\Rightarrow t = 4, \ \frac{4}{5}$

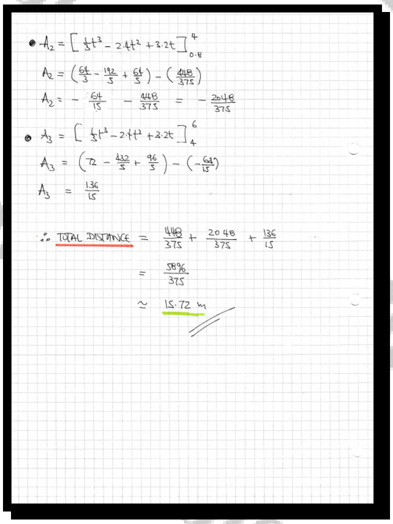
c) SKETCHING THE VELOCITY TIME GRAPH

- $\int v \, dt = \int t^2 - 4.8t + 3.2 \, dt = \frac{1}{3}t^3 - 2.4t^2 + 3.2t + C$

- $A_1 = \left[\frac{1}{3}t^3 - 2.4t^2 + 3.2t\right]_0^{0.8}$

$A_1 = \left(\frac{64}{375} - \frac{192}{125} + \frac{64}{25}\right) - (0)$

$A_1 = \frac{448}{375}$



- $A_2 = \left[\frac{1}{3}t^3 - 2.4t^2 + 3.2t\right]_{0.8}^{4}$

$A_2 = \left(\frac{64}{3} - \frac{192}{5} + \frac{64}{5}\right) - \left(\frac{448}{375}\right)$

$A_2 = -\frac{64}{15} - \frac{448}{375} = -\frac{2048}{375}$

- $A_3 = \left[\frac{1}{3}t^3 - 2.4t^2 + 3.2t\right]_4^6$

$A_3 = \left(72 - \frac{432}{5} + \frac{96}{5}\right) - \left(-\frac{64}{15}\right)$

$A_3 = \frac{136}{15}$

$\therefore$ TOTAL DISTANCE $= \frac{448}{375} + \frac{2048}{375} + \frac{136}{15} = \frac{5896}{375} \approx 15.72$ m

Question 13 (****)

A particle P is moving on the x axis and its acceleration a ms^{-2}, t seconds after a given instant, is given by

$$a = 4t - 9, \quad t \geq 0.$$

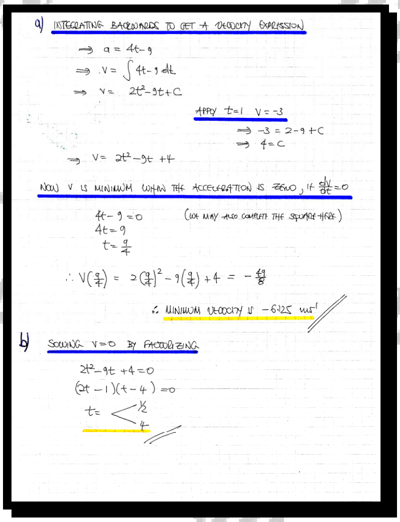
When $t = 1$, P is moving with a velocity of -3 ms^{-1}.

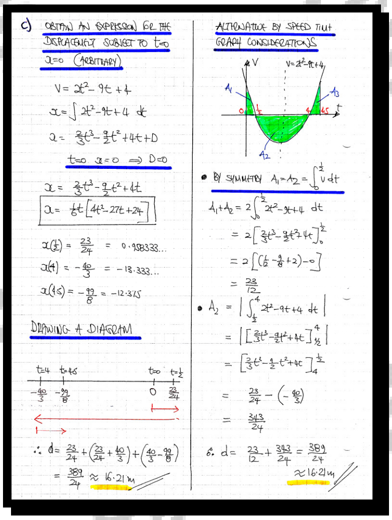
a) Find the minimum velocity of P.

b) Determine the times when P is instantaneously at rest.

c) Find the distance travelled by P in the first $4\frac{1}{2}$ seconds of its motion.

[ ], $v_{\text{min}} = -6.125 \text{ ms}^{-1}$, $t = \frac{1}{2}, 4$, $d = \frac{389}{24} \approx 16.21 \text{ m}$





Question 14 (****)

A particle is moving in a straight line, so that its velocity, v ms^{-1}, at time t s satisfies

$$v = 2t + kt^2, \qquad 0 \le t \le 10,$$

where k is a non zero constant.

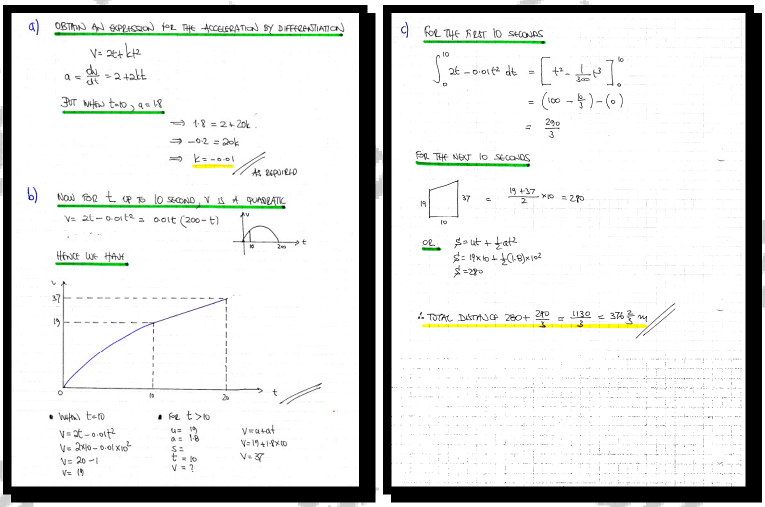
When $t = 10$, the particle reaches an acceleration of 1.8 ms^{-2}, which it maintains for a further 10 s.

a) Show that $k = -0.01$.

b) Sketch a detailed velocity time graph, which describes the motion of this particle, for $0 \le t \le 20$.

c) Find the distance travelled by the particle for $0 \le t \le 20$.

$\boxed{}$, $\boxed{d = 376\frac{1}{3} \text{ m}}$



Question 15 (****)

Russel is driving through the countryside, along a straight horizontal road at a constant speed of 22.5 ms^{-1}.

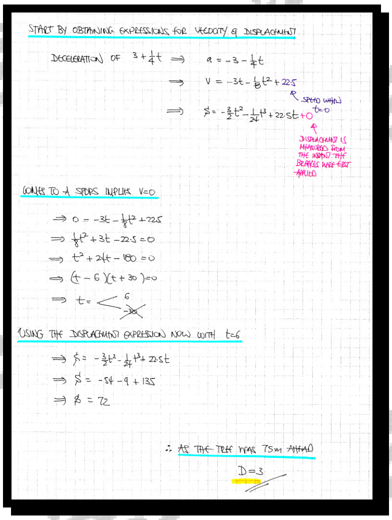
He sees a fallen tree blocking the road ahead, at a distance of 75 m ahead, so he immediately applies the brakes trying to stop his car before it hits the fallen tree.

The way he applies the brakes is such so that the **deceleration** of his car is given by $\left(3+\frac{1}{4}t\right) \text{ ms}^{-2}$, where t is measured since the instant he first applied the brakes.

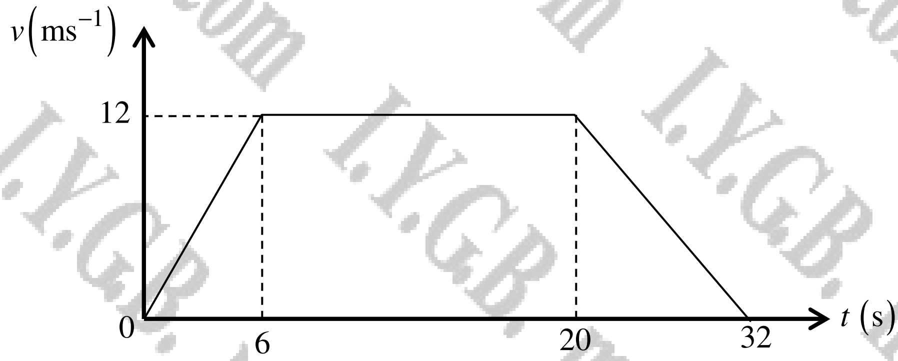
Russel's car stops D m **before** he hits the tree.

Determine the value of D.

$\boxed{X}$, $\boxed{D=3}$



Question 16 (****)



The figure above shows the speed time graph (t, v) of a car travelling along a straight horizontal road between two sets of traffic lights.

The car starts from rest at the first set of lights and accelerates uniformly for 6 s, reaching a speed of 12 ms^{-1}.

This speed is maintained for 14 s, before the car decelerates uniformly for 12 s, coming to rest as it reaches the second set of lights.

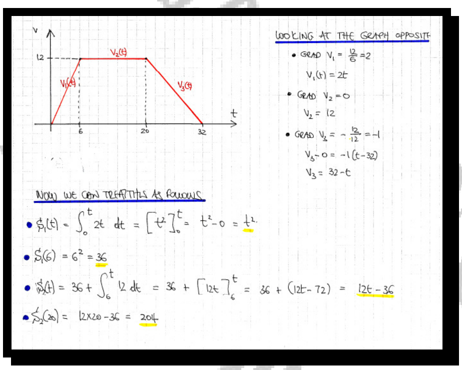
The distance of the car, $s(t)$, measured from the first set of traffic lights is given by

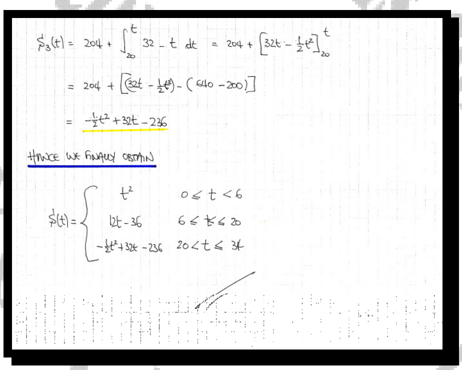
$$s(t) = \begin{cases} f_1(t) & 0 \le t < 6 \\ f_2(t) & 6 \le t < 20 \\ f_3(t) & 20 \le t < 32 \end{cases}$$

where $f_1(t)$, $f_2(t)$ and $f_3(t)$ are functions of t.

Determine simplified expressions for $f_1(t)$, $f_2(t)$ and $f_3(t)$.

$\boxed{f_1(t) = t^2}$, $\boxed{f_2(t) = 12t - 36}$, $\boxed{f_3(t) = \frac{1}{2}t^2 + 32t - 236}$





Question 17 (****+)

A particle P is moving on the x axis and its velocity v ms^{-1}, t seconds after a given instant, is given by

$$v = \begin{cases} 6t - t^2 & 0 \le t \le 5 \\ 25 - 4t & t > 5 \end{cases}$$

The particle is initially at the origin O.

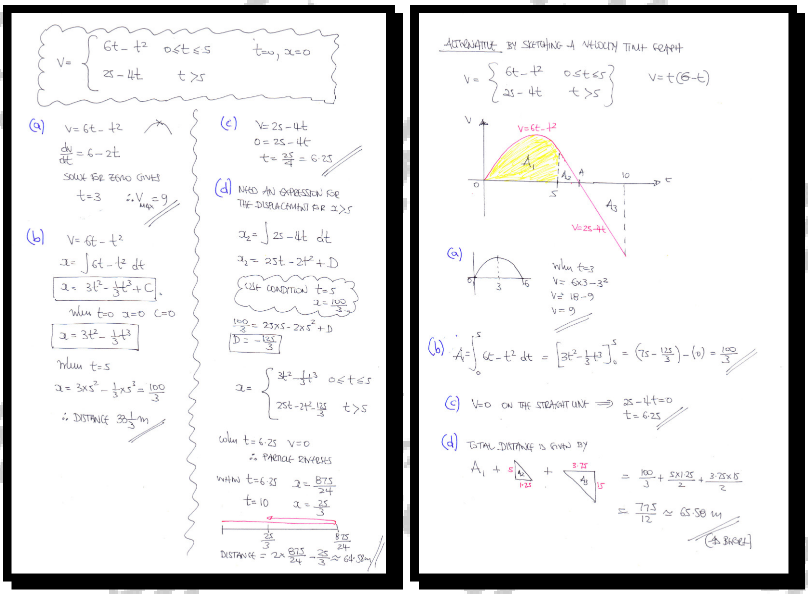
a) Find the greatest speed of P for $0 \le t \le 5$.

b) Show that the distance of P from O when $t = 5$ is $33\frac{1}{3}$ m.

c) State the time at which P is instantaneously at rest for $t > 5$.

d) Hence determine the **total distance** travelled by P during the first 10 seconds of its motion.

$\boxed{v_{max} = 9\,\text{ms}^{-1}}$, $\boxed{t = \frac{25}{4} = 6.25\text{ s}}$, $\boxed{d = \frac{775}{12} \approx 64.58\text{ m}}$



Question 18 (****+)

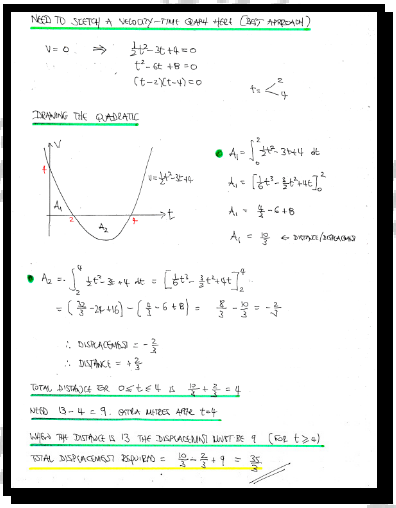
A particle P is moving on the x axis and its velocity v ms^{-1} in the positive x direction, t seconds after a given instant, is given by

$$v = \frac{1}{2}t^2 - 3t + 4, \quad t \geq 0.$$

The particle is passing through the origin when $t = 0$

Determine the displacement of the particle from the origin, when it has covered a **total distance** of 13 m.

$\boxed{}$, $\boxed{x = \frac{35}{3}}$



Question 19 (****+)

A car moving on a straight road is modelled as a particle moving on the x axis, and its acceleration a ms^{-2}, t seconds after a given instant, is given by

$$a = \begin{cases} 4 - \frac{1}{2}t & 0 \le t \le 8 \\ 0 & t > 8 \end{cases}$$

The car starts from rest at the origin O.

a) Find a similar expression for the velocity of the car, as that of its acceleration.

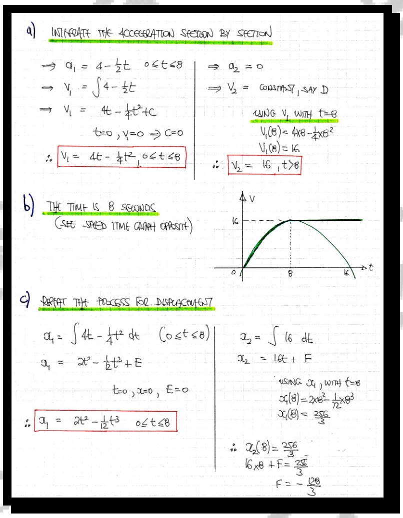
b) State the time it takes for the car to reach its maximum speed.

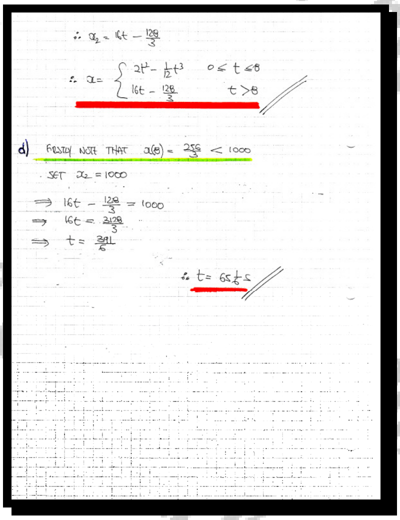
c) Show that the displacement of P from O is given by

$$x = \begin{cases} 2t^2 - \frac{1}{12}t^3 & 0 \le t \le 8 \\ 16t - \frac{128}{3} & t > 8 \end{cases}$$

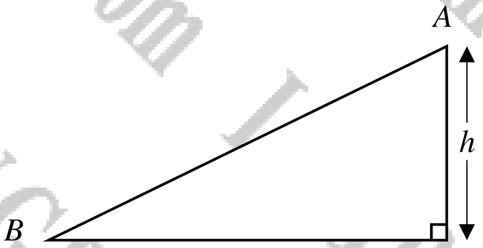
d) Calculate the time it takes the car to cover the first 1000 m.

$$v = \begin{cases} 4t - \frac{1}{4}t^2 & 0 \le t \le 8 \\ 16 & t > 8 \end{cases}, \quad t = 8 \text{ s}, \quad t = 65\tfrac{1}{6} \text{ s}$$





Question 20 (****+)



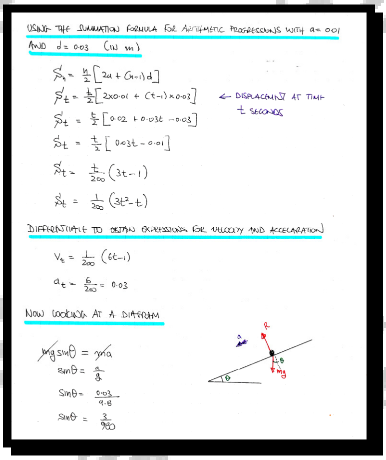
A particle is sliding down the line of greatest slope of a **smooth** plane inclined at a fixed angle to the horizontal. The particle experiences no other resistances.

The particle is released from rest from a point A at the top of the plane and takes 12 seconds to slide down to a point B on the plane. Point A lies at a vertical distance of h above the level of B, as shown in the figure above.

The particle slides down by 1 cm during the first second of its motion, and in each subsequent second it slides down by an extra 3 cm than in the previous second.

Show that $h = 6\tfrac{3}{7}$, measured in millimetres.

proof



USING THE SUMMATION FORMULA FOR ARITHMETIC PROGRESSIONS WITH $a = 0.01$ AND $d = 0.03$ (IN m)

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_t = \frac{t}{2}[2 \times 0.01 + (t-1) \times 0.03]$ ← DISPLACEMENT AT TIME t SECONDS

$S_t = \frac{t}{2}[0.02 + 0.03t - 0.03]$

$S_t = \frac{t}{2}[0.03t - 0.01]$

$S_t = \frac{t}{200}(3t - 1)$

$S_t = \frac{1}{200}(3t^2 - t)$

DIFFERENTIATE TO OBTAIN EXPRESSIONS FOR VELOCITY AND ACCELERATION

$V_t = \frac{1}{200}(6t - 1)$

$a_t = \frac{6}{200} = 0.03$

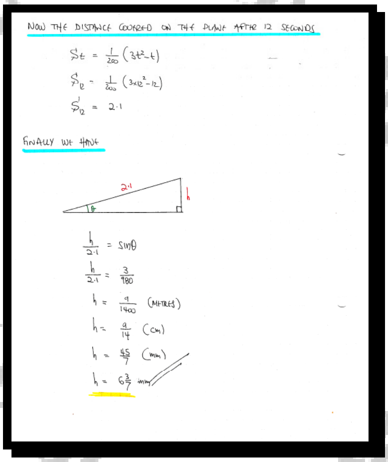
NOW LOOKING AT A DIAGRAM

$mg\sin\theta = ma$

$\sin\theta = \frac{a}{g}$

$\sin\theta = \frac{0.03}{9.8}$

$\sin\theta = \frac{3}{980}$



NOW THE DISTANCE COVERED ON THE PLANE AFTER 12 SECONDS

$S_t = \frac{1}{200}(3t^2 - t)$

$S_{12} = \frac{1}{200}(3 \times 12^2 - 12)$

$S_{12} = 2.1$

FINALLY WE HAVE

$\frac{h}{2.1} = \sin\theta$

$\frac{h}{2.1} = \frac{3}{980}$

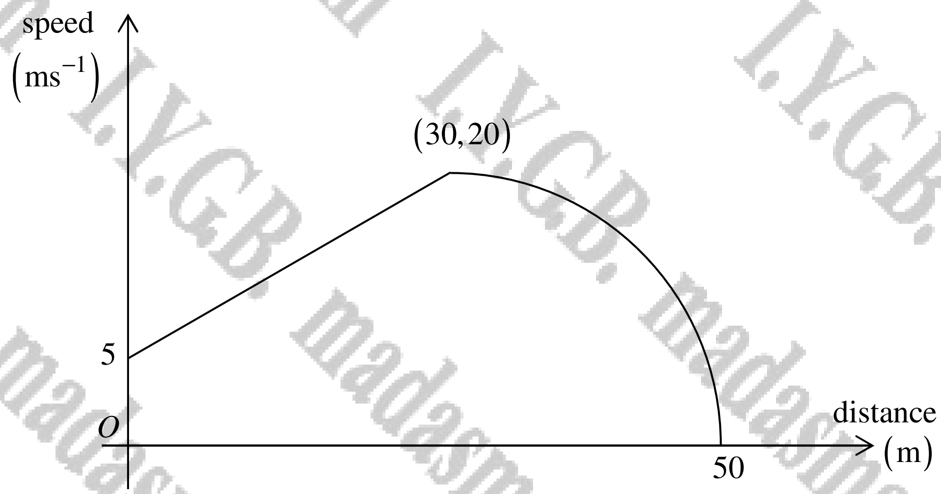
$h = \frac{9}{1400}$ (METRES)

$h = \frac{9}{14}$ (cm)

$h = \frac{45}{7}$ (mm)

$h = 6\frac{3}{7}$ mm

Question 21 (*****)



The speed distance graph of the journey of a particle is shown above.

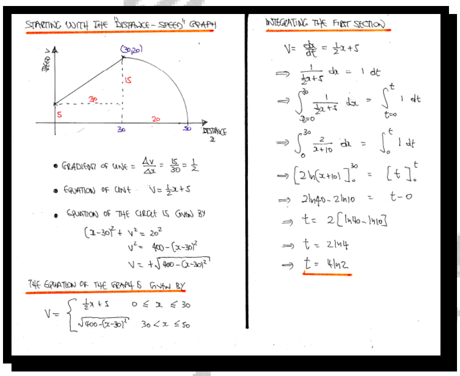
It consists of a straight line segment joining the point $(0,5)$ to $(30,20)$, joined to a quarter circle of radius 20. The total distance covered by the particle is 50 m.

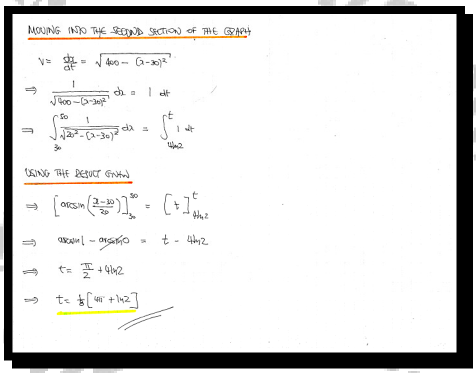
Determine in exact form the total journey time of the particle.

You may assume without proof that

$$\int \frac{1}{\sqrt{a^2-(u-b)^2}}\,du = \arcsin\left(\frac{u-b}{a}\right)+\text{constant}$$

$\boxed{}$, $\boxed{t=\left(\frac{1}{2}\pi+4\ln 2\right)\text{ s}}$





CALCULUS KINEMATICS IN VECTOR FORM

Question 1 (**)

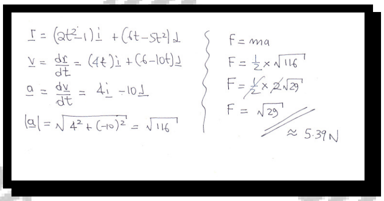
The position vector, $\mathbf{r}$ m, of a particle, t seconds after a given instant is given by

$$\mathbf{r} = \left(2t^2 - 1\right)\mathbf{i} + \left(6t - 5t^2\right)\mathbf{j}, \ t \geq 0,$$

where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors pointing due east and due north, respectively.

Given that the mass of the particle is 0.5 kg, determine the magnitude of the resultant force acting on the particle.

$\boxed{F = \sqrt{29} \approx 5.39 \text{ N}}$


$\underline{r} = (2t^2-1)\underline{i} + (6t-5t^2)\underline{j}$

$\underline{v} = \frac{d\underline{r}}{dt} = (4t)\underline{i} + (6-10t)\underline{j}$

$\underline{a} = \frac{d\underline{v}}{dt} = 4\underline{i} - 10\underline{j}$

$|\underline{a}| = \sqrt{4^2 + (-10)^2} = \sqrt{116}$

$F = ma$

$F = \frac{1}{2} \times \sqrt{116}$

$F = \frac{1}{2} \times 2\sqrt{29}$

$F = \sqrt{29}$

≈ 5.39 N

Question 2 (**)

The position vector, $\mathbf{r}$ m, of a particle P, t s after a given instant is given by

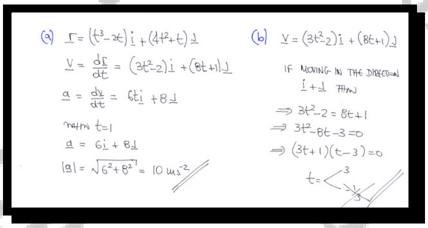
$$\mathbf{r} = \left(t^3 - 2t\right)\mathbf{i} + \left(4t^2 + t\right)\mathbf{j}, \ t \geq 0,$$

where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors pointing due east and due north, respectively.

a) Find the magnitude of the acceleration of the particle, when $t = 1$.

b) Determine the value of t when P is moving parallel to the vector $\mathbf{i} + \mathbf{j}$.

$\boxed{a = 10 \text{ ms}^{-2}}$, $\boxed{t = 3}$


(a) $\underline{r} = (t^3-2t)\underline{i} + (4t^2+t)\underline{j}$

$\underline{v} = \frac{d\underline{r}}{dt} = (3t^2-2)\underline{i} + (8t+1)\underline{j}$

$\underline{a} = \frac{d\underline{v}}{dt} = 6t\underline{i} + 8\underline{j}$

WHEN $t=1$

$\underline{a} = 6\underline{i} + 8\underline{j}$

$|\underline{a}| = \sqrt{6^2+8^2} = 10 \text{ ms}^{-2}$

(b) $\underline{v} = (3t^2-2)\underline{i} + (8t+1)\underline{j}$

IF MOVING IN THE DIRECTION $\underline{i} + \underline{j}$ THEN

$\Rightarrow 3t^2 - 2 = 8t + 1$

$\Rightarrow 3t^2 - 8t - 3 = 0$

$\Rightarrow (3t+1)(t-3) = 0$

$t = 3$ or $-\frac{1}{3}$

Question 3 **(**+)**

The velocity, $\mathbf{v}$ ms^{-1}, of a particle P, t seconds after a given instant is given by

$$\mathbf{v} = (4t-3)\mathbf{i} + (2t+3)\mathbf{j}, \; t \geq 0,$$

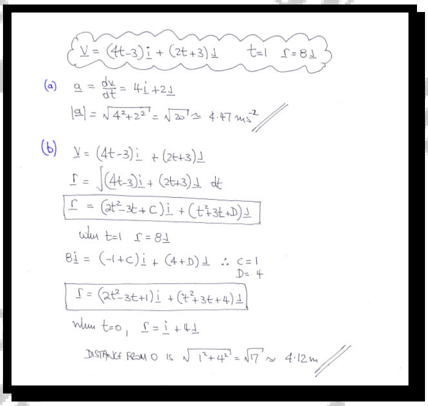
where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors pointing due east and due north, respectively.

a) Find the magnitude of the acceleration of P.

When $t=1$, the position vector of P is $8\mathbf{j}$ m.

b) Determine the **initial distance** of P from the origin O.

$a = \sqrt{20} \approx 4.47 \, \text{ms}^{-2}$, $d = \sqrt{17} \approx 4.12 \, \text{m}$



$\underline{v} = (4t-3)\underline{i} + (2t+3)\underline{j}$ $\quad t=1 \quad \underline{r} = 8\underline{j}$

(a) $\underline{a} = \frac{dv}{dt} = 4\underline{i} + 2\underline{j}$

$|\underline{a}| = \sqrt{4^2+2^2} = \sqrt{20} \simeq 4.47 \, \text{ms}^{-2}$

(b) $\underline{v} = (4t-3)\underline{i} + (2t+3)\underline{j}$

$\underline{r} = \int (4t-3)\underline{i} + (2t+3)\underline{j} \, dt$

$\underline{r} = (2t^2 - 3t + C)\underline{i} + (t^2 + 3t + D)\underline{j}$

when $t=1$ $\underline{r} = 8\underline{j}$

$8\underline{i} = (-1+C)\underline{i} + (4+D)\underline{j}$ $\therefore C = 1$, $D = 4$

$\underline{r} = (2t^2 - 3t + 1)\underline{i} + (t^2 + 3t + 4)\underline{j}$

when $t=0$, $\underline{r} = \underline{i} + 4\underline{j}$

DISTANCE FROM O IS $\sqrt{1^2+4^2} = \sqrt{17} \approx 4.12 \, \text{m}$

Question 4 (**+)

The velocity, $\mathbf{v}$ ms^{-1}, of a particle of mass 2 kg, t s after a given instant is given by

$$\mathbf{v} = 6t^2\mathbf{i} - 6t^{\frac{3}{2}}\mathbf{j}, \ t \geq 0,$$

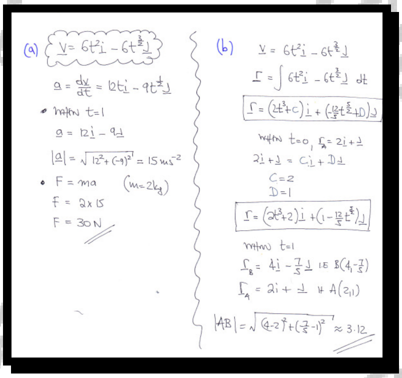
where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors pointing due east and due north, respectively.

a) Find the magnitude of the resultant force acting on the particle, when $t = 1$.

When $t = 0$, the particle is at the point A whose position vector is $(2\mathbf{i} + \mathbf{j})$ m and when $t = 1$ the particle is at the point B.

b) Determine the distance AB.

$\boxed{F = 30\,\text{N}}$, $\boxed{|AB| \approx 3.12\,\text{m}}$



Question 5 (+)**

The velocity , $\mathbf{v}$ ms^{-1}, of a particle of mass 5 kg, t s after a given instant is given by

$$\mathbf{v} = \left(12t^2 - 2\right)\mathbf{i} + \left(2t - 3t^2\right)\mathbf{j},\ t \geq 0,$$

where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors pointing due east and due north, respectively.

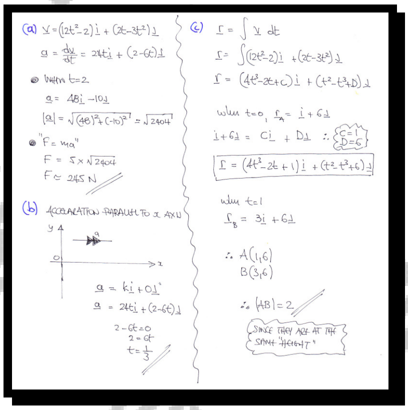
a) Find the magnitude of the resultant force acting on the particle, when $t = 2$.

b) Find the value of t when the particle's acceleration is parallel to the x axis.

When $t = 0$, the particle is at the point A with position vector $(\mathbf{i} + 6\mathbf{j})$ m and when $t = 1$, the particle is at the point B.

c) Determine the distance AB.

$\boxed{F \approx 245\text{ N}}$, $\boxed{t = \frac{1}{3}}$, $\boxed{|AB| = 2\text{ m}}$



Question 6 (***)

The position vector, $\mathbf{r}$ m, of a particle of mass 0.5 kg, t s after a given instant satisfies

$$\mathbf{r} = \left(3t^2 - 7t + 2\right)\mathbf{i} + \left(2t^2 - 5t + 2\right)\mathbf{j}, \; t \geq 0,$$

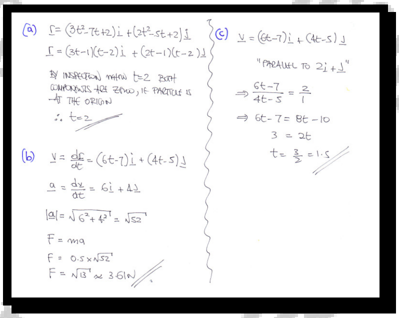
where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors pointing due east and due north, respectively.

a) Find the value of t when the particle is at the origin.

b) Determine the magnitude of the resultant force acting on the particle.

c) Find the value of t when the particle is moving parallel to the vector $2\mathbf{i} + \mathbf{j}$.

$\boxed{t = 2}$, $\boxed{F = \sqrt{13} \approx 3.61\text{N}}$, $\boxed{t = 1.5}$


(a) $\underline{r} = (3t^2-7t+2)\underline{i} + (2t^2-5t+2)\underline{j}$

$\underline{r} = (3t-1)(t-2)\underline{i} + (2t-1)(t-2)\underline{j}$

BY INSPECTION WHEN $t=2$ BOTH COMPONENTS ARE ZERO, IE PARTICLE IS AT THE ORIGIN

$\therefore t=2$

(b) $\underline{v} = \frac{d\underline{r}}{dt} = (6t-7)\underline{i} + (4t-5)\underline{j}$

$\underline{a} = \frac{d\underline{v}}{dt} = 6\underline{i} + 4\underline{j}$

$|\underline{a}| = \sqrt{6^2+4^2} = \sqrt{52}$

$F = ma$

$F = 0.5 \times \sqrt{52}$

$F = \sqrt{13} \approx 3.61\text{N}$

(c) $\underline{v} = (6t-7)\underline{i} + (4t-5)\underline{j}$

"PARALLEL TO $2\underline{i}+\underline{j}$"

$\Rightarrow \frac{6t-7}{4t-5} = \frac{2}{1}$

$\Rightarrow 6t-7 = 8t-10$

$3 = 2t$

$t = \frac{3}{2} = 1.5$

Question 7 (***+)

The acceleration $\mathbf{a}$ ms^{-2} of a particle P of mass 0.2 kg, t s after a given instant is given by

$$\mathbf{a} = (2t-4)\mathbf{i} + 3\mathbf{j}, \; t \geq 0,$$

where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors pointing along the positive x axis and along the positive y axis, respectively.

a) Find the magnitude of the resultant force acting on P, when $t = 4$.

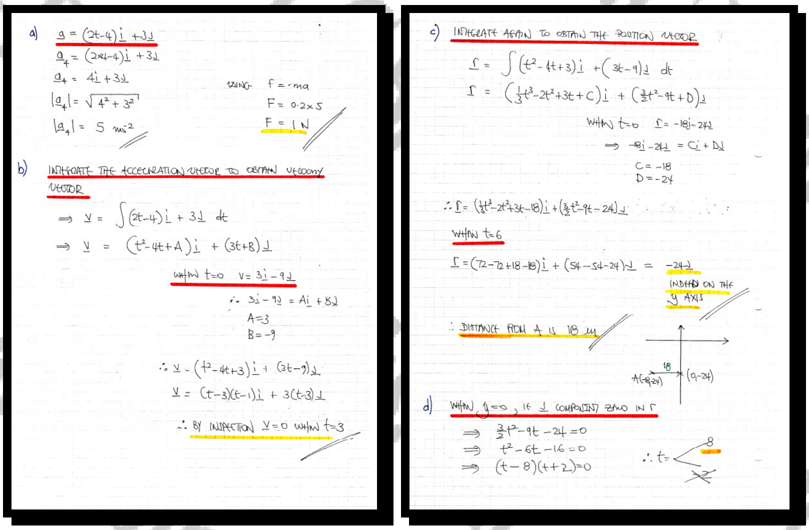
It is further given that when $t = 0$, P is at the point A with position vector $(-18\mathbf{i} - 24\mathbf{j})$ m and has velocity $(3\mathbf{i} - 9\mathbf{j})$ ms^{-1}.

b) Find the value of t when the particle is at rest.

c) Show that when $t = 6$, P is on the y axis and state its distance from A.

d) Determine the value of t when the particle is on the x axis.

$\boxed{}$, $\boxed{F = 1\text{N}}$, $\boxed{t = 3}$, $\boxed{18\text{ m}}$, $\boxed{t = 8}$



Question 8 (****)

The position vector, velocity and acceleration of a particle P, t s after a given instant are denoted by $\mathbf{r}$ m , $\mathbf{v}$ ms^{-1} and $\mathbf{a}$ ms^{-2}.

When $t=1$, $\mathbf{r}=9\mathbf{i}+2\mathbf{j}$ and $\mathbf{v}=13\mathbf{i}+\mathbf{j}$, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors pointing due east and due north, respectively.

It is further given that P has a constant acceleration of $6\mathbf{i}$ ms^{-2}.

a) Determine the distance of P from the origin O, when $t=3$.

b) Show that P is moving on the curve with equation

$$x=3y^2+y-5.$$

$\boxed{}$, $\boxed{\approx 47.17 \text{ m}}$